**Title:** Dynamic Coupling of Boundary-Fitted Coordinate System with the Navier-Stokes Equations

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**Abstract:**
This research effort concerned the incorporation of a self-adjusting boundary-fitted coordinate system into a numerical solution of the time-dependent, incompressible Navier-Stokes equations. In the resulting code, the coordinate system continually adjusts itself in time to automatically concentrate lines in regions of developing flow gradients. The code is applicable to arbitrary two-dimensional bodies. This code is based on the velocity-pressure formulation with an algebraic turbulent eddy viscosity model. The solution uses second-order, central differences and is done by SOR iteration with a variable acceleration parameter field that is calculated automatically from the local velocity and grid spacing.
DYNAMIC COUPLING OF BOUNDARY-FITTED COORDINATE SYSTEM WITH THE NAVIER-STOKES EQUATIONS

Final Report

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Problem Statement

This research effort concerned the incorporation of a self-adjusting boundary-fitted coordinate system into a numerical solution of the time-dependent, incompressible Navier-Stokes equations. In the resulting code, the coordinate system continually adjusts itself in time to automatically concentrate lines in regions of developing flow gradients. The code is applicable to arbitrary two-dimensional bodies. This code is based on the velocity-pressure formulation with an algebraic turbulent eddy viscosity model. The solution uses second-order, central differences and is done by SOR iteration with a variable acceleration parameter field that is calculated automatically from the local velocity and grid spacing.

Numerical grid generation has now become a fairly common tool for use in the numerical solution of partial differential equations on arbitrarily-shaped regions. This is especially true in computational fluid dynamics, but the procedures are equally applicable to all physical problems that involve field solutions.

Numerical grid generation is basically a procedure for the orderly distribution of observers over a physical field in a way that efficient communication among the observers is possible and all physical phenomena on the entire continuous field may be represented with sufficient accuracy by this finite collection of observations. This technique frees the computational simulation from restriction to certain boundary shapes and allows general codes to be written in which the boundary shape is specified simply.
by input. The boundaries may also be in motion, either as specified externally or in response to the developing physical solution. Similarly, the observers may adjust their positions to follow gradients developing in the evolving physical solution. In any case, the numerically generated grid allows all computation to be done on a fixed square grid in the computational field. (Computational field refers to the space of the curvilinear coordinates, i.e., where these coordinates serve as independent variables, rather than the cartesian coordinates. This field is always rectangular by construction.)

The area of numerical grid generation is relatively young in widespread practice, although its roots in mathematics are old. This area involves the engineer's feel for physical behavior, the mathematician's understanding of functional behavior, and a lot of imagination. The physics of the problem at hand must ultimately direct the grid points to congregate so that a functional relationship on these points can represent the physical solution with sufficient accuracy. The mathematics controls the points by sensing the gradients in the evolving physical solution, evaluating the accuracy of the discrete representation of that solution, communicating the needs of the physics to the points, and, finally, by providing mutual communication among the points as they respond to the physics.

The basic techniques involved then are as follows:

(1) a means of distributing points over the field in an orderly fashion, so that neighbors may be easily identified and data can be stored and handled efficiently.

(2) a means of communication between points, so that a smooth distribution is maintained as points shift their positions.

(3) a means of representing continuous functions by discrete values on a collection of points with sufficient accuracy, and a means for
evaluation of the error in this representation.

(4) a means for communicating the need for a re-distribution of points in the light of the error evaluation, and a means of controlling this re-distribution.

It should be borne in mind that the requirements, e.g., smoothness, orthogonality, etc., that must be met by the grid are ultimately determined by the numerical algorithm to be run on the grid. Thus, at the same time that effort is made to generate better grids, a like effort should be made to develop hosted algorithms that are more tolerant of the grids.

Considerable progress has been made in the past decade, especially in the last few years, toward the development of these techniques and toward casting them in forms that can be readily applied. A comprehensive survey of procedures and applications through 1981 has been published (Ref. 1), and two conferences specifically on the area of numerical grid generation have been held, the proceedings of which have been published (Ref. 2 & 3). Some expository papers are included in the latter proceedings (Ref. 3) which can serve as an introduction to the area.

There is now a rapidly increasing interest in the dynamic coupling of the grid with the physical solution so that the grid lines continually move to achieve concentration in regions of strong variation of the physical solution and alignment with shocks, flame fronts, etc. Such adaptive coordinate systems have been developed using a variety of techniques for sensing and reacting to the developing regions of concentration and/or alignment. A summary and discussion of adaptive grid techniques is given in Ref. 4.

The Navier-Stokes system of equations used in the present work consists of the two momentum equations and a Poisson equation for the pressure.
The pressure equation is obtained as usual by taking the divergence of the vector momentum equation. At each time step, the pressure is calculated assuming conservation of mass is satisfied at that time step. The effects of turbulence are simulated by the inclusion of an eddy viscosity coefficient based on the algebraic turbulence model of Baldwin and Lomax (Ref. 6).

The coordinate system is generated from the numerical solution of an elliptic system of partial differential equations as discussed in Ref. 5. These equations contain control functions which are used to control the coordinate line spacing and orientation in the field. In the adaptive grid algorithm, the attraction parameters in these coordinate control functions are made dependent on the gradient of the velocity magnitude, so that coordinate lines will concentrate in the boundary layer region where the largest changes occur. At each time step, the flow solution is obtained from the Navier-Stokes equations. New values of the coordinate control functions are then calculated from the velocity gradient, and a new coordinate system is obtained from the solution of the elliptic generation system. The grid point speeds are then calculated from the change in location of the grid points for use in the Navier-Stokes solution at the next time step.
Summary of Results

Some Computational results were obtained for flow over a NACA 663-018 airfoil at zero angle of attack and a Reynolds number of 2,000,000. The velocity profiles, although qualitatively correct, are somewhat thick due to the use of artificial viscosity for stability. Although a steady-state solution was not obtained, the feasibility of the adaptive-grid algorithm is demonstrated and lends sufficient encouragement for future work.

The results of this study demonstrate the feasibility of using an adaptive grid algorithm based on modifying the coordinate attraction control parameters to obtain a solution of the Navier-Stokes equations. Although the results are limited, they are reasonable and indicate the promise of the method. It is difficult to say that this adaptive-grid algorithm produces a better solution than that on a fixed grid since a final steady-state solution was not obtained in the course of the present effort.

The algorithm demonstrates a certain robustness since a reasonable solution is obtained with significant grid movement. The final computational parameters were obtained after numerous starts and restarts. The solution survived even though the Navier-Stokes results were restricted to a maximum of fifty SOR iterations per time step. In generating each new coordinate system, the number of iterations was also held to a maximum of fifty.

Further work on this technique is needed and has, in fact, been started. Because of coordinate line skewness in the region near the trailing edge, a new initial coordinate system should be generated with more nearly orthogonal grid lines for use in future work. It is expected that this would eliminate the oscillations experienced in the velocity solution. It is also conceivable that with this better coordinate system a solution could be obtained without using artificial viscosity.
List of Publications


Scientific Personal

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Dr. L. Michael Freeman (Ph.D. degree earned under this effort, December 1982)
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