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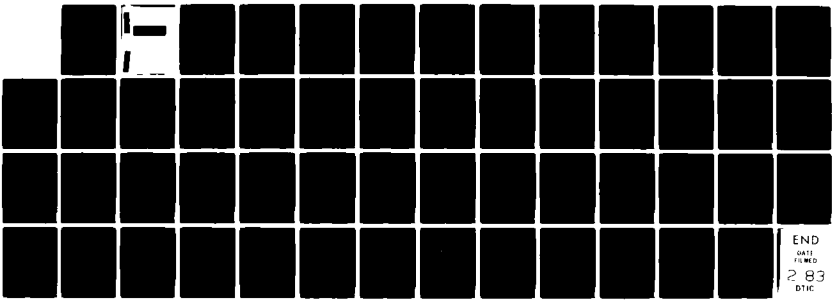

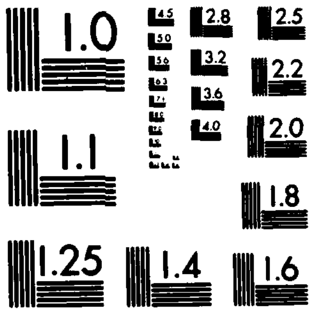
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Numerous solid propellant motor developments have encountered instability which could not be fully explained by linear pressure-coupling alone. This includes also laboratory-scale motors with rather simple interior flowfields. It has been therefore inferred that instability might likewise emanate from dynamic interaction (continued . . .)

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between the fluid-dynamic processes in the chamber, notably nonsteady velocity, and combustion. Attempts to characterize this phenomenon by a linear acoustic admittance function, similar to the pressure acoustic admittance (or response) have not been successful; there are strong indications that velocity coupled combustion instability is nonlinear. The objectives for the present phase of research are (1) critical literature survey, to determine the mechanisms which could lead to velocity coupled instability, and the various existing methods, analytical and experimental, related to its investigation, and (2) perform order of magnitude analyses to determine which of the proposed driving mechanisms is plausible and warrants further analysis. Ultimately, the objective is to develop a comprehensive analytical model, capable of prediction of this type of instability in some relatively simple cases. This study is coupled to an experimental investigation by UTC/CSD, headed by Dr. Robert Brown, to simulate nonsteady internal flowfields, in both controlled cold-flow, and combustion environments. The literature survey of velocity coupled instability has been completed. Preliminary order of magnitude analyses of two distinct driving mechanisms of instability, namely (1) acoustic/core-combustion coupling, and (2) viscous/acoustic interaction have also been completed. A similar study on the effect of acoustic/small scale turbulence is underway, and will be reported (along with its full related literature) following completion of the present phase of study. The preliminary analysis indicates the frequency-dependent surface heat feedback component, due to viscous/acoustic coupling, has both phase and amplitude ranges which would enable driving of acoustic vibrations; its amplitude tends to increase as the mean coreflow Mach number and the frequency become higher.

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1. INTRODUCTION

Numerous solid propellant motor developments have encountered instability which could not be fully explained by linear pressure-coupling alone. This includes also laboratory-scale motors with rather simple interior flowfields. It has been therefore inferred that instability might likewise emanate from dynamic interaction between the fluid-dynamic processes in the chamber, notably nonsteady velocity, and combustion. This has been pointed out already in the early works of Hart and McClure, Culick and others. Attempts to characterize this phenomenon by a linear acoustic admittance function, similar to the pressure acoustic admittance (or response) have not been successful; there are strong indications that velocity coupled combustion instability is nonlinear.

The conditions leading to velocity coupled instability, the physical interactions and the exact mode of manifestation still remain unclear. This study is therefore motivated by the desire to obtain physical insight into the phenomenon and its possible driving mechanisms.

The objectives for the present phase of research are (1) critical literature survey, to determine the mechanisms which could lead to velocity coupled instability, and the various existing methods, analytical and experimental, related to its investigation, and (2) perform order of magnitude analysis to determine which of the proposed driving mechanisms is plausible and warrants further analysis. Ultimately, the objective is to develop a comprehensive analytical model, capable of prediction of this type of instability in some relatively simple cases.

This study is coupled to an experimental investigation by UTC/CSD, headed by Dr. Robert Brown, to simulate nonsteady internal flowfields, in both controlled cold-flow, and combustion environments (Contract No. F49620-81-C-0027).

The literature survey of velocity coupled instability has been completed; within this task a large amount of published work has been accumulated and reviewed. The preliminary order of magnitude analyses of two distinct driving mechanisms of instability, namely (1) acoustic/core-combustion coupling, and (2) viscous/acoustic interaction have also been completed. A similar study on the effect of acoustic/small scale turbulence is underway, and will be reported (along with its full related literature) in our report following the present phase of study. Interaction with the cold-flow experiments by Dr. Brown at UTC/CSD has begun recently, based on the findings herein, regarding viscous/acoustic coupling.

2. MECHANISMS OF VELOCITY-COUPLED COMBUSTION INSTABILITY - OVERVIEW

Coupling between velocity oscillations and the combustion zone of solid propellants was recognized in the early combustion stability studies. Hart and McClure[1] discussed this coupling as a possible source of acoustic energy in their analytical studies. Experimental studies by Price[2] demonstrated increased instability which was consistent with these predictions. In addition, Crump[3] and Nadaud[4] demonstrated increases in mean burning rates occur in the presence of acoustic velocity. These changes were consistent with the increased mean chamber pressure observed in motors with low frequency axial mode instabilities. Thus, these studies established that both linear and nonlinear properties are important in describing the burning rate response to velocity oscillations.

The first experiments to measure the linear response function were made by Stepp[5] in a modified T-burner. Further improvements were made by Beckstead[6] and by Micheli[7] to permit testing with aluminized propellants. Micheli also explored the use of a sonic end vented burner for these measurements. More recently, Brown[8] has explored the dual rotating valve approach, while Micci[9] et al. has examined the modulated throat motor as a test device for linear velocity response functions. While these studies have reported velocity coupling response functions, no comparisons between the various methods employed at the different laboratories have been made. These studies have developed secondary evidence for nonlinear velocity coupling (mostly pressure oscillations having high harmonic content) but no conclusive measurements of the nonlinear velocity reponse have been reported. Part of the difficultly in defining the proper response is the lack of an adequate fundamental understanding of the basic processes involved.

Price[10] has proposed a heuristic model for nonlinear coupling to account for acoustic energy generation, nonlinear wave forms, and burning rate changes. This model assumes oscillatory heat transfer to the propellant surface is the dominant mechanism. By analogy to erosive burning, a threshold velocity is assumed to exist for velocity coupling. These concepts led to consideration of mean flow effects, rectification of the burning rate response by the heat transfer, and flow reversal when the velocity oscillations exceed the local mean flow. Subsequently, Dehority[11] studied the properties of this model parametrically.

Culick[12] proposed a model for the linear response of the burning rate to velocity oscillations. This model uses the fundamental assumptions of the pressure coupled response model proposed by Dennison and Baum.[13] This model would apply when the total velocity (mean plus oscillatory velocity) exceeds the threshold, although subsequent applications have ignored this restriction. Lengelle[14] used a turbulent boundary layer analysis to predict the oscillatory heat flux to the propellant surface and developed an expression which is identical to Culick's result in functional form. Micheli[7] also proposed slight modifications to Culick's basic model while Condon[15] has recently included particle size effects in the basic functional form. Srivastava[16] as recently added reacting boundary layer effects into the basic heat transfer approach. It should be noted that all these models assume the principal time lags occur in the solid phase and that the gas phase is quasi-steady. This precludes the effects of the acoustic boundary layer near the propellant surface.

Recently, Price[17] noted a number of limitations and inconsistencies in our current understanding of velocity coupling. For example, motor stability predictions treat the velocity response as a propellant property which is independent of position in the motor. This implies the response is independent of the mean flow environment. On the other hand, the dominant mechanisms is assumed to be oscillatory heat transfer from the gas phase to the solid surface. Most models further assume that turbulent boundary layer behavior characterizes these heat flux oscillations. Hence, it seems inconsistent to assume simultaneously that the response is related to flow behavior while being independent of position in the motor.

Expanding on the inconsistency, recent theoretical[18] and experimental[19] evidence suggests that classical turbulent boundary layer approaches may be inappropriate. These studies indicate an inviscid rotational flow model for the flow correlates the mean velocity and turbulence intensity profiles more accurately than a conventional turbulent boundary layer model. One would then question the use of conventional turbulent boundary layer approaches for velocity coupling.

Discussions at the recent JANNAF Workshop on Velocity Coupling[20] focused on yet another difficulty in the turbulent boundary layer model for velocity coupling. Participants in the workshop agreed that current experience, particularly in laboratory burners, suggests that the threshold velocity for velocity coupling is significantly different from the threshold for erosive burning. In fact, the threshold for velocity coupling may be zero. This suggests the "acoustic boundary layer" differs significantly from the "turbulent boundary layer." Furthermore, this raises a question regarding the assumption of a quasi-steady state gas phase.

2.1 Dynamic Erosive Burning or Acoustic Erosivity.

The presence of an acoustic field imposed on the mean coreflow may cause generation of small-scale turbulence within the gaseous combustion region close to the propellant surface. It has been shown experimentally that pipe flow with periodic velocity oscillations can become turbulent under certain favorable conditions, regarding the oscillation frequency and the vibrational Reynolds number. This would bring about enhancement (or decrease) in the heat transfer and chemical species diffusion rates within the combustion region, and may lead to a stationary (DC) shift in the propellant burning rate.

Acoustic erosivity has been considered in an early study by Bird, Hart and McClure[21], although its cause was associated with large amplitude velocity oscillations in the coreflow, and rectification of the propellant burning response (i.e., only the magnitude, not the direction, of the oscillatory tangetial velocity matters). The nonlinear nature of this coupling was clearly pointed out.

Other works have more directly inferred the cause of acoustic erosivity from small scale turbulence, such as Lengille[14], Matveev[22], Medvedev and Revyagin[23] and Vilyunov[24]. Gostintsev[25] and Medvedev[23] have correlated the onset of acoustic erosive burning and found that the threshold velocity was lowered by a factor of two to four relative to stationary (non oscillatory) erosive burning.

The generation of small-scale turbulence near the burning propellant surface might not be the only explanation for a DC-shift in the mean burning rate during acoustic excitation. Another possible mechanism is acoustic streaming, associated with acoustic/viscous interaction in the acoustic boundary layer (or Stokes layer) as will be discussed further in the following sections. This interaction leads to generation of a steady velocity component, generally pointing toward acoustic velocity nodes; in parallel, a DC component of surface heat transfer appears, which may account for the burning rate modification.

2.2 Core-Combustion/Acoustic Coupling

Residual combustion in the coreflow of a solid propellant motor might occur due to particular types of propellants, such as nitramines and double base (DB), which are observed to obtain extended gaseous flames under rocket operating conditions. Otherwise, residual exothermicity in the core may be due to burning aluminum particles, in aluminized propellant configurations. The actual heat release required locally to interact with the acoustic modes is quite small: as will be shown later, 1% of the total propellant energy may provide quite strong acoustic driving. The condition for this coupling is, of course, the sensitivity of the residual heat release to local pressure oscillations. Further, the chemical relaxation timescales should be comparable to the acoustic timescales available to the chamber modes. For a chemical reaction in the core, the residence time and the reactant concentration depend on both flow history (at a given position) and on local fluid dynamic variables. Hence the relevance to velocity-coupled instability.

This subject is treated in detail in the following section. Order of magnitude analysis indicates its importance as a strong velocity coupling instability mechanism, although its application is limited to certain propellant families, as mentioned earlier.

2.3 Viscous/Acoustics Interaction

The presence of an acoustic field outside a viscous boundary layer is known to generate oscillations in both shear stress and heat transfer to the surface. As the frequency of attendant (acoustic) perturbation increases, the amplitude of the velocity oscillations near the surface tends to increase, and a phase shift relative to the external perturbation develops. Measurements of oscillatory heat transfer in impervious pipes, with periodically perturbed mean flow, show that the heat transfer augmentation (relative to the mean flow Nusselt number) can reach over 200%.

The amplitudes and phase shift in heat transfer indicate that this mechanism may couple with the relaxation of the thermal wave in the condensed phase, as well as appreciably modify the gaseous combustion zone near the surface, as its region of effectiveness is the acoustic boundary layer (or Stokes layer), which is comparable to thin combustion flames (typical of ammonium perchlorate propellants) at frequencies of the order of 1000 cps.

This mechanism is studied in detail in Section 4 herein to point out the effects of frequency and mean flow properties upon the amplitude and phase of the surface heat transfer. Again this is demonstrated as a powerful velocity coupling instability mechansim, which warrants further detailed investigation.

2.4 Other Velocity-Coupled Instability Mechanisms

A prominent mechanism which has been proposed recently is vortex shedding within the coreflow of the solid propellant motor. This may occur due to the presence of baffles, slots, segmented propellant - but may also occur naturally following the head-end region where the coreflow begins to assume a predominantly axial direction. The subject has been investigated by Flandro[26] and recently verified experimentally by Brown et al[27], who demonstrated pressure oscillations due to vortex shedding in a cold flow simulation including segmented porous-wall tubes with baffles. This particular mechanism will not be treated within the scope of the percent study. Considerable work is currently in progress on this subject.

Additional velocity-coupling instability mechanisms and interactions may prevail, beyond the four plainsible mechanisms mentioned herein. One of the objectives for the ongoing literature analysis is to try and find indicatons to their existence. Further modes of dynamic interaction leading to instability may be found from comprehensive analysis and the associated experiments, currently under way at UTC/CSD.

2.5 Nonlinear Considerations

The physical phenomena under consideration is clearly nonlinear. It may involve relatively high amplitudes of perturbation. steep-fronted travelling waves, triggering (i.e., appearence at a certain critical perturbation intensity), interaction between oscillatory modes and mean properties in the motor (pressure, flow velocities), multiple time scales (dependence of amplitudes on total phase), and nonlinear damping (appearance of limit-cylce oscillatory behavior). These observations lead to the following conclusions.

(1) Intrinsic or global acoustic admittance functions for velocity coupling, based on propellant properties alone, are expected to be inadequate or of limited utility for description of this type of instability.

(2) A comprehensive description of the internal flowfield is necessary, at least in 2 dimensional, unsteady form. This is extremely important to obtaining physically meaningful results, and would ultimately lead to numerical simulation.

(3) Further complication is posed by the necessity to incorporate the thin Stokes layer processes, where combustion prevails in all cases of practical interest. For cases of appreciable amplitude of perturbation, timewise condensed phase solutions should be generated at each point of interest along the ports; this particular element is not new, and methods of solution are readily available.

The foregoing points indicate the necessity for nonlinear analysis, and serve to motivate the following phase of this study, involving comprehensive numerical modeling.

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3. ACOUSTIC-COMBUSTION INTERACTION

3.1 Physical Considerations

One mechanism of velocity-coupled instability investigated within this framework concerns interaction between residual combustion and acoustic modes in the coreflow of a solid propellant rocket motor. Such interaction may occur in motors incorporating the important classes of double base or nitramine-type propellants, which have been observed to exhibit extended flame zones under rocket-operating pressures; this means that under cross-flow conditions such as in an interior-burning grain, some reactants would enter the coreflow to react further and release heat. Another possible source for core exothermicity are burning propellant particles or aluminum particles. In all cases the extent of heat release in the core is expected to be relatively small (that is, excluding motor pathologies), involving of the order of one percent variation in total specific impulse, yet controlled by some pressure-sensitive reaction rate. Further, since there is good reason to expect turbulence over certain portions of the coreflow region, the aforementioned reaction rates would also depend upon fluid-dynamic variables, in addition to their "laminar" dependence on local state variables (p,T).

The actual mechanism through which such coupling is manifested, resulting in enhancement (or damping) of acoustic vibrations, can be inferred from the Rayleigh criterion, as explained in the remainder of this section.

The Rayleigh criterion[1] is of particular interest as it deals directly with thermal excitation of acoustic oscillations. It postulates that heat addition made at the proper point in space (such as a pressure antinode), in the proper time (heat added when the pressure is maximum, and extracted when the pressure is minimal) may lead to local enhancement of acoustic vibrations. The same mechansim could lead to suppression of oscillations, as heat is extracted when pressure is maximal and added when pressure is minimal. Thus, in order to generate acoustic oscillations, the net heat flux, g_v, into a local control volume, should have an oscillatroy component in phase with the pressure perturbation. Stated somewhat less strictly, acoustic oscillations would be enhanced for time-wise phase angles $Arg(q_v,p')<\pi/2$, and damped for $\pi/2<Arg(q_v,p')<\pi$, where p' denotes the complex pressure perturbation. Satisfying this condition does not insure that oscillations will persist locally with ever-growing amplitude, since nonlinear damping may take over as the amplitude increases. Neither does it imply that the amplitude of oscillations would increase over the entire field following a local heat perturbation. Indeed, Glushkov and Kareev[2] have shown recently that the Rayleigh criterion consists of a necessary (not sufficient) condition for acoustic instability in the presence of dissipative processes (viscosity, diffusion); their work concerns a perturbed gas with zero mean flow.

Utilization of the Rayleigh criterion in works related to solid propellant combustion instability is quite rare. Smith and Sprenger[3], in an early work concerning the tangential (spinning) acoustic modes in tubular solid propellant cavities, have stated it without formulation as a possible cause of "sonance" burning, i.e., the self-enhancement of acoustic vibrations by non-steady heat release (from gaseous flame tone near the propellant surface).

Cheng[4] considered the effect of heat addition in the main chamber flow upon acoustic stability, within the framework of solid propellant combustion with reactive additives (e.g., aluminum powder). The Rayleigh criterion was invoked for the purpose of an explanation of the coupling between nonsteady, distributed heat sources in the chamber flow and the acoustic field. The model is based upon modifications to previous instability analyses by the same author[5], employing the Crocco time-lag concept[6].

In this conjunction, it is interesting to point out that finite-rate chemical relaxation processes, in the neighborhood of the equilibrium state within the solid propellant coreflow, were considered strictly dissipative in the textbook by Williams, Barrere and Huang[7], the acoustic analysis allows only small departures from equilibrium. Owing to the assumption of zero entropy perturbation, the resulting wave equation (first order) is homongeneous; although expected to be valid in the neighborhood of the equilibrium point, the assumption is rather inadequate for a chemically reacting flow, with the reaction time appearing in the zeroth order formulation.

The Rayleigh mechanism is only one of several possible means by which excitation or attenuation of acoustic vibration is possible. Salant and Toong[8] discuss the effects upon the acoustic field of nonuniform mean flow properties (du/dx, dp/dx, and ds/dx, in a one dimensional, inviscid case), in addition to the influence of nonsteady distributed mass and heat sources. Dispersive (or energy-conservative), as well as dissipative coupling mechanisms were identified in the theory. In a recent work by Cummings[9] the effect of temperature gradient upon the acoustics of a quiescent gas were investigated experimentally, showing amplification of a pressure wave travelling in the direction of $dT/dx < 0$, at a frequency of 4.5kHz. Conservation of acoustic energy has been indicated; the results are in line with the theory of Salant and Toong[8].

3.2 Background

The fact that double base propellants burn with extended flame zone (several mm at 20 atm) under typical rocket pressures is well known [10,11]. More recently, the experimental data of Kubota [12] demonstrated that nitramine propellants behave in a similar way. There is, therefore, reason to expect residual combustion in the coreflow within a propellant cavity in both cases. The extent of this core exothermicity, and, more importantly, its ability to interact with the acoustic chamber mode, remains still to be demonstrated experimentally, and hence is open to argument at present.

Direct observations of nonsteady behavior associated with double base propellant rocket motors were reported by Trubridge and Badham[13] at the Summerfield Research Center (England). A systematic study of motors with diameters ranging between 6 and 19 inches was made. The various effects of operating pressure, propellant composition, ambient propellant temperature, length, and shape of propellant grain, and acoustic instability suppression, devices (inert rods) were tested. Two major types of nonsteady behavior were reported: (1) one, roughly close to ignition time, is an irregular, very high amplitude, low frequency (several Hz) phenomenon; (2) more orderly, periodic, small amplitude pressure oscillations in the estimated 1-10 kHz range. In all cases, instability was spontaneous, without external stimulation. The various effects pertaining to initial propellant temperature, propellant composition, operating pressure and scaling (motor internal diameter) were summarized elsewhere[14]; also, several tubular grain samples, extinguished after undergoing high frequency instability, were found to have a region of enhanced absolute burning rate near the grain center (the region of velocity antinode for the fundamental axial mode, and possible the region of highest spinning-mode velocity amplitude).

In a somewhat earlier publication by Angelus[15], a series of experiments utilizing a small, laboratory-scale, interior burning tubular grain was reported. The effects of propellant ambient temperature and operating pressure upon regular (acoustic) and irregular (high amplitude) instability were investigated. The frequency and amplitude analyses presented tend to support the effects described in Ref. 13, but with more quantitative detail. In addition, a reduction in the motor specific impulse was observed whenever appreciable nonsteady behavior occurred; the effect was attributed to possible incomplete combustion of the gases upon leaving the chamber. Unfortunately, the effect of operating pressure change is given through the ratio of burning surface to throat area, and it is not clear whether different interior grain diameters were used. This could have a strong bearing upon the extent of erosive burning present in the various configurations.

Recent experiments by the group at Thiokol[16], regarding irregular instability involving nitramine propellants, bear a striking qualitative resemblance to the instability behavior described above for double base propellants. Similar nonsteady p-t curves, which show both low and high frequency content were recently observed at Naval Ordnance Station, involving modified-grain, double base rocket motors[17-19].

Finite rate gaseous reactions in nonsteady solid propellant combustion analyses are traditionally excluded by invoking the quasi-steady assumption regarding the gas phase region adjacent to the propellant surface. Motivated by the analysis of Hart and McClure[20], two notable exception to the above statement exist in the works of Williams[21] and of Tien[22].

Hart and McClure,[20] in a study aimed at obtaining analytical expression for the burning rate response function, assumed that (a) the entire gas phase reaction occurs within a thin flame sheet at a distance from the propellant surface, and (b) the reactant flow region from the propellant to the flame, termed induction zone, is entirely convective, namely, diffusion and reaction in this region are assumed negligible. The resulting response function, although containing implicity the effect of finite reaction zone thickness, is not adequate for describing the influence of finite rate reaction and diffusion upon the response function. These deficiencies were pointed out by the authors.

The investigation by Williams[21] goes one step further in terms of formulation. The full diffusive-convective-reactive model of the gaseous flame zone was employed. The nonsteady first order perturbation problem was rigorously posed, but the analysis stops short of solution, which could have been obtained numerically. The final results were limited to small values of the imposed perturbation frequency [however, since the typical time scale in the gas phase used may be shown to be $O(0.1 \mu sec)$, actual frequencies of the order of 10^4 cps should still fall within that category] and to very high values of the gas phase activation energy. Under the foregoing fundamental constraints, the acoustic admittance function was derived, and its real part,

A_R, plotted against dimensionless frequency; this was shown to rise from zero to a maximum, then fall monotonously to negative values, as frequency increases. Unfortunately, the alternating (negative) portions of A_R occur at the high frequency region, outside the domain of validity afforded by the simplifying assumptions. A quantity related to the surface-gasification activation energy served as parameter in these calculations; at higher values of this parameter, the peak in A_R is higher, and shifted to higher frequencies.

Interestingly, as pointed out by the author, the admittance function derived does not exhibit any explicit dependence upon the thermochemical data of the distributed flame zone reaction, unlike analyses which are closer to employing the collapsed flame surface approximation, e.g., the work of Dension and Baum[23], where kinetic parameters appear explicitly in the results. This may be directly attributed to the elimination of the inhomogeneous reaction terms in the first order (perturbation) formulation, by the assumption of high activation energy.

Ten years later, apparently independent of the work by Williams, Tien[22] presented a study of the same problem. In this instance, the zeroth order (steady state) and first order (nonsteady perturbation) equations were solved numerically, after assuming for the dimensionless perturbation quantities the form $\Phi'(x,t) = \Phi(x) \exp(i\Omega t)$, which reduces the perturbed system to ordinary differential equation form. No limitations were made a priori as to the actual magnitude of frequency or the gas phase activation energy. Two comments regarding this analysis are in order: (1) The boundary condition at the flame outer edge was $Ds/Dt(y = +\infty, t) = 0$, namely, entropy is conserved for a fluid particle leaving the flame. Consequently, the resulting perturbed boundary condition is expected to be valid only for large enough frequencies, such that $1/\Omega^*$ is much smaller than typical thermal relaxation times, as pointed out by Krier and Summerfield[24]. (2) The fast gas phase time scale was used for the solution of the nonsteady heat conservation equation in the condensed phase. This gives rise to a coefficient $a^* = (cond)/(gas, y = \infty) >> 1$, multiplying the time derivative. At high enough frequency, this results in a singular problem for the perturbed condensed phase, implying a thin, high-frequency "boundary layer" within the solid near the surface. The real part of the acoustic admittance function, A_R, was calculated numerically and plotted against the dimensionless frequency $\Omega = \Omega^* t(gas)$, with a^* and the characteristic time scale ratio, $b^* = t(gas)/t(cond)$, as parameters. For ($a^* = 1000$, $b^* = 0.001$), the A_R curve has a positive peak at low frequencies (resembling the well known quasi-steady gas phase results, cf Dension and Baum[23]) and a negative minimum at high frequency, indicating attenuation at higher frequencies. With smaller a^* and larger b^* values, the positive peak is displayed to higher frequencies somewhat reduced, and the negative part vanishes.

Of course, the foregoing works[21,22] are studies of nonsteady combustion of propellants, not rocket motor grains; effects of cross-flow and dependence of local fluid-dynamic variables in the core are precluded. Nevertheless, they point out the coupling between finite-rate combustion kinetics and acoustic perturbations, in a linearized sense.

A theoretical study of core combustion/acoustic instability was carried out by Ben-Reuven[14,25], for nitramine-type propellants. Numerical solutions were obtained for a one dimensional (axial), nonsteady formulation, employing a unique erosive burning model[14,26] to determine the actual burning rate, as well as the mass fraction of residual reactant entering the coreflow at each axial location. Evolution and enhancement (as well as damping) of acoustic vibrations was demonstrated, with pronounced dependence upon local mean flow properties, axial position, and L/d. The prominent frequencies at which such enhancement was obtained were relatively high, close to twice the fundamental axial mode, in line with the analysis by Cheng[4]. It should be emphasized that the analysis[25] employed a rather crude turbulence correlation (turbulence was considered only in the neighborhood of the wall layer) and was strongly dependent upon the aforesaid erosive burning model[26], the validity of which remains to be proven. Nevertheless, the results indicate that instability can evolve due to this mechanism (condensed phase thermal relaxation was assumed instantaneous); further, this instability can not be characterized in terms of a global "acoustic admittance" function, namely, it may not be determined by propellant properties alone, independent of the actual motor environment.

3.3 Order of Magnitude Analysis

The purpose of the simplified calculations herein is to demonstrate the importance of core combustion/acoustic coupling, relative to the so-called pressure coupling instability. In order not to encumber the results by any particular combustion model, no inferences are made from the more detailed analysis carried out previously[25].

To demonstrate the plausibility of reactants entering the coreflow, a comparison is made between the viscous sublayer thickness and typical chemcial reaction length scales, as follows. In Fig. 1, a typical turbulence intensity profile is shown schematically, with the peak intensity, Ip, indicated. The coordinate y^+ is made dimensionless by use of the frictional velocity and the kinematic viscosity; the viscous sublayer thickness at $y^+ = 50\text{-}60$ is superimposed. Similar turbulence intensity profiles were measured by Yamada[27] in cold flow tests. Some of these results pertaining to peak intensity and its proximity to the surface are plotted in Fig. 2, against the ratio of mean channel mass flux to injected mass flux, G/m. The trends are obvious: the peak intensity tends to increase and approach the surface as G/m (or x/d) increases.

The length scales associated with two typical secondary reactions encountered in many propellant formulations, are plotted against pressure in Fig. 3; these are NO and ONO and hydrocarbon reactions; the associated chemical kinetics data have been reported elsewhere[14]. The concentrations used for calculation of the actual reaction rates were taken from a theoretical analysis of nitramine deflagration[14], at the end of the primary decompostion zone in the gas phase; the temperature, however, was taken close to typical final flame temperatures, at 300K. The mass burning rates employed are from reported strand burner data of HMX. For comparison, two viscous sublayer thickness scales are overplotted: for unblown cases (impervious wall) and for blown wall (with mass injection), both at specific values of (Re, Mach No.) and (G/m, channel diameter), respectively. The detailed analysis leading to the blown viscous layer thickness calculation is given elsewhere[26], and follows in general the work of Tennekes[28]. Now the scale comparison of Fig. 3 demonstrates that oxidation reactions such as NO and ONO might extend beyond the viscous sublayer. The prevalence of peak turbulence intensity in the neighborhood of the viscous sublayer edge can therefore lead to appreciable burning rate modification (in both DC shift as well as oscillatory manner) and carry reactants into the coreflow region, by the action of eddies.

For a notion of the frequencies involved, the same two reaction rates are used to calculate inverse timescales vs pressure in Fig. 4. Over the range from 1-10 MPa (10-100 atm) the frequency range is between 100 and 10,000 cps.

Finally, a highly simplified comparison between residual core exothermicity and acoustic energy is made. A typical propellant formulation is considered, having a total specific chemical energy of Q_R = 1000 cal/g. Supposing that only 1% of this quantity is involved in acoustic coupling in the coreflow, then

$$\delta Q_R = 0.01 \times 1000 \times 4184 = 4 \times 10^4 \text{ J/kg}$$

Now for an acoustic oscillation of 10% of the mean pressure, the energy stored over one half cycle is

$$\delta p/\bar{\rho} = 0.10 \times R_u T/W = 8 \times 10^4 \text{ J/kg}$$

where T = 3000K and W = 30 g/mol have been assumed. This indicates that the residual core combustion and acoustic energies are comparable, even if only 1% of residual exothermicity is involved in acoustic interaction in the core.

3.4 Conclusion

Based on the foregoing observations, the mechanism of core combustion/ acoustic coupling seems quite important. Conservative estimates indicate that both magnitude of enthalpy perturbations as well as the associated characteristic frequencies are in the right range for such coupling to prevail in actual rocket motor configurations.

This type of instability can not be characterized by intrinsic propellant properties alone. Theoretical study would therefore require comprehensive core flow field analysis (involving both turbulence and combustion), as well as detailed modeling of the flow and combustion processes near the propellant surface (in the wall layer region). Evidently, a major analytical and computational effort is required for such an analysis. Further, for verification, new experiements should be devised, incorporating combustion and non-intrusive (or flow-visualization) measurements.

It is recognized that this mode of combustion instability is confined to a particular (although important) propellant family, or to aluminized propellant configurations. Thus, the associated analytical undertaking, with all its complexity, might not be supported by wide practical interest at this time. Such practical interest may however evolve, with the increasing use of nitramine/double base propellant configurations in a wide variety of rocket motors.

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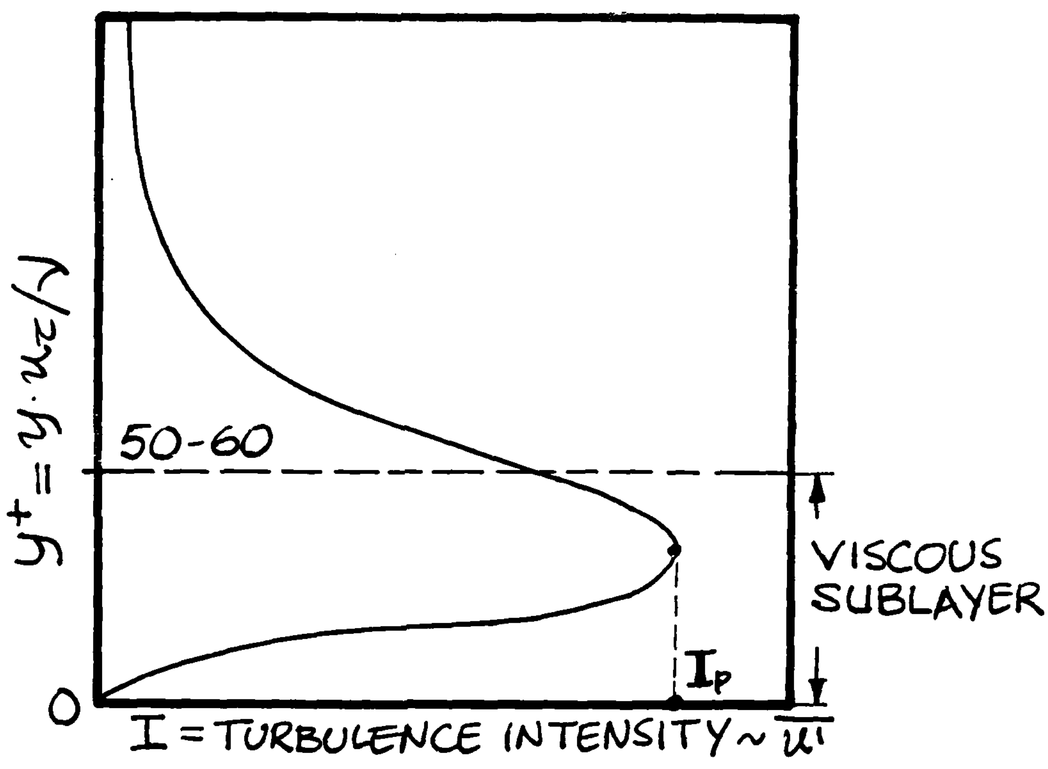

Fig. 1 Schematic diagram of turbulence intensity vs dimensionless distance perpendicular to the wall, y^+, showing the peak intensity. Overplotted is the thickness of the viscous sublayer (without mass injection, after Hinze) y^+ = 50-60.

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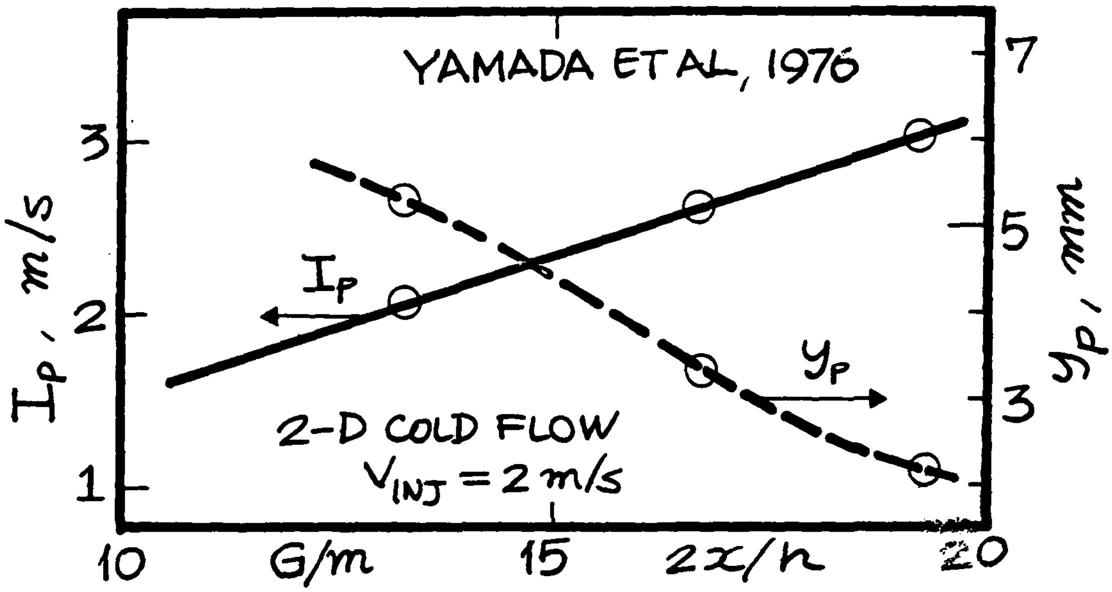
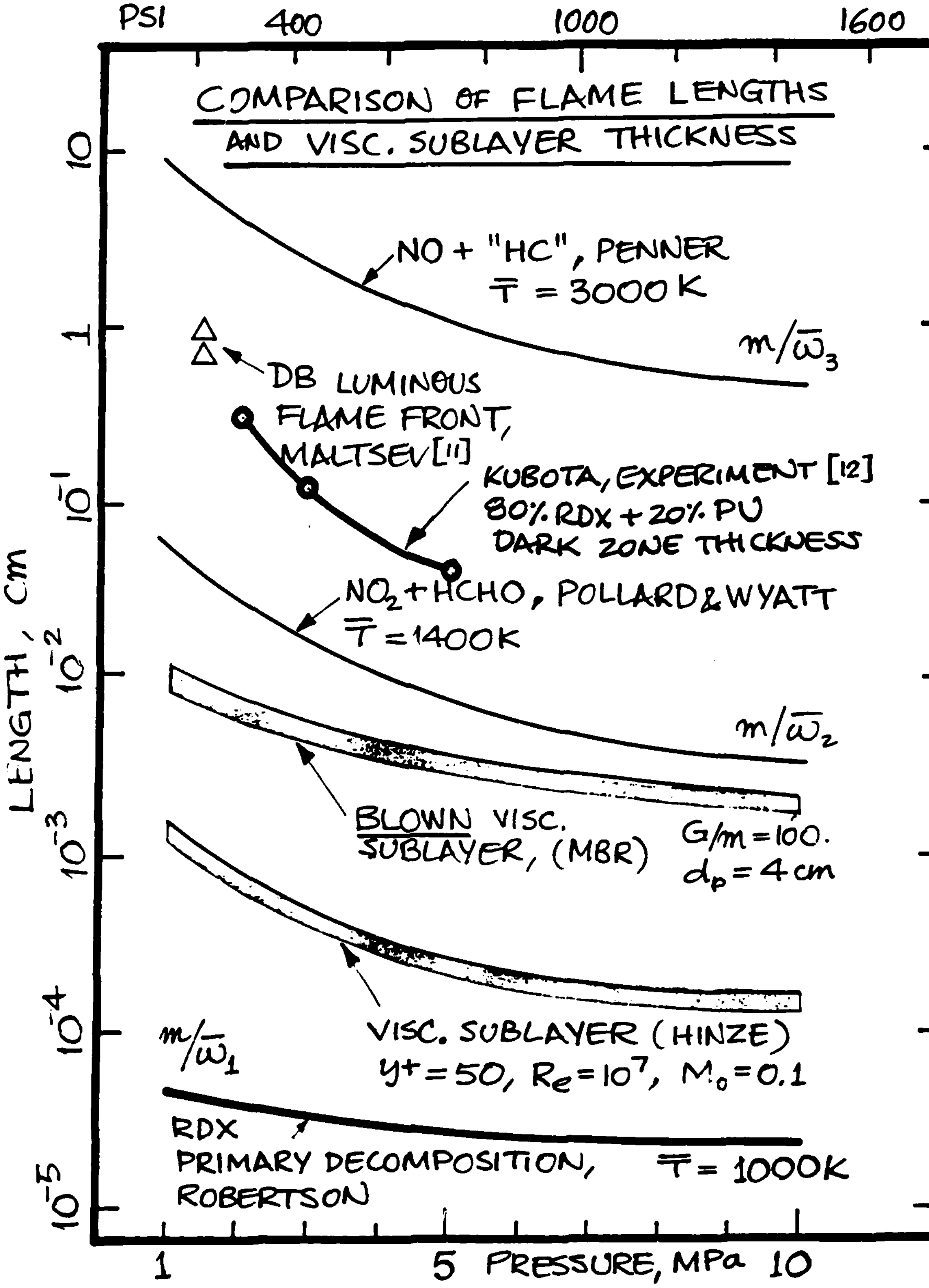


Fig. 2 Plots of peak turbulence intensity and its locus relative to the surface, I_p and y_p respectively, vs the ratio of mean mass flux in the channel, G = , and the injected mass flux, m. The three points shown were taken from cold flow measurements in a porous-walled channel by Yamada[27]. Clearly, the peak turbulence intensity increases and its proximity to the wall decreases as axial distance downstream increases.


PSI
400
1000
1600
COMPARISON OF FLAME LENGTHS
AND VISC. SUBLAYER THICKNESS
NO + "HC", PENNER
T = 3000 K
m/ω3
DB LUMINOUS FLAME FRONT, MALTSEV [11]
KUBOTA, EXPERIMENT [12]
80% RDX + 20% PU
DARK ZONE THICKNESS
NO2 + HCHO, POLLARD & WYATT
T = 1400 K
m/ω2
BLOWN VISC. SUBLAYER, (MBR)
G/m = 100.
dp = 4 cm
VISC. SUBLAYER (HINZE)
y+ = 50, Re = 10^7, M0 = 0.1
m/ω1
RDX
PRIMARY DECOMPOSITION, ROBERTSON
T = 1000K
LENGTH, cm
10
1
10^-1
10^-2
10^-3
10^-4
10^-5
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5
PRESSURE, MPa
10

Fig 3

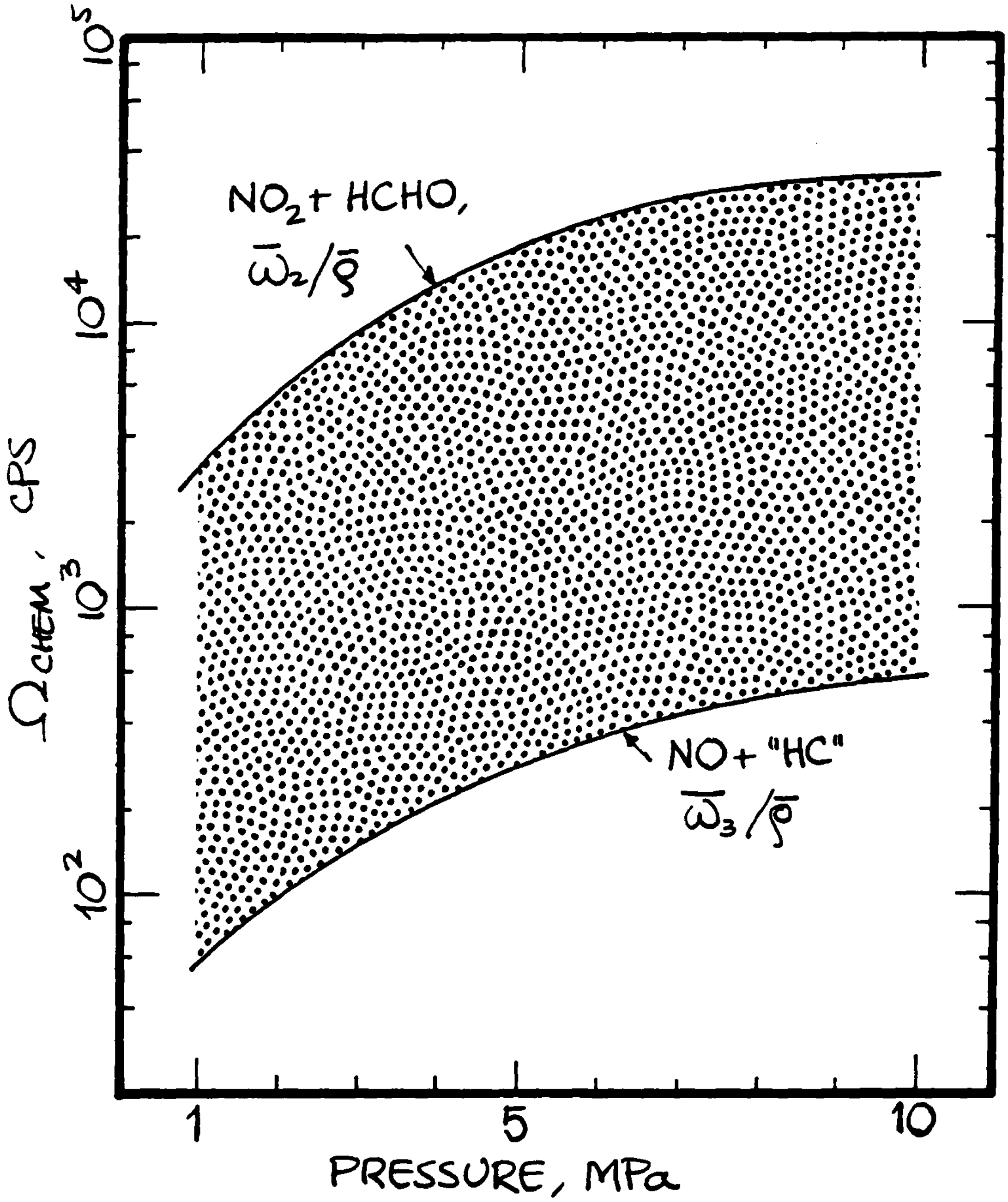

Fig. 4 Characteristic frequencies (or inverse Damkohler timescales) for the NO and ONO reactions vs pressure. The same reaction rates as in Fig. 3 were used in the calculation, showing coverage of a broad frequency range from 100 to over 10^4 cps in the rocket pressure range. This indicates that coupling with coreflow acoustic modes is plausible.

4. ACOUSTIC-WALL LAYER COUPLING

4.1 Physical Considerations

The second plausible mechanism of velocity coupled instability investigated herein pertains to acoustic/wall-layer or acoustic/viscous coupling near the propellant surface. The phenomenon leading to consideration of this mechanism is the powerful sound produced in the Rijke tube, as described by Rayleigh[1]. In that experiment, a steel wire mesh is placed in a long vertical tube (open at both ends) in the neighborhood of the fundamental pressure antinode. The mesh is then heated until glowing-hot, and the heat source or burner removed abruptly. After a short while, a strong audible sound is produced, which may last for 10 seconds or so. The evolution and sustenance of these acoustic vibrations is connected to the special mode of heat transfer from the hot wire mesh to the gas stream, in which both axial location and, in particular, temporal phase relative to the acoustic motions are important.

Of special concern in the present framework is a different manifestation of the same physical interactions operating in the Rijke tube: acoustic vibrations in the core flowfield of a solid propellant cavity, can produce periodic changes (as well as a DC-shift) in the heat transfer to the surface, with a timewise phase shift relative to the attendant perturbation.

An excellent physical explanation of these dynamic interactions is provided in the pioneering paper by Lighthill[2]. Consider a boundary layer produced by a steady flow, over which a periodic small perturbations is imposed, as shown schematically in Fig. 1. At very low perturbation frequency, the entire velocity profile would be quasi steady and react instantaneously to the external perturbation. As the frequency is increased, a phase shift would evolve at the portions of the layer close to the surface. Contributions to phase lag would be from fluid element inertia and dissipative processes (viscosity); contributions to phase advance (or negative lag) would be from the rapid acceleration of the realtively-slow fluid elements adjacent to the surface, producing "rapid convective motions" in Lighthill's[2] terminology. Note that the pressure perturbation is assumed instantaneous from the outer side of the boundary layer to the surface (i.e., pressure is uniform at any instant throughout the layer). Thus, the same attendant lengthwise pressure differential accelerates both outer flow (high velocity) and inner flow (low velocity). Neglecting for the moment any density differences, the result of this uniform accelerating force is a much larger percent change in the velocity of the inner region near the wall. It will be shown later that indeed the balance of phase-lag/advance effects results ultimately in a phase lead for velocity at high enough frequencies, and hence also for the fluid dynamic shear stress at the surface.

Similar considerations apply to the temperature profile, although here compressibility plays an important role, as hinted above. Phase lags can evolve due to thermal inertia, and conductive dissipation; phase lead is influenced by the same rapid convective motions mentioned earlier. These effects directly influence the heat transfer to the surface.

As frequency becomes higher, the amplitudes of both shear stress and heat transfer at the surface would depart from that of the external perturbation, and the configuration near the surface would begin to resemble one without mean flow[2].

The Stokes layer thickness indicated in Fig. 1 is the range (perpendicular to the surface) over which timewise changes can be balanced by viscous forces. This scale naturally arises from the parabolic diffusion equation for mementum (or heat), and is decreasing inversely with the square root of frequency.

Following Batchelor[3], an explanation of the associated steady "streaming" or DC phenomenon can be advanced as follows. The flow configuration at two levels in the boundary layer is schematically depicted in Fig. 2. For incompressible flow, the perturbed tangential velocity, u, induces a normal velocity component; from continuity,

$$v = -\int_0^y (\partial u/\partial x)dy'$$

Now when is not out of phase with u, viz, the relative phase Arg(u,v)≠90°, then a net x-momentum flux occurs over a period of oscillation, since a fluid element with velocity u_1 is carried vertically to a region where the velocity is $u_2 \neq u_1$. Thus, a steady flow component is produced by the pure periodic perturbation. This streaming component can be approximated[4,5] as

$$u_s \sim -\overline{u\partial u/\partial x}/\Omega$$

where overbar indicates average over a period of oscillation. Evidently, the magnitude of the streaming effect involves products of perturbed quanitites, and would call for second order analysis. Aside from the steady velocity component, higher frequency periodic terms are likewise introduced in this instance. With use of the u_s - equation, Fig. 3 shows a simple example of acoustic streaming in a standing wave configuration. Shown in Fig. 3 is the envelope of the acoustic velocity fluctuations over a period of oscillation; the signs of u and u_x consistently yield streaming outward from the center. The actual flow pattern in the Kundt's tube is somewhat more complex[4].

Considering now the oscillatory heat transfer component with its phase shift relative to the attendant velocity perturbation, one can readily recognize the relevance of this mechanism to the combustion instability problem: modulation of the heat transfer to the surface can cause directly modulation of the gas generation; the aforementioned phase shift may combine with that due to thermal relaxation in the condensed phase (and that due to the combustion process within the thin wall layer, when finite-rate kinetics are considered) to enhance or retard acoustic vibrations in the coreflow, through this perturbation of the mass injection rate.

4.2 Background

The detailed theoretical treatment by Lighthill[2] has been mentioned earlier. A first-order analysis on the effects of acoustics upon skin friction and heat transfer was carried out, concerning the oscillatory incompressible flowfield about an impervious flat plate. The analysis covers both high and low ranges of the frequency parameter $S = \Omega L/U$ (or Strouhal number). The important conclusions concerning the high frequency asymptote were that as S approaches very high values, (1) skin friction tends to attain a phase lead of 45 degrees and its amplitude tends to grow out of bounds and (2) heat transfer to the surface, tends to attain a 90 degree phase lag, while its amplitude tends to zero. This dissimilar behavior was recognized as a departure from the Reynolds analogy.

The aforementioned analysis was somewhat preceded by Moore[6], who addressed the question of nonsteady, aperiodic motions about a flat plate in compressible flow. Regions of "quasi-steady" boundary layer response and of "starting from rest" (i.e., accelerating) behavior were discerned.

Additional theoretical treatment by C.C. Lin[7] of the oscillatory boundary layer at high frequencies, provided a unique averaging method (over an oscillation period), particularly suited for higher-order anlaysis[4].

The effects of compressibility have been incorporated in the analysis by Illingworth[8], for oscillatory flow (travelling waves) about a flat plate; the analysis was carried out up to first order, and the results regarding dimensionless skin friction and heat transfer were plotted against the Strouhal number (for both high and low values, separately). The results-after some corrections - are surprising, as they appreciably depart at high frequencies from those of Lighthill, regarding the dimensionless heat transfer; these results show that when compressibility is incorporated, the Reynolds analogy between shear stress and heat transfer is recovered. Further discussion is provided in the following section.

A problem is easily identified in the work by Illingworth[8], that is, the solution for velocity at large values of y = distance from the wall, does not match the imposed external velocity. This shortcoming has been resolved in the works of Rott and Rosenzweig[9] and later by Lam and Rott[10], using matched asymptotic expansions to generate smoothly-jointed, uniformly valid solutions for both small values of S (within the thin stokes layer, at high frequency) and large S. Several years later the same methodology was used by Gersten[11] to carry out incompressible solutions (following Lighthill[2]) up to second order, the observations were similar to those of Lighthill.

Two points should be highlighted regarding the works of Lighthill[2] and Illingworth[8], as follows. The first order solutions obtained for large and for small frequency parameters are quite distinct, and no attempt has been made to match them. On the other hand, the overlap domain of common validity for these solutions is quite narrow, as explained by Lam[10], and the solutions are expected to hold at values of S away from this overlap domain. Secondly, the motivation for including compressibility effects in Illingworth's work[8] has been to enable calculation of cases where the difference between the wall temperature and the free stream temperature is large, as would give rise to large density differences, although the free stream Mach numbers were kept low. However, as will be shown in the following section, there is additional merit to incorporation of compressible theroy, at the high frequency range.

An integral method was used by Miller[12, 13] to solve the general unsteady (incompressible) thermal boundary layer over a flat plate. Solutions up to second-order[13] were generated for the case of free steam with small periodic perturbations in the standing wave mode. Steady second order streaming contribution, expressed in terms of a single parameter, was found to decrease the extent of heat feedback to the plate at low values of $S(X) = \Omega x/U$; however, this contribution falls steeply to zero as $S(x) = 1.2$ is approached. Earlier, first-order results for nonsteady heat transfer[12] compared favorably with the experimental heat transfer data of Hill and Stenning[14].

A number of additional theoretical works on this subject have been reviewed, chiefly concerning analytical solutions to the perturbed incompressible boundary layer about a flat plate or oscillating cylinder; notably, the works by Cheng[15, 16], Ishigaki[17, 19], Ghoshal[20], and Sarma[21]. An additional work by Sarma[22] incorporated blowing or suction at the boundary of a body in a steady external flow, to create the nonsteady perturbation; the configuration considered compressibility, and utilized the Howarth transform for solution. Unfortunately, no calculated results were presented.

Huerre and Karamcheti[23] investigated the effects of viscous action upon sound propagation in ducts without mean flow or injection. Their "high frequency, wide tube mode" results resemble the rocket motor configuration; the real part of the pressure profile obtained was uniform in radial direction, while the imaginary part shows a phase lag as the tube wall is approached. The real part of the axial velocity, however, is uniform over most of the tube radius, with a distinct boundary layer near the wall; here the positive imaginary part shows phase advance near the wall (relative to the centerline), as expected from Lighthill's analysis[2]. The induced radial velocity was plotted as well, with both amplitude and phase lead largest near the surface. This way, the combination of radial and axial velocities is expected to lead to streaming, as observed in numerous experiments[4].

Additional perturbed pipe flow analysis was performed recently by Barnett[24], for turbulent, fully developed flow with periodic pressure perturbation. The calculated results show that the axial nonsteady velocity component near the boundary may well exceed the flow rate at the centerline, creating the well-known "annular effect" due to Richardson and Tyler[25]; this effect was confined closer to the wall as the vibrational Reynolds number, $Rev = \Omega R^2/\nu$, is increased, as expected. Good agreement was reported with published data for laminar flow in tubes, and vibrations in tubes without mean flow.

All of the foregoing works involved analytical solutions to a large extent. In contrast, a numerical finite-difference technique was employed by Farn and Aparci[26] to solve directly the partial differential system of of momentum and continuity equations for an oscillatory (incompressible) Blasuis flow. Only the external velocity was allowed a periodic perturbation, while the pressure was assumed constant. The results, for amplitude and phase of the nonsteady tangential velocity component were plotted vs distance from the surface; likewise, the maximal velocity phase (near the surface) vs $S(x)$ in the range $S = 1\text{-}10$ was plotted. In both instances good agreement was found with the aforementioned experimental data of Hill and Stenning[14]. No attempt was reported[26] to calculate skin friction values, and the energy equation was precluded from the study.

Another work concerned with nonsteady heat transfer through boundary layers, which employed numerical solutions is due to Kang and Levatin[27], who considered a developing, turbulent (compressible) hot flow, not periodic oscillations. Their solution method was alternating-direction-implicit (ADI), and employed the Cebeci-Smith eddy viscosity model. The particular version of the numerical algorithm employed is based on the development of Beam and Warming[28]. Results for axial velocity and thermal energy profiles (turbulent) were plotted, with timewise variation shown. Likewise, the evolution of wall temperature vs axial distance was shown, displaying rapid increase as local turbulence evolved.

Only few of the works on this subject have incorporated combustion and mass injection over a propellant surface. One such study was made by Williams[29, 30], where a flat fuel plate in a nonsteady oxidizing stream was analyzed, in both standing and travelling wave modes. Combustion was assumed to occur through a diffusion flame, with infinite kinetics rate. The acoustic admittance components for pressure and velocity were calculated. For low values of the frequency parameter S(x), it was found that when the layer was quasi-steady no instability occured, and similarly for the pressure response, for obvious reasons (the heat feedback from the diffusion flame is not pressure sensitive). However, instabilty could occur for the fully nonsteady velocity response (due to heat transfer), and more so when the flame resides within the layer. For a different approach to a similar problem, regarding nonsteady fuel droplet combustion, the reader is referred to the work by Strahle[31].

An interesting additionaly work on droplet combustion instability has been reported by T'ien and Sirignano[32], subsequent to the aforementioned works. T'ien has investigated the burning flat plate problem with perturbed outer flow, but, unlike Williams[29] and Strahle[31], has chosen to incorporate the boundary layer and the associated gas phase combustion in a quasi-steady manner; whereby, of course, all dynamic contributions from viscous-acoustic and nonsteady combustion effects were precluded, while instabilty has been investigated in terms of thermal wave relaxation in the condensed phase.

At least one analytical study involving solid propellant combustion with finite-rate chemical kinetics (premixed) is currently underway, by Flandro[33]. The preliminary conclusions point to the importance of the mechanism of viscous-acoustic coupling to instability.

The experimental work of Hill and Stenning[14] was previously mentioned. This work contains also analytical solutions, and comparisons were made with Lighthill's[2] solutions over the range of frequency parameter S = 1-10. The solutions for phase and amplitude of axial velocity perturbation vs distance from the wall, agree well with the experimental data generated in the low and the high frequency ranges. Intermediate frequency data were not simulated as well. It should be pointed out that the actual velocities in the experiment were quite low (6-10 ft/s), at ambient temperature; these allow for simulation at the limit of M = 0 or incompressible flow. No heat transfer measurements were reported.

Experimental measurements were performed at the Guggenheim Laboratories, Princeton University, within the framework of liquid propellant instability, under Professor L. Crocco[34, 35]. The work by Marec and Harrje[34] involved extensive measurements of heat transfer from the wall of a circular pipe, through which a mean flow was passed, modulated periodically (in the range of 200 1400 cps) by a downstream siren. Periodic wall heat transfer enhancement was measured directly and plotted as the Nusselt number ratio, Nu (nonsteady)/Nu (steady) at an axial location. In cases of flow reversal (or rectification), obtained for high-amplitude standing acoustic modes, enhancement of up to 200% could be discerned. The periodic heat transfer enhancement was found to be better correlated by the peak axial velocity perturbation, rather than the associated intensity, u'(rms). Greater such enhancement was found near velocity antinodes (for obvious reasons), but the wave shape, i.e., front steepness etc was also found important. In this respect, the enhancement was more effective near the velocity nodes, where the effect of high-frequency and travelling waves is pronounced. Overall (mean DC effect averaged over the test section length) heat transfer increase in the duct was also reported, which appeared to attain a maximum of 90% at intermediate frequencies of 1000 cps. It should be noted that the Mach number range employed in the experiment was quite low (at 0.03), and the heated-element temperature range was 350-550K. Thus compressibility effects were not pronounced.

The foregoing work was extended by Bogdanoff[35] to investigate heat transfer due to pure standing acoustic modes (with low Mach number mean flow imposed). Additional velocity profile and heat transfer data were obtained; the results clearly demonstrate enhanced heat transfer in the neighborhood of velocity antinodes along the tube, as well as the "annular effect" mentioned earlier (for the axial velocity profiles obtained). Some effects of turbulence interaction with the nonsteady heat transfer were also discussed, although no clear conclusions could be reached.

Evans[36, 37] has investigated nonsteady heat transfer to a flat plate at low frequencies both theoretically[36] and experimentally[37]. The theoretical study[36] considered compressible, perturbed flow and used the Howarth transform to obtain a stream-function configuration. The experiments[37] were performed at very low actual frequencies (1-60 cps) where the plate was oscillated parallel to the mean flow. The range of frequency parameters, S, covered is 0.01 to 20, the higher values probably due to very low actual velocities. Reasonable agreement (in terms of relative heat transfer amplitude and phase vs S) was shown with the incompressible, M = 0, limits calculated both by Evans[36] and Illingworth[8], at low and at high S-values; the intermediate-frequency data, 0.1 <S <1, show considerable scatter and was not correlated.

The effects of nonsteady heat transfer under resonant (or standing acoustic mode) conditions have also been investigated by Purdy, Jackson and others[38, 42]. These experimental and analytical works have been reviewed in detail by Bogdanoff[35].

4.3 Order of Magnitude Analysis

Several problems are addressed in this Section, regarding the relative physical importance of various terms in the formulation. The major interest herein is in the high frequency regime, and the following treatment is accordingly limited. These calculations serve as an extension of the section dealing with physical interactions, where some of the inferences made are given quantitative value.

4.3.1 Momentum Considerations

At high frequencies, the characteristic timescales are short. Thus, for moderate velocities and excluding very steep wavefronts,

$$\left| \frac{\partial u}{\partial t} \right| \gg \left| \underset{\sim}{u} \cdot \nabla \underset{\sim}{u} \right| \qquad (1)$$

where u is the tangential velocity (axial) in the layer. this means that the linear acceleration term dominates. In terms of typical values, the forefoing relationship yields

$$\Omega U \gg U^2/L$$

where U is the mean outer flow velocity and L a characteristic axial distance, over which velocity may change appreciably; Ω is the frequency. Thus, by the last inequality one arrives at the particular requirement:

$$S = \Omega L/U \gg 1 \qquad (2)$$

viz, the Strouhal number must be large. Now the high frequency asymptotc for the wall shear stress can be derived, following Batchelor[3] and Lighthill[2]. The momentum equation in the wall layer, retaining only dominant terms, is

$$\partial u/\partial t = \partial U/\partial t + \nu\, \partial^2 u/\partial y^2 \qquad (3)$$

$$u(y=0) = 0\,, \quad u(y \to \infty) = U \qquad (4)$$

where the first term on the right hand side of Eq. (3) represents the induced pressure gradient. The following perturbations are considered, in the standing wave mode.

$$U' = \varepsilon U_1(x) e^{i\Omega t}, \quad u' = \varepsilon u_1(x,y) e^{i\Omega t} \qquad (5)$$

with the perturbation quantity $o < \varepsilon << 1$; thus, substitution in Eq. (3) yields

$$i\Omega(u_1 - U_1) = \nu\, \partial^2 u_1/\partial y^2 \qquad (6)$$

At the high frequency limit, therefore, the nonsteady first-order component of the wall shear stress is given by

$$\tau_w' = \mu\, \partial u'/\partial y(y=0) = \varepsilon \sqrt{\frac{i\Omega}{\nu}}\, \mu U_1 e^{i\Omega t} \qquad (7)$$

which has a positive phase-lead over the outer velocity perturbation due to the square root of i. Two important conclusions can be made: (1) the skin friction asymptote has a phase-lead of 45 degrees relative to the imposed outer flow velocity perturbation, and (2) its amplitude increases as the square root of frequency.

4.3.2 Compressibility Effects

Often, compressibility is omitted from the analytical model due to low Mach numbers in the coreflow of rocket motor, $M_o = 0$. However, as will be shown here, the particular frequency regime under consideration has a strong influence upon the relative importance of the compressibility terms.

Following Batchelor[3], the condition for the flowfield to be approximately solenoidal (i.e., obtaining a vanishing velocity divergence) is realized as

$$\nabla \cdot \underset{\sim}{u} \doteq 0, \quad \left| \frac{1}{\rho} \frac{D\rho}{Dt} \right| \ll U/L \qquad (8)$$

where L denotes again an axial distance along which the velocity U changes appreciably. Now an equation of state is introduced,

$$\rho = \rho(p, s) \qquad (9)$$

where s is the specific entropy; for isentropic flow,

$$\left| \frac{1}{\rho} \frac{D\rho}{Dt} \right| = \left| \frac{1}{\rho a^2} \frac{Dp}{Dt} \right| \ll U/L \qquad (10)$$

the adiabatic (frozen) speed of sound being a. For periodic motions, the induced pressure variation can be approximated as

$$\delta p \sim \rho U \cdot \Omega L$$

Hence, from Eq.(10), the requirement for compressibility to be negligible amounts to

$$L^2 \Omega^2 / a^2 \ll 1, \text{ OR } \Omega L / a \ll 1 \qquad (11)$$

which, obviously, may not be satisfied at arbitrary high frequencies.

The foregoing result has an important bearing upon the energy balance at high frequencies. Compressibility or mass accumulation is important to the phase lag/advance considerations in this case, as follows. Recall that the same induced pressure differential (due to perturbation, at low Mach numbers) applies, at any particular axial location, to both outer (just outside the boundary layer) and inner (near wall) boundary layer regions, as discussed by Lighthill[2] and in the foregoing section. Now timewise, this pressure differential acts to cause compression/expansion. When the outer layers are considerably hotter than the inner layers (as would apply to a combustion situation in the rocket motor), the same pressure differential applied to both layers would cause a higher percent density change in the high-temperature, low density region. Similarly, a higher percent temperature change is expected to obtain at the low-temperature region near the wall. This obviously leads to direct influence upon energy dissipation, and will be shown in the following sections to recover the Reynolds analogy (between the rate of strain and heat transfer) at the high frequency regime. This goes contrary to the high frequency heat transfer asympotote prediction of Lighthill[2].

4.3.3 The High Frequency Energy Asymptote

Lighthill[2] posed the energy balance in the boundary layer at the high frequency limit, as

$$\lambda \frac{\partial^2 T_1}{\partial y^2} - i\Omega T_1 = u_1 \frac{\partial T_0}{\partial x} + v_1 \frac{\partial T_0}{\partial y} \qquad (12)$$

where u_1, v_1, T_1 denote perturbed boundary layer variables, cf Eq. (5), and to the outer flow temperature. Obviously, the dissipative term (later neglected) and the inertial term, on the left hand side, are balanced by the two advective terms; subsequently, Lighthill dropped the dissipative term and imposed a linear $T_0(y)$ to facilitate solution. The nonsteady heat flux at the surface is therefore,

$$\lambda \frac{\partial T'}{\partial y}(y=0) \sim \frac{\lambda}{i\Omega} \varepsilon e^{i\Omega t} \qquad (13)$$

which shows a phase lag of 90 degrees and an amplitude decrease as the inverse of frequency.

Now when compressibility is accounted for, the dominant terms in boundary layer energy equation (written in terms of thermal enthalpy) are

$$\rho \frac{\partial h'}{\partial t} - \frac{\partial p}{\partial t} = \frac{\lambda}{C_p} \frac{\partial^2 h'}{\partial y^2} \qquad (14)$$

with $h' = (h' - h'_w)/(h'_o - h'_w)$, and $h'(y = 0) = 0$, $h'(y \to \infty) = 1$ (15)

The inertial and dissipative terms are balanced by the pressure variation rate (due to compressibility), all of which are expected to be of the same order of magnitude. Furthermore, Eq. (14) is formally analogous to the momentum equation, cf Eq. (7), derived earlier. Consequently the high frequency asymptote for the surface heat flux perturbation is,

$$\frac{\lambda}{C_p} \frac{\partial h'}{\partial y}(y=0) \sim \frac{\lambda}{C_p} \sqrt{i\Omega}\, \varepsilon e^{i\Omega t} \qquad (16)$$

which obtains a 45 degree phase lead, and its amplitude increases as square root of frequency. Thus the Reynolds analogy is obviously recovered at the high frequency limit, and compressibility considerations are shown to lead to distinct differences regarding nonsteady heat feedback to the surface.

4.3.4 Nonsteady Surface Heat Transfer: Calculated Results

The solutions by Illingworth[8], for laminar, compressible boundary layer flow at the high frequency regime, are utilized. These solutions originally contained several algebraic errors which were corrected in the course of this study[43]. The results correspond to fixed mean temperature ration, To/Tw, and impervious walls; the general case of travelling waves was treated.

Combustion within the Stokes layer and mass injection at the wall (both capable of dynamic respone to the attendant perturbations) are expected to appreciably influence the results, as noted by Flandro[33]. However, in a linearized sense, the calculations herein serve to indicate the phase and amplitude of the "open control loop" surface heat transfer perturbations.

The perturbed outer quantities are as follows:

$$U = U_0(1+E), \quad p = p_0(1+\gamma M E), \quad T = T_0[1+(\gamma-1)ME] \qquad (17)$$

where $E = \varepsilon \exp[i\Omega t - iM\,S]$ . . . . (18)

M is the free stream Mach number U_0/a_0, and $S = x/U_0$; E denotes a perturbation wave travelling in the positive x direction. The nonsteady surface heat transfer component derived by Illingwoth[8] is,

$$q'_w = q_w(t)/q_w(ss) - 1 = (C + iD)E \qquad (19)$$

where the real and imaginary parts of the complex factor are

$$C \equiv M[g_0(\theta)\sqrt{S} + g_1(\theta) + g_2(\theta)/\sqrt{S}] \qquad (20)$$

$$D \equiv b_3(\theta)/S + M[g_0(\theta)\sqrt{S} - g_2(\theta)/\sqrt{S} + g_3(\theta)/S] \qquad (21)$$

and $\theta = T_w/T_0$.

The coefficients are (after correction)

$$g_0 = -0.8118\,\theta/(\theta-1) \qquad (22.a)$$

$$g_1 = -0.5410\,\theta \qquad (22.b)$$

$$g_2 = 0.2464(\theta-1) \qquad (22.c)$$

$$g_3 = (0.3946\,\theta - 0.4227)\,\theta/(\theta-1) \qquad (22.d)$$

$$b_3 = -0.3946\,\theta \qquad (22.e)$$

The phase and amplitude relative to the external velocity perturbation are, respectively,

$$Amp = (C^2 + D^2)^{1/2} \qquad (23)$$

$$\phi = tg^{-1}(D/C) \qquad (24)$$

These parameters were plotted against Strouhal number, S, in Figs. 4 and 5. The results clearly demonstrate that Illingworth's[8] solutions (after correction), regarding high frequency wall heat transfer, recover the asymptote derived herein. Mach number of the outer mean flow, M, serves as a parameter in these calculations, and the mean temperature ratio, $T_w/T_0 = 0.2$ is typical of combustion in a solid propellant motor. The effect of M is shown to increase the amplitude at a given frequency parameter; more pronounced is its effect upon phase, Fig. 5: at higher M, the phase change from negative (-90 degrees) to positive values is affected at lower S, and is steeper. Note that the limit when M = 0 is identical to the compressible asymptote derived by Lighthill[2], cf Eqs. (20), (21), and (23), (24): the ratio D/C tends to minus infinity (and hence the phase of - 90 degrees), while the amplitude decreases like 1/S as the frequency is increased. However, small values of the Mach number already show significant departure from this behavior.

An approximate calculation was also made to show the results in terms of actual frequencies depicted in Figs. 6 and 7. The ratio x/U was approximated as for a porous cylindrical tube with uniform mass injection,

$$x/U \doteq \rho d/4 m_o(p) = f(p,d) \qquad (25)$$

where $m_0(p)$ is the injection mass flux (taken from nitramine data), d = 10 cm is the port diameter, and U in this manner represents the mean mass flux in the tube. The density was calculated by assuming T_0 = 3000K and a mean molecular weight of 25 g/mol. The amplitude and phase angle are plotted against frequency [which can now be calculated for any given S from Eq. (25)], with fixed Mach number, and pressure as parameter. The same trends are recovered as with the effect of M. These calculations are admittedly quite crude, but still give an idea of the frequency domain of interest in practical solid propellant motors, at rocket operating pressures.

4.4 Conclusions

In summary, the foregoing approximate calculations indicate that nonsteady wall heat transfer due to viscous/acoustic coupling is strongly influenced by compressibility, even at low coreflow Mach numbers. Its phase relative to the attendant velocity perturbation is from -90 to +45 degrees and its amplitude tends to increase with frequency. Both Mach number and pressure seem to bring about higher amplitudes and more rapid transition from phase lag to phase advance. The minimum obtained in the amplitude vs. S curves, Figs. 4 and 6, indicate that a "threshold" effect may be operative.

Both phase and amplitude of the nonsteady wall heat transfer obtained, as well as the range of frequencies indicate that viscous/acoustic effects may couple with other processes,(e.g., combustion within the Stokes layer, thermal relaxation in the condensed phase) and modify the overall dynamics. In this respect, the consideration of a "thin, quasi-steady gaseous combustion zone" near the propellant surface, is therefore physically questionable.

It should be emphasized that the effects of viscous/acoustic heat transfer, and combustion within the Stokes layer, can not be simply superimposed, as noted by Flandro[33]. Different levels within the boundary layer obtain different phase and amplitude of nonsteady heat transfer, as explained previously, and particularly near resonant conditions. When finite-rate chemical knietics are considered in the combustion zone (even if it is confined in the Stokes layer), the overall gaseous flame structure <u>at each instant</u> would not resemble the associated steady state flame at the given instantaneous conditions. This means that the resultant (combined) heat transfer to the surface would depend <u>nonlinearly</u> upon its contributing effects, viscous/acoustic and combustion. The result of such coupling is that the burning rate at each instant is a function of both state variables (pressure) and fluid dynamic variables of the <u>mean</u> coreflow (as shown herein). This would unfortunately preclude construction of intrinsic "acoustic admittance" functions, which depend on properties of the propellant alone.

The foregoing literature survey and preliminary analysis strongly indicate that the mechanism of viscous/acoustic interaction is quite powerful, obtaining dynamic interaction over a broad range of phase angles and frequencies. It definitely warrants additional analytical study (incorporating finite-rate chemical kinetics of combustion in the Stokes layer and mass injection), and experiments (cold flow and combustion), within the framework of velocity-coupled instability.

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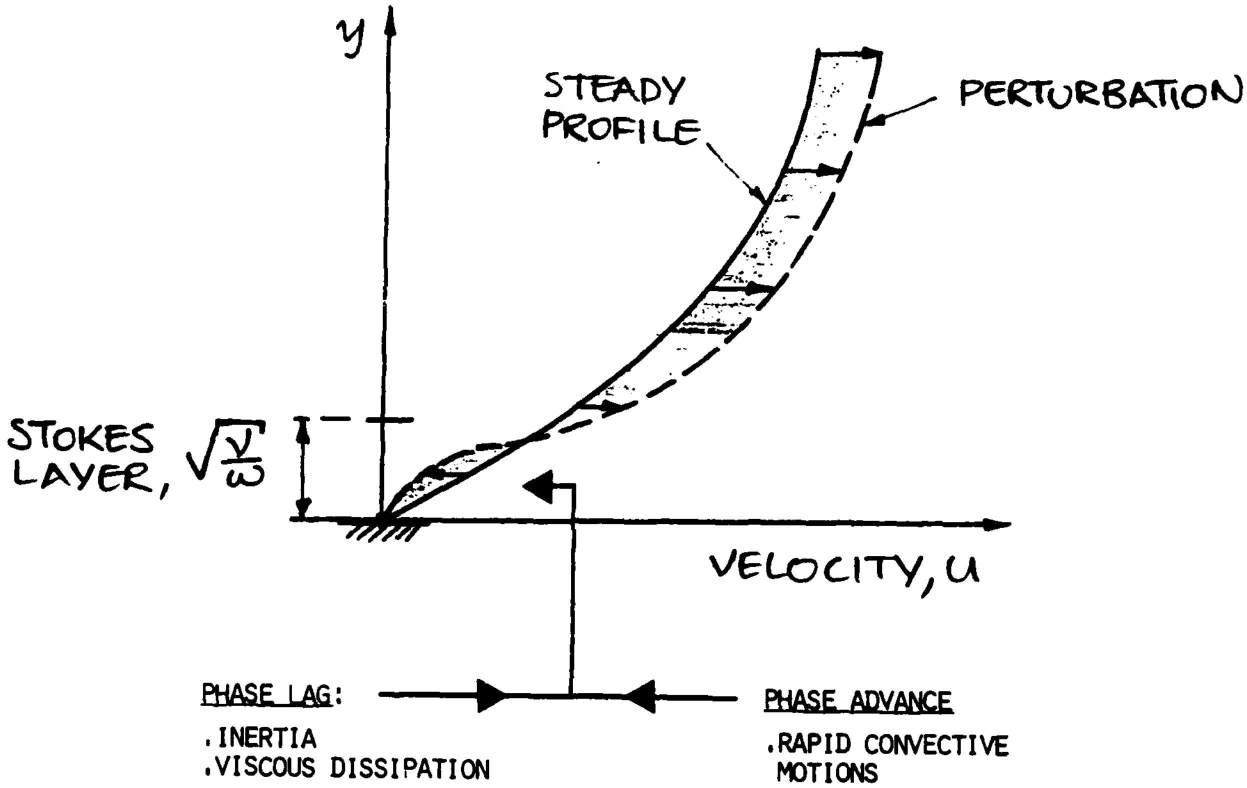

Fig. 1 Schematic representation of viscous/acoustic coupling for perturbed velocity in the boundary layer. Effects leading to phase lag/advance indicated (phase lead dominates at high frequencies). The amplitude in the Stokes layer may become larger than that due to the attendant perturbation, (and independent of the mean flow) under resonant conditions.

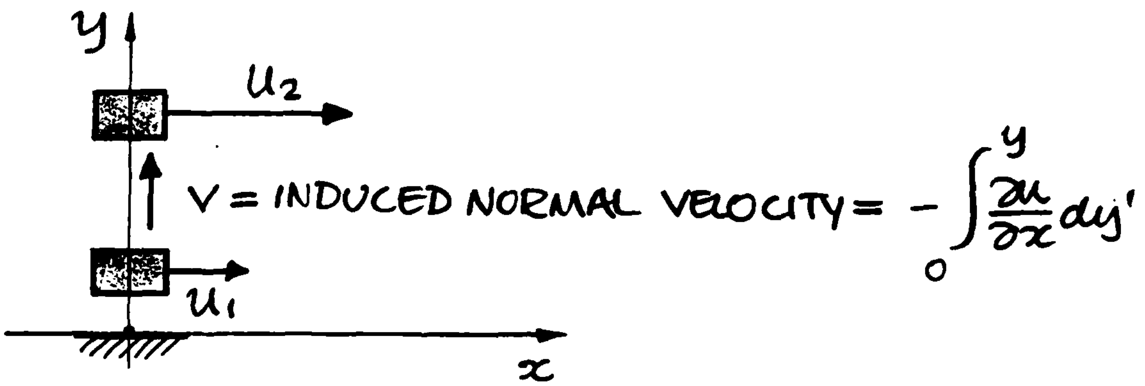

Fig. 2 Schematic diagram for explanation of the steady (DC) acoustic streaming effect due to acoustic/viscous coupling. When the induced normal velocity perturbation is not out of phase with u, net x-momentum flux is created over a period of oscillation. The streaming is a second-order effect, can penetrate to large distance from the wall, independent of viscosity, and depends strongly on the perturbation amplitude.

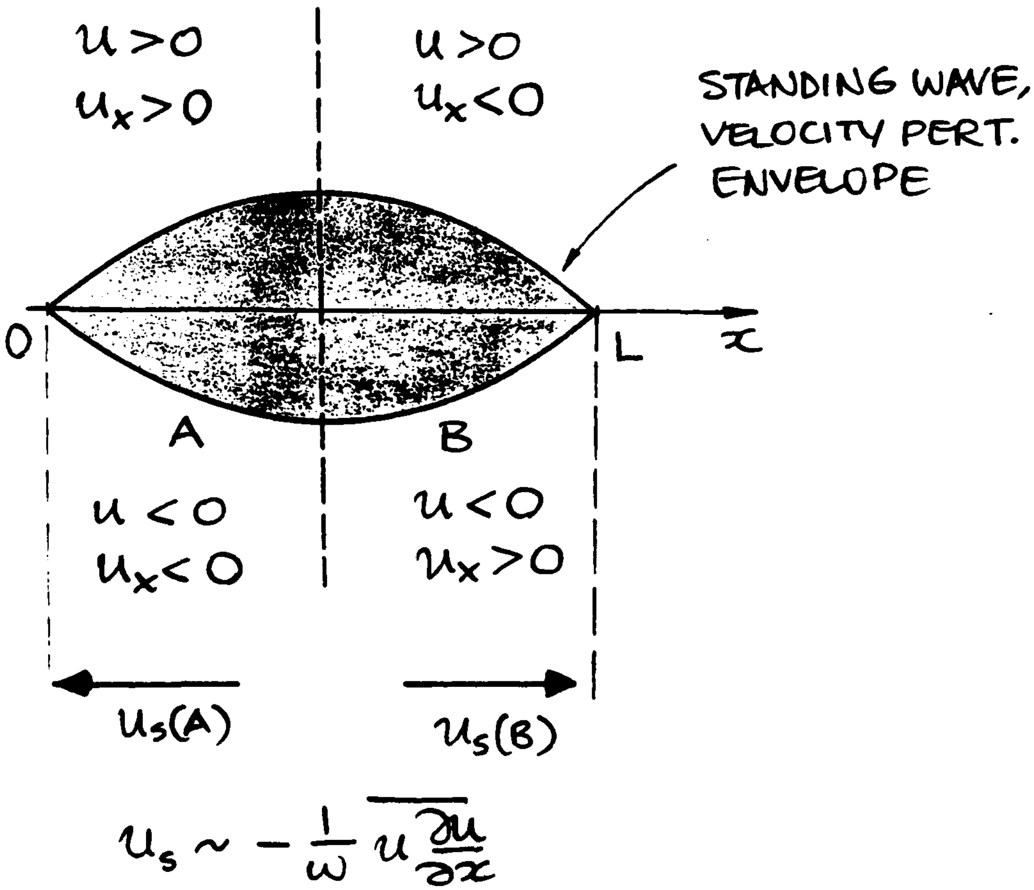

Fig. 3 Acoustic streaming shown conceptually for a standing acoustic mode. The direction of the steady streaming is demonstrated to point toward velocity nodes, and is expected to become weaker at high frequencies, yet steep-fronted travelling waves would enhance this effect.

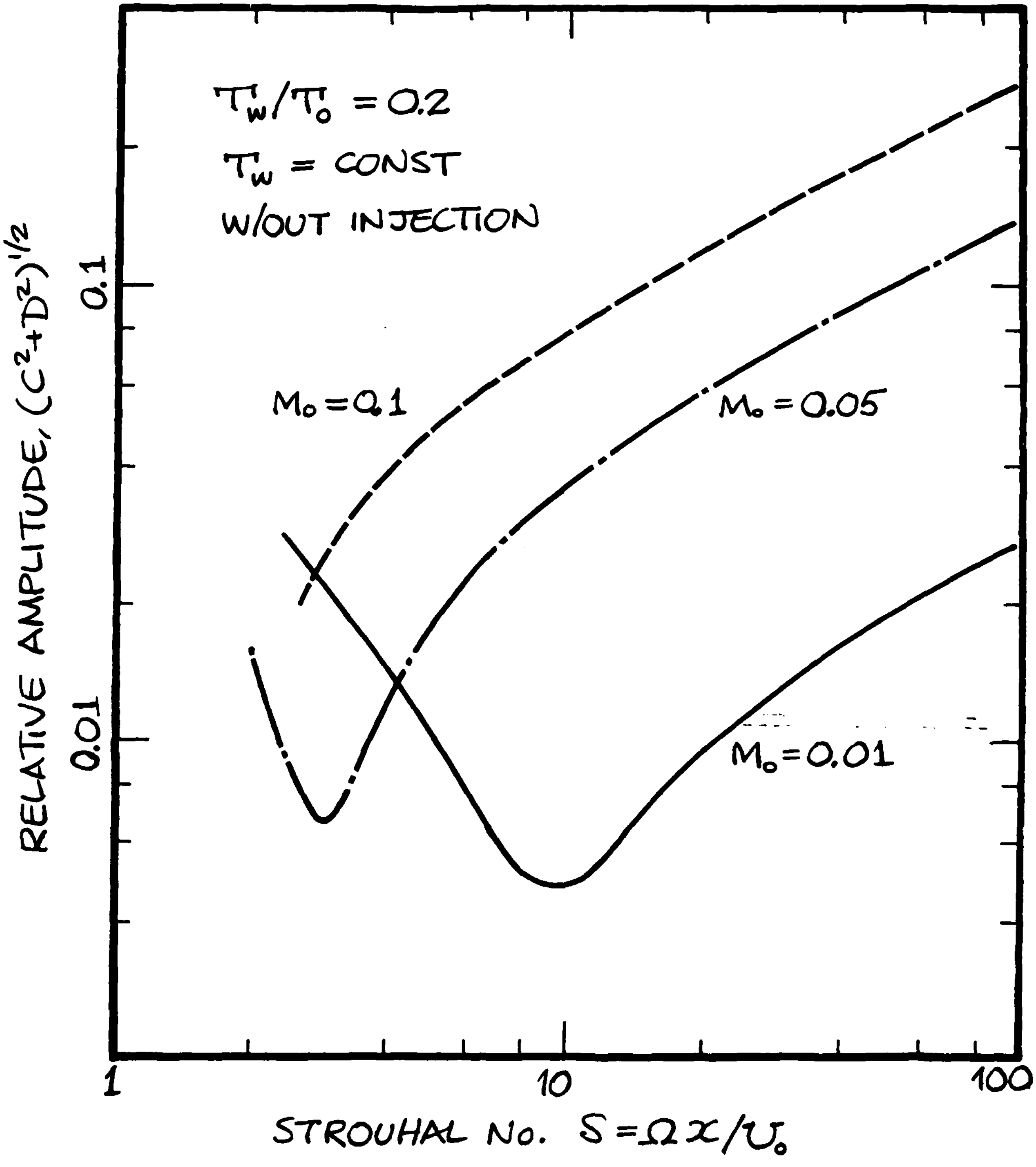

Fig. 4 Amplitude of nonsteady wall heat transfer (relative to velocity perturbation) is shown vs Strouhal number, with mean outer flow Mach number as parameter. Calculations follow the compressible analysis by Illingworth[8].

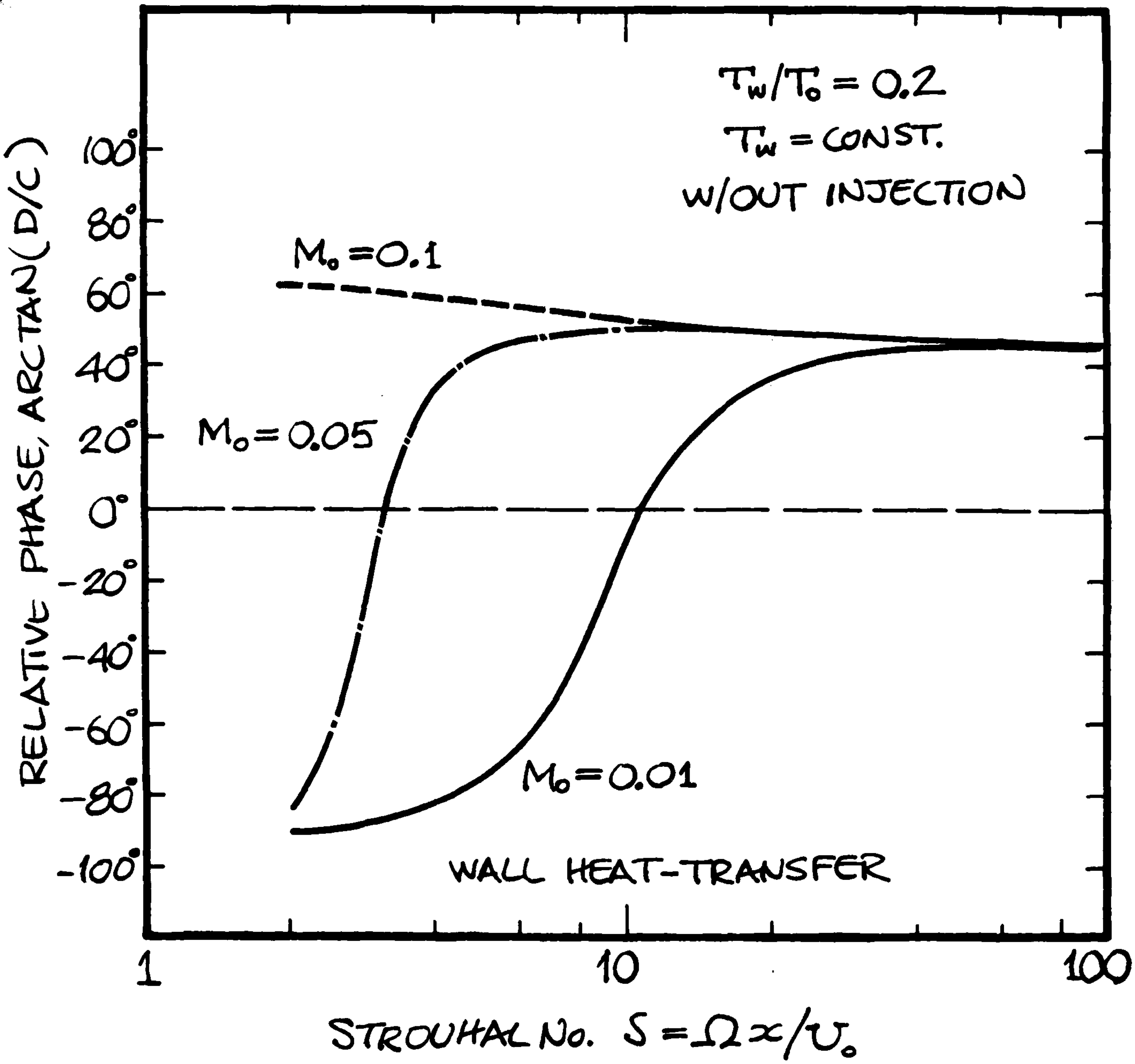

Fig. 5 Calculated phase angle of nonsteady wall heat transfer (relative to velocity perturbation) vs Strouhal number, with mean outer flow M as parameter. Note strong effect of M in the range S = 2 - 4, and the asymptotic recovery of the 45 degree phase lead.

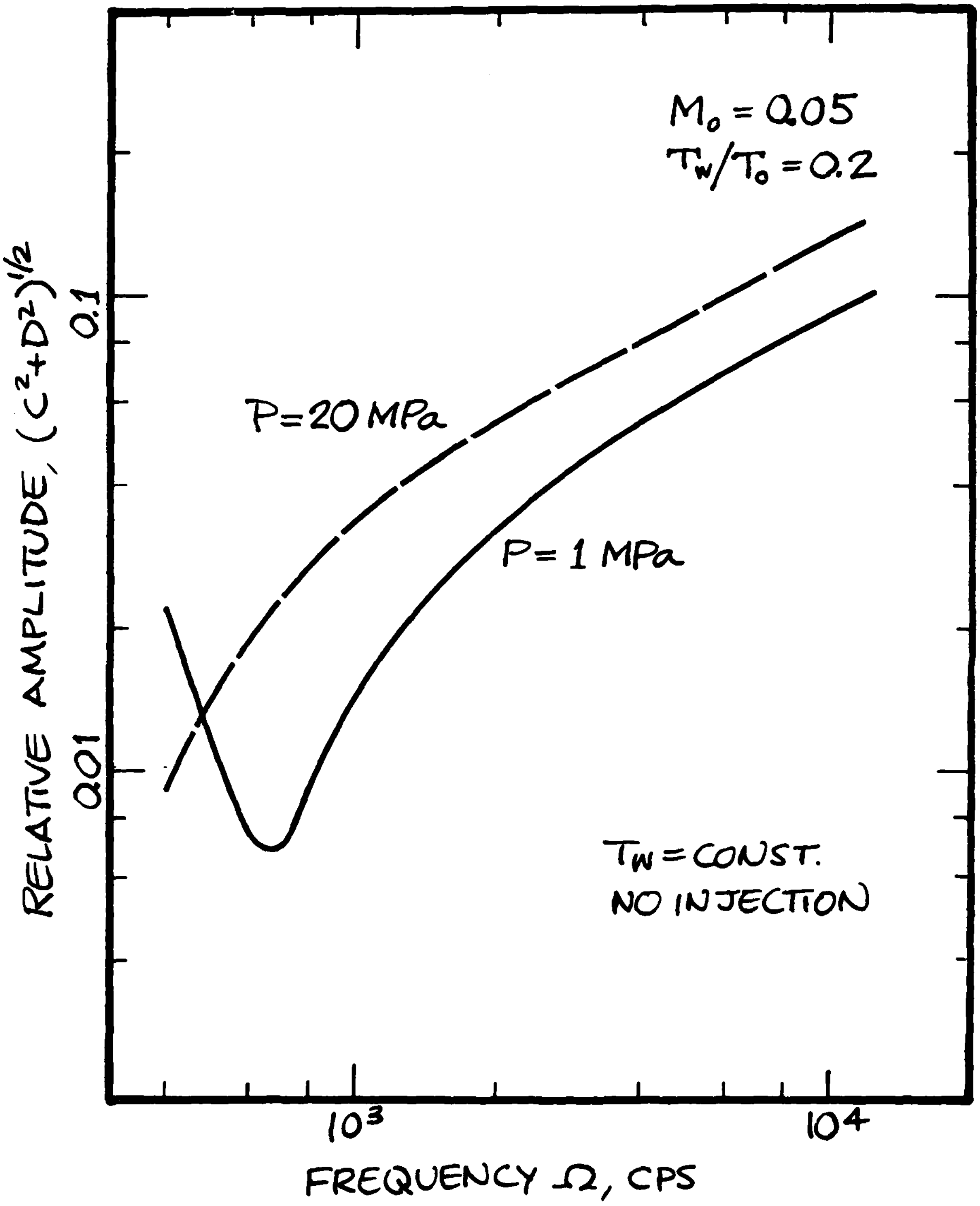

Fig. 6 Amplitude of nonsteady wall heat transfer (relative to velocity perturbation) vs frequency. Approximate calculation for fixed $x/u = f(p)$ allows conversion from Strouhal number to frequency, and the effect of pressure is shown, similar to that of M in Fig. 4.

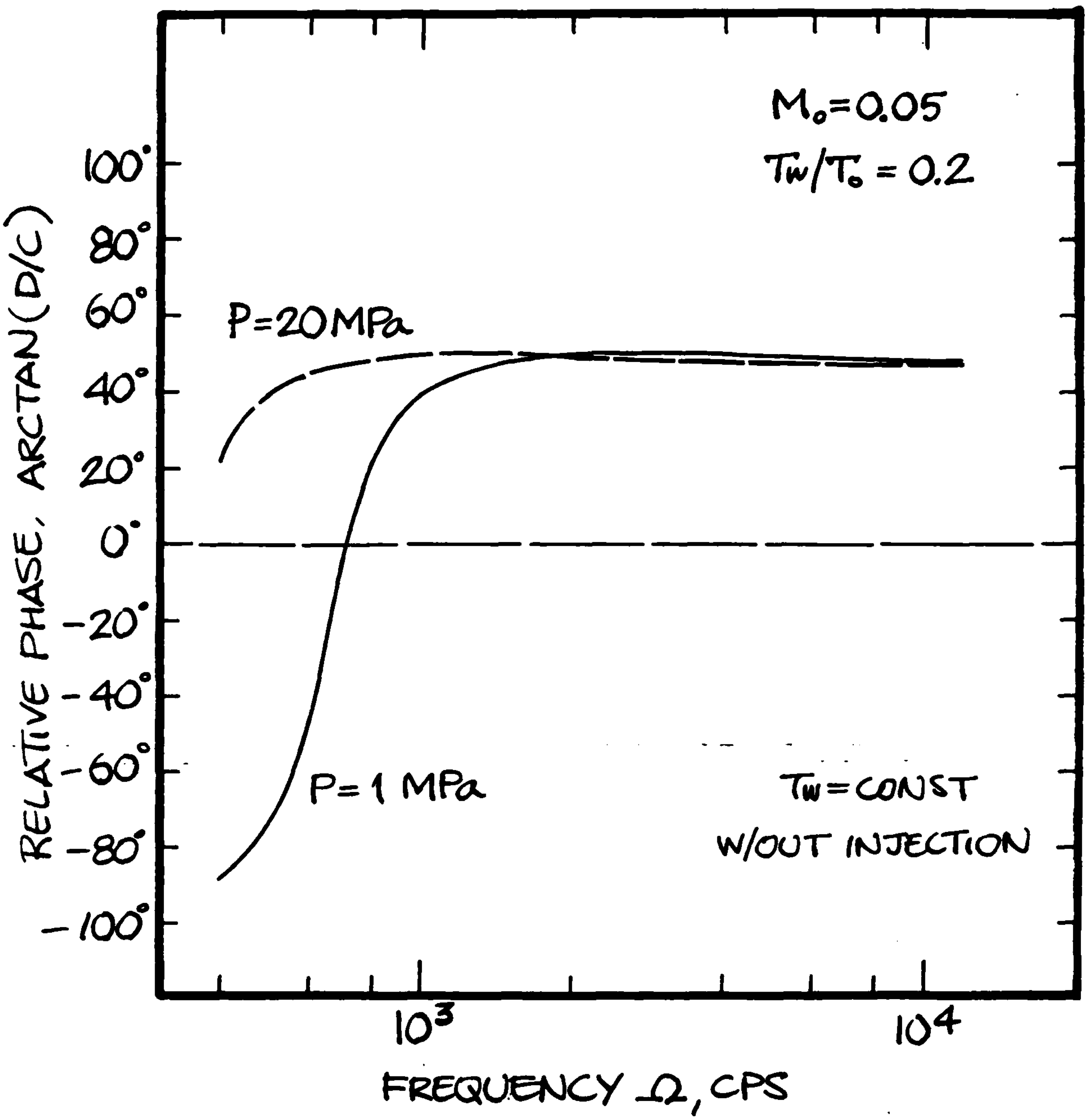

Fig. 7 Phase angle of nonsteady wall heat transfer (relative to velocity perturbation) vs frequency. The same approximation discussed in Fig. 6 used; again, the effect of pressure is shown similar to that of M in Fig. 5.

5. Suggested Measurements and Comprehensive Analysis

The suggested measurements regarding the ongoing cold-flow experiments at UTC/CSD are based entirely on the findings of preliminary analysis herein, regarding viscous/acoustic interaction.

The foregoing analysis in Section 4 indicates that the following parametric ranges are important to discern visous/acoustic interation:

(1) Frequencies in the range 10^3 - 10^4 cps
(2) Mean flow Mach numbers above 0.1
(3) pressures above 1 MPa (150 psia)

Of these, items (1) and (2) could be readily obtained in the cold-flow device setup at UTC/CSD. Pressures between 40-100 psia are achievable at present, owing to the well known limitations of creating a coreflow through porous-wall injection. This, however, may be readily overcome by the relatively large coreflow Mach numbers obtainable, over 0.6; recall that the effects of p and M were show quite similar regarding flows without chemical reactions.

Ideally, one would desire both velocity and heat flux measurements from the coreflow, through the Stokes layer and down to the wall. Unfortunately, this can not be fully realized, since the Stokes layer thickness at a frequency of 1000 cps (cold) would be of the order of 100 micrometers; present velocity probing (hot wire anemometry) may get no closer than about this distance from the surface.

Thus, detailed probing for velocity amplitudes and phases within the Stokes layer are precluded from the time being, although they are (1) highly desirable even with cold flow, and (2) would be highly recommended for isolated experiments with combustion, with high frequency boundary layer perturbation.

Data regarding nonsteady heat transfer to the surface can still be obtained, by use of the high frequency response heat flux film gage. It is suggested to cross-correlate the velocity data outside of the stokes layer with the nonsteady heat transfer data (via fast Fourier transform algorithm) and discern phase differences; entrainment effects and dominant frequencies. To obtain a higher resolution, an electrically (or radiatively) heated porous segment of the pipe is recommended, which may be configured at various axial positions along the port (or just the aft end), to traverse a range of mean flow Mach numbers.

The actual interaction between the present analysis and the cold flow experiment would come about in the following phase of the study, when a comprehensive 2-D (axisymmetric), nonsteady numerical algorithm is developed. The first order of analysis would be the pressure, velocity and heat transfer data obtained from the UTC/SCD cold flow experiments. It should be emphasized that due to the importance of compressibility effects and heat transfer, the formulation will incorporate all three conservation equations for mass, momemtum and energy, and will enable sufficient resolution in the Stokes layer region.