Annual Report

For

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"Wave Propagation—Nonlinear Boundary Value Problems"

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Work was completed on a paper entitled "Perturbed Bifurcation of Stationary Striations in a Contaminated, Non-Uniform Plasma." It was accepted by SIAM APPLIED MATH and is currently in press. The paper is attached.

On-going research in collaboration with J. Magnan (Northwestern) continues with respect to moving situations or modulated travelling waves in reaction diffusion systems.
Moving Striations

Problem

\[ \frac{\partial N}{\partial t} = D \frac{\partial^2 N}{\partial z^2} + \frac{\mu + \beta}{\mu - \epsilon} \left( \frac{P}{N^2} \frac{\partial N}{\partial z} - \frac{1}{N} \frac{\partial P}{\partial z} \right) + F, \]

\[ \frac{\partial P}{\partial t} = D' \frac{\partial^2 P}{\partial z^2} + \frac{\mu + \beta}{\mu - \epsilon} \left( \frac{P}{N^2} \frac{\partial N}{\partial z} - \frac{1}{N} \frac{\partial P}{\partial z} \right) + G, \]

\[ \frac{\partial M}{\partial t} = D' \frac{\partial^2 M}{\partial z^2} + H, \]

where it has been assumed that \( \mu - N > \mu + P \)

B.C. (see Fig. 1)

\[ N(0) - \alpha \frac{\partial N(0)}{\partial z} = A, \quad N(L) + \alpha \frac{\partial N(L)}{\partial z} = A \]

\[ P(0) - \beta \frac{\partial P(0)}{\partial z} = B, \quad P(L) + \beta \frac{\partial P(L)}{\partial z} = B \]

\[ M(0) - \gamma \frac{\partial M(0)}{\partial z} = C, \quad M(L) + \gamma \frac{\partial M(L)}{\partial z} = C \]

I.C.

\[ N(z, 0) = N_0. \]
\[ P(z, 0) = P_0. \]
\[ M(z, 0) = M_0. \]
Scaled equations

\[ \frac{\partial \eta}{\partial t} = \epsilon \frac{\partial^2 \eta}{\partial \xi^2} + \frac{\partial^2 \eta}{\partial \eta^2} - \frac{1}{\eta} \frac{\partial \phi}{\partial \xi} + f \]

\[ \frac{\partial \phi}{\partial t} = \epsilon \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} - \frac{1}{\eta} \frac{\partial \phi}{\partial \xi} + g \]

\[ \frac{\partial \eta}{\partial \xi} = \epsilon \epsilon^2 \frac{\partial^2 \eta}{\partial \xi^2} + h \]

B.C.

\[ \eta(0, \xi) = \eta(0, \xi) + \epsilon \frac{\partial \eta}{\partial \xi} = a = \eta(\xi, 0) + \epsilon \frac{\partial \eta}{\partial \xi} \]

\[ \phi(0, \xi) = \phi(0, \xi) + \epsilon \frac{\partial \phi}{\partial \xi} = b = \phi(\xi, 0) + \epsilon \frac{\partial \phi}{\partial \xi} \]

I.C.

\[ \eta(\xi, 0) = 1 \]

\[ \phi(\xi, 0) = 0 \]

\[ \psi(\xi, 0) = 1 \]
where

\[ z = \left( \frac{L^2}{D_-} \right) \frac{\mu_+}{\mu_-} \frac{J}{eN_0} \xi, \]

\[ t = \frac{L^2 \eta}{D_-}, \]

\[ \epsilon = \left( \frac{D_- N_0}{L} \right)^2 \left( \frac{\mu_+}{\mu_-} \frac{J}{e} \right)^2 = \left( \frac{\Gamma_D}{\Gamma_+} \right)^2, \]

\[ N = nN_0, \quad P = pN_0, \quad M = mM_0, \]
\[ A = aN_0, \quad B = bN_0, \quad C = eM_0, \]
\[ \alpha = \beta = \gamma = \sqrt{\epsilon} L, \]
\[ \delta = \left( \frac{D_+}{D_-} \right)^2, \quad \theta = \left( \frac{D_+}{D_-} \right)^2, \quad \xi = \sqrt{\epsilon} \]
Xform to homogeneous B.c.

let $n = n' + a$
$p = p' + b$
$m = m' + c$

substitute into the scaled equations and drop the primes.

\[
\frac{\partial n}{\partial t} = \varepsilon \frac{\partial^2 n}{\partial x^2} + \frac{(p+b)}{(n+a)^2} \frac{\partial n}{\partial x} - \frac{1}{(n+a)} \frac{\partial p}{\partial x} + f
\]

\[
\frac{\partial p}{\partial t} = \varepsilon \frac{\partial^2 p}{\partial x^2} + \frac{(p+b)}{(n+a)^2} \frac{\partial n}{\partial x} - \frac{1}{(n+a)} \frac{\partial p}{\partial x} + g
\]

\[
\frac{\partial m}{\partial t} = \frac{\partial^2 m}{\partial x^2} + h
\]

BC:
$n(0,t) - \varepsilon \frac{\partial n(0,t)}{\partial x} = 0 = n(l,t) + \varepsilon \frac{\partial n(l,t)}{\partial x}$

$p(0,t) - \varepsilon \frac{\partial p(0,t)}{\partial x} = 0 = p(l,t) + \varepsilon \frac{\partial p(l,t)}{\partial x}$

$m(0,t) - \varepsilon \frac{\partial m(0,t)}{\partial x} = 0 = m(l,t) + \varepsilon \frac{\partial m(l,t)}{\partial x}$

IC:
$n(\xi,0) = 1 - a$
$p(\xi,0) = 1 - b$
$m(\xi,0) = 1 - c$
Try a solution

Let

\[ n(t,x) = e^{-\frac{\phi(t,x)}{2\xi}}, \]
\[ P(t,x) = e^{-\frac{\phi(t,x)}{2\xi}} y(t,x), \]
\[ M(t,x) = e^{-\frac{\phi(t,x)}{2\xi}} z(t,x). \]

Substitute into equations and get to order \( \frac{1}{\xi} \):

\[ \phi_t + \frac{1}{2} \phi_{xx}^2 = 0 \]

with solution

\[ \phi(t,x) = \frac{1}{2} \left( \frac{t - \xi_0}{\xi} \right)^2. \]

\((x_0, y_0, z_0)\) will satisfy the 1st order unperturbed problem (hence a travelling wave). We will seek "modulated" travelling wave solutions.