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ANALYSIS AND COMPUTER STUDIES OF
MAGNETOSTATIC WAVE TRANSDUCERS

University of Lowell

l. Jacob Weinberg

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    - This report contains the results obtained for magnetostatic wave transducers which were outlined in the Interim Technical Report.

    The analysis is presented for obtaining magnetostatic wave power, radiation resistance, radiation reactance and insertion loss for forward volume waves and backward volume waves. Included for consideration are ground planes, apodization of the fundamental mode, isolated independent conduc-
tors, multistrip meander line and parallel grating, for a uniform current distribution without variable coupling.

Also presented are the results for the dispersion relation and group delays for volume waves with ground planes for arbitrary orientation of the biasing field. We also include the results of attempting to obtain non-dispersive, electronically tunable time delay elements by combining two volume waves with the same specified biasing field orientation but with different magnitudes of the magnetic field intensity.

We also present the results of incorporating a non-uniform, hyperbolic current distribution in the basic magnetostatic surface wave program.

For each of the tasks described above there are presented the analysis, the computer program with detailed documentation enabling its use by others and computer produced graphical results for cases of interest.
# Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Basic Theory - Forward and Backward Volume Waves.</td>
<td>3</td>
</tr>
<tr>
<td>Dispersion Relation - Generalized Volume Waves.</td>
<td>10</td>
</tr>
<tr>
<td>Nondispersive Time Delay</td>
<td>13</td>
</tr>
<tr>
<td>Hyperbolic Current Distribution.</td>
<td>14</td>
</tr>
<tr>
<td>Computer Programs.</td>
<td>16</td>
</tr>
<tr>
<td>Basic Theory - Forward and Backward Volume Waves.</td>
<td>16</td>
</tr>
<tr>
<td>Dispersion Relation - Generalized Volume Waves.</td>
<td>26</td>
</tr>
<tr>
<td>Hyperbolic Current Distribution - Surface Waves.</td>
<td>33</td>
</tr>
<tr>
<td>Figures</td>
<td>52</td>
</tr>
<tr>
<td>References</td>
<td>69</td>
</tr>
</tbody>
</table>
Introduction

The purpose of this report is to present the results obtained for magnetostatic wave transducers under contract number F19628-80-C-0029 from the Air Force ESD RADC/EEA Hanscom AFB, MA since obtaining the results presented in the Interim Technical Report [1].

In the earlier report the analysis, computer programs and graphical results were displayed for the flat field, basic theory magnetostatic surface wave program, the microstrip model for surface waves and the dispersion relation for generalized surface waves. There was also presented the analysis for complex impedance for surface waves in free space.

We here present the analysis and computer program for obtaining the dispersion relation, group delay, magnetostatic wave power, radiation resistance, radiation reactance and insertion loss for forward volume waves and backward volume waves. Included are the consideration of ground planes, apodization of the fundamental mode, multistrip meander line and parallel grating and isolated independent conductors for a uniform current distribution without variable coupling. The analysis, computer program and its use together with graphical results are presented.

The analysis for obtaining the dispersion relation and group delay for volume waves with arbitrary orientation of the biasing field is presented here. The computer program and graphical results for cases of interest for the generalized
dispersion relation for volume waves including ground planes are presented. Also included in the report are the results of attempting to obtain nondispersive, electronically tunable time delay elements by combining the group delays for volume waves of two different biasing field magnitudes in the same specified orientation.

We also present the analysis of incorporating a non uniform, hyperbolic current distribution in the basic magnetostatic surface wave program. The computer program accomplishing this task is also presented.
Basic Theory - Forward and Backward Volume Waves

The basic theory leading to the dispersion relation, group delay, magnetostatic wave power, radiation resistance, radiation reactance and insertion loss for forward and backward volume waves for a transducer consisting of a YIG region sandwiched between two finite dielectrics (see Figure 1) is here outlined. We follow the previous work on surface waves [1][4]. We start with Maxwells equations

\[ \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} ; \quad \nabla \cdot \vec{B} = 0 \quad (1) \]

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} ; \quad \nabla \cdot \vec{D} = 0 \]

and the constitutive relations in each region

\[ \vec{B} = \mu_0 (\vec{H} + \vec{M}) \quad (2) \]

\[ \vec{D} = \varepsilon \vec{E} \]

where \( \vec{M} \) is non zero in the YIG region only. We utilize the gyromagnetic relation in the YIG region

\[ \frac{\partial \vec{M}}{\partial t} = -\gamma \vec{M} \times \vec{H} \quad (3) \]

and retain first order terms only.

We assume the time dependence of all physical quantities to be \( e^{j\omega t} \), we also assume the magnetostatic approximation

\[ H_z = E_x = E_y = 0 \quad (4) \]

\[ \omega \varepsilon E_z = 0 \]

and no variation of any physical quantity in the z direction.
We have

\[ \frac{\partial E_z}{\partial y} = -j \omega B_x \]  
\[ \frac{\partial E_z}{\partial x} = j \omega B_y \]

Also

\[ B_x = \mu_0 H_x \]  
\[ B_y = \mu_0 H_y \]

in the non YIG regions, while

\[ \begin{pmatrix} B_x \\ B_y \end{pmatrix} = \mu_0 \begin{pmatrix} \mu_{11} & -j \mu_{12} \\ j \mu_{21} & \mu_{22} \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix} \]  

in the YIG region, where, for forward and backward volume waves,

\[ \mu_{12} = \mu_{21} = 0 \]  

and, for forward volume waves, (\( \theta = 0 \) in Figure 1),

\[ \mu_{11} = \left( 1 + \frac{\gamma^2 H_o \cdot 4\pi M_o}{\gamma^2 H_o^2 - f^2} \right) \]  
\[ \mu_{22} = 1 \]

and, for backward volume waves, (\( \theta = 90^\circ, \phi = 0 \) in Figure 1)

\[ \mu_{11} = 1 \]  
\[ \mu_{22} = \left( 1 + \frac{\gamma^2 H_o \cdot 4\pi M_o}{\gamma^2 H_o^2} \right) \]

where

\[ \gamma = 2.8 \text{ mhz/oe} \; ; \; 4\pi M_o = 1750 \text{ oe} \; ; \; f = \frac{\omega}{2\pi} \]
We define

\[ \alpha^2 = -\frac{\mu_{11}}{\mu_{22}} \]  

(12)

so that

\[
\alpha^2 = \begin{cases} 
\left(1 + \frac{\gamma^2 H_O 4\Pi M_O}{\gamma^2 H_O^2 - f^2}\right) \quad \text{forward volume waves} \\
\left(1 + \frac{\gamma^2 H_O 4\Pi M_O}{\gamma^2 H_O^2 - f^2}\right)^{-1} \quad \text{backward volume waves}
\end{cases}
\]  

(13)

and, in order to have volume waves,

\[ \alpha^2 > 0 \]  

(14)

Solutions are sought which satisfy boundary conditions \( B_y = 0 \) at the ground planes and continuity conditions on \( H_x \) and \( B_y \) at the region junctions except, that at \( y = 0 \) we require \( l_x \) to be discontinuous by the amount of the surface current density, \( J(x) \), supplied there.

Assuming a solution in the form of a potential function

\[ \psi = R(y)e^{j(\omega t - kx)} \]  

(15)

where

\[ H_x = \frac{\partial \psi}{\partial x} \quad \text{and} \quad H_y = \frac{\partial \psi}{\partial y} \]  

(16)

we find the form of \( R(y) \) in the non YIG regions as

\[ R(y) = A_i e^{\kappa y} + B_i e^{-\kappa y} \quad i=1,3 \]  

(17)

and in the YIG region

\[ R(y) = A_2 \cos \kappa y + B_2 \sin \kappa y \]  

(18)

Attempting to resolve the boundary and continuity conditions results in a quantity \( F(k) \) appearing in the expressions for the constants in (17) and (18).
For forward volume waves

\[ F(k) = \frac{\sin \alpha |k|d e^{-|k|d}}{\alpha} \left\{ -\alpha \cot \alpha |k|d \left[ (\coth |k|t_1 + 1) - e^{-2|k|d} \right] \\
(\coth |k|t_1 - 1) + (\alpha^2 \coth |k|t_1) e^{-2|k|d} (\alpha^2 \coth |k|t_1 + 1) \right\} \]

(19)

and, for backward volume waves

\[ F(k) = \frac{\sin \alpha |k|d e^{-|k|d}}{\alpha} \left\{ -\alpha \cot \alpha |k|d \left[ (\coth |k|t_1 + 1) - e^{-2|k|d} \right] \\
(\coth |k|t_1 - 1) + (\alpha^2 - \coth |k|t_1) e^{-2|k|d} (\alpha^2 + \coth |k|t_1) \right\} \]

(20)

where \( l, d, t_1 \) are the widths of the three regions (see Figure 1).

If we require that \( F(k) \) vanishes we obtain the dispersion relation

\[ \tan \alpha |k|d = \alpha \left[ (\coth |k|t_1 + 1) - e^{-2|k|d} (\coth |k|t_1 - 1) \right] \]

\[ \left\{ \frac{(\alpha^2 \coth |k|t_1 - 1) e^{-2|k|d} (\alpha^2 \coth |k|t_1 + 1)}{(\alpha^2 - \coth |k|t_1) e^{-2|k|d} (\alpha^2 + \coth |k|t_1)} \right\}^{-1} \] forward waves

\[ \left\{ \frac{(\alpha^2 \coth |k|t_1 + 1) e^{-2|k|d} (\alpha^2 \coth |k|t_1 - 1)}{(\alpha^2 - \coth |k|t_1) e^{-2|k|d} (\alpha^2 + \coth |k|t_1)} \right\}^{-1} \] backward waves

(21)

which is solved numerically, by iteration, to obtain \( k \) as a function of \( f \).

There are an infinite number of solution modes of (21) corresponding to the multiplicity of the inverse tangent function. For each mode the solution for positive \( k \) (\( s = |k|/k = 1 \)) is the same as for negative \( k \) (\( s = |k|/k = -1 \)) since only the magnitude of \( k \) appears in the dispersion relation.

The bandwidth for which the solution is obtained is given by

\[ \gamma H_0 < f < \gamma \sqrt{H_0 \left( H_0 + 4\pi M_0 \right)} \]  

(22)
We can now find the group delay from the dispersion relation curves, $k$ vs $f$, by numerically applying

$$v_g = \frac{\partial \omega}{\partial k} \tag{23}$$

We can find all the constants in the solution and proceed to find the field equations as for surface waves \[4\]. The magnetostatic wave power is then obtained from \[4\]

$$P = \frac{1}{2} \int_{-(\ell+d)}^{t+1} E_z H_y \, dy \tag{24}$$

utilizing (5).

We then find

$$P = \omega \omega_0 \frac{G^2}{4|k|^2} \, A \tag{25}$$

where $k$ is obtained from the dispersion relation, and

$$G = \frac{e^{-|k|d}}{k} \frac{\tilde{J}(k)}{\tilde{J}'(k) \tilde{P}(k')} \Bigg|_{k'=k} \tag{26}$$

For independent conductors

$$|\tilde{J}(k)| = \left| \sum_{i=1}^{N} \text{sinc} \frac{a_i k}{2\pi} \eta i \sqrt{l_{1i}} e^{-jkp_i} \right| \tag{27}$$

where $l_{1i}$, $a_i$, $p_i$ are the strip lengths, strip widths and center to center spacings for the $N$ strips and $\eta=1$ for a parallel grating while $\eta = -1$ for a meander line and

$$\text{sinc} \ x = \frac{\sin \pi x}{\pi x} \tag{28}$$

For uniform $N$ strips we can write

$$|\tilde{J}(k)| = \sqrt{l_1} \text{sinc} \frac{ak}{2\pi} \left| \frac{1-\eta N}{1-\eta} e^{jkp} \right| \tag{29}$$
For the quantity $A$ in (25) we have, for forward volume waves,

$$A = 2e^{-2|\mathbf{k}|^2} (\sinh 2|\mathbf{k}|^2 - 2|\mathbf{k}|^2) + (\alpha^2 + 1)|\mathbf{k}|^2 - \frac{\alpha^2 - 1}{2\alpha} \sin 2\alpha|\mathbf{k}| + \cos 2\alpha|\mathbf{k}|^2 - 1$$

$$+ \left( \sinh 2|\mathbf{k}|^2 - 2|\mathbf{k}|^2 \right) \frac{\alpha^2 - 1}{2\alpha} - \frac{\alpha^2 - 1}{2} \cos 2\alpha|\mathbf{k}| + \alpha \sin 2\alpha|\mathbf{k}|$$

$$+ 2e^{-2|\mathbf{k}|^2} \left( \alpha^2 - 1 \right)|\mathbf{k}|^2 - \frac{\alpha^2 - 1}{2\alpha} \sin 2\alpha|\mathbf{k}| + \frac{\sinh 2|\mathbf{k}|^2 - 2|\mathbf{k}|^2}{2 \sinh^2 |\mathbf{k}|^2} \left( \frac{\alpha^2 - 1}{2} - \frac{\alpha^2 - 1}{2} \cos 2\alpha|\mathbf{k}| + 2 \sin 2\alpha|\mathbf{k}| \right)$$

and, for backward volume waves,

$$A = 2e^{-2|\mathbf{k}|^2} (\sinh 2|\mathbf{k}|^2 - 2|\mathbf{k}|^2) - (\alpha^2 + 1)|\mathbf{k}|^2 - \frac{\alpha^2 - 1}{2\alpha} \sin 2\alpha|\mathbf{k}| + \cos 2\alpha|\mathbf{k}|^2$$

$$- 1 + \left( \sinh 2|\mathbf{k}|^2 - 2|\mathbf{k}|^2 \right) \frac{\alpha^2 - 1}{2\alpha} + \frac{\alpha^2 - 1}{2\alpha} \cos 2\alpha|\mathbf{k}| + \frac{\sin 2\alpha|\mathbf{k}|}{\alpha}$$

$$+ 2e^{-2|\mathbf{k}|^2} \left( \alpha^2 - 1 \right)|\mathbf{k}|^2 + \frac{\alpha^2 + 1}{2\alpha} \sin 2\alpha|\mathbf{k}| - \frac{\sinh 2|\mathbf{k}|^2 - 2|\mathbf{k}|^2}{2 \sinh^2 |\mathbf{k}|^2} \left( \frac{\alpha^2 - 1}{2} + \frac{\alpha^2 - 1}{2} \cos 2\alpha|\mathbf{k}| \right)$$

$$+ e^{-4|\mathbf{k}|^2} \left[ 1 - (\alpha^2 + 1)|\mathbf{k}|^2 - \frac{\alpha^2 - 1}{2\alpha} \sin 2\alpha|\mathbf{k}| + \cos 2\alpha|\mathbf{k}|^2 \right]$$

(31)
\[
+ \left( \frac{\sinh 2|k|t_1 - 2|k|t_1}{2 \sinh^2 |k|t_1} \right) \left( \frac{a^2 + 1}{2a^2} + \frac{a^2 - 1}{2a^2} \cos 2a|k|d - \sin \frac{2a|k|d}{a} \right)
\]

Proceeding as for surface waves\textsuperscript{[1]}, the radiation resistance is

\[
R = \frac{4|p|}{(1-\eta) + (1+\eta)N^2}
\]  
(32)

for one wave.

The total radiation resistance is

\[
R_m = 2R
\]  
(33)

and the radiation reactance is obtained from

\[
X_m(f) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{r_m(f') df'}{f'-f}
\]  
(34)

by numerical integration.

The insertion loss is then given by

\[
IL = 20 \log_{10} \frac{4 \frac{R}{R_g} \frac{R_g}{(R_g + R_m + R_L)^{2} + (X_m + X_L)^{2}}}{\Delta H \Delta r} - \frac{76.4 \times 10^6}{3\omega / \partial k} \frac{\Delta H \Delta r}{\omega}
\]

where \(R_g\) is source resistance, \(R_L\) is conduction loss and \(X_L\) is a series matching impedance. \(\Delta H\) is linewidth representing material loss and \(\Delta r\) is the propagation distance.

We thus have relations for all the quantities of interest for both forward volume waves and backward volume waves.

In figures 8-17 are presented graphical displays of the dispersion relation, group delay, radiation resistance, radiation reactance and insertion loss for forward volume waves and backward volume waves.
Dispersion Relation - Generalized Volume Waves

In this section we obtain the dispersion relation for volume waves including ground planes when the biasing field can be in an arbitrary direction (see Figure 1). We have the permeability tensor components appearing in (7) as \([1],[5]\)

\[
\begin{align*}
\mu_{11} &= 1 + \gamma^2 H_0 (4\pi M_0) \left( \frac{\sin^2 \theta \sin^2 \phi + \cos^2 \theta}{\gamma^2 H_0^2 - f^2} \right) \\
\mu_{22} &= 1 + \gamma^2 H_0 (4\pi M_0) \frac{\sin^2 \theta}{\gamma^2 H_0^2 - f^2} \\
-j\mu_{12} &= j\gamma (4\pi M_0) \frac{\sin \theta (f \sin \phi + j\gamma H_0 \cos \phi \cos \theta)}{\gamma^2 H_0^2 - f^2} \\
j\mu_{21} &= -j\gamma (4\pi M_0) \frac{\sin \theta (f \sin \phi - j\gamma H_0 \cos \phi \cos \theta)}{\gamma^2 H_0^2 - f^2}
\end{align*}
\]

where

\[
\begin{align*}
\gamma &= 2.8 \text{ mHz/oe} , \quad 4\pi M_0 = 1750 \text{ oe} , \quad f = \frac{\omega}{2\pi}
\end{align*}
\]

The solution is in the form of a potential function such as

\[
\psi = e^{j(\omega t-kx)} (Ae^{j|k|y} + Be^{-j|k|y})
\]

in the non YIG regions while, in the YIG region, we have the form for the solution as

\[
\psi = e^{j(\omega t-kx)} (A \cos c|k|y + B \sin c|k|y)e^{-jkby}
\]

where

\[
c^2 = \frac{-\mu_{21} \mu_{12}^2 - 4\mu_{11} \mu_{22}}{4\mu_{22}^2}
\]
\[ b = -j(\mu_{21} - \mu_{12}) \frac{2}{\mu_{22}} \]

and, for volume waves,

\[ c^2 > 0. \quad (41) \]

Here we note the additional propagation term containing \( b \) in (39) which vanishes for standard forward volume waves (\( \theta = 0 \)) and backward volume waves (\( \theta = 90^\circ, \phi = 0 \)).

By attempting to satisfy the continuity conditions and boundary conditions we obtain the dispersion relation

\[
[(\coth |k| t_1 + 1) - e^{-2|k| \ell} (\coth |k| t_1 - 1)] \mu_{22} \cot c|k|d = \\
\coth |k| t_1 \left[c^2 \mu_{22}^2 - (j\mu_{21} + b\mu_{22})^2 \right] \\
-1 + j \left[ \frac{|k|}{\ell} \right] (\coth |k| t_1 - 1) (j\mu_{21} + b\mu_{22}) \\
+ e^{-2\ell} \left[ \coth |k| t_1 \left[c^2 \mu_{22}^2 - (j\mu_{21} + b\mu_{22})^2 \right] \\
+ 1 - j \left[ \frac{|k|}{\ell} \right] (\coth |k| t_1 + 1) (j\mu_{21} + b\mu_{22}) \]
\]

There are an infinite number of solution modes for this equation corresponding to the multiplicity of the inverse tangent function. For each mode there is one solution for positive \( k(|k|/k = s = 1) \) and another solution for negative \( k(|k|/k = s = -1) \).

This dispersion relation reduces to the ones obtained for forward volume waves and backward volumes by specializing the \( \theta \) and \( \phi \) values.

The bandwidth for the existence of volume waves is known to be \([2]\)
\[ \gamma \sqrt{H_0 (H_0 + 4\pi M_0 \sin^2 \theta \sin^2 \phi)} < f < \gamma \sqrt{H_0 (H_0 + 4\pi M_0)} \]

(43)

Dispersion relation curves are presented in Figures 3-6.

For the group delay we evaluate numerically

\[ v_g = \frac{\omega}{\partial k} \]

(44)
Non-dispersive Time Delay

The dispersion relation curves obtained in the plane defined by \( \phi = 0 \) consist of a backward volume wave curve with a forward volume wave curve as shown in Figures 5 and 6. The frequency at which the changeover from backward volume wave to forward volume wave occurs is given by \(^[2]\)

\[
f_c = \gamma \sqrt{H_o (H_o + 4\pi M_o \sin^2 \theta)}
\]  

(45)

This critical frequency is at the midpoint of the frequency spectrum when \( \theta \) is approximately \( 43^\circ \) as is shown in Figure 6.

It then appeared feasible that, if two volume waves with different biasing fields were joined at this angle \( \theta \), we may be able to obtain regions of constant time delay. Figure 7 shows the results of such attempts, one curve is the joining of volume waves with biasing fields of 2500 and 2125 (oe) and the other curve is the joining of volume waves with biasing fields of 2500 and 2200 (oe), all at the same position \( \phi = 0^\circ, \theta = 43.19^\circ \).

The results obtained can be compared with those obtained from the joining of a surface wave with a backward volume wave \(^[3]\).
Hyperbolic Current Distribution

To be considered here is an alternative to the flat field current distribution \( J(x) \) which will now lead to a replacement for the transform function \( \tilde{J}(k) \) of equations (27) or (29).

With \( \ell_1, a, p \) being the strip length, strip width and center to center spacing for the \( N \) transducer strips, the current distribution now under consideration is

\[
J(x) = \frac{I_o}{a} \frac{\cosh \frac{x-(i-1)p}{\delta}}{\sinh \frac{a/2\delta}{a/2\delta}} \eta^{i-1} \quad i=1,2,\ldots,N \tag{46}
\]

\(-\frac{3}{2} + (i-1)p < x < \frac{3}{2} + (i-1)p\)

(see Figure 2).

where

\[
\delta = \frac{\delta_1}{1+j} \tag{47}
\]

for some given \( \delta_1 ; j \) being \( \sqrt{-1} \).

With \( I_o =1 \), the Fourier transform gives

\[
|\tilde{J}(k)| = \left| \frac{\sqrt{\frac{2}{\ell_1}}}{\sinh \frac{a/2\delta}{a/2\delta}} \left( 1 - N e^{j kpN} \right) \frac{[A_1 e^{a/2\delta} + A_2 e^{-a/2\delta} - j(A_3 e^{a/2\delta} + A_4 e^{-a/2\delta})]}{a^2(k^2 + \frac{j2}{\delta^2})} \right| \tag{48}
\]

where

\[
A_1 = d_1 \left( \cos c_1 + \cos f_1 \right) + f_1 \sin c_1 + c_1 \sin f_1
\]

\[
A_2 = -d_1 \left( \cos c_1 + \cos f_1 \right) + f_1 \sin c_1 + c_1 \sin f_1 \tag{49}
\]

\[
A_3 = d_1 \left( \sin f_1 - \sin c_1 \right) + f_1 \cos c_1 - c_1 \cos f_1
\]

\[
A_4 = d_1 \left( \sin f_1 - \sin c_1 \right) - f_1 \cos c_1 + c_1 \cos f_1
\]
and

\[ d_1 = \frac{a}{2\delta} , \quad c_1 = \frac{a}{2} (k + \frac{1}{\delta}) , \quad f_1 = \frac{a}{2} (k - \frac{1}{\delta}) \quad (50) \]

When apodization is considered we obtain, instead of (48),

\[
|\tilde{J}(k)| = \left| \sum_{i=1}^{N} \frac{N^{1/2} \eta_i e^{-jk\eta_i}}{\text{sinh} \frac{a_i/\delta}{\delta}} \left[ A_{1i} e^{a_i/\delta} + A_{2i} e^{-a_i/\delta} + j(A_{3i} e^{a_i/\delta} + A_{4i} e^{-a_i/\delta}) \right] \right| \frac{a_i^2 (k^2 + j \frac{2}{\delta})}{a_i/\delta} \quad (51)
\]

There is no consideration for a truncated array of normal modes here. When equations (48) or (51) are employed in the basic theory program we have the basic theory for a hyperbolic current distribution.
A. Basic Theory - Forward and Backward Volume Waves

There is a computer program operational on the CDC 6600 at Hanscom AFB, MA which incorporates the results of the basic theory for both forward volume waves and backward volume waves. The program produces plots on the Calcomp plotter at Hanscom AFB depicting the various physical quantities as functions of frequency. Plots as well as printouts are obtained for wave number, group delay, radiation resistance, radiation reactance and insertion loss.

Flexibility is designed into the program so that one can choose the solution mode for (21) and whether we have uniform conducting strips or apodization in strip length, strip width and/or center to center spacing. Additionally, the program provides the relevant frequency range from (22).

Now follows a detailed description of the input cards with details on the use of the above described features. Columns 1-72 may be used on the first 5 cards and columns 1-70 may be used on the last three cards.

Card 1-H, t, d, 2, mode number, option

These six quantities are here supplied separated by commas. Lengths are in meters. Mode number is 0 for the fundamental mode of the dispersion relation (21). Option is 0 for forward volume waves while option is 1 for backward volume waves.

*FORTRAN
Here are entered these five quantities as defined earlier.

Card 3 - first $l_1$, $\Delta l_1$, $l_1$ option
Card 4 - first $a$, $\Delta a$, $a$ option
Card 5 - first $p$, $\Delta p$, $p$ option

In cards 3, 4 and 5 three items are entered for each of the quantities $l_1$, $a$, $p$. For each card, if the option (third item) is 0 then the dimension is entered for the first strip (first item) and an increment is entered for the remaining strips (second item), but if the option (third item) is 1 then the dimension is entered for the first strip (first item) and the increment (second item) increments the next $\frac{N-1}{2}$ strips and is then a decrement for the last $\frac{N-1}{2}$ strips.

The number of strips, $N$, must be odd if the option is 1. When there is no apodization the increment (second item) is 0 as is the option (third item).

Card 6 - heading for plots
Card 7 - heading for plots
Card 8 - heading for plots

These three lines will appear as headings, in the same order, on several plot frames.
FUNCTION COTH(CA)
COTH = 1./TANH(CA)
END

FUNCTION COT(CA)
COT = 1./TAN(CA)
END

FUNCTION FT(CA)
COMMON EL, T1, 5, C, UY, PI, ENM, FCA
A = 2.*CA+T1
L = A3S(A)
IF (A > 61.675) GO TO 5
A1 = COTH(CA*EL)+1.*EXP(-L.*CA*FT1) (COTH(CA*EL)-1.)
GO TO 6
A1 = COTH(CA*EL)+1.
CONTINUE
3 + COTH(CA*EL)* (C*UY)**2
IF (A > 61.675) GO TO 2
C1 = EXP(-2.*CA*FT1)* (-1.)
Y = COTH((C*EL)* (C*UY)**2)
CC TO 3
G1 = 1.
2 CONTINUE
Fi = ATAN(C*UY* A1/(B1-C1)) /C/C
Fi = (FT *L1) * FT = FT + A3S(PI/C/D)
FT = FT + A3S(ENM*FT1/C/L)
F(A = 41.*C*UY)**2 (T1*C*CA+D)-31+C1
RETURN
END

FUNCTION CSCH(CA)
CSCH = 1./SINH(CA)
END

FUNCTION SINC(CA)
PI = 3.14159265
SINC = SIN(F1*CA)/(PI*CA)
RETURN
END
FUNCTION F(CA)
COMPLEX E,C
COMMON D,J,L,T,S,C,UYY,PI,EN,E,EN,FC,OPN,EL,L1,EF,E
X=COS(PI-CA)*SIN(EL)+I*AN(CA)*SIN(EL)
X1=-SIN(PI-CA)*SIN(EL)+I*AN(CA)*SIN(EL)
Y=EXP(2*PI*I/EL1)
END

FUNCTION G(CA)
COMPLEX D,J,L,T,S,C,UYY,PI,EN,E,EN,FC
G=APB(AAY(CA) +*XPI(-CA*L)/FT1(CA))
END
20 CONTINUE
F=Fi-g
x(1)=x(1)+0.5*fi+fcalc

21 CONTINUE
F=FNC-FUEL
x(1)=x(1)+0.5*fi+fcalc

22 CONTINUE
F=FNC-FUEL
x(1)=x(1)+0.5*fi+fcalc

23 CONTINUE
F=FNC-FUEL
B. Dispersion Relation - Generalized Volume Waves

A computer program has been implemented on the CDC 6600 at Hanscom AFB, MA which obtains the dispersion relation and group delay for volume waves of arbitrary orientation of the biasing field. Plots on the Calcomp plotter at Hanscom AFB as well as computer printouts are produced for the wave number and group delay as functions of frequency. One can choose the solution mode for the dispersion relation (42). The frequency range is determined by the program from (43).

The use of the input cards to this program now follows:

Card 1 - \( H_0, t_1, d, \theta, \ell, \phi, \) mode number

Seven quantities, separated by commas, are supplied here. All lengths are measured in meters. Angles are measured in degrees. Their orientations can be determined from figure 1. For the fundamental mode the mode number is 0. Columns 1-72 may be used.

Card 2 - heading for plots

Card 3 - heading for plots

Here, there are two lines supplied, in order, which are used as headings on several plot frames. Columns 1-70 may be used on each card.
IF (CA .LE. 0.) GO TO 35
FF(1)=E0
CAP(I)=CA
I=I+1
PRINT *, "CA =", (I, "F=", IF.
L=2
GO TO 31
C 20 C=CA
IF (APS(FCA), GT, 1.) GC TO 35
IF (CA .LT. 0.) GO TO 39
FM(J)=EF
CM(J)=CA
J=J+1
C 20 PRINT *, "CA =", CA, "F=", EF
IF (J .EQ, 1) GC TO 15
IF (J .GT. 1) GO TO 31
I=1
K=K-1
31 K=K-1
IF (K .LE. NF) GC TO 50
PRINT 60
I=I-1
J=J-1
IF (I .LE. 0.) GO TO 15
PC=0.2
C CONTINUE:
C 60 J=1,11
C 60 FOR J=1,11, I=1,11, I=1,11,5
FM(J)=0
ELAP(I)=0
CAP(J)=0
C 60 PRINT 60, (FM(J), CAP(J), J=1, J1, 5)
C CONTINUE
C PRINT 60, (FM(I), VMG(I), I=1, J1, 5)
XM==2300.
XTN==42.".
XM==42.".
C==108.
C==207.
XM==*.
IY==*.
IC==1,5
CC=1, I=1,11
IF (CAP(I) .GT. 1,E6) CAP(I)=1.E6
IF (ELAP(I) .GT. 1,E6) ELAP(I)=1.E6
VCM(T)=A5*(VMG(J))
13 IF (VMG(J) .GT. 1,E6) VMG(J)=1.E6
IF (CM(J) .GT. 1,E6) CM(J)=1.E6
IF (CM(J) .LT. 1,E6) CM(J)=-1.E6
VCM(J)=A5*(VMG(J))
14 IF (VMG(J) .GT. 1,E6) VMG(J)=1.E6
7=11,
X=1.
Y=10.
CALL FLOT(1,5,0,0,3)
C. Hyperbolic Current Distribution – Surface Waves

There is a surface wave program operational on the CDC 6600 at Hanscom AFB which employs the hyperbolic current distribution described earlier. The program produces plots and printout for quantities of interest for surface waves. The option for a truncated array of normal modes is eliminated from this program. Otherwise, the program is similar to the basic theory surface wave program described in the Interim Technical Report. The number of input cards is the same as in the earlier program and the use of most of them is the same. We here describe only those cards whose functions differ from the earlier version.

Card 5 - $\Delta H$, $\Delta r$, $\delta_1$, option, $R_L$

Here, there are five items to be input, with the first two, $\Delta H$ and $\Delta r$, already described. The third item is $\delta_1$ which enters (47). The fourth item is an option. If the option is set 1 then (48) is used for the transform of the current distribution. If the option is 0 then (51) is used for the transform of the current distribution. The fifth item is conduction loss which is now to be input at this point.

Card 6 - heading for plots

This is the first line for the heading for several plot frames. Columns 1-70 may be used. There is no indication now for an array of normal modes.
$C^2 = \frac{EF}{(2 \times 0.175C_2)}$
$L_{11} = \frac{1 - OMH}{(CM^2 - 2CMH^2)}$
$L_{22} = U_{11}$
$L_{12} = CM/(OM^2 - 2OMH^2)$

```plaintext
37 IF (L EQ. 2) GO TO 2
1 L = 1
2 CONTINUE
3 AL1 = L22 * 2
4 AL2 = L22 * 2
5 IF (LGT. 1) GC TO 53
6 CAO = 2 * SQRT(L22/L11) * ALOG(1 + 4 * SQRT(L11 + L22))
7 Y = 2 * 2 - (SQRT(U11 * L22) + 1.1 + 2)
8 CAO = CAO/C
9 CONTINUE
10 IF (I EQ. 1) GO TO 51
11 IF (L EQ. 1) CAO = CAP(I) - 1
12 IF (L EQ. 2) CAO = CAM(J - 1)
13 P = 1
14 IF = CAT * CAO
15 CAO = CAO + DEL
16 CAC = CAO - DEL
17 CAC = CAO * D
18 CAC = ABS(CAO)
19 CAO = ABS(CAO * G)
20 IF (CAO GT. 0.0E4) GO TO 35
21 IF (CAO GT. 0.9E4) GO TO 35
22 FC = FT(CAO)
23 IF (CAO GT. CAO) GO TO 35
24 IF (CAO GT. CAO) GO TO 35
25 IF (CAO (CA1 - CAO) / CAO) LT. 0.011) GC TO 10
26 FT = CAO
27 IF (PGT. 1.0) GO TO 35
28 GC TO 5
29 IF (L EQ. 2) GO TO 20
30 IF = CAO
31 IF (ABS(FTC1) GT. 1.0) GC TO 35
32 IF (CAO LT. 0.1) GC TO 35
33 FF = FT(1) = EF
34 IF (I EQ. 1) CAO = CA
35 I = I + 1
36 L = 2
37 GC TO 35
38 CA1 = CAO
39 IF (ABS(FTC1) GT. 1.0) GC TO 35
40 IF (CAO LT. 1.1) GC TO 35
41 IF = FT(1) = EF
42 IF (J EQ. 1) CAO = CA
43 J = J + 1
44 K = K - 1
45 IF (K LE. 0) GC TO 50
46 PRINT 60
47 I = I - 1
48 J = J - 1
49 GC TO 2
50 PRINT * "ITERATION DOES NOT CONVERGE." = "", EF, S = ", S
51 IF (L EQ. 2) GO TO 15
52 L = 2
53 GC TO 2
```
CALL FLCT(10., 0., -3)
CALL AXIS(9., 9., 9., 10., 0.1, YMIN, DX, YMIN, CY, 0.6)
CC = 27.
CALL LINE (FP, VM, JI, 1, J1, XMIN, DX, YMIN, CY, 0.6)

CONTINUE
CC = 26.
CALL FLCT(17., 0., -3)
CALL AXIS(9., 9., 9., 10., 0.1, XMIN, DX, YMIN, CY, 0.6)
CC = 26.
CALL LINE (FP, VM, JI, 1, J1, XMIN, DX, YMIN, CY, 0.6)

CONTINUE
CC = 25.
CALL FLCT(17., 0., -3)
CALL AXIS(9., 9., 9., 10., 0.1, XMIN, DX, YMIN, CY, 0.6)
CC = 25.
CALL LINE (FP, VM, JI, 1, J1, XMIN, DX, YMIN, CY, 0.6)

CONTINUE
CC = 24.
CALL FLCT(17., 0., -3)
CALL AXIS(9., 9., 9., 10., 0.1, XMIN, DX, YMIN, CY, 0.6)
CC = 24.
CALL LINE (FP, VM, JI, 1, J1, XMIN, DX, YMIN, CY, 0.6)

CONTINUE
CC = 23.
CALL FLCT(17., 0., -3)
CALL AXIS(9., 9., 9., 10., 0.1, XMIN, DX, YMIN, CY, 0.6)
CC = 23.
CALL LINE (FP, VM, JI, 1, J1, XMIN, DX, YMIN, CY, 0.6)

CONTINUE
CC = 22.
CALL FLCT(17., 0., -3)
CALL AXIS(9., 9., 9., 10., 0.1, XMIN, DX, YMIN, CY, 0.6)
CC = 22.
CALL LINE (FP, VM, JI, 1, J1, XMIN, DX, YMIN, CY, 0.6)

CONTINUE
CC = 21.
CALL FLCT(17., 0., -3)
CALL AXIS(9., 9., 9., 10., 0.1, XMIN, DX, YMIN, CY, 0.6)
CC = 21.
CALL LINE (FP, VM, JI, 1, J1, XMIN, DX, YMIN, CY, 0.6)

CONTINUE
CC = 20.
CALL FLCT(17., 0., -3)
CALL AXIS(9., 9., 9., 10., 0.1, XMIN, DX, YMIN, CY, 0.6)
CC = 20.
CALL LINE (FP, VM, JI, 1, J1, XMIN, DX, YMIN, CY, 0.6)

CONTINUE
CC = 19.
CALL FLCT(17., 0., -3)
CALL AXIS(9., 9., 9., 10., 0.1, XMIN, DX, YMIN, CY, 0.6)
CC = 19.
CALL LINE (FP, VM, JI, 1, J1, XMIN, DX, YMIN, CY, 0.6)

CONTINUE
CC = 18.
CALL FLCT(17., 0., -3)
CALL AXIS(9., 9., 9., 10., 0.1, XMIN, DX, YMIN, CY, 0.6)
CC = 18.
CALL LINE (FP, VM, JI, 1, J1, XMIN, DX, YMIN, CY, 0.6)

CONTINUE
CC = 17.
CALL FLCT(17., 0., -3)
CALL AXIS(9., 9., 9., 10., 0.1, XMIN, DX, YMIN, CY, 0.6)
CC = 17.
CALL LINE (FP, VM, JI, 1, J1, XMIN, DX, YMIN, CY, 0.6)

CONTINUE
CC = 16.
CALL FLCT(17., 0., -3)
CALL AXIS(9., 9., 9., 10., 0.1, XMIN, DX, YMIN, CY, 0.6)
CC = 16.
CALL LINE (FP, VM, JI, 1, J1, XMIN, DX, YMIN, CY, 0.6)
FLNCTION GAY(CA)
COMPLEX DC, EL, AL1, AL2, E, O, T1, G, S, ETA, EN, F, A, Y, A
X, LMCCES, JEL, JOPT
P1 = 2.3141592654
N = EN
N = N
IF (JOPT .EQ. 1) M = 1
C = CMPLX(C, .EQ. 1)
CC = .5*DEL1*CMPLX(1., -1.)
LC 1 I = 1, M
K = AA(I)
E1 = E(I)
F1 = F(I)
C1 = .5*W/CC
C1R = REAL(D1)
C1I = AIMAG(D1)
C1 = .5*M*(CA+1./CC)
F1 = .5*M*(CA-1./CC)
A1 = C1*(CCOS(C1) + CCOS(F1)) + C1*CSIN(C1) + CSIN(F1) + C1
A2 = -C1*(CCOS(C1) + CCOS(F1)) + C1*CSIN(C1) + CSIN(F1) + C1
A3 = C1*(CSIN(C1) - CSIN(F1)) + C1*CCOS(C1) - C1*CCOS(F1)
A4 = C1*(CSIN(C1) - CSIN(F1)) + C1*CCOS(C1) + C1*CCOS(F1)
C1 = CMPLX(C1, (C1 + F1) + 1.
CS = CMPLX(C1, (C1 + F1) - 1.
IF (1 .LT. ETA*CMPLX(C1, 1.), ETA*CMPLX(C1, 1.) + 1.
X (1. - ETA*CMPLX(C1, 1.), ETA*CMPLX(C1, 1.))
C = C + SCPT(E1)*CS*
X(A1 - CEXP(-D1) + A2*CEXP(-D1)*CMPLX(0., -1.) + (A3*CEXP(C1) + A4*CEXP(-D1)
X) ) / X(CMPLX(C1, 1) + CMPLX(C1, 1) + CMPLX(C1, 1)
CONTINUE
GAY = CMPLX(C1, 1)
RETURN
END

FLNCTION R1(CA)
COMPLEX EL, AL1, AL2, E, O, T1, G, S, ETA, EN, F, A, Y, A
X, LMCCES, JEL, JOPT
RETURN
END

FLNCTION R2(CA)
COMPLEX EL, AL1, AL2, E, O, T1, G, S, ETA, EN, F, A, Y, A
X, LMCCES, JEL, JOPT
RETURN
END
SLFCTINE HTRAN(R,X,FEEG,FEND)
DIMENSION F(3),X(3)
FI=1.F3666555555
FDEL=(FEND-FEEG)/(N-1)
F=FEEG+.5*FDEL
INC=MOD(N,2)
I1=1+INC-1
N1=N-1
N1C=N1-2
CC 33 I1=1,N1
X(I)=.
IF (I .EQ. 1) RX=(3.*R(I)+6.*R(I+2)-R(I))/8.
IF (I .EQ. NM1) RX=(-N(N-2)+6.*R(NM1)+3.*R(INC))/8.
IF (I .EQ. 1) OF=I + INC, EQ. NM1 GO TO 20
R=I-R(I-1)+9.*R(I)+9.*R(I+1)-R(I+2))/16.

20 CONTINUE
FI=FEEG
CC .& IP=1,N1M2,2
X(I)=X(I)+4.*(R(I+1)-RX)/(FI+FDEL)**2-F**2
X(I)=2.*X(I)+2.*(R(IP)-RX)/FI**2-F**2
FI=FI+2.*FDEL

28 CONTINUE
FEN=FEND
IF (INC .EQ. 0) FEN=FINC-FDEL
X(I)=X(I)+4.*(R(NI)-Rx)/(FEN**2-F**2)
X(I)=FDEL/3.*X(I)
IF (INC .EQ. 1) GO TO 30
X(I)=X(I)+3.*FDEL*(R(NI)-Rx)/(FEN**2-F**2)
X(I)=X(I)+2*(X(I)-Rx)/(FEN**2-F**2)

30 CONTINUE

33 CONTINUE
X(I)=I-EQ.*X(I-1)-X(I-2)*X(I)-X(I+1))/16.
X(I)=X(I)+X(I-1)*X(I-2)+9.*X(I)-X(I+1))/16.
X(I+1)=X(I)+X(I-1)*X(I-2)+9.*X(I)-X(I+1))/16.
X(I+2)=X(I)+X(I-1)*X(I-2)+9.*X(I)-X(I+1))/16.
X(IN)=15.*X(NM1)-1.5*X(NM2)+3.*X(N-3))/6.
X(N-1)=3.*X(NM1)+6.*X(NM2)-X(N-3))/6.
X(N-2)=X2
X(N-3)=X1
FI=LFEN
FEND
Figure 2 - Hyperbolic Current Distribution
Figure 3 - Dispersion Relations - Generalized Forward Volume Waves
VOLUME WAVES
H=2500(0E10) D=10(UM), L AND T I INFINITE

Figure 4 - Dispersion Relations - Generalized Backward Volume Waves
VOLUME WAVES
H=2500(0E)
D=10(UM)
L AND T, INFINITE

\[ \phi = 0^\circ, \theta = 60^\circ \]
\[ \phi = 0^\circ, \theta = 30^\circ \]
\[ \phi = 0^\circ, \theta = 43^\circ \]

Figure 5 - Dispersion Relations - \( \phi = 0 \)
Figure 6 - Dispersion Relation - $\phi=0, \theta=43^\circ$
Figure 7 - Non Dispersive Time Delay Elements - $\phi=0$, $\theta=43.1^\circ$
Figure 8 - Dispersion Relation - Forward Volume Waves
Figure 9 - Group Delay - Forward Volume Waves
Figure 12 - Insertion Loss - Forward Volume Waves
Figure 4 - Group Delay - Backward Volume Waves
Figure 15 - Radiation Resistance - Backward Volume Waves
Figure 16 - Radiation Reactance - Backward Volume Waves
Figure 17 - Insertion Loss - Backward Volume Waves
References


