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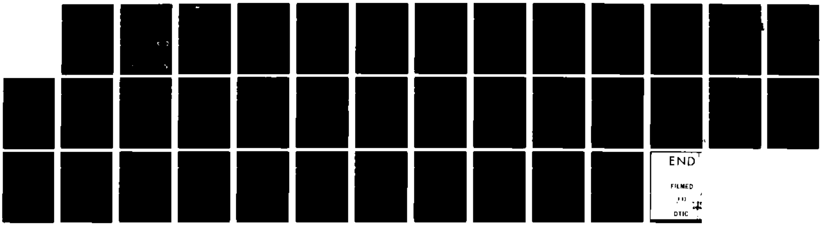

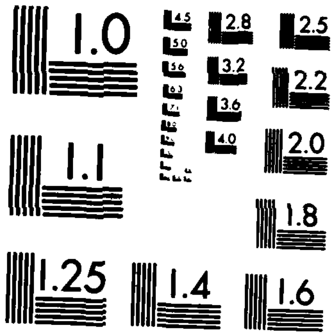

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MULTIACCESS OF A SLOTTED CHANNEL USING A CONTROL MINI-SLOT

Submitted to:

Air Force Office of Scientific Research
Bolling Air Force Base
Washington, DC 20332

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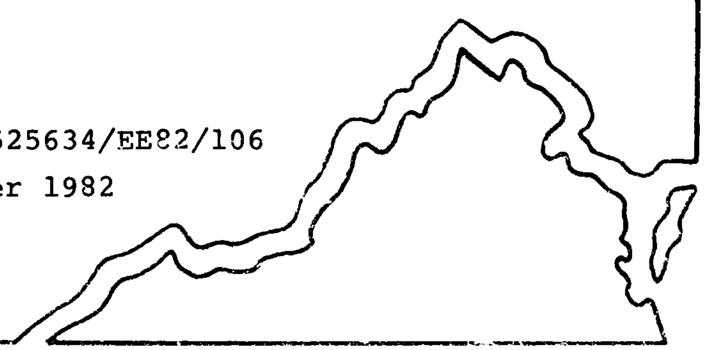

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MULTIACCESS OF A SLOTTED CHANNEL USING A CONTROL MINI-SLOT

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Abstract

The multiaccess problem as characterized by an infinite user population and a time-slotted channel using a control mini-slot is examined. The input rate stability region of the proposed algorithm is determined and compared to the random access algorithm with the greatest known efficiency for the Poisson multiple access model without additional information. A break-even point is given.

1. Introduction

The multiple access problem is a communication problem that has received much attention during the past decade. By multiple access (or multiaccess) problem we mean the problem of organizing or coordinating a population of users so that they may efficiently share the resources of a single communication channel.

The various models for the user population and communication channel that have appeared in the literature generally have the following common features. The users are geographically distributed and generate messages in an independent random fashion. There is no other way of communication among the users except the single common channel. The channel is such that only one user at a time can successfully transmit a message. Some form of feedback to the users is associated with the message transmissions. This feedback has typically ranged from no feedback (e.g., TDMA) to each individual user determining the outcome of only his own transmission attempts (e.g. Aloha [1]) to every user determining after some given delay whether there are 0, 1, or $\geq$ 2 messages being transmitted on the channel (e.g. Tree [2], [3]).

A model that has received considerable attention is specified by the following idealized conditions:

(i) The forward channel is a time-slotted collision-type channel, but is otherwise noiseless. The transmitters can transmit only in "packets" whose duration is one slot. A "collission" between two or more packets is always detected as such, but the individual packets cannot be reconstructed.

(ii) The feedback channel is a noiseless broadcast channel that informs the transmitters immediately at the end of each slot whether (a) that slot was empty, or (b) that slot contained one packet (which was thus successfully transmitted), or (c) that slot contained a collision of two or more packets (which must be retransmitted at later times).

(iii) Propagation delays are negligible, so that the feedback information for slot i can be used to determine who should transmit in the following slot.

(iv) The number of users is infinite. The cumulative input traffic is a Poisson point process with intensity λ.

We refer to conditions (i) - (iv) as the Poisson Multiple Access Model (PMAM).

For the PMAM, Capetanakis [2, 3], proposed a tree searching technique for the resolution of collisions. The Capetanakis collision resolution algorithm (CCRA) is stable in the sense of finite average packet delay, and there is a static and a dynamic version of it. In the static version, after each collision, each one of the collided packets retransmits with probability $p = .5$. In the dynamic version, each one of an optimally chosen subset of the collided packets retransmits with probability $= .5$. The throughput attained by the static and dynamic version is .346 and .429 correspondingly.

The general concept of tree algorithm was previously pioneered by Hayes [4]. In [4], many of the basic ideas of [2], [3] appeared in the development of polling schemes.

Massey [5] improved Capetanakis's algorithms by observing that if a collision slot is followed by an empty slot, one slot can be saved by repeating the random retransmission before a certain to occur collision. Massey's modified algorithms (MCCRA) induce a throughput equal to .375 in the static case, and .462 in the dynamic case.

Both the (CCRA) and (MCRA) algorithms were independently introduced by Tsybakov and Mikhailov [6].

Observing the equivalence between random retransmissions and subdivisions of the arrival time axis, Gallager [7] introduced a different algorithm by decoupling transmission times from arrival times. Gallager's algorithm realizes a maximum stable throughput of .4871 and has first come-first serve characteristics. Mosely [8] refined this approach to obtain a maximum stable throughput of .48785.

The question of determining the maximum achievable throughput for all stable protocols, without necessarily constructing a realizable one, has also received considerable attention. Due to the highly complicated nature of the problem, the channel capacity, or maximum throughput, is an elusive quantity, and to this date only a set of upper bounds is known.

Several investigators have developed upper bounds on the throughputs induced by the class of all stable protocols. Pippenger [9] used information-theoretic arguments to show that all realizable algorithms are unstable for $\lambda > .744$. This bound was sharpened to .704 by Humblet, to .6731 by Molle [10] (using a "magic genie" argument), to .6125 by Cruz and Hajek and to .587 by Tsybakov and Mikhailov [11]. The bounding techniques in [9] - [11] do not provide constructive algorithms for attaining high throughput performance.

Lately there have been proposed some realizable algorithms which achieve higher throughput and stability. The bounds of [9] - [11] do not apply to the more recent schemes because the latter ones deviate from condition (ii) of the basic model by assuming some additional feedback information available to the system. Several recent efforts of developing constructive algorithms of high throughput have culminated in the work of Papantoni-Kazakos and Georgiadis [12], and Papantoni-Kazakos and Marcus [13], [14]. In [12] it is assumed that after each collision the number (up to an upper maximum limit) of the packets involved is revealed to all users, through a bank of energy detectors.

In the present paper we are concerned with the multiaccess problem as characterized by the User-Channel model described in Section 2. In particular, we assume that selected users transmit, along with their regular packet, a "control bit" in a "control mini-slot". Thus we introduce additional feedback information which can be used by the contenting users to resolve conflicts. In section 3 we give the description of the algorithm. The general operation of the Random Access Algorithm using a Control Mini-Slot (RAA-CMS) is a modification of Gallager's general algorithm. The modification consists of incorporating the additional information provided by the control mini-slots into the collision resolution procedure. In section 4 we evaluate the induced maximum stable throughput. We found that the RAA-CMS has a capacity of .56 $(1+r)^{-1}$ packets per packet slot, where r is the ratio of the length of a control mini-slot over the length of a packet. We should note that the information transmitted (if any) in each control mini-slot is minimal--just absence or presence of any message. Consequently, the length of a control mini-slot can be quite short compared to the length of a packet, resulting in $r \ll 1$. Finally, in section 5 we consider the case of reduced feedback information (binary) for the control mini-slot. The idea of using a small part of the channel capacity for transmitting additional feedback information that facilitates the resoultion of conflicts, was also utilized in a different context in [13], [14]. The protocol in [13], [14] utilizes the user signature for transmitting user state information.

2. The Channel-User Model

In this section we introduce the channel and user models considered. There is an infinite number of identical, packet transmitting, bursty users. The cumulative input traffic is modeled as a homogeneous Poisson point process. The channel time is divided into disjoint slots of identical length. The intensity of the Poisson process is λ messages per slot. All messages from all users are transmitted in packets of identical length.

For the implementation of the RAA-CMS, described in section 3, each channel slot or simply slot is divided into two parts as shown in Figure 1. The first part is a control mini-slot (CMS) or "control-bit" and is used by the user for transmission of control information. The second part is used for the transmission of the message and has a length equal to the length of a packet. This second part of the channel slot is called message-slot (MS).

Figure 1

The transmission of a "control-bit" and/or a regular packet by each user during a particular slot is governed by the RAA-CMS. If none, one or more than one users attempt transmission of a "control-bit" this results in an empty, successful or collided control mini-slot respectively. Similarly, if none, one or more than one users attempt transmission of a regular packet this results in an empty, successful or collided message-slot respectively.

We assume that if only one packet is transmitted in a given message-slot, it is received without error (noiseless channel), whereas if two or more packets are transmitted in the same message-slot, the packets "collide" and no information about any of the messages is received.

We assume finally that at the end of a channel slot each user detects the outcome of both the control mini-slot and the message slot, i.e. propagation delays are considered negligible.

3. The RAA-CMS General Operation

We consider an arrival time axis and a channel time axis, where the channel time axis is measured in slots. The interval $[i, i+1]$, $i \varepsilon Z_+$ on the channel axis designates the i^{th} slot.

Each user keeps track of two common parameters, the system lag d and the length μ of the current transmission interval. Let T_c designate the current time. Each user with a packet to transmit also keeps track of the time δ since that packet arrived, i.e. δ is the packet's delay. The user transmits that packet in the message-slot of a given slot if, at the beginning of the slot, $\delta \varepsilon [d, d-\mu)$, i.e. if the packet's delay belongs to the *current transmission* interval. If, in addition, $\delta \varepsilon [d, d-\mu/2)$ the user also transmits a "control-bit" in the control mini-slot of the same given slot. In other words users in the left-half of the current transmission interval on the arrival time axis (left users) transmit both their packets and the "control-bit", whereas users in the right half of the current transmission interval (right users) transmit only their packets.

By the beginning of the next slot, all users know the outcome of the transmissions in the previous slot. Let $E_c (E_m)$ be a variable assuming the letter values I, S or C if the control mini-slot (message-slot) is empty, successful or collided respectively. We denote the outcome of a given slot by $E = E_c E_m$. There are six possible different outcomes, namely II, IS, SS, IC, SC and CC.

If E = II, an empty or idle slot occurs, since there are no users in the current transmission interval.

If E = IS or SS, the only packet in the current transmission interval is successfully transmitted.

If E = IC, a collision in the message-slot occurs and since there is no "left user" there must be at least two "right users".

If E = SC, a collision in the message-slot occurs and since there is one "left user" there must be at least one "right user".

If E = CC, both message-slot and control mini-slot are collided and since there are at least two "left users" no additional information on the number of "right users" is available.

In the last three cases the collided packets are retransmitted according to the RAA-CMS. The algorithm uses the feedback information that is provided by the outcome of the previous transmission to maximize the fraction of slots devoted to exactly one message (i.e., neither idle nor wasted because of a collision). In the event of a message collision, a smaller transmission interval is specified next time, and so on, until the collision is resolved.

The statement of the algorithm is given in Figure 2. The main features of the RAA-CMS are best illustrated by going through an example. Figure 3.1 shows the slotted channel axis. At current time $T_c = i$ the system is in a renewal state. By this we mean that all messages generated before the point R_1

(Figure 3.2) have been successfully transmitted, and nothing is known about the message distribution beyond R_1 (except that it is Poisson). At a renewal state the algorithm provides a transmission interval of length μ_o beginning at point R_1. μ_o is a parameter of the system and will be chosen so as to maximize the throughput. If $d < \mu_o$ at a renewal point, we assume that the system either transmits in an interval of length d, or waits until $d \geq \mu_o$ before transmitting. In Figure 3.2 there is only one message transmitted, and thus the system progresses to another renewal state. One slot has elapsed and the system variables are updated as follows:

$$d_{new} = d_{old} + 1-\mu_o$$

$$\mu_{new} = \mu_o$$

In Figure 3.3 the point R_2 indicates the new renewal point. If the channel had been idle during this slot the algorithm would have continued identically. No message would have been transmitted in this case and again R_2 would have been the new renewal point.

In the next slot transmission (Figure 3.3) the outcome is IC, i.e. a packet conflict occurs, but the control minislot is empty. The algorithm states that in resolving a conflict we define the next transmission interval as the first half of the conflict interval. At this point it is known system wide that the left half of the previous transmission interval is empty and there are at least two messages in the right half. Thus the conflict interval is the left one. Hence

$$d_{new} = d_{old} + 1 - \mu_o/2$$

$$\mu_{new} = \mu_{old}/4 = \mu_o/4$$

In the next slot transmission (Figure 3.4) the outcome is CC, i.e. a conflict occurs in both the message slot and the control mini-slot. At this point all users know that there are at least two messages in the left half of the previous transmission interval. No additional information is known about the number of messages in the right half of the interval except that is a Poisson distributed random variable. Thus, the right half interval can be merged into the unexamined portion of the arrival time axis rather than explored as determined by continuation of the algorithm. The unexamined interval is defined as the set of times that will not be part of a transmission interval before the system has passed through at least one renewal state. In Figure 3.5 the unexamined interval begins at point U and continues to the current time T_c = i+3. We note that any portion of the so defined unexamined interval has a Poisson message distribution, due to the memoryless and independent increments property of this distribution.

In this case the system variables are updated as follows:

$$d_{new} = d_{old} + 1$$

$$\mu_{new} = \mu_{old}/2 = \mu_o/8$$

In the next slot (Figure 3.5) the outcome is SC, i.e. a message conflict occurs but the control mini-slot is successful. At this point all users know that there is only one message in the left half and at least one message in the right half of the previous transmission interval. The algorithm proceeds to successfully transmit the only message in the left half. Hence

$$d_{new} = d_{old} + 1$$

$$\mu_{new} = \mu_{old}/2 = \mu_o/16$$

The outcome IC at the next slot (Figure 3.6) reassures the expected successful transmission and we are left with an interval that contains at least one message. The algorithm states that in this case we send the entire interval. Hence

$$d_{new} = d_{old} + 1 - \mu_{old}$$

$$\mu_{new} = \mu_{old} = \mu_o/16$$

A successful transmission occurs during this slot (Figure 3.7) and we have reached a new renewal state indicated by the point R_3.

The renewal state incorporated into the algorithm is important. This implies that the system repeatedly comes to a point in the algorithm where the channel history is independent of the statistics of any interval that will be transmitted in the future. Due to the existence of the renewal state, time can be thought of as consisting of a series of epochs. An epoch begins at a renewal point and ends at the next renewal point. It consists of either (1) one algorithm step if there is no transmission or one successful packet transmission, as in Figure 3.2, or (2) if a collision occurs, all of the steps required until (and including) the first subsequent successful packet transmission, as in figures 3.3-3.7.

1. Users with a message delay in the interval $[d, d-\mu/2)$ transmit a "control bit" in the control mini-slot and the message in the message-slot.

Users with a message delay in $[d-\mu/2, d-\mu)$ transmit the message in the message-slot.

Sense the Channel:

if E = IC

$d \leftarrow d + 1 - \mu/2$

$\mu \leftarrow \mu/4$

go to step 2

if E = SC

$d \leftarrow d + 1$

$\mu \leftarrow \mu/2$

go to step 2

if E = CC

$d \leftarrow d + 1$

$\mu \leftarrow \mu/4$

go to step 2

else

$d \leftarrow d + 1 - \mu_o$

$\mu \leftarrow \min(\mu_o, d + 1 - \mu_o)$

go to step 1

2. Users with a message delay in the interval $[d, d - \mu/2]$ transmit a "control bit" in the control mini-slot and the message in the message-slot.

Users with a message delay in $[d - \mu/2, d - \mu)$ transmit the message in the message-slot.

Sense the Channel:

if E = IC

$d \leftarrow d + 1 - \mu/2$

$\mu \leftarrow \mu/4$

go to step 2

if E = SC

$d \leftarrow d + 1$

$\mu \leftarrow \mu/2$

go to step 2

if E = CC

$d \leftarrow d + 1$

$\mu \leftarrow \mu/4$

go to step 2

if E = 11

$d \leftarrow d + 1 - \mu$

$\mu \leftarrow \mu/2$

go to step 2

else

$d \leftarrow d + 1 - \mu$

$\mu \leftarrow \mu$

go to step 1

Figure 2. Statement of the algorithm

4. Algorithm Analysis

4.1. Maximum stable throughput evaluation

Let $N(t)$ denote the random number of messages generated up to time t by all the users combined. We assume that $\{N(t), t \geq 0\}$ is a homogeneous Poisson process with intensity λ, measured in packets per slot.

Let η denote the output rate or throughput of the system, i.e. η is the long-run average number of successful transmissions per slot. Also, let δ_n denote the random delay between the instant at which the n^{tj} message is generated and the instant at which its packet is successfully transmitted.

A random-access algorithm is called stable if $\limsup_{n\to\infty} E\{\delta_n\}$ is finite, assuming that the limit exists. This means that for a stable algorithm the delay of a packet will remain finite with probably 1 or equivalently by Little's result that the average number of users with a mesasge which has not been successfully transmitted remains finite. It can be proved (see [ ] for a rigorous treatment) that the stability condition given above is equivalent to the following: $\lambda<\eta$

Let η_{max} denote the maximal value of the output rate η. If λ is less than η_{max}, then the throughput is λ and the system is stable. If λ exceeds η_{max}, then the throughput is at most η_{max}, thus the average delay becomes unbounded and the system is unstable. In other words

$$\eta_{max} = \sup\{\eta: \text{ the algorithm is stable}\}$$

We call η_{max} the efficiency of the given algorithm

We have seen in the previous section that the stochastic evolution of the system under the RAA-CMS defines a renewal process in time, which consists of a series of epochs. Moreover, as a result of the memoryless property of the Poisson process, the lengths of epochs are statistically independent and identically distributed. Therefore, the mean ergodic theorem implies that

$$\eta = S/U$$

where

$$U \triangleq E\{\text{Number of slots used during an epoch}\},$$

and

$$S \triangleq E\{\text{Number of successful packet transmissions during an epoch}\}.$$

We concern ourselves now with the evaluation of U and S. First we present the following notation.

(1) U_k: Average number of slots used during an epoch with k messages in the initial transmission interval.

(2) S_k: Average number of successful transmissions during an epoch with k messages in the initial transmission interval.

(3) C_k: Average number of slots used during an epoch with k messages in the current transmission interval when it is a priori known that $k \geq 2$.

(4) P_i^k: Probability that i out of k users in a given transmission interval reside in the left half of the interval.

It is relatively straightforward to obtain the following recursive expressions for the conditional expectation U_k and C_k:

$$U_k = 1 + P_1^k(1 + U_{k-1}) + \sum_{i=2}^{k} P_i^k C_i + P_o^k C_k; \qquad k \geq 2 \quad (1.a)$$

$$C_k = P_o^k(1 + C_k) + P_1^k(1 + U_{k-1}) + \sum_{i=2}^{k} P_i^k U_i; \quad k \geq 2 \quad (1.b)$$

$$U_o = U_1 = 1$$

The derivation of the above equations can be found in the Appendix.

In a similar manner we obtain the following recursive expressions for the conditional expectation S_k:

$$S_k = (P_o^k + P_k^k)S_k + P_1^k(1 + S_{k-1}) + \sum_{i=2}^{k-1} P_i^k S_i; \qquad k \geq 2 \qquad (2)$$

$$S_o = 0$$

$$S_1 = 1$$

In the above equations $P_i^k = \binom{k}{i} p^i(1-p)^{k-i}$, where $p = .5$.

Under the Poisson model assumption, we have

$$U = U(x) = \sum_{k=0}^{\infty} q_k(x)U_k$$

and

$$S = S(x) = \sum_{k=0}^{\infty} q_k(x)S_k$$

where

$$q_k = x^k e^{-x}/k! \quad \text{and } x = \lambda\mu_o.$$

It follows that the throughput as a function of x is

$$\eta(x) = \frac{\sum_{k=0}^{\infty} q_k(x) S_k}{\sum_{k=0}^{\infty} q_k(x) U_k} \tag{3}$$

The values of S_k and U_k for $0 \le k \le 10$, as obtained from (1) and (2), are given in table 1. They were used, together with numerical methods, to find the maximum in (3). It turned out to be equal to $\eta_{max} = .56$ -- and is attained for $x \simeq 1.5$.

Thus, the algorithm is stable for input rates less than .56 messages per slot.

4.2 Effective throughput

We have seen in the previous section that if the intensity of the Poisson input process is measured in messages per slot the RAA-CMS is stable iff $\lambda < \eta_{max} = .56$. Since we are using a part of the channel resources for the control mini-slots the above efficiency has to be normalized to yield the effective efficiency.

Let b(B) be the length in number of bits of the control mini-slot (message-slot) respectively. Also, let r=b/B. It follows that $\lambda' = (1+r)^{-1}\lambda$ is the Poisson intensity measured in messages per message-slot. We define the effective efficiency of the algorithm $\bar{\eta}_{max}$, as follows:

$$\bar{\eta}_{max} = (1+r)^{-1}\eta_{max}$$

Hence the algorithm is stable if

$$\lambda' < \bar{\eta}_{max}$$

Several realizable algorithms treating the multiple-access conflict resolution problem have appeared in the literature. For the case of Poisson message statistics, the best to date algorithm that performs without the use of any side information (such as the control mini slot of the present algorithm) achieves $\eta_B = .4878$ [8]. Comparing the two algorithms we find the break-even point for values of r that guarantee better throughput:

$$\bar{\eta}_{max} \geq \eta_B \text{ if and only if } r \leq (\bar{\eta}_{max}/\eta_B) - 1 \simeq .15$$

We should note that the information transmitted in each mini-slot is minimal--just absence or presence of any message. No signature information is needed. Consequently, the length b of a mini-slot could be quite short compared to B, resulting in $r \ll 1$.

5. Binary Feedback for the Control Mini-Slot

The algorithm described in section 3 assumes that the feedback information for both the control mini-slot (E_c) and the message-slot (E_m) is ternary. Thus the feedback information variable E assumes the values II, IS, SS, IC, SC and CC. In cases where the feedback is supplied to the users from a central facility, the feedback information variable needed is five-valued, i.e. E=I, or S, or IC, or SC or CC, since in the case of an empty or successful message-slot the outcome of the associated control mini-slot is not used by the algorithm and therefore is redundant.

For reasons of robustness in the presence of channel noise and/or easiness of implementation, especially in packet radio environment, one could consider different types of reduced feedback information.

We shall consider here the case of binary feedback for the control mini-slot and ternary feedback for the message-slot. The binary feedback informs the users about whether or not the previous control mini-slot was empty. Thus only "no left users" (E_c = I) and "at least one left user" ($E_c = \bar{I}$) can be distinguished. This type of binary feedback has been called Something/Nothing feedback (notation suggested by Mehravari, and Berger [16]).

In this case, there are five possible different outcomes, namely, II, IS, $\bar{I}$S, IC, and $\bar{I}$C. The algorithm to be described below uses the four-valued feedback:

$$E = \begin{cases} J & \text{if no packet in MS} \\ S & \text{if one packet in MS} \\ IC & \text{if} \geq 2 \text{ packets in MS and no control-bit in CMS} \\ \overline{I}C & \text{if} \geq 2 \text{ packets in MS and} \geq 1 \text{ control-bit in CMS} \end{cases}$$

The operation of the algorithm under binary feedback for the CMS is identical to the one described in section 3 except from the fact that some more slots are wasted due to collisions that would have been avoided if ternary feedback had been available. For example, if the outcome of the previous slot is $\overline{I}C$, we send the entire left half of the interval under consideration, since we cannot distinguish between one message ($\overline{I}$ = S) and more than one message ($\overline{I}$ = C). The later case results in a collision. Under ternary feedback, this collision is "certain-to-occur," since we know that $\overline{I}$ = C, and is avoided by a priori dividing the left half interval into two.

The definitions of η, U, S, U_k, S_k, C_k, P_i^k are the same as those used in section 3. The recursive formulas for C_k and S_k are identical to (1.b) and (2). For U_k, we have:

$$U_k = 1 + P_o^k C_k + P_1^k(1 + U_{k-1}) + \sum_{i=2}^{k} P_i^k U_i; \quad k \geq 2 \qquad (4)$$

where

$$U_0 = U_1 = 1$$

$$P_i^k = \binom{k}{i} P^i (1-p)^{k-i} \quad \text{and } p = .5$$

Substitution of (1.b) into (3) gives:

$$U_k = 1 + \frac{1}{2^k-2}\left(\frac{1}{2^k} + 2k\, U_{k-1} + \sum_{i=0}^{k-2} \binom{k}{i} U_i\right); \qquad k \geq 2 \tag{5}$$

where

$$U_0 = U_1 = 1$$

The values of U_k for $0 \leq k \leq 10$, as obtained from (5), are given in the fourth column of table 1 under the name U_k'. They were used, together with numerical methods, to find the maximum in (3). It turned out to be equal to $\eta_{max} = .522$ and is attained for $x \approx 1.4$.

Hence, the RAA-CMS with binary feedback is stable for input rates below .522 messages per channel-slot or $(1+r)^{-1}$ .522 messages per message-slot.

6. Conclusions

In this paper, we presented a random access algorithm which utilizes an additional feedback information for the resolution of collisions, through the use of a control mini-slot. We called this algorithm RAA-CMS, and we found that its stability region is obtained for input rates below $.56(1+r)^{-1}$. Where r is the ratio of the length of the control mini-slot used over the length of a packet. Since the information transmitted in each mini-slot is minimal (absence or presence of any message) r can be very small.

We compared the RAA-CMS to the algorithm with the greatest known efficiency without additional information and we found that under the assumed idealized conditions the former achieves higher throughput if the length of the control mini-slot is less than 15% of the length of a packet.

The corresponding properties of the RAA-CMS using binary feedback for the control mini-slot were also studied.

It should be noted that the analysis presented here is an effort towards more practical access algorithms that achieve higher throughputs. The basic problem of finding the capacity of the best random access algorithm under the Poisson model defined in the introduction remains open.

Appendix

Derivation of the Recursive Equations for U_k and C_k

Consider a new transmission interval which contains K message arrivals. If K = 0 or 1 the epoch lasts only one slot and $U_0 = U_1 = 1$. If $K \geq 2$, each of the K messages has equal probability of arriving anywhere within the current transmission interval independently of the other messages, i.e. the K arrival times are independent and uniformly distributed. This is due to the fact that the arrival process within the given transmission interval is Poisson conditioned on the number K of arrivals. Hence, the number i of messages in the left half of the given transmission interval is binomially distributed with probability

$$P_i^k = \binom{k}{i} p^i (1-p)^{k-i} \quad \text{and } p = .5$$

Upon studying the algorithm, we consider these subcases, i=0, i=1 and $i \geq 2$. We define U_i^k to be the expected duration of an epoch in which there are K arrivals of which i reside in the left half of the transmission interval. In all cases there is an initial one unit of time due to the initial collision.

a) i=0: In this case the epoch includes the initial collision and C_k slots to resolve the right half interval which contains K messages and it is known that $K \geq 2$. Hence $U_0^k = 1 + C_k$, $k \geq 2$.

b) $i=1$: In this case the epoch includes the initial collision, one slot to successfully transmit the only message in the left half and U_{k-1} slots to resolve the remaining k-1 messages residing in the right half of the transmission interval. Hence
$U_1^k = 2 + U_{k-1}$, $k \geq 2$

c) $i \geq 2$: In this case, the epoch includes the initial collision and C_i slots to resolve the i messages residing in the left half of the transmission interval. Note that the epoch will end without examining the remaining right half because it is known that $i \geq 2$. Hence $U_i^k = 1 + C_i$, $k \geq 2$; $2 \leq i \leq k$

Removing the conditioning on i gives the following equation.

$$U_k = \sum_{i=0}^{k} P_i^k U_i^k = P_0^k(1 + C_k) + P_1^k(2 + U_{k-1}) + \sum_{i=2}^{k} P_i^k(1 + C_i)$$

Rearranging this yields

$$U_k = 1 + P_1^k(1 + U_{k-1}) + \sum_{i=2}^{k} P_i^k C_i + P_0^k C_k; \quad k \geq 2 \qquad (1.a)$$

Similarly, to derive the recursive equation for C_k we define C_i^k to be the expected duration of an epoch in which there are K arrivals of which i reside in the left half of the transmission interval and is a priori known that $K \geq 2$. It is not difficult to see that the following equations hold:

$$C_0^k = 1 + C_k$$

$$C_1^k = 1 + U_{k-1}$$

$$C_i^k = U_i, \qquad k \geq 2; \quad 2 \leq i \leq k$$

Removing the conditioning on i gives

$$C_k = \sum_{i=0}^{k} P_i^k \, C_i^k = P_o^k(1 + C_k) + P_1^k(1 + U_{k-1}) + \sum_{i=2}^{k} P_i^k \, U_i;$$

$$k \geq 2 \tag{1.b}$$

TABLE 1

K	S_K	U_K
0	0	1
1	1	1
2	2	3.4
3	2.5	4.7400
4	2.5714	5.0958
5	2.5238	5.1303

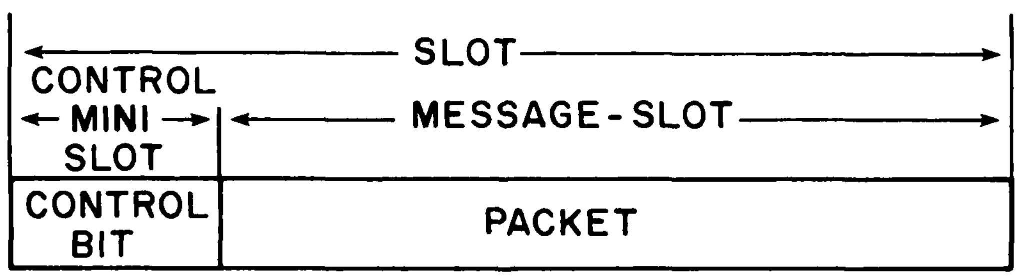

FIG. 1

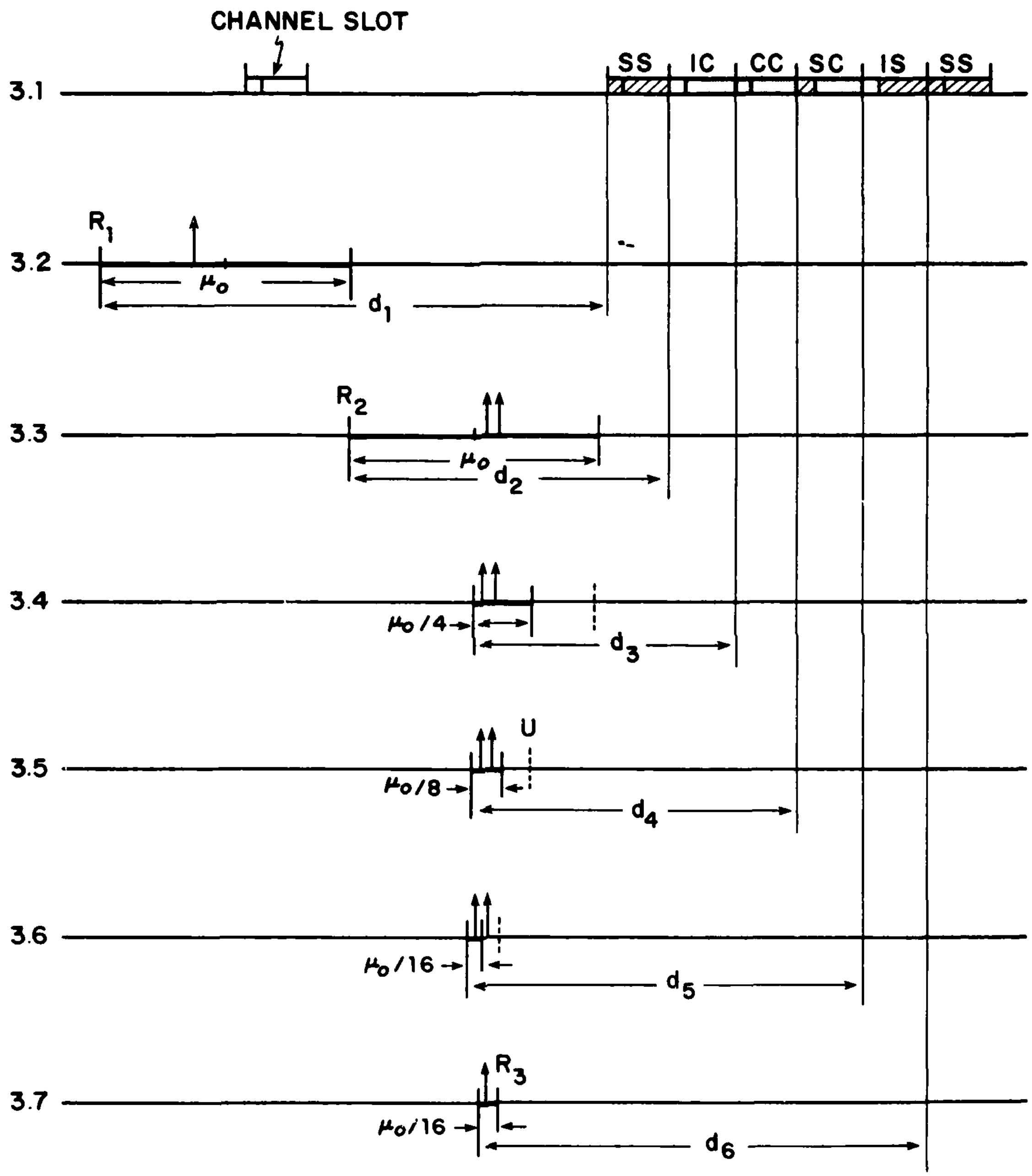
CHANNEL SLOT
SS
IC
CC
SC
IS
SS
3.1
R1
3.2
μo
d1
R2
3.3
μo
d2
3.4
μo/4
d3
U
3.5
μo/8
d4
3.6
μo/16
d5
R3
3.7
μo/16
d6

FIG. 3

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