ADAPTIVE CANCELLATION OF SCATTERED INTERFERENCE (U)

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L E BRENNAN ET AL. DEC 82 N00019-81-C-0519

UNCLASSIFIED F/G 17/4 NL
ADAPTIVE CANCELLATION OF SCATTERED INTERFERENCE

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December 1982

The Final Report

Submitted to

The Naval Air Systems Command

by

Adaptive Sensors, Inc.
216 Pico Blvd., Suite 8
Santa Monica, CA 90034

on

Contract N00019-81-C-0519

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I. INTRODUCTION

This is the final report on a 1-year study of adaptive arrays in a scattered jamming environment. It is assumed that the jamming signals are scattered into the main receiving beam of a radar or communication system. The scattering medium could be terrain, rain, or chaff. Conventional adaptive nulling which uses angular discrimination cannot be used against this type of interference without also nulling the desired signal, since both the scatterers and signal source are in the main beam. A method of cancelling the scattered jamming signals is outlined in Sec. 2 of this report. The theory of this type of adaptive cancellation, presented in earlier reports on this contract, is also reviewed in Sec. 2.

The technique for cancelling scattered jamming uses delayed replicas of the jamming signal. Since the area or volume of scatterers may be large, the resulting adaptive canceller may require a large number of adaptive weights (e.g., 1000 or more). This suggests a complicated adaptive system and the importance of devising methods of simplifying the adaptive weight computation. It has been shown that the required number of adaptive degrees of freedom depends on the frequency spectrum of the jamming. With wideband noise jamming, this frequency spectrum depends on the receivers which can be designed to facilitate scatter cancellation. In earlier reports, two frequency spectra have been investigated, the rectangular bandlimited spectrum and the Gaussian spectrum. These results are reviewed in Secs. 3 and 4 of this report. During the last quarter, a bandlimited cosine spectrum was investigated. These recent results are contained in Sec. 5. It is shown that the cosine spectrum requires fewer adaptive degrees of freedom, so that simpler implementations can provide 20 to 30 dB of scatter cancellation.
During this study, the case of two jammers was also analyzed. It was shown that the same technique for scatter cancellation can be used against two jammers when both are illuminating a scattering medium in the main beam. Results for the two jammer cases are contained in the first quarterly report on this study [1].
2. BACKGROUND

Adaptive array antennas can provide a significant improvement in the performance of communication and radar systems when jamming is present. The weights on the array elements are adaptively controlled to place antenna pattern nulls in the directions of jammers, thus improving the system signal-to-noise ratio. This adaptive array technology has been experimentally verified and is currently being incorporated in many radar and communication systems.

When a system utilizes a low sidelobe receiving antenna plus adaptive nulling, a high degree of immunity to direct line-of-sight jamming is achieved. In these cases, a second type of interference will sometimes limit the system performance, viz, jamming scattered from terrain or chaff into the main receiving beam. This interference is received from the desired look direction and cannot be rejected by angular nulling.

A method of cancelling scattered jamming, which has been described in detail in earlier ASI reports, is summarized in Fig. 1. A jammer is illuminating a scattering area or volume in the main beam of a receiving antenna. The scattered jamming is delayed by \((R_1+R_2)/c\), which extends over a time interval depending on the geometry as illustrated in Fig. 1. This same jamming signal, delayed by \(R_d/c\), is received by an auxiliary antenna element located near the receiving antenna. A delay line is used to equalize the two delay paths. When the scattering region is extended in area, an interval of delay is covered by taps on the delay line. These taps correspond to an interval bracketing the interval of \((R_1+R_2-R_d)/c\), and are spaced by
Figure 1 -- Scatter Cancellation

\[ Z(t) = y(t) - \sum_{n=1}^{N} W_n^* E_n(t) \]
roughly 1/B, where B is the jammer bandwidth. The weights $w_1, w_2, \ldots, w_N$ are selected to minimize the jamming residue in the output.

As in Fig. 1, let $y$ denote the output of the main antenna and $z$ the final output after cancellation of the scattered interference. The tapped delay line outputs are denoted by the column vector,

$$E = (E_1, E_2, \ldots, E_N)^T.$$  

Let $W = (w_1, w_2, \ldots, w_N)^T$, denote the set of adaptive weights applied to the delay line taps. Then,

$$Z = y - WE$$  \hspace{1cm} (1)

It has been shown that the weights which minimize the scattered jammer power in the output $z$ are

$$W = M^{-1} \overline{Ey}^*$$  \hspace{1cm} (2)

where: $M = \overline{EE}^*$ = covariance matrix of tap outputs  
$\overline{Ey}^*$ = column vector with component $E_nY^*$

With the optimum weights of (2), the output power is

$$|Z|^2 = |y|^2 - \overline{Ey}^* M^{-1} \overline{Ey}$$  \hspace{1cm} (3)

Let $\rho(\tau)$ denote the autocorrelation function of the interference signal.

$$\rho(\tau) = f(t)f^*(t-\tau)$$  \hspace{1cm} (4)
Then, the covariance matrix of tap outputs is

\[
M = \begin{bmatrix}
\rho(0) & \rho(\Delta) & \rho(2\Delta) & \cdots & \rho((N-1)\Delta) \\
\rho(\Delta) & \rho(0) & \rho(\Delta) \\
\vdots & \vdots & \vdots \\
\rho((N-1)\Delta) & \rho(0) \\
\end{bmatrix}
\]

(5)

where \( \Delta \) is the spacing between taps. This matrix is Toeplitz and Hermitian. In most cases, the spectrum is symmetric about the carrier frequency so \( M \) is real and symmetric. The Toeplitz property [i.e., that \( M_{mn} \) is a function only of \( (m-n) \)] can be exploited in inverting the covariance matrix.

The output of the main antenna is a function of the jamming signal \( f(t) \) and of the scattering process. Representing the scattering by a large set of independent scatterers, each with a complex reflection coefficient \( \alpha_j \) and a delay \( \tau_j \),

\[
y(t) = \sum_{j=1}^{J} \alpha_j f(t-\tau_j)
\]

(6)

The corresponding component of the \( Ey^* \) vector of (2) are

\[
E_n y^* = E_n(t) y^*(t)
\]

(7)

\[
= \sum_{j=1}^{J} \alpha_j^* \rho(\tau_j - n\Delta)
\]
For a given set of scatterers represented by \( \{\alpha_j, \tau_j\} \), the power in the main antenna output before cancellation is

\[
|y|^2 = \sum_{j,k} \alpha_j \alpha_k^* \rho(\tau_k - \tau_j) \quad (8)
\]

This result follows from (4) and (6). Note that this is also the first term in the equation (3) for the output power after cancellation. Combining (3), (7), and (8), the power residue in the output is

\[
|z|^2 = \sum_{j,k=1}^J \alpha_j \alpha_k^* \left[ \rho(\tau_k - \tau_j) - S_j^* M^{-1} S_k \right], \quad (9)
\]

where \( S_j \) is a column vector associated with the \( j^{th} \) scatterer,

\[
S_j = \begin{bmatrix}
\rho(\tau_j - \Delta) \\
\rho(\tau_j - 2\Delta) \\
\vdots \\
\rho(\tau_j - N\Delta)
\end{bmatrix} \quad (10)
\]

The Monte Carlo technique of selecting the scattering field randomly and running the program many times would yield an estimate of performance, using (8) and (9) for the power before and after cancellation, respectively.
A more efficient method, however, is to assume that the scatterers are equally spaced with independent random coefficients \( \{a_j\} \). Then \( a_j^*a_k = C \), a real constant for \( j=k \) and 0 for \( j \neq k \). In this case,

\[
|y|^2 = Cj_\rho(0) \tag{11}
\]

and

\[
|z|^2 = C \sum_{j=1}^{J} \left[ j_\rho(0) - s_j^* M^{-1} s_j \right] \tag{12}
\]

The cancellation ratio is

\[
C.R. = \frac{|z|^2}{|y|^2} = 1 - \frac{1}{J\rho(0)} \sum_{j=1}^{J} s_j^* M^{-1} s_j 
\]

\[
= 1 - \frac{1}{J} \sum_{j=1}^{J} \sum_{m,n=1}^{N} (M^{-1})_{mn} H_{mn}(j),
\]

where

\[
H_{mn}(j) = \rho(\tau_j-m\Delta)\rho(\tau_j-n\Delta)/\rho(0) \tag{14}
\]

The performance of an adaptive scatter canceller depends on the spectrum of the scattered jamming signal. This spectrum depends on both the spectrum radiated by the jammer and the frequency response of the receivers in the receiving system. The simplest case to analyze is the bandlimited rectangular spectrum, i.e., where the jammer signal is strictly limited to the bandwidth $B$ with a constant spectral density over the band. The tap spacing is assumed to be $\Delta = 1/B$. In this case, the autocorrelation function of the jamming signal is

$$
\rho_1(\tau) = f(t)f^*(t-\tau) = \frac{\sin(\pi \tau/\Delta)}{(\pi \tau/\Delta)} \rho_1(0)
$$

The components of the $E$ vector are delayed replicas of this jammer signal, independent from tap to tap of the delay line, so the covariance matrix $M$ is diagonal and can be inverted analytically. In this strictly bandlimited case, the residue after cancellation is

$$
|Z|^2 = \sum_{j,k=1}^J \alpha_j \alpha_k^* \left[ \rho_1(\tau_k - \tau_j) - \sum_{n=1}^N \rho_1(\tau_k - n\Delta) \rho_1(\tau_j - n\Delta) \rho_1(0) \right]
$$
It was shown in an earlier report [1] that the residue of (16) goes to zero as the number of taps \( N \) becomes very large in the bandlimited case where \( \rho_1(\tau) \) is given by (15). However, it has also been shown [2] that a large number of taps extending beyond the scatter delay interval is required in this bandlimited case to achieve a large cancellation ratio. This is illustrated in Table 1. For example, when the scattering region extends over two resolution cells, a total of 79 taps yields only 25.9 dB of cancellation.

Table 1. Variation of Cancellation Ratio with Number of Taps - Bandlimited Noise with Rectangular Frequency Spectrum

<table>
<thead>
<tr>
<th>Number of Taps ((2N+1))</th>
<th>Width of Scattering Region ((2L))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>11.3</td>
</tr>
<tr>
<td>7</td>
<td>15.3</td>
</tr>
<tr>
<td>15</td>
<td>18.7</td>
</tr>
<tr>
<td>27</td>
<td>21.2</td>
</tr>
<tr>
<td>43</td>
<td>23.3</td>
</tr>
<tr>
<td>59</td>
<td>24.6</td>
</tr>
<tr>
<td>79</td>
<td>25.9</td>
</tr>
</tbody>
</table>
4. **GAUSSIAN SPECTRUM** [3,4]

Another spectrum which is often assumed in analyzing systems is the Gaussian spectrum, where the noise spectral density has the form

\[ S(f) = e^{-\beta f^2} \]  

(17)

The spectral density has a value of 1/2 of its peak value of unity at \( f = \frac{0.8325}{\sqrt{\beta}} \). The 3 dB bandwidth is

\[ B = \frac{1.665}{\sqrt{\beta}} \]  

(18)

The auto correlation function of the noise process corresponding to the Gaussian spectrum (17) is

\[ \rho(\tau) = 2 \int_{0}^{\infty} S(f) \cos 2\pi f \tau df = \sqrt{\frac{\pi}{\beta}} e^{-\left(\frac{\pi \tau}{\beta}\right)^2} \]  

(19)

If the delay line taps (Fig. 1) are spaced by the reciprocal bandwidth, \( \Delta = \frac{1}{B} = \sqrt{\beta}/1.665 \). The performance of the scatter canceller has been calculated for a variety of different tap spacings. The parameter \( \alpha \) is the ratio of assumed tap spacing to the reciprocal bandwidth, i.e.,

\[ \Delta = \frac{\alpha}{B} = \frac{\alpha \sqrt{\beta}}{1.665} \]  

(20)
For a given $\alpha$ and tap spacing, the elements of the covariance matrix $M$ are

$$M_{mn} = \sqrt{\pi} \exp \left\{ - \left[ \pi (m-n) \alpha / 1.665 \right]^2 \right\},$$

(21)

where the parameter $\beta$ has been assumed equal to unity. This expression was used to obtain the covariance matrix of (5).

The elements of the column vector $S_j$ of (10) are

$$\rho(\tau_j-n\Delta) = \sqrt{\pi} \exp \left\{ - \left[ \pi (\tau_j-n\alpha) / 1.665 \right]^2 \right\},$$

(22)

This equation was used to obtain the $H_{mn}(j)$ in (14).

Results for the Gaussian distribution were discussed in [4], which contains a series of tables showing the dependence of cancellation ratio on the number of taps and the tap spacing. One set of results is shown in Table 2, illustrating the effect of the number of delay line taps on cancellation. In each case, the tap spacing is equal to the width of a resolution cell. These results show that a few extra taps on each side of the scattering interval provides good cancellation. With the scattering medium extending over three resolution cells of delay, a total of 7 to 9 taps yields 30 dB of cancellation.
Table 2. Effect of Number of Taps on Cancellation
Gaussian Spectrum

<table>
<thead>
<tr>
<th>N(Taps)</th>
<th>L(Cells)</th>
<th>K(Scat/Cell)</th>
<th>α</th>
<th>CR(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>5</td>
<td>.3333</td>
<td>19.4</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>5</td>
<td>.3333</td>
<td>27.1</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>5</td>
<td>.3333</td>
<td>29.9</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>5</td>
<td>.3333</td>
<td>31.0</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>5</td>
<td>.3333</td>
<td>31.5</td>
</tr>
</tbody>
</table>

Table 3 illustrates the effect of tap spacing on cancellation. In the first set of 4 examples, the width of the scattering region and the interval covered by the taps are the same, viz, an interval of one reciprocal bandwidth. For example, with $\alpha = .5$ the tap spacing and width of one scatter cell are both 1/2 of the reciprocal bandwidth. Hence, two taps and two cells are required to cover one delay interval equal to a reciprocal bandwidth. Note in this series of 4 examples that the cancellation ratio improves significantly as $\alpha$ decreases. The next set of 4 cases are the same as the first four cases, except that two additional delay line taps are included.
These results show that small tap spacings must be used with a Gaussian spectrum, e.g., $\alpha < 1/3$ to achieve $\sim 30$ dB of cancellation. In a system cancelling scattered interference from a large scattering region, this requirement for closely spaced taps is a serious problem. Many extra adaptive degrees of freedom would be required.

Table 3. Effect of Tap Spacing ($\alpha$) on Cancellation

<table>
<thead>
<tr>
<th>N(Taps)</th>
<th>L(Cells)</th>
<th>K(Scat/Cell)</th>
<th>$\alpha$</th>
<th>CR(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>4.3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>5</td>
<td>.5</td>
<td>11.2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>5</td>
<td>.3333</td>
<td>19.4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>5</td>
<td>.25</td>
<td>28.1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>4.7</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>5</td>
<td>.5</td>
<td>14.9</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>5</td>
<td>.3333</td>
<td>27.1</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>5</td>
<td>.25</td>
<td>39.8</td>
</tr>
</tbody>
</table>
5. BANDLIMITED COSINE SPECTRUM

Next, consider the frequency spectrum

\[ S(f) = \frac{\pi}{2B} \cos\left(\frac{\pi f}{B}\right) \quad |f| \leq B/2 \]  
\[ = 0 \quad |f| > B/2 \]  

(23)

The corresponding autocorrelation function is

\[ R(\tau) = 2 \int_0^{B/2} S(f) \cos(2\pi f \tau) \, df \]  
\[ = \frac{\pi}{B} \int_0^{B/2} \cos\left(\frac{\pi f}{B}\right) \cos(2\pi f \tau) \, df \]  

(24)

From (24), it can easily be shown that

\[ R(\tau) = \frac{\cos(\pi B \tau)}{1-(2B \tau)^2} \]  

(25)

Since this spectrum is strictly limited to a bandwidth B, it suffices to take samples at an interval \( \Delta = 1/B \). With samples spaced 1/B apart, the exact waveform at points between the sample points can be reconstructed using the sampling theorem. The elements of the corresponding covariance matrix (5) are

\[ M_{mn} = \frac{\cos[\pi(m-n)]}{1-[2(m-n)]^2} = \frac{(-1)^{m-n}}{1-[2(m-n)]^2} \]  

(26)
The column vector $S_j$ for the $j^{th}$ scatterer is

$$S_j = \begin{bmatrix} R(\tau_j - \Delta) \\ R(\tau_j - 2\Delta) \\ \vdots \\ R(\tau_j - N\Delta) \end{bmatrix}$$

(27)

Again, it is assumed that the delay line taps bracket the delay interval of the scattered jamming.

The performance of the scatter canceller with a cosine spectrum was investigated using the FORTRAN program listed in Appendix I of this report. The cancellation ratio of (13) was computed for a set of different cases. In (13), the covariance matrix $M$ was obtained from (26), and $H_{mn}$ from

$$H_{mn}(j) = \frac{R(\tau_j - m\Delta) R(\tau_j - n\Delta)}{R(0)},$$

(28)

where $R(\tau)$ is given by (25).

First, a series of cases were run to determine the number of scatterers per resolution cell required to simulate a continuously distributed scattering medium. It was found that 3 to 5 scatterers per cell are sufficient. A series of examples are shown in Table 3 to illustrate this point.
Table 3. Effect of Number of Scatterers per Cell on Cancellation Ratio

<table>
<thead>
<tr>
<th>No. Taps</th>
<th>No. Cells</th>
<th>Scatterers/Cell</th>
<th>CR/dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>5</td>
<td>1</td>
<td>17.1</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>2</td>
<td>19.9</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>3</td>
<td>20.0</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>4</td>
<td>20.0</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>5</td>
<td>20.0</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>10</td>
<td>20.0</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>1</td>
<td>20.5</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>2</td>
<td>23.4</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>3</td>
<td>23.5</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>4</td>
<td>23.5</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>5</td>
<td>23.5</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>10</td>
<td>23.5</td>
</tr>
</tbody>
</table>

The correlation function for a cosine distribution (25) goes to zero for large $\tau$ more quickly than the corresponding sinc function for the rectangular spectrum (15). One would expect fewer taps bracketing the scatterering region to suffice for the cosine distribution. The Gaussian correlation function (19) approaches zero more rapidly than the correlation function of (25), so more bracketing taps should be required with the cosine spectrum than with the Gaussian spectrum. These expected results were confirmed by the computer analyses of the 3 cases.

Table 4 shows the cancellation ratio as a function of the number of taps for the case where all the scatterers are within a single delay cell
Table 4. Effect of Number of Taps on CR with One Scatterer Cell

<table>
<thead>
<tr>
<th>No. Taps</th>
<th>No. Cells</th>
<th>Scatterers/Cell</th>
<th>CR/(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>5</td>
<td>14.1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>5</td>
<td>19.1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>5</td>
<td>22.3</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>5</td>
<td>24.6</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>5</td>
<td>26.5</td>
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<tr>
<td>12</td>
<td>1</td>
<td>5</td>
<td>27.9</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
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<td>29.2</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>5</td>
<td>30.4</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>5</td>
<td>31.4</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>5</td>
<td>32.3</td>
</tr>
</tbody>
</table>

and the cosine spectrum. Between 4 and 6 taps are required for 20 dB of cancellation, and 14 to 16 taps for 30 dB cancellation. Approximately 7 taps on each side of the scattering region are required to achieve 30 dB of cancellation in the case of Table 1.

When the scattering region is extended over several resolution cells, fewer extra cells on each side of the scatter delay interval are required. This is illustrated in Table 5 for the cases of 4 and 5 cells of scatterers. In these cases 5 to 6 extra taps on each side of the scatter interval yield 30 dB of cancellation. This difference from the results of Table 1 is expected, since with more scatter delay cells, the scattering from the central cells is cancelled more exactly. For these central cells there
Table 5. Effect of Number of Taps on CR

<table>
<thead>
<tr>
<th>No. Taps</th>
<th>No. Cells</th>
<th>Scatterers/Cell</th>
<th>CR(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>5</td>
<td>3</td>
<td>20.0</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>3</td>
<td>23.5</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
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<td>25.7</td>
</tr>
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<td>12</td>
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<td>18</td>
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</tr>
<tr>
<td>22</td>
<td>4</td>
<td>3</td>
<td>33.0</td>
</tr>
</tbody>
</table>

are, in effect, more taps on each side of the cells which can be used to replicate the scattered jamming signal. This point is further illustrated in Table 6, where 10 taps yield 19.6 dB of cancellation when the scatter delay extends over 10 cells. With 8 cells of scatter delay, 4 to 5 overlap cells on each side of the scatter region yields 30 dB of cancellation.
Table 6. Results for Larger Number of Scattering Cells

<table>
<thead>
<tr>
<th>No. Taps</th>
<th>No. Cells</th>
<th>Scatterers/Cell</th>
<th>CR(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>3</td>
<td>19.6</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td>3</td>
<td>24.9</td>
</tr>
<tr>
<td>14</td>
<td>10</td>
<td>3</td>
<td>27.3</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>3</td>
<td>22.1</td>
</tr>
<tr>
<td>11</td>
<td>8</td>
<td>3</td>
<td>25.3</td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td>3</td>
<td>22.4</td>
</tr>
<tr>
<td>15</td>
<td>8</td>
<td>3</td>
<td>28.9</td>
</tr>
<tr>
<td>17</td>
<td>8</td>
<td>3</td>
<td>30.2</td>
</tr>
</tbody>
</table>
6. CONCLUSIONS

In an adaptive scatter canceller, the achievable performance depends on the frequency spectrum of the signals at the canceller, the number of delay taps used, and the spacing between taps. The complexity of the adaptive system depends on the number of delay taps, which may be large in some cases of interest due to the range extent of the scattering medium. This frequency spectrum is determined by the frequency response of the receivers in the usual case of broad band noise jamming, and the receiver frequency response can be selected to facilitate the implementation of a scatter canceller.

Three different frequency spectra have been investigated, i.e., the achievable cancellation calculated as a function of tap spacing and the number of taps overlapping the scatter delay interval. It was shown that a large number of overlapping taps are required with a rectangular band-limited spectrum. With a Gaussian spectrum, the required tap spacing is roughly $1/3$ of the reciprocal bandwidth, thus requiring a large number of taps. With a cosine spectrum, a tap spacing of the reciprocal bandwidth can be used and a few overlapping taps yield 20 to 30 dB of scatter cancellation. The cosine spectrum will be assumed in continuing studies of the adaptive scatter canceller. These studies will address the important problem of designing a practicable adaptive weight computer in systems where the scatter delay interval extends over a large number of resolution cells.

In the first quarterly report, it was shown that the adaptive scatter canceller can be used when two jammers are illuminating the scattering region in the main beam. This requires the use of two auxiliary antennas which receive the direct path jamming signals[1].
7. REFERENCES


APPENDIX I. SCATTER CANCELLATION WITH COSINE SPECTRUM

FORTRAN IV

PROGRAM MAIN

C NN=NUMBER OF TAPS
C LL=NUMBER OF SCATTER CELLS
C KL=NUMBER OF SCATTERERS PER CELL

DIMENSION B(50,50)
DOUBLE PRECISION A(30,30),H(30,30),C,R,T,A1,A2
WRITE(7,100)
ACCEPT 200,NN,LL,KL
PI=4.*ATAN(1.)
NM=NN
DO 150 II=1,NM
NN=LL+2*(II-1)

DO COVARIANCE MATRIX

DO 110 M=1,NN
A(M,M)=1.
B(M,M)=A(M,M)
IF(M.EQ.NN) GO TO 110
DO 110 N=M+1,NN
A(M,N)=COS(PI*(M-N))/(1.-4.*(M-N)**2)
A(N,M)=A(M,N)
B(N,M)=A(N,M)
110 CONTINUE

INVERT COVARIANCE MATRIX

CALL MATR(A,NN,1,30)

COMPUTE H(M,N)

T1=FLOAT(NN-LL+1)/2.
N1=LL*KL
DO 120 M=1,NN
DO 120 N=1,NN
H(M,N)=0.
120 CONTINUE
TAU=T1+(FLOAT(J)-.5)/KL
A1=DCOS(PI*(TAU-M))/(1.-4.*(TAU-M)**2)
A2=DCOS(PI*(TAU-N))/(1.-4.*(TAU-N)**2)
H(M,N)=H(M,N)+A1*A2
WRITE(6,800) M,N,H(M,N),A(M,N),B(M,N)
CONTINUE
C COMPUTE CANCELLATION RATIO
C
0034  C2=1./FLOAT(N1)
0035  CR=1.
0036  DO 140 M=1,NN
0037  DO 140 N=1,NN
0038  140 CR=CR-C2*A(M,N)*H(M,N)
0039  IF(CR.LT.0.) CR=0.0
0041  CRDE=-10.+LOG10(CR)
0042  WRITE(6,500) NN,LL,KL
0043  WRITE(6,700)CRDE
0044  150 CONTINUE
0045  100 FORMAT(/X,'INPUT # TAPS,#SCAT CELLS,#SCAT/CELL IN 315')
0046  200 FORMAT(315)
0047  500 FORMAT(/X,'#TAPS=',I4,5X,'#CELLS=',I4,5X,'#SCAT/CELL=',I4)
0048  700 FORMAT(/X,'CANCELLATION RATIO(DB)=',F10.4)
0049  800 FORMAT(X,2I10,10X,3F15.5)
0050  END