MOTION DETECTION AT CORNERS

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ABSTRACT

A method for directly computing image motion at corners is presented. The method is based on temporal intensity gradients along lines parallel to the sides of the corner. The results of applying the method to two time-varying images are discussed.

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1. **Introduction**

The earliest problem that arises in the analysis of time-varying images is the detection of moving image elements (edge, regions) and the computation of the image velocity (optic flow) of those elements. A variety of computational schemes have been proposed to solve this problem. In a recent survey, Ullman [1] broadly classifies these as intensity-based and token-matching schemes.

An important class of intensity-based schemes takes advantage of the relationship between the temporal and spatial gradient of any continuous and differentiable image property which is invariant to small changes in perspective. For example, if we assume that the intensity, \( I \), satisfies these properties, the relationship

\[-I_t = uI_x + vI_y\]

can be used to determine velocity. Here, \( I_t \) is the temporal intensity gradient, \( I_x \) and \( I_y \) the x and y components of the spatial intensity gradient, and \( u \) and \( v \) the x and y components of image velocity. Measuring \( I_t, I_x, I_y \) from an image sequence establishes a linear constraint on the x and y velocity components. A single velocity estimate can be computed by spatially combining the constraints using e.g., Hough transforms [2], least-squares methods [3] or minimization techniques [4]. All of these techniques suffer from certain disadvantages. The Hough-transform and minimization techniques assume that image velocity is uniform over large parts of
the image, and the least-squares method further assumes that the constraint equations determined for nearby points are independent - an assumption that is violated by the spatial integration required to compute spatial derivatives. An alternative to these approaches would be to compute multiple constraint equations of a single point based on several invariant image properties - e.g., intensity, derivatives of intensity, color. This approach is described in Thrift et al. [5].

This is the first in a series of reports which describes an approach to image velocity estimation and subsequent moving object extraction and tracking. Although the approach utilizes some of the gradient-based motion estimation techniques mentioned above, it has significant differences from those techniques.

First, we initially compute the motion estimates at only a small subset of image points. These are points at which it is possible, in principle, to determine image velocity with little or no integration of spatial information. In this way we avoid making any unnecessary assumptions about even the local distribution of image velocities. Furthermore, the actual estimated velocities are subject to certain statistical and heuristic confidence tests, which can further reduce the initial set of points to which motion vectors
are attributed. This paper is concerned with this first step and describes techniques for measuring image motion at corners. Corners have the property that their motion can be directly computed based only on measurements made at the corner. (in practice, of course, one must examine a small neighborhood of the corner). Another important property of corners is that they can be safely regarded as projections of scene features whose general appearance is invariant to rigid motion - e.g., an image corner may be the projection of the vertex of a polyhedron, or of a curvature discontinuity on the boundary of a surface marking. Thrift et al. [5] describes a complementary approach in which several image surface attributes are combined at a single point to compute a motion vector at that point. They also describe how the confidence of that estimate can be evaluated.

The second step involves propagating these velocity estimates to a larger number of picture points. In Wu et al [6] we will describe one such method which propagates velocity vectors along image contours. The principal difficulties with developing and applying such techniques are guaranteeing that the propagation technique is formally correct (at least for rigid, planar motions) and avoiding propagation across object boundaries. The resulting pattern
of velocity vectors may be loosely regarded as a "mosaic" of Glass patterns [7]. Davis and Narayanan [8] will consider the problem of segmenting such Glass pattern mosaics. Subsequent reports will consider the integration of motion information across many frames.
2. Estimating motion at corners

The motion of a corner can be computed based on temporal intensity changes along lines parallel to the sides of the corner. We first describe velocity computation for the case of translation motion, and then consider translation combined with rotation.

Suppose that a corner simply translates from point $C_0$ to $C_1$ between two frames $t_0$ and $t_1$ (see Figure 1). Let $0'$ be a point on the bisector of $PC_0R$ and let $O'A$ and $O'B$ be lines parallel to $C_0P$ and $C_0R$, respectively at some unit distance from $C_0P$ and $C_0R$. Suppose that $|O'A| = |O'B| = 1 + m$, for some constant $m$. Finally, assume that the intensity inside the corner is 1 and outside the corner is 0.

Now, at time $t_0$, the average intensity along line segments $O'A$ and $O'B$ is

$$I_{O'A}(t_0) = I_{O'B}(t_0) = 1/(1 + m)$$

If $\Delta x'$ and $\Delta y'$ are the components of the translation in the directions of the lines $O'A$ and $O'B$, then

$$I_{O'A}(t_1) = (1 + \Delta x')/(1 + m)$$
$$I_{O'B}(t_1) = (1 + \Delta y')/(1 + m)$$

assuming that $m$ is chosen large enough so that $\max(\Delta x', \Delta y') < m$. Finally, $\Delta x'$ and $\Delta y'$ can be computed from

$$\Delta I_{O'A} = I_{O'A}(t_1) - I_{O'A}(t_0) = \Delta x'/(1 + m)$$
$$\Delta I_{O'B} = I_{O'B}(t_1) - I_{O'B}(t_0) = \Delta y'/(1 + m)$$

Once $\Delta x'$ and $\Delta y'$ are computed, the components of the velocity in the original image coordinate system can be recovered easily:
Figure 1
The practical success of this technique depends on our ability to compute several corner parameters accurately. These parameters are

1. corner location at \( t_0 \),
2. corner shape (angles \( \alpha \) and \( \beta \)), and
3. corner contrast (assumed here to be 1)

Section 3 discusses the computation of these parameters.

Next, we extend the previous simple analysis to include rotation as well as translation. We will treat this case as a translation from \( C_0 \) to \( C_1 \) followed by a rotation about \( C_1 \) through a clockwise angle \( \gamma \) (see Figure 2). Since translation and rotation are specified by a total of three parameters, we could extend the above analysis using only a third line segment parallel to either \( O'A \) or \( O'B \). Instead, we consider two pairs of parallel line segments, and compute the displacements in the directions \( O'A \) and \( O'B \) rather than directly computing the angle \( \gamma \).

Let \( \Delta x'_t \) and \( \Delta y'_t \) be the translational components of the motion in the \( O'A \) and \( O'B \) directions, and \( \Delta x'_r \) and \( \Delta y'_r \) the corresponding rotational components. Then

\[
\begin{align*}
\Delta x'_t + \Delta x'_r &= (1+m) \Delta I_{O'A} \quad (1) \\
\Delta y'_t - \Delta y'_r &= (1+m) \Delta I_{O'B} \quad (2)
\end{align*}
\]

From Figure 3, we see that
\[
\frac{\Delta x'_r}{1+\Delta y'_t} = \frac{\Delta x'_r - \Delta x''_r}{\delta}
\]  
where \(\delta\) is the distance between the parallel line segments O'A and CD. Similarly

\[
\frac{\Delta y'_r}{1+\Delta x'_t} = \frac{\Delta y'_r - \Delta y''_r}{\delta}
\]

Also

\[
\Delta x'_r - \Delta x''_r = (1+m)[I_{O'_A}(t_1) - I_{CD}(t_1)]
\]

\[
\Delta y'_r - \Delta y''_r = (1+m)[I_{O'_B}(t_1) - I_{EF}(t_1)]
\]

Substituting (5) and (6) into (3) and (4) and simplifying, we obtain

\[
\Delta x'_r - C_1\Delta y'_t = C_1
\]

\[
C_2\Delta x'_t - \Delta y'_r = -C_2
\]

where

\[
C_1 = \frac{(1+m)}{\delta} [I_{O'_A}(t_1) - I_{CD}(t_1)]
\]

\[
C_2 = \frac{(1+m)}{\delta} [I_{EF}(t_1) - I_{O'_B}(t_1)]
\]

Solving for \(\Delta x'_t\) and \(\Delta y'_t\) we obtain

\[
\Delta x'_t = \frac{(1+m)(\Delta I_{O'_A} - C_1\Delta I_{O'_B}) - C_1(1+C_2)}{1+C_1C_2}
\]

\[
\Delta y'_t = \frac{(1+m)(C_2\Delta I_{O'_A} - C_1\Delta I_{O'_B}) - C_2(1-C_1)}{1+C_1C_2}
\]

Substituting (7) and (8) into (9) and (10), we can also compute \(\Delta x'_r\) and \(\Delta y'_r\), which gives us a complete description of the motion of the corner.
3. **Applications**

The corner motion model described in Section 2 has been applied to two image sequences containing two frames each (Figures 4-5). Corners are initially detected using the corner detector described in Kitchen and Rosenfeld [9]. Next, a small window around each corner is analyzed to obtain a more accurate description of the corner. Based on the assumption that the corner locally contrasts with its surround, a local thresholding procedure (Milgram [10]) is used to segment the window. The corner is then relocated to a maximum curvature boundary point in the thresholded window. The slopes of the line segments meeting at the corner are computed using a one-dimensional (slope) Hough transform procedure (only slope need be computed since the lines are constrained to pass through the corner point.) The corners detected by this procedure are marked with dark crosses in Figures 4a and 5a.

To overcome the effects of various sources of error on the motion estimation, several quadruples of line segments are used to compute estimates of \( \Delta x'_t, \Delta y'_t, \Delta x'_r \) and \( \Delta y'_r \), with the final motion estimate taken as the average.

The results are displayed in Tables 1 and 2. The estimated motion vectors were obtained by the authors' examination of digital enlargements of the images.

For most of the corners, the results are quite satisfactory. The error in corner 8 in Table 1 is due to a hole in the object
Figure 4. First motion sequence. Detected corners marked with dark crosses in frame 1.

Figure 5. Second motion sequence. Detected corners marked with crosses in frame 1.
### Table 1. Motion vectors for corners in Figure 4a.

<table>
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<th>x</th>
<th>y</th>
<th>alpha</th>
<th>beta</th>
<th>dx</th>
<th>dy</th>
<th>x</th>
<th>y</th>
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<td>8</td>
<td>227</td>
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### Table 2. Motion vectors for corners in Figure 5a.

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<tr>
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<td>292.6</td>
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</tr>
</tbody>
</table>
near the corner which violates the homogeneity assumptions made by our segmentation algorithm. We should also point out that for more rounded corners, the estimates are poor because of errors in locating the corner (corner 1 in Table 2).
4. Conclusions

We have shown how the velocity of a corner can be reliably estimated based only on very simple measurements of temporal intensity changes in the neighborhood of the corner. The success of the proposed technique depends mostly on our ability to reliably detect and describe corners, but also to a great extent on our being able to identify a sufficiently large neighborhood of the corner which contains only the corner and one component of the background, and no other moving objects. In cases where such a neighborhood does not exist, the method gives poor results. The need for very high resolution near the corners for motion estimation suggests that for reasons of efficiency the corner motion estimation problem should be reconsidered based on a multiresolution image representation. The uses of pyramids for motion detection in general, and corner motion detection in particular, will be discussed in a subsequent report.
References


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