MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A
SCATTERING COEFFICIENT ESTIMATION: AN
EXAMINATION OF THE NARROW-BEAM
APPROXIMATION

by

Y.S. Kim
R.K. Moore
R.G. Onstott

Remote Sensing Laboratory
Center for Research, Inc.
The University of Kansas
Lawrence, Kansas 66045

RSL Technical Report
RSL TR 331-23

August 1982

Supported by:

OFFICE OF NAVAL RESEARCH
Department of the Navy
800 N. Quincy Street
Arlington, Virginia 22217

Contract N00014-76-C-1105

DISTRIBUTION STATEMENT A
Approved for public release:
Distribution Unlimited
ABSTRACT

One quick and easy way of estimating scattering coefficient is to assume that the antenna beam is confined in a finite cone with constant gain inside and the range to the antenna is constant for all the cells inside the illuminated area. Also the variation of the $\sigma^0$ is assumed to be small enough inside the illuminated area so one can estimate the $\sigma^0$ with only the power return measurements.

The above may not be true depending on the antenna gain shapes or the target characteristic. In this report, the so-called narrow-beam approximation is examined using a little more realistic antenna gain functions and range variation and also a specific $\sigma^0$ variation function. The numerical integration shows that the narrow beam approximation can give as much as 3.6 dB error for normal incidence to the target with rapidly varying $\sigma^0$ when the beamwidths are $\frac{\lambda}{\lambda}$ wide. For the incidence angles of $10\frac{\lambda}{\lambda}$ or larger, the error is in the order of a dB or less depending on the beamwidths and target characteristics.
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>11</td>
</tr>
<tr>
<td>LIST OF FIGURES AND TABLES.</td>
<td>iv</td>
</tr>
<tr>
<td>1.0 BACKGROUND</td>
<td>1</td>
</tr>
<tr>
<td>2.0 NARROW-BEAM APPROXIMATION.</td>
<td>2</td>
</tr>
<tr>
<td>3.0 EFFECT OF ANTENNA GAIN AND RANGE VARIATION INSIDE THE INTEGRAL</td>
<td>6</td>
</tr>
<tr>
<td>3.1 Gaussian Antenna Gain</td>
<td>6</td>
</tr>
<tr>
<td>3.2 ((\sin x/x)^4) Antenna Gain</td>
<td>8</td>
</tr>
<tr>
<td>3.3 Range Variation and the Effect of the Bandpass Filter</td>
<td>10</td>
</tr>
<tr>
<td>3.4 Integration Limits</td>
<td>13</td>
</tr>
<tr>
<td>3.5 Results</td>
<td>14</td>
</tr>
<tr>
<td>4.0 EFFECT OF (\sigma^0) VARIATION INSIDE THE INTEGRAL</td>
<td>15</td>
</tr>
<tr>
<td>4.1 Iteration</td>
<td>22</td>
</tr>
<tr>
<td>5.0 SUMMARY</td>
<td>24</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>25</td>
</tr>
<tr>
<td>APPENDIX: Program Listing</td>
<td>26</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Effective Beamwidth and Half-Power Beamwidth.</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Effective Illuminated Area.</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>Coordinate System for the Antenna Gain Functions.</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{\sin x}{x} ) Antenna Gain Projected on the Ground.</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>Bandpass Filter Function.</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>Integration Limits.</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>Local Incidence Angle</td>
<td>18</td>
</tr>
</tbody>
</table>

LIST OF TABLES

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ERROR1</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>ERROR2</td>
<td>21</td>
</tr>
<tr>
<td>3</td>
<td>( \circ ) Data, Original and Up to 4th Refined</td>
<td>23</td>
</tr>
</tbody>
</table>
1.0 BACKGROUND

Accurate estimation of scattering coefficient for an area-extensive target using the average return power is not a simple task because it involves an integral equation. Hence, it is usually simplified by using several approximations and assumptions. The purpose of this report is to investigate the validity of these commonly used approximations and to evaluate the possible error when these simplifying assumptions are used to calculate the scattering coefficient, for example antenna beamwidths and target scattering coefficient angular variations.

Axline [1] developed a matrix inversion technique to invert the integral equation for various cases of antenna gain shapes and $\sigma^0$ variation functions. This technique requires power return measured as a function of incidence angle at a small angular interval and is not readily applicable when this is not true.

Stiles et al. [2] described an empirical procedure to estimate the error, when the narrow-beam approximation is used for small incidence angles, by initially hypothesizing the theoretical $\sigma^0(\theta)$ curve.

Moore [3] suggested an iteration scheme to estimate the scattering coefficient by using the constant-$\sigma^0$ approximation to get a first estimate of the $\sigma^0$ variation function and then continuously refining that function to get better estimate of $\sigma^0$. This method is used in this report, and the integral is evaluated numerically using the real system parameters of the HELOSCAT III FM-CW scatterometer and a data set taken from sea ice.
2.0 NARROW-BEAM APPROXIMATION

Consider the radar equation

\[ P_r = \int \int_{\text{area illuminated}} \frac{P_t G^2 \lambda^2 \sigma^0}{(4\pi)^3 R^4} \, dA \]  

where identical transmit- and receive-antenna gains are used.

The first problem here is how to define the illuminated area. Actually the antennae have lobes everywhere, so the area has to be from \(-\infty\) to \(+\infty\) in the plane of illumination. Usual practice for this is to assume that the antenna beam is confined in a finite cone within which the maximum value of gain applies, with zero gain elsewhere. The effective beamwidth, \(\theta_{\text{eff}}\), is defined in such a way that the pattern solid angles for the two-way real antenna beam and the cone-shaped beam are the same (see Figure 1). Therefore,

\[ \int \int_{4\pi} G^2(\theta, \phi) \, d\Omega = \int \int_{0}^{\pi} G^2(\theta, \phi) \sin \theta d\theta d\phi \]

\[ \Delta = \int \int_{0}^{2\pi} \left( \frac{\theta_{\text{eff}}}{2} \right)^2 G_0^2 \sin \theta d\theta d\phi \]  

where \(G_0^2\) is the maximum gain for the two-way pattern. This is not a simple task unless \(G^2(\theta, \phi)\) is completely known. If the gain function is assumed to be a Gaussian and if it is \(\phi\)-independent, then

\[ 2\pi \int_{0}^{\pi} G_0^2 e^{-2.773g^2/\theta_{3dB}^2} \sin \theta d\theta \Delta = 2\pi \int_{0}^{\theta_{\text{eff}}/2} G_0^2 \sin \theta d\theta\]  

\[ 2\pi \int_{0}^{\pi} G_0^2 e^{-2.773g^2/\theta_{3dB}^2} \sin \theta d\theta = 2\pi \int_{0}^{\theta_{\text{eff}}/2} G_0^2 \sin \theta d\theta \]
Figure 1: Effective Beamwidth and Half-Power Beamwidth
which reduces to

\[ \beta_{\text{eff}} = 2 \cos^{-1}(1 - \int_0^{\frac{\pi}{2}} e^{-2.7738^2/3\text{dB}^2 \sin \theta d\theta}) \]  

(4)

This equation can be solved numerically and for the \( \beta_{3\text{dB}} \) of up to 15\(^\circ\), the following relation can be used.

\[ \beta_{\text{eff}} = 1.2 \beta_{3\text{dB}} \]  

(5)

Antenna beamwidths are usually measured in azimuth and elevation planes. When the measured patterns are considerably different from each other, separate effective beamwidths are defined.

\[ \beta_{a,\text{eff}} = 1.2 \beta_{a,3\text{dB}} \]  

(6)

\[ \beta_{e,\text{eff}} = 1.2 \beta_{e,3\text{dB}} \]  

(7)

The illuminated area is then the ground projection (Figure 2) of the conical beam and the area of the ellipse is approximately

\[ A_i = \frac{\pi h^2}{2 \cos^2 \theta_p} [\tan(\theta_p + \beta_{e,\text{eff}}) - \tan(\theta_p - \beta_{e,\text{eff}})] \tan(\beta_{a,\text{eff}}) \]  

(8)

If the beamwidths are narrow enough, \( R^4 \) inside the integral in Eq. (1) can be treated to be constant; and if \( \sigma^0 \) varies slowly inside the illuminated area given by Eq. (8), one can remove the \( \sigma^0 \) from the integral and solve for \( \sigma^0 \).

\[ \sigma^0 = \frac{(4\pi)^3}{\lambda^2} \left( \frac{P_t}{G_o} \right)^4 \frac{R^4}{A_i} \]  

(9)

This is the normal practice. The variations in transmitted power, \( P_t \) and \( G_o^2 \) are found using a delay line calibration scheme and standard point-target measurements, respectively. However, these approximations of constant-gain,
Figure 2: Effective Illuminated Area
finite-beam, constant-range, constant-\( \sigma^0 \) inside the integral may not be true for a wide-beam, an antenna pattern with high sidelobes, near-nadir or near grazing angles of incidence.

3.0 EFFECT OF ANTENNA GAIN AND RANGE VARIATION INSIDE THE INTEGRAL

As a first attempt, the effects of a more realistic antenna gain and \( R^4 \) inside the integral are evaluated. The effect of \( \sigma^0 \) variation across the illuminated area will be treated in Section 4.0.

If a full set of antenna patterns in all directions is available, these numerical values can be entered as a gain function. But this is not usually the situation and some kind of functional approximation of the antenna gain is inevitable.

3.1 Gaussian Antenna Gain

If the antenna gain can be represented as a Gaussian, then

\[
G^2 = G^2 \left( \theta_a, \phi_a \right) = G_o^2 \exp \left(-2.773 \left( \theta_{a, \text{dB}} - \theta_{\text{p}} \right)^2 / \beta_{e, \text{dB}}^2 \right) \exp \left(-2.773 \phi_{a, \text{dB}}^2 / \beta_{a, \text{dB}}^2 \right)
\]

where:

- \( \theta_{\text{p}} \) = antenna boresight pointing angle off vertical (in x-z plane)
- \( \theta_a \) = angle off vertical in elevation plane for observation direction
- \( \phi_a \) = angle off x-z plane for observation direction
- \( \beta_{e, \text{dB}} \) = two-way 3 dB beamwidth in elevation plane
- \( \beta_{a, \text{dB}} \) = two-way 3 dB beamwidth in azimuth plane

For point \( P(x, y) \) on the ground (see Figure 3),

\[
\theta_a = \tan^{-1} \left( x / h \right)
\]
Figure 3: Coordinate System for the Antenna Gain Functions
\[ \phi_a = \tan^{-1}(y/\sqrt{x^2+h^2}) \]  

(12)

This coordinate system is not a spherical coordinate system, but this is used because the antenna patterns are available in these azimuth and elevation planes.

### 3.2 \((\sin x/x)^4\) Antenna Gain

One-way antenna gain can also be represented by \((\sin x/x)^2\). In terms of one-way 3 dB beamwidth, \(\beta_{3\text{dB}}^e\),

\[ G(\theta) = G_0 \frac{\sin(2.78310/\beta_{3\text{dB}}^e)}{2.78310/\beta_{3\text{dB}}^e} \]

(13)

Therefore, in the coordinate system shown in Figure 3, two-way antenna gain can be represented as

\[ G^2(\theta_a,\phi_a) = G_0^2 \left( \frac{\sin \theta_a}{\chi_\theta} \right) \left( \frac{\sin \phi_a}{\chi_\phi} \right)^4 \]

(14)

where:

\[ \chi_\theta = 2.7831 (\theta_a - \theta_p)/\theta_{a,3\text{dB}} \]

\[ \chi_\phi = 2.7831 \phi_a/\phi_{a,3\text{dB}} \]

\(\beta_{e,3\text{dB}}\) = one-way 3 dB beamwidth in elevation plane

\(\beta_{a,3\text{dB}}\) = one-way 3 dB beamwidth in azimuth plane

\(\theta_a,\phi_a\) = given in Eqs. (11) and (12)

This two-way gain function given by Eq. (14) has many sidelobes and nulls, with the highest sidelobes 26.5 dB down from mainlobe (see Figure 4). The location of the first sidelobes will be about 1.6 \(\beta_{3\text{dB}}^e\) away from the center of the mainlobe.
Figure 4: \( \left( \frac{\sin x}{x} \right)^4 \) Antenna Gain Projected on the Ground
3.3 Range Variation and the Effect of the Bandpass Filter

The range $R$ from the antenna to the infinitesimal area element $dA$ on the ground is

$$R = \sqrt{x^2 + y^2 + h^2}$$  \hspace{1cm} (15)

In the actual HELOSCAT III system (an FM radar) the range to the target is limited by the bandpass filter in the final stage. Each area element on the ground has different range to the antenna and thereby gives a different IF frequency. Therefore a bandpass filter with finite bandwidth would limit the range from which the radar receives the scattered field. In the narrow-beam approximation, the bandpass filter plays a role only when the effective beamwidth is very wide and the incidence angle is large enough. In such a case, the filter cuts the area of the ellipse from which $\sigma^0$ is calculated.

The transfer function of the bandpass filter shown in Figure 5 can be represented as a function of range as follows.

$$f(R) = \begin{cases} 
1 & \text{when } R_o \left(1 - B^\prime/2f_o\right) \leq R < R_o \left(1 + B^\prime/2f_o\right) \\
\frac{165}{4} \left(\frac{R}{R_o}\right)^2 & \text{when } R < R_o \left(1 - B^\prime/2f_o\right) \\
-30 \left(\frac{R}{R_o}\right)^2 & \text{when } R > R_o \left(1 + B^\prime/2f_o\right) \\
\end{cases}$$  \hspace{1cm} (16)

where:

- $R_o$ is the distance to the center of the illuminated area
- $B^\prime$ is the bandwidth of the filter (Figure 5),
- $f_o$ is the center frequency of the filter.
Theoretically in a FM-CW radar, the range $R$ to the target and the IF frequency output ($F_{\text{if}}$) of the mixer have the following relationship.

$$F_{\text{if}} = \frac{4R\Delta F F_m}{c}$$  \hspace{1cm} (17)

where:

- $\Delta F = \text{RF frequency sweep width}$
- $F_m = \text{modulating frequency}$
- $c = \text{velocity of light}$

A sample of the transmitted waveform long before it goes out of the antenna is usually used as the local oscillator signal to be mixed with the received signal. Hence the actual IF frequency is different from what one expects from Eq. (17), especially when long antenna cables are used. The discrepancy in range should be obtained experimentally as a system characteristic. When this additional range, $R_c$, is not considered to be negligible, Eq. (17) and the bandpass filter function given by Eq. (16) should be modified accordingly (see Figure 5).

$$F_{\text{if}} = \frac{4(R+R_c)\Delta F F_m}{c}$$  \hspace{1cm} (18)

$$f(R) = 1, \quad \text{when } R_0 (1-B''/2f_o)-(B''/2f_o)R_c < R \leq (1+B''/2f_o)R_o + (B''/2f_o)R_c$$

$$= \frac{145}{4} \left( \frac{R+R_c}{R_0+R_c} \right) - (1-B''/2f_o), \quad \text{when } R \leq (1-B''/2f_o)R_o - (B''/2f_o)R_c$$

$$= 10^{-30} \left( \frac{R+R_c}{R_0+R_c} \right) - (1+B''/2f_o), \quad \text{when } R \geq (1+B''/2f_o)R_o + (B''/2f_o)R_c$$  \hspace{1cm} (19)

Examining Eq. (19), one can see that having some additional range, $R_c$, has effectively widened the bandwidth of the filter.
Figure 5: Bandpass Filter Function
3.4 Integration Limits

In this section, $\sigma^0$ will be assumed to be constant across the integration limits and evaluated using the following equation.

$$\sigma^0 = \frac{(4\pi)^3}{\lambda^2} \frac{P}{P_t} \frac{1}{\int_{R_0}^{R} \frac{G^2(\theta_0, \phi_0) f(R)}{R^4} dA}$$

(20)

In view of the narrow beam approximation given by Eq. (9), one can rewrite this equation as

$$\sigma^0 = \frac{(4\pi)^3}{\lambda^2} \frac{P}{P_t} \frac{1}{\int_{R_0}^{R} \frac{G^2(\theta_0, \phi_0) f(R)}{G_0^2 (R/R_0)^4} dA}$$

(21)

where $G^2(\theta_0, \phi_0) = G_0^2 g^2(\theta_0, \phi_0)$.

The integral can be treated as a weighted area [4] compared to the illuminated area described in Section 2.0.

$$A_w \Delta = \int_{R_0}^{R} \frac{g^2(\theta_0, \phi_0) f(R)}{(R/R_0)^4} dA$$

(22)

The integration is done using a rectangular coordinate system. Therefore the infinitesimal area $dA = dx dy$ and the integration limits should be the whole ground plane and the filter function will effectively set the limit.

To simplify the numerical integration, the integration limits were set to be the points where the antenna gain function reduces to small enough numbers (about -30 dB) for the Gaussian gain, and for the $(\sin x/x)^4$ gain function the limits were up to the first sidelobes. In this case the integral is also evaluated for the mainlobe only to see the effect of sidelobes. Figure 6 shows the integration limits (square area marked -30 dB on four sides).
together with the elliptical (shaded) illuminated area normally used for the narrow-beam approximation. Also, the equal-range circles which effectively limit the area by the filter function can be seen.

The weighted area now becomes

$$A_w = \int_{y^-}^{y^+} \int_{x^-}^{x^+} \frac{g^2[a(x,y),\phi(x,y)]f[R(x,y)]}{[R(x,y)/R_o]^4} \, dx \, dy \quad (23)$$

where $g^2, R$ and $f$ are given by Eqs. (10-12), (14), (15), (19) and

$$x^\pm = h \tan(\theta_p \pm 1.6 \beta_{e,3dB}) \text{ for Gaussian gain}$$

$$= h \tan(\theta_p \pm 3.2 \beta_{e,3dB}) \text{ for } (\sin x/x)^4, \text{ up to first sidelobes}$$

$$= h \tan(\theta_p \pm 1.6 \beta_{e,3dB}) \text{ for } (\sin x/x)^4, \text{ mainlobe only}$$

and

$$y^\pm = R_o \tan(\pm 1.6 \beta_{a,3dB}) \text{ for Gaussian gain}$$

$$= R_o \tan(\pm 3.2 \beta_{a,3dB}) \text{ for } (\sin x/x)^4, \text{ up to first sidelobes}$$

$$= R_o \tan(\pm 1.6 \beta_{a,3dB}) \text{ for } (\sin x/x)^4, \text{ mainlobe only}$$

and

$$R_o = h / \cos \theta_p$$

3.5 Results

In this section, the possible error when using constant range and constant gain across the effective beamwidth is defined as follows.

$$\text{ERROR1} = A_1 / A_w \quad (24)$$

where:
A_w is the weighted area given by Eq. (23) and
A_i is the area of the ellipse given by Eq. (8)
then
\[ \text{ERROR}_I \ (\text{dB}) = 10 \log(A_i / A_w) \] (25)
and this ERRORI (dB) should be added to the \( \sigma^0 \) (dB) calculated using a narrow-beam approximation.

The numerical integration was done using Gauss' formula [5] and a total of 1600 (40x40) points inside the integration limit shown in Figure 6. Because the number of points evaluated was fixed for all the incidence angles between 0° to 70° while the actual integration limits in x- and y-coordinate grew tremendously from 0° to 70°, the accuracy of the numerical integration tends to get worse at large incidence angles.

Table I is a summary of ERRORI for various antenna beamwidths and antenna gain functions. As can be seen in the table, the constant-range and constant-gain across the effective beamwidths are not bad approximations to use up to the pointing angle of 60°, except when the two-way beamwidths are as large as 9°, if the variations of \( \sigma^0 \) inside the integral can be neglected as assumed here.

4.0 **EFFECT OF \( \sigma^0 \) VARIATION INSIDE THE INTEGRAL**

Up to now, \( \sigma^0 \) was assumed to be constant within the limits of integration, so \( \sigma^0 \) was outside the integral. Actually, \( \sigma^0 \) itself is a function of the local incidence angle and therefore a function of x- and y-coordinates of the point evaluated (see Figure 7).
Figure 6: Integration Limits
<table>
<thead>
<tr>
<th>Pointing Angle $\theta_p(\degree)$</th>
<th>Two-Way 3dB Beamwidths</th>
<th>( \beta_e = 9.0\degree )</th>
<th>( \beta_e = 6.3\degree )</th>
<th>( \beta_e = 5.0\degree )</th>
<th>( \beta_e = 2.4\degree )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.055(dB)</td>
<td>0.025(dB)</td>
<td>0.017(dB)</td>
<td>0.004(dB)</td>
<td>( \beta_a = 9.0\degree )</td>
</tr>
<tr>
<td>10</td>
<td>0.057</td>
<td>0.025</td>
<td>0.018</td>
<td>0.004</td>
<td>( \beta_a = 5.7\degree )</td>
</tr>
<tr>
<td>20</td>
<td>0.060</td>
<td>0.027</td>
<td>0.019</td>
<td>0.004</td>
<td>( \beta_a = 5.0\degree )</td>
</tr>
<tr>
<td>30</td>
<td>0.069</td>
<td>0.031</td>
<td>0.021</td>
<td>0.005</td>
<td>( \beta_a = 2.4\degree )</td>
</tr>
<tr>
<td>40</td>
<td>0.095</td>
<td>0.038</td>
<td>0.026</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.208</td>
<td>0.063</td>
<td>0.035</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>0.591*</td>
<td>0.222</td>
<td>0.094</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>0.366*</td>
<td>0.616</td>
<td>0.588</td>
<td>0.033</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.059</td>
<td>0.029</td>
<td>0.022</td>
<td>0.008</td>
<td>( \frac{\sin x}{x} )</td>
</tr>
<tr>
<td>10</td>
<td>0.060</td>
<td>0.029</td>
<td>0.022</td>
<td>0.008</td>
<td>up to</td>
</tr>
<tr>
<td>20</td>
<td>0.067</td>
<td>0.031</td>
<td>0.023</td>
<td>0.008</td>
<td>40</td>
</tr>
<tr>
<td>30</td>
<td>0.076</td>
<td>0.037</td>
<td>0.026</td>
<td>0.009</td>
<td>First</td>
</tr>
<tr>
<td>40</td>
<td>0.094</td>
<td>0.046</td>
<td>0.033</td>
<td>0.010</td>
<td>sidelobes</td>
</tr>
<tr>
<td>50</td>
<td>0.228</td>
<td>0.070</td>
<td>0.044</td>
<td>0.012</td>
<td>60</td>
</tr>
<tr>
<td>60</td>
<td>0.583*</td>
<td>0.206</td>
<td>0.091</td>
<td>0.021</td>
<td>70</td>
</tr>
<tr>
<td>70</td>
<td>0.484*</td>
<td>0.484</td>
<td>0.528</td>
<td>0.042</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.079</td>
<td>0.049</td>
<td>0.043</td>
<td>0.030</td>
<td>( \frac{\sin x}{x} )</td>
</tr>
<tr>
<td>10</td>
<td>0.080</td>
<td>0.050</td>
<td>0.043</td>
<td>0.030</td>
<td>mainlobe</td>
</tr>
<tr>
<td>20</td>
<td>0.084</td>
<td>0.052</td>
<td>0.044</td>
<td>0.030</td>
<td>only</td>
</tr>
<tr>
<td>30</td>
<td>0.092</td>
<td>0.056</td>
<td>0.047</td>
<td>0.030</td>
<td>40</td>
</tr>
<tr>
<td>40</td>
<td>0.111</td>
<td>0.063</td>
<td>0.051</td>
<td>0.031</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.208</td>
<td>0.080</td>
<td>0.060</td>
<td>0.033</td>
<td>60</td>
</tr>
<tr>
<td>60</td>
<td>0.561*</td>
<td>0.213</td>
<td>0.102</td>
<td>0.038</td>
<td>70</td>
</tr>
<tr>
<td>70</td>
<td>0.340*</td>
<td>0.582</td>
<td>0.557</td>
<td>0.052</td>
<td></td>
</tr>
</tbody>
</table>

* These values may not be very accurate due to the large spread between points actually evaluated.
Figure 7: Local Incidence Angle
\[ \sigma^0 = \sigma^0(\theta) = \sigma^0(x,y) \]

where \( \theta \) is the local incidence angle and

\[ \theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{h} \]  \hspace{1cm} (26)

One way of evaluating \( \sigma^0 \) suggested in [3] is to apply Eq. (21) to a set of measurements to get a rough estimate of \( \sigma^0 \) variation with incidence angle, then use this function in the integral (Eq. (1)), and continuously refine the \( \sigma^0 \) variation function to get better estimate of \( \sigma^0 \).

Variation of \( \sigma^0 \) with incidence angle will take many forms depending on the kind of target. In many cases, including sea ice, it fits quite well with the exponential function used here. From Eqs. (1) and (21),

\[ P_r = \frac{\lambda^2}{(4\pi)^3 P_t} \frac{G_o^2}{R_o^4} \int \int \frac{g^2(\theta, \phi) f(R) \sigma^0(\theta)}{(R/R_o)^4} \, dA \]  \hspace{1cm} (27)

and \( \sigma^0(\theta) \) for some particular angle of incidence can be evaluated as,

\[ \sigma^0_2(\theta_p) = \frac{(4\pi)^3}{\lambda^2} \frac{P_r}{P_t} \frac{R_o^4}{G_o^2} \int \int \frac{1}{g^2 f [\sigma_1(\theta)/\sigma_1(\theta_p)]} \frac{1}{(R/R_o)^4} \, dA \]  \hspace{1cm} (28)

where \( \sigma_1(\theta) \) is the function one gets by applying Eq. (21) to a set of measurements. The next iteration becomes

\[ \sigma^0_3(\theta_p) = \frac{(4\pi)^3}{\lambda^2} \frac{P_r}{P_t} \frac{R_o^4}{G_o^2} \int \int \frac{1}{g^2 f [\sigma_2(\theta)/\sigma_2(\theta_p)]} \frac{1}{(R/R_o)^4} \, dA \]  \hspace{1cm} (29)

and so on.

If we assume the form \( \sigma^0(\theta) = ae^{-b\theta} \), then
and \( \sigma^0 \) variation function around the pointing angle, \( \theta_p \), can be defined as

\[
S_n(\theta, \theta_p) = \frac{\sigma_n(\theta)}{\sigma_n(\theta_p)} = e^{-b_n(\theta-\theta_p)}
\]

and the \( n \)-th refined \( \sigma^0 \) becomes

\[
\sigma_n(\theta_p) = \frac{(4\pi)^3}{2} \frac{P}{\lambda^2} \frac{R_4}{(\theta_p - \theta)^2} \frac{g^2 f S_{n-1}(\theta, \theta_p)}{(R/R_0)^4} \int dA
\]

Hence the integral can be treated as another weighted area similar to that of Eq. (23).

\[
A_{ws} = \int g^2 f S_{n-1}(\theta, \theta_p) \frac{1}{(R/R_0)^4} dA
\]

The possible error when the \( \sigma^0 \) variation function is neglected is defined as

\[
\text{ERROR2}(dB) = 10 \log(A_w/A_{ws})
\]

and numerically evaluated (Table 2) for several antenna gain functions, beamwidths and \( \sigma^0 \) variation across the integration limits (assuming that the variation is known in advance). In Table 2, the functions of \( e^{-\theta/12.9^\circ} \) and \( e^{-\theta/9.4^\circ} \) were derived from linear regression (in dB scale) of the data points obtained from sea ice using the narrow beam approximation and the steep variation function of \( e^{-\theta/5^\circ} \) is appropriate to some open water.

In the next section, the iteration will be tried. The error caused by not considering steep \( \sigma^0 \) variation can be as large as 3.6 dB for normal
## TABLE 2

ERROR2

<table>
<thead>
<tr>
<th>Pointing Angle $\theta_p$ (°)</th>
<th>$\sigma^o = \theta/5^o$</th>
<th>$\sigma^o = \theta/12.9^o$</th>
<th>$\sigma^o = 3$</th>
<th>$\sigma^o = \theta/9.4^o$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_e = 9.0^o$</td>
<td>$\beta_e = 6.3^o$</td>
<td>$\beta_e = 5.0^o$</td>
<td>$\beta_e = 2.4^o$</td>
</tr>
<tr>
<td></td>
<td>$\beta_a = 9.0^o$</td>
<td>$\beta_a = 5.7^o$</td>
<td>$\beta_a = 5.0^o$</td>
<td>$\beta_a = 2.0^o$</td>
</tr>
<tr>
<td>0</td>
<td>3.647 (dB)</td>
<td>1.040 (dB)</td>
<td>2.140 (dB)</td>
<td>0.532 (dB)</td>
</tr>
<tr>
<td>10</td>
<td>-0.484</td>
<td>0.004</td>
<td>-0.193</td>
<td>-0.010</td>
</tr>
<tr>
<td>20</td>
<td>-0.994</td>
<td>-0.060</td>
<td>-0.314</td>
<td>-0.020</td>
</tr>
<tr>
<td>30</td>
<td>-1.310</td>
<td>-0.108</td>
<td>-0.401</td>
<td>-0.029</td>
</tr>
<tr>
<td>40</td>
<td>-1.456</td>
<td>-0.133</td>
<td>-0.438</td>
<td>-0.033</td>
</tr>
<tr>
<td>50</td>
<td>-1.386</td>
<td>-0.190</td>
<td>-0.505</td>
<td>-0.038</td>
</tr>
<tr>
<td>70</td>
<td>-0.398</td>
<td>-0.184</td>
<td>-0.548</td>
<td>-0.050</td>
</tr>
</tbody>
</table>

**Gaussian Gain**

|                               | 0         | 10         | 20         | 30         |
|                               | 3.613     | -1.015     | -1.330     | -1.405     |
|                               | 3.647     | -1.015     | -1.330     | -1.405     |
|                               | 3.574     | -1.063     | -1.177     | -1.339     |

**Sidelobes**

|                               | 0         | 10         | 20         | 30         |
|                               | 3.574     | 1.012      | -0.966     | -0.904     |
|                               | 3.647     | 1.040      | 0.003      | 0.055      |
|                               | 3.574     | 1.012      | -0.966     | -0.904     |

**Mainlobe only**

|                               | 0         | 10         | 20         | 30         |
|                               | 3.574     | 1.012      | -0.966     | -0.904     |
|                               | 3.647     | 1.040      | 0.003      | 0.055      |
|                               | 3.574     | 1.012      | -0.966     | -0.904     |
incidence and in the order of 1 dB for other incidence angles when the beamwidths are 9°. When the beamwidths are 5°, the error is 2.1 dB for normal incidence and less than 0.5 dB for other incidence angles. When the $\sigma^0$ variation is not so rapid ($e^{-8/12.9^\circ}$ or $e^{-8/9.4^\circ}$), the error reduces to 1 dB or less for normal incidence and becomes very small for other incidence angles. Recall from Section 3.5 that the numerical integration may have some error for large incidence angles. Table 2 also shows that the $(\sin x/x)^4$ gain function has such a low sidelobe level (-26.5 dB) that integrating up to first sidelobes did not differ significantly from integrating only the mainlobe. Furthermore, the $(\sin x/x)^4$ antenna gain function and the Gaussian gain function did not show very much difference.

4.1 Iteration

As explained in the previous section, iteration is tried with real data and some arbitrary data which fits well with the exponential function. Because the real data points are from 10° to 70°, only these points are evaluated to get an exponential fit and then iterated several times. Table 3 is a summary of the result. The first row shows the $\sigma^0$ calculated using the narrow beam approximation ($\sigma^0_{nb}$) and the second row is the modified $\sigma^0$ computed using gain function and range variations ($\sigma^0_{nb} + \text{ERROR1}$). These points are regressed to get an exponential fit, and this exponential function is included in the integration to give the third row ($\sigma^0_{nb} + \text{ERROR1} + \text{ERROR2}$). From these points we get new slope and therefore new ERROR2, and for the fourth row, $\sigma^0 = \sigma^0_{nb} + \text{ERROR1} + \text{ERROR2}$, and so on.
**TABLE 3**

DATA, ORIGINAL AND UP TO 4th REFINED

1. 13.6 GHz Sea Ice Data \( \sigma^0 = e^{-8/9.4^\circ} \)
   \( \beta_e = 2.4^\circ \) \( \beta_a = 2.0^\circ \)

<table>
<thead>
<tr>
<th>ANGLE ( \sigma^0 )</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
<th>70°</th>
<th>SLOPE_CORRELATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian Gain</td>
<td>2.70</td>
<td>2.60</td>
<td>1.70</td>
<td>-6.30</td>
<td>-10.00</td>
<td>-14.90</td>
<td>-26.00</td>
<td>-0.4621 ( -0.9643 )</td>
</tr>
<tr>
<td></td>
<td>2.71</td>
<td>2.61</td>
<td>1.69</td>
<td>-6.79</td>
<td>-9.99</td>
<td>-14.88</td>
<td>-25.96</td>
<td>-0.4617 ( -0.9644 )</td>
</tr>
<tr>
<td></td>
<td>2.70</td>
<td>2.59</td>
<td>1.72</td>
<td>-6.82</td>
<td>-10.02</td>
<td>-14.92</td>
<td>-26.00</td>
<td>-0.4622 ( -0.9645 )</td>
</tr>
<tr>
<td></td>
<td>2.70</td>
<td>2.59</td>
<td>1.72</td>
<td>-6.82</td>
<td>-10.02</td>
<td>-14.92</td>
<td>-26.00</td>
<td>-0.4622 ( -0.9645 )</td>
</tr>
</tbody>
</table>
| \( \sin^2 \theta \) \( \frac{4}{\theta} \) \uparrow \!
| up to first \sidelobe | 2.69 | 2.58 | 1.72 | -6.82 | -10.03 | -14.93 | -26.02 | -0.4623 \( -0.9645 \) |
|                     | 2.69 | 2.58 | 1.72 | -6.82 | -10.03 | -14.93 | -26.02 | -0.4623 \( -0.9645 \) |
|                     | 2.69 | 2.58 | 1.72 | -6.82 | -10.03 | -14.93 | -26.02 | -0.4623 \( -0.9645 \) |
|                     | 2.69 | 2.58 | 1.72 | -6.82 | -10.03 | -14.93 | -26.02 | -0.4623 \( -0.9645 \) |

2. 4.8 GHz Sea Ice Data \( \sigma^0 = e^{-8/12.9^\circ} \)
   \( \beta_e = 6.3^\circ \) \( \beta_a = 5.7^\circ \)

<table>
<thead>
<tr>
<th>ANGLE ( \sigma^0 )</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
<th>70°</th>
<th>SLOPE_CORRELATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian Gain</td>
<td>-2.70</td>
<td>-4.90</td>
<td>-10.50</td>
<td>-12.30</td>
<td>-14.80</td>
<td>-18.00</td>
<td>-24.00</td>
<td>-0.3571 ( -0.9986 )</td>
</tr>
<tr>
<td></td>
<td>-2.67</td>
<td>-4.87</td>
<td>-10.47</td>
<td>-12.26</td>
<td>-14.74</td>
<td>-17.78</td>
<td>-23.58</td>
<td>-0.3309 ( -0.9993 )</td>
</tr>
<tr>
<td></td>
<td>-2.67</td>
<td>-4.93</td>
<td>-10.55</td>
<td>-12.36</td>
<td>-14.87</td>
<td>-17.96</td>
<td>-23.56</td>
<td>-0.3304 ( -0.9991 )</td>
</tr>
<tr>
<td></td>
<td>-2.67</td>
<td>-4.93</td>
<td>-10.55</td>
<td>-12.37</td>
<td>-14.87</td>
<td>-17.97</td>
<td>-23.56</td>
<td>-0.3304 ( -0.9991 )</td>
</tr>
<tr>
<td></td>
<td>-2.67</td>
<td>-4.93</td>
<td>-10.55</td>
<td>-12.37</td>
<td>-14.87</td>
<td>-17.97</td>
<td>-23.56</td>
<td>-0.3304 ( -0.9991 )</td>
</tr>
</tbody>
</table>
| \( \sin^2 \theta \) \( \frac{4}{\theta} \) \uparrow \!
| up to first \sidelobe | -2.66 | -4.93 | -10.55 | -12.36 | -14.85 | -17.97 | -23.72 | -0.3341 \( -0.9996 \) |
|                     | -2.66 | -4.93 | -10.55 | -12.36 | -14.85 | -17.97 | -23.72 | -0.3341 \( -0.9996 \) |
|                     | -2.66 | -4.93 | -10.55 | -12.36 | -14.85 | -17.97 | -23.72 | -0.3341 \( -0.9996 \) |

3. Arbitrary Data \( \sigma^0 = e^{-8/5^\circ} \)
   \( \beta_e = 9^\circ \) \( \beta_a = 9^\circ \)

<table>
<thead>
<tr>
<th>ANGLE ( \sigma^0 )</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
<th>70°</th>
<th>SLOPE_CORRELATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian Gain</td>
<td>-8.70</td>
<td>-17.00</td>
<td>-26.60</td>
<td>-33.00</td>
<td>-42.00</td>
<td>-53.00</td>
<td>-60.00</td>
<td>-0.8618 ( -0.9987 )</td>
</tr>
<tr>
<td></td>
<td>-8.64</td>
<td>-16.94</td>
<td>-26.53</td>
<td>-32.91</td>
<td>-41.79</td>
<td>-52.41</td>
<td>-59.63</td>
<td>-0.8542 ( -0.9993 )</td>
</tr>
<tr>
<td></td>
<td>-9.11</td>
<td>-17.00</td>
<td>-27.67</td>
<td>-34.17</td>
<td>-43.22</td>
<td>-53.78</td>
<td>-60.62</td>
<td>-0.8636 ( -0.9999 )</td>
</tr>
<tr>
<td></td>
<td>-9.13</td>
<td>-17.00</td>
<td>-27.69</td>
<td>-34.20</td>
<td>-43.25</td>
<td>-53.80</td>
<td>-60.63</td>
<td>-0.8636 ( -0.9999 )</td>
</tr>
<tr>
<td></td>
<td>-9.13</td>
<td>-17.00</td>
<td>-27.69</td>
<td>-34.30</td>
<td>-43.25</td>
<td>-53.80</td>
<td>-60.63</td>
<td>-0.8636 ( -0.9999 )</td>
</tr>
</tbody>
</table>
| \( \sin^2 \theta \) \( \frac{4}{\theta} \) \uparrow \!
| up to first \sidelobe | -9.08 | -17.00 | -27.83 | -34.29 | -43.11 | -53.77 | -60.14 | -0.8577 \( -0.9987 \) |
|                     | -9.08 | -17.00 | -27.83 | -34.29 | -43.11 | -53.77 | -60.14 | -0.8577 \( -0.9987 \) |

-23-
For all the cases in Table 3, the data seems to converge after three iterations, so it did not require further iteration. Also from Table 1 and Table 2, it can be seen that the ERROR1 and ERROR2 tend to cancel out each other to make $\sigma_{nb}^0$ a tolerable estimation at least for the incidence angles of 10° to 70° and when the beamwidths are not very wide.

5.0 SUMMARY

Three kinds of so-called illuminated areas are treated in this report:

1. The area of an ellipse with constant gain and constant range to the antenna for all the cells within the ellipse, $A_i$, (Eq. (8)).

2. The area weighted with antenna gain and range, $A_w$ (Eq. (23)).

3. The area weighted with antenna gain and range and also $\sigma^0$ variation, $A_{ws}$, (Eq. (32)).

The possible error when using $A_i$ instead of $A_w$ was defined as ERROR1 and summarized in Table 1. When the $\sigma^0$ variation with incidence angle is small, the result shows that the constant range and constant gain approximation gives a result within 0.6 dB or less.

Next, $\sigma^0$ variation is included and the possible error of using $A_w$ instead of $A_{ws}$ was defined as ERROR2 and listed in Table 2. For normal incidence, one can have up to 3.6 dB error when the beamwidth is 9° for rapidly varying $(e^{-B/5^0})\sigma^0$. The big error at normal incidence is always present and one must consider this problem when $\sigma^0$ for normal incidence is evaluated. The error will be bigger than the values obtained here if the real $\sigma^0$ variation near vertical is steeper than the exponential function, which is often true for very smooth targets.
The more realistic $\sigma^0$ values corrected with these estimated errors were further refined using iterative scheme. Actually, however, the iteration did not show any further improvement because it converged almost instantly for the exponential $\sigma^0$ variation function assumed in this report. This may not be true for other kinds of $\sigma^0$ variation functions and can be further investigated using the technique described in this report.

REFERENCES


APPENDIX
PROGRAM LISTING

THIS PROGRAM IS TO EVALUATE THE POSSIBLE ERROR WHEN USING THE NARROW BEAM APPROXIMATION IN CALCULATING THE SCATTERING COEFFICIENT.

ANTENNA GAIN FUNCTIONS GAUSSIAN AND (SIN(X)/X)**4, AND REAL RANGE AND TRAPEZOIDAL BAND PASS FILTER FUNCTION AND THE EXPONENTIAL SIGMA VARIATION FUNCTION IS EVALUATED IN THE INTEGRAL USING GAUSS* FORMULA. 40 POINT BY 40 POINT TOTAL 1600 POINTS ARE EVALUATED IN THE INTEGRATION LIMIT.

FUNCTIONS REFERENCED
GAIN1=GAUSSIAN ANTENNA GAIN
GAIN2=(SIN(X)/X)**4 GAIN
BPFI=BANDPASS FILTER FUNCTION
ASIGMA=EXPONENTIAL SIGMA VARIATION FUNCTION
REGRES=LINEAR REGRESSION ROUTINE
AVG= AVERAGING ROUTINE
AREA=EFFECTIVE AREA CALCULATING ROUTINE FOR THE NARROW BEAM APPROXIMATION CASE

VARIABLES USED
BE=TWO WAY 3 DB ELEVATION BEAMWIDTHS
BA=TWO WAY 3 DB AZIMUTH BEAMWIDTHS
ANGLE=POINTING ANGLE
THEO=ANGLE IN RADIANS
W(40),X(40)=GAUSS* CONSTANTS FOR INTEGRATION
RD=RANGE TO THE BEAM CENTER
UPX,UPY=UPPER LIMITS IN X AND Y COORDINATES
LOWX,LOWY=LOWER LIMITS
H=ANTENNA HEIGHT
ERROR1=ERROR FOR NOT COUNTING GAIN AND RANGE VARIATION
ERROR2=ERROR FOR NOT COUNTING SIGMA
AREA=EFFECTIVE AREA OF THE ELLIPSE
WARE=GUID AND RANGE WEIGHTED AREA
RAREA=SIGMA*GAIN*RANGE WEIGHTED AREAS
AL=CONST TO SET THE LIMIT OF INTEGRATION

COMMON BE, BA
REAL BE, BA
REAL ANGLE, THEO, PI, W(40), X(40), RD, UPX, LOWX, UPY, LOWY
REAL ISUM2, JSUM2, PROD2, ERROR2(7), ERROR1(7), RAREA
REAL WARE, EAREA, ABE, ABA, EBE, EBA
REAL FILTER, BPFO, BPFI
REAL BCONST, CCONST, SIGMA, AL, ASIGMA
REAL SIGMA0(7), SIGMA1(7)
INTEGER I, J, K

DATA (W(I),I=1,20)=.07750594,.07703981,.07611036,.07472316,
.07288658,.07051166,.06691240,.06480401,
.06130624,.05743976,.05327284,.04869580,
.04387090,.03878216,.03346019,.02793700,
.02224584,.01642105,.01049828,.00452127,
.00223640,.00111605,.00043199,.00014094,
.00004735,.00001403,.00000432,.00000141,
.00000009,.00000003,.00000001,.00000000,
.00000000

DATA (X(I),I=1,20)=.03872721,.11653407,.19269758,.26815218,
.33419409,.41377920,.48307560,.54966712,
.61255388,.67195668,.72731825,.77830565,
.82461223,.86595930,.90209880,.93281280,
.95918518,.97725994,.99072623,.99823770,
.99968830,.99999999

SYSTEM CONST FOR INTERNAL RANGE
DATA RCONST/2.6/
**CONSTANT TO CONVERT SLOPE IN DB SCALE INTO EXPONENTIAL SLOPE**

```
BBB=10.*ALOG10(EXP(1.))
PI=4.*ATAN(1.D)
```

**TWO WAY 3 DB BEAMWIDTHS IN DEGREES**

```
ABE=9.0
ABA=9.0
```

**BE=ABE*PI/180.**

**BA=ABA*PI/180.**

**TWO WAY EFFECTIVE BEAMWIDTHS IN DEGREES**

```
EBE=1.201*ABE
EBA=1.201*ABA
```

**INPUT SIGMAO AND OUTPUT THESE INPUT PARAMETERS**

```
DATA SIGMAO/-8.7,-17.7,-26.6,-35.5,-42.5,-50.7/-60.7/
```

**CALL REGRES(SIGMAO,N,SLOPE,CORR)**

**WRITE(**6,699)**

```
699 FORMAT(/1X,'SIGMAO DATA: ORIGINAL AND UP TO 4TH REFINED')
```

**WRITE(6,698)**

```
698 FORMAT(/1X,'ANGLE 10 20 30 40 50 60')
```

**SLOPE CORRELATION**

```
697 FORMAT(4X,7F7.2,1X,7F7.4,2X,7F7.4)
```

**ASSIGN INTEGRATION CONSTS**

```
DO 100 I=1,40
W(I)=W(I-20)
X(I)=-X(I-20)
100 CONTINUE
```

**DO UNTIL 4TH ITERATION**

```
DO 89 II=1,4
SLOPE=-BBB/SLOPE
1000 DO 90 K=1,7
ANGLE=FLOAT(K)*10.,THEO=PI*ANGLE/180.
1030 C
1040 C
RD=H/COS(THEO)
1050 C
1060 C
LIMITS UP TO FIRST SIDELOBES OF (SIN(X)/X)**4
1080 AL=PI*1.414/2.,7831
1090 ALU=AL
1100 IF(K.EQ.6.OR.K.EQ.7) ALU=1.5
1110 UPX=H*(TAN(THEO+ALU*BE))
1120 LOWX=H*(TAN(THEO-AL*BE))
1130 UPHY=RD*TAN(AL+BA)
1140 LOWY=UPY
1150 HALFX=(UPX-LOWX)/2.
1160 HALFY=(UPY-LOWY)/2.
1170 AVGX=(UPX+LOWX)/2.
1180 AVGY=(UPY+LOWY)/2.
1190 C
1200 ISUM1=0.
```
ISUM=0.
DO 101 I=1,20
   YY=X(I)*HALFY+AVGY
   JSUM=0.
   JSUM1=0.
   DO 102 J=1,40
      XX=X(J)*HALFX+AVGX
      CALL GAIN1(XX,YY,H,THEO,G)
      RANGE=SQRT((XX**2+YY**2+H**2)
      FILTER=BPF1(RANGE,RO,RCONST)
      RANGE4=RANGE**4
      SIGMA=ASIGMA(XX,YY,H,ANGLE,SLOPP)
      PROD=G*FILTER*SIGMA/RANGE4
      JSUM1=JSUM1+W(J)*PROD
   102 CONTINUE
   JSUM=JSUM1+W(I)*JSUM
   ISUM=JSUM+W(I)*JSUM
101 CONTINUE
INTGRL=2.*HALFX*HALFY*ISUM
ISUM=ISUM+W(I)*JSUM
ISUM1=ISUM+W(I)*JSUM
101 CONTINUE
WEIGHTED AREA USING 3DB BEAMWIDTHS WITHOUT SIGMA VARIATION
   WAREA=RO**4*INTGRL
   WEIGHTED AREA INCLUDING SIGMA VARIATION
   RAREA=2.*HALFX*HALFY*ISUM1+RO**4
   EFFECTIVE AREA USING EFFECTIVE BEAMWIDTHS
   AND CONST GAIN AND CONST RANGE
   AREA=AEBE(EBE,EBASE,ANGLE,RCONST)
   ERROR1(K)=10.*ALOG10(WAREA/EAREA)
   ERROR2(K)=10.*ALOG10(RAREA/WAREA)
   WRITE(6,999) ANGLE,ERROR1,ERROR2
999 FORMAT(1X,ANGLE=',F5.1,2X,'ERROR1=',F6.3,' DB',2X,
   'ERROR2=',F6.3,' DB')
   WRITE(6,997) EAREA,WAREA,RAREA
90 CONTINUE
90 CONTINUE
IF(II .NE. 1) GC TO 112
DO 111 I=1,7
   SIGMA1(I)=SIGMA0(I)-ERROR1(I)
111 CONTINUE
CALL REGRES(SIGMA1,N,SLOPE,CORR)
WRITE(6,997) (SIGMA1(I),I=1,7),SLOPE,CORR
GO TO 112
112 CONTINUE
DO 113 I=1,7
   SIGMA0(I)=SIGMA1(I)-ERROR2(I)
113 CONTINUE
114 CALL REGRES(SIGMA0,N,SLOPE,CORR)
115 WRITE(6,697) (SIGMA0(I),I=1,N),SLOPE,CORR
116 114 CONTINUE
117 89 CONTINUE
118 STOP
END

1810 1820 C CONTINUE
1830 1840 CALL REGRES(SIGMA0,N,SLOPE,CORR)
1850 1860 WRITE(6,697) (SIGMA0(I),I=1,N),SLOPE,CORR
1870 1880 114 CONTINUE
1890 1870 STOP

1900 C***************
1910 1920 SUBROUTINE GAIN1(X,Y,HH,TO,GG)
1930 1940 GAUSSIAN ANTENNA GAIN FUNCTION
1950 1960 WITH TWO WAY BEAMWIDTHS=BE,BA
1970 1980 COMMON BE,BA
1990 2000 REAL X,Y,HH,TO,GG
2010 2020 REAL THEA,PHIA,G1,G2,AA
2030 2040 THEA=ATAN(X/HH)
2050 2060 PHIA=ATAN(Y/(SQRT(X*X+HH*HH)))
2070 2080 AA=4.4*ALOG(2.)
2090 2100 G1=EXP(-AA*((THEA-TO)**2)/(BE**2))
2110 2120 G2=EXP(-AA*((PHIA)**2)/(BA**2))
2130 2140 GG=G1*G2
2150 2160 RETURN
END

2100 C***************
2110 2120 SUBROUTINE GAIN2(X,Y,HH,TO,GG)
2130 2140 (SIN(X)/X)**4 TWO WAY ANTENNA GAIN-PATTERN
2150 2160 WITH TWO WAY BEAMWIDTHS=BE,BA
2170 2180 COMMON BE,BA
2190 2200 REAL X,Y,HH,TO,GG
2210 2220 REAL THEA,PHIA,G1,G2,BEONE,BAONE,A1,A2,G11,G22
2230 2240 ONE WAY 3DB BEAMWIDTHS
2250 2260 BEONE=BE*SQRT(2.)
2270 2280 BAONE=BA*SQRT(2.)
2290 2300 THEA=ATAN(X/HH)
2310 2320 PHIA=ATAN(Y/(SQRT(X*X+HH*HH)))
2330 2340 A1=2.7831*(THEA-TO)/BEONE
2350 2360 A2=2.7831*PHIA/BAONE
2370 2380 G1=ABS(SIN(A1))/A1
2390 2400 IF(G1.LE.(1.OE-10)) GO TO 400
2410 2420 G11=G1**4
2430 2440 G2=ABS(SIN(A2))/A2
2450 2460 IF(G2.LE.(1.OE-10)) GO TO 400
2470 2480 G22=G2**4
2490 2500 GG=G11*G22
2510 2520 RETURN
END

2380 2390 400 CONTINUE
2400 2410 GG=0.
2420 2430 RETURN
REAL FUNCTION BPF1(R,RO,CONST)

TRAPEZOIDAL BANDPASS FILTER FUNCTION

REAL R,RO,CONST
REAL FILE, BW1, CF, LOWR, HIGHR, RRO, HF, LF
DATA BW1, CF/12.0, 50.0/

FIL= BW1/(2.*CF)
HF= 1. +FIL
LF= 1. -FIL
LOWR= RO*LF -FIL*CONST
HIGHR= RO*HF +FIL*CONST
RRO= (R*CONST)/(R+CONST)

IF(R.GT.HIGHR .AND. RRO.LE.(1.2*HF)) BPF1=10.**(-30.* (RRO-HF))
IF(RRO.GT.(1.2*HF)) BPF1=0.
IF(R.LE.HIGHR .AND. R.GE.LOWR) BPF1=1.
IF(R.LT.LOWR) BPF1=10.**(-145.*(RRO-LF)/4.)

RETURN

END

REAL FUNCTION BPFD(R,RO,CONST)

REAL R,RO,CONST, FIL
REAL BW, CF, LOWR, HIGHR
DATA BW, CF/1.5, 5.0/

FIL= BW/(2.*CF)
LOWR= RO*(1.-FIL) -FIL*CONST
HIGHR= RO*(1. +FIL) +CONST*FIL

IF(R.GE.LOWR .AND. R.LE.HIGHR) GOTO 800
BPFD= 0.
RETURN
800 CONTINUE
BPFD= 1.0
RETURN
END

REAL FUNCTION ASIGMA(X, Y, HH, ANGL, B)

THIS FUNCTION IS THE SIGMA VARIATION FUNCTION FOR POINT P(X,Y) WITH POINTING ANGLE=ANGL AND SLOPE=B.

REAL X, Y, HH, ANGL, B
REAL THETA, ATHT, PI
PI= 4.*ATAN(1.)
THETA= ATAN(SQRT(X*X+Y*Y)/HH)
ATHT= THETA*180./PI
ASIGMA= EXP(-((ATHT-ANGL)/B))
RETURN
END

SUBROUTINE REGRES(Y, HN, SLOP, COR)
THIS SUBROUTINE CALCULATES THE SLOPE AND THE CORRELATION COEFFICIENT OF AN ARRAY Y(10). THE X-VARIABLE IS THE INCIDENCE ANGLE FROM 10 TO 10*NN DEGREES.

INTEGER NN
REAL Y(NN), SLOPE, SLOPE, COR, X(10), SY(10), SX(10)
REAL SXY(10), AVG, AVX, AVY

DO 10 I=1, NN
X(I)=FLOAT(I)*10.
10 CONTINUE

AVX=AVG(X,NN)
AVY=AVG(Y,NN)

DO 11 I=1, NN
SY(I)=(Y(I)-AVY)**2
SX(I)=(X(I)-AVX)**2
SXY(I)=(X(I)-AVX)*(Y(I)-AVY)
11 CONTINUE

SLOP=AVG(SXY,NN)/AVG(SX,NN)
COR=SLOP*SQRT(AVG(SX,NN)/AVG(SY,NN))

RETURN

END

REAL FUNCTION AVG(A, N)
INTEGER N
REAL A(N), SUM
SUM=0.
DO 12 I=1, N
SUM=SUM+A(I)
12 CONTINUE
AVG=SUM/FLOAT(N)
RETURN
END

REAL FUNCTION AREA(ELB, AZB, ANGL, H, RCONST)
INTEGER ELB, AZB, ANGL, H, RCONST
REAL PI, H

FUNCTIONS REFERENCED
REAL LAREA1, LAREA2

DATA FC, BW/50.0*13.5/
PI = 3.1415926

CONVERT BEAMWIDTHS TO RADIANs

ELBR = PI * ELB/180.
AZBR = PI * AZB/180.

IF(IFIX(ANGL) .EQ. 0) GO TO 301

THE = ANGL*PI/180.
R = H/COS(THE)
FIL = BW/(2.*FC)

ALPA1 = R*(1. - FIL) - RCONST*FIL
ALPA2 = R*(1. + FIL) + RCONST*FIL
GAM = H*(TAN(THE-ELBR/2.))
RA = 2.*R*(TAN(AZBR/2.))
RE = H*(TAN(THE+ELBR/2.)) - GAM

A = RE/2.
B = RA/2.

K = GAM + A
L1 = GAM
L2 = RE-L1

IF(ALPA1-H) 120, 120, 122
120 X1 = C
GO TO 124
122 X1 = C - SQRT(ALPA1*ALPA1-H*H)
CONTINUE
124

X2 = SQRT(ALPA2**2-H*H)-C

CHECK IF THERE IS ANY BEAM LIMITING DUE TO NARROW BPF
AND IF ANY GO TO APPROPRIATE CASES

CASE 1, NO LIMITING OF THE BEAM
AREA = PI*A*B
RETURN

CASE 2, NEAR SIDE OF THE BEAM IS LIMITED BY THE FILTER
AREA = PI*A*B-LAREA1(A, B, C, K, X1)
RETURN

CASE 3, FAR SIDE OF THE BEAM IS LIMITED BY THE FILTER
AREA = LAREA2(A, B, C, K, X2)
RETURN

CASE 4, BOTH SIDES OF THE BEAM IS LIMITED BY THE FILTER
AREA = LAREA2(A, B, C, K, X2) - LAREA1(A, B, C, K, X1)
RETURN
4210 301 CONTINUE
4220 AREA=PI*H*TAN(ELBR/2.)*TAN(AZBR/2.)
4230 RETURN
4240 END
4250 C
4260 C
4270 REAL FUNCTION INTSEC(A1,A2,A3,A4)
4280 C
4290 C
4300 C
4310 C
4320 REAL A1,A2,A3,A4
4330 C LOCAL VARIABLES
4340 REAL B1,B2,B3,B4,B5,B6
4350 B1=(A2+A1)*(A2-A1)
4360 B2=(A4+A2)*(A4-A2)
4370 B3=(A1+A2*A3)**2
4380 B4=A2*A2*A3
4390 B5=A1*A1
4400 B6=SQRT(B3-B5*B1*B2)
4410 INTSEC=(B4-B6)/B1
4420 RETURN
4430 END
4440 C
4450 C
4460 REAL FUNCTION MISC(THETA)
4470 C
4480 C
4490 C
4500 C
4510 C
4520 C
4530 C
4540 C
4550 C
4560 REAL FUNCTION THEX(MG,MP)
4570 C
4580 C
4590 C
4600 C
4610 C
4620 C
4630 C
4640 C
4650 C
4660 C
4670 C
4680 C
4690 C
4700 C
4710 C
4720 C
4730 REAL AA,BB,CC,KK,XX1
4740 REAL TH1,INT1,K<MP,THE2,INT2,GG,PPS
4750 REAL THEX,MISC,INTSEC
4760 REAL GG=CC-XX1
4770 PPS=INTSEC(AA,BB,KK,GG)
4780 C
4790 C INTEGRATE ALONG THE CIRCLE
TTH1 = THEX(GG * PSI)
INT1 = GG * GG * MISC(TTH1)

INTEGRATE ALONG THE ELLIPSE
KKMP = KK - PSI
TTHE2 = THEX(AA, KKMP)
INT2 = AA * BB * MISC(TTHE2)

LAREA1 = 2. * (INT1 + INT2)
RETURN

REAL FUNCTION LAREA2(AA, BB, CC, KK, XX2)

THIS FUNCTION CALCULATES THE AREA FOR THE CASE WHEN
THE FAR SIDE OF THE BEAM IS LIMITED BY THE FILTER

REAL AA, BB, CC, KK, XX2
REAL GPSI, THE1, THE3, INT1, INT2, PI, KMP

DATA PI/3.1415926/,
G = CC + XX2
PSI = INTSEC(AA, BB, KK, G)

THE1 = THEX(G, PSI)
INT1 = G * G * MISC(THE1)

IF(KK - PSI) 300, 301, 302
KMP = PSI - KK
THE3 = THEX(AA, KMP)
INT2 = AA * BB * (.5 * PI - MISC(THE3))
GO TO 303

INT2 = PI * AA * BB / 4.
GO TO 303

THE3 = THEX(AA, KK - PSI)
INT2 = AA * BB * MISC(THE3)

LAREA2 = 2. * (INT1 + INT2)
RETURN
END
END
FILMED