A Fast Passive Integrator

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Integrator
Sub-nanosecond
Passive

We describe a passive integrator capable of correctly giving outputs with risetimes as short as \(~250\) ps. The integrator can drive a 50 \(\Omega\) load and is intended to exploit the capabilities of the 7A29 preamp in the Tektronix 7104 oscilloscope \((\tau_r \approx 380\) ps). The decay time for the integrator is \(~250\) ns.
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A FAST PASSIVE INTEGRATOR

I. INTRODUCTION

This article describes a passive integrator designed to exploit the risetime capabilities of the 7A29 preamp in the Tektronix 7104 oscilloscope. Our present application for this device is to integrate the signals from current and voltage probes on the NRL electron autoaccelerator.

This device addresses three problems encountered with simple RC integrators (Fig. 1): First, the parasitic capacitance across the resistor causes the output to overshoot when given a fast rising input. Second, the integrating capacitor will display a self resonance because of its inductance. Third, a preamp with a high input impedance is needed to avoid shortening the integrator decay time. We solve these problems by compensating the parasitic capacitances, keeping the resonances fast and damping them, and resistively coupling to the preamp.

II. PHYSICAL DESCRIPTION

Figure 2 is a schematic of the cross section of the integrator. The construction is coaxial with a brass outer conductor and BNC connectors on each end. The short sections of low impedance coaxial line (a) are compensating capacitors and use 3 mil mylar for insulation. The resistors (b) are 1 KΩ, 1/4 Watt Allen Bradley resistors. These are molded composition resistors but for this value and size the skin depth does not equal the radius until ~100 GHz. The integrating capacitor (c) is a longer section of very low impedance line insulated with 1 mil mylar. The center conductor in this section is made of carbon which provides damping of the reflections between the ends of the capacitor. (This refinement is unnecessary for the compensating capacitors because they are very short and have very high resonant frequencies as do all other cavities within the device.) Each resistor goes through a double aperture. These apertures are brass and provide partial electrostatic shielding between the center conductors of the compensating and integrating capacitors.

III. ELECTRICAL BEHAVIOR

Figure 3 shows an equivalent circuit for the integrator (neglecting capacitor inductance). A qualitative understanding of the behavior of the device is aided by dividing the circuit into three regions. Region I includes the first compensating capacitor and the 50 Ω input circuit. (The length of our signal lines allows us to use an unterminated line and increase the gain of the integrator by a factor of 2. A change in the value of the compensating capacitor would permit termination of the line if reflections posed a problem.) The line impedance (50 Ω) and the capacitor constitute an integrator for the faster incoming signals (r < RC = 1 ns). The subsequent sections (II, III) act as capacitive voltage dividers on this time scale. Furthermore they do not load section I.

The time constants for Section I and the resistor, parasitic capacitor branch of Section II are set equal. Thus the second section takes over the job of integration at the same frequency for which the first section fails to integrate. The voltage divider effect of Section II keeps the overall gain equal for fast and slow signals. Section III is a compensated voltage divider and allows the 50 Ω preamp (represented by the 50 Ω resistor) to be driven without loading the integrator.

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A formal approach to the analysis of this circuit is provided by finding the \( s \) domain transfer function. The transfer function is defined as the ratio of the Laplace transforms of the input and output voltages:

\[
H(s) = \frac{V_0(s)}{V_i(s)}.
\]  

(1)

The response of an initially uncharged circuit to an applied signal, \( V_i(t) \), is given by the transform of the product of \( H(s) \) and \( V_i(s) \)

\[
V_0(t) = \mathcal{L}^{-1}\{H(s)V_i(s)\}.
\]  

(2)

In particular the impulse response \( V_i(t) = \delta(t) \) of a passive integrator should be a decaying step \( e^{-at}u(t) \).

The transfer function for this circuit is readily found by a ladder analysis. We demonstrate this analysis in Fig. 4. The following set of consistent units is convenient for this analysis

- time - nanoseconds
- voltage - gigavolts
- current - megamps
- resistance - kilo ohms
- capacitance - picofarads

A unity output voltage is assumed. This leads to a current \( I \) which is the sum of the currents in the resistor and the capacitor

\[
I = 20 + 20s.
\]  

(3)

The voltage at the next node is then

\[
V = 1 + (20 + 20s)\left(\frac{1}{1 + s}\right) = 21.
\]  

(4)

The currents and voltages for each branch and node may be similarly determined back to the input. The transfer function is then given by

\[
H(s) = \frac{1}{V_i(s)} = \frac{9 \times 10^{-5}(1 + s)}{s^2 + s + 3.8 \times 10^{-3}}.
\]  

(5)

A partial fraction expansion yields

\[
H(s) = \frac{9 \times 10^{-5}}{(s + 3.8 \times 10^{-3})(s + 1)} - \frac{3.6 \times 10^{-7}}{(s + 1)}.
\]  

(6)

It is thus evident that the \( \delta \) function response of this circuit is a step with an \( \sim 250\text{-ns} \) decay (first term) and an infinitesimal \( \sim 1/250 \) of the main pulse) undershoot lasting \( \sim 1 \) ns. This transfer function closely approximates that which would result if the parasitic and compensating capacitors were removed from the circuit (Fig. 5)

\[
H(s) \approx \left( \frac{1}{RC} \right) \left( \frac{R_0}{R} \right) \left( \frac{1}{s + \frac{2}{RC}} \right).
\]  

(7)

where we assume \( R >> R_0 \).
IV. TRANSIT TIME EFFECTS

An equivalent circuit of the type shown in Fig. 3 (lumped parameter) is valid for frequencies for which the device is small compared to a wavelength.

Because each separate section within the device does not load the preceding sections, in this particular case, each section must be small compared to the signal wavelength. The section of transmission line being used as the integrating capacitor (Fig. 6a) is long enough to warrant consideration of transit time effects. If this section of line is lossless, a pulse can bounce between its ends. An equivalent circuit for this resonance is given in Fig. 6b where the line capacitance is split into two halves and is joined by the line impedance. The resonant frequency corresponds (slow by $\pi/2$) to the double transit time. (Both ends appear open at any frequency of interest given the size of the parasitic capacitances, $\sim 1$ pf, and the line impedance, $\sim 2$ $\Omega$.) If this circuit is critically damped by including a series resistance, the charge distribution between the two ends evens out in $\sim 1/2$ cycle i.e.: one transit time ($\tau = 125$ ps). The required resistance is

$$R_{\text{crit}} = 2\sqrt{\frac{L}{C}} \approx 1\Omega.$$  

(8)

A center conductor made of carbon has a resistance at the resonant frequency of $\sim 0.5\, \Omega$ and was used in the existing integrators as the most readily available material.

V. CONSTRUCTION DETAILS

Figure 7 shows the actual assembly used in the integrator. The center conductors of the compensating capacitors and the resistors are first soldered to a BNC fitting. These are then inserted in brass shells with a step in their diameter. The electrostatic shields are a separate piece and are slipped over the resistors and held in place with a spot of epoxy. The brass shells are slotted at each end and are held in place by hose clamps. This allows the compensating capacitors to be adjusted by sliding the shells.

A fitting was made which allows each resistor, capacitor combination to be tested as a voltage divider and adjusted before it is plugged into the center section of the integrator. The center section has the carbon center conductor and a brass shell with slotted ends. The slotted ends slip over the electrostatic shields while holes in the carbon center conductor engage multifingered pins soldered onto the resistors. The assembled integrator is then calibrated with a square pulse of known voltage and duration (Fig. 8).

VI. ACKNOWLEDGMENT

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VII. REFERENCES


2. Ibid, p. 198.
Fig. 1 — A schematic diagram of a simple RC integrator showing the $R$ with its parasitic capacitance $C_p$ and the capacitor $C$.

Fig. 2 — A diagram showing the cross-section through the fast passive integrator.
Fig. 3 — The equivalent circuit for the fast passive integrator being driven by a 50 Ω line and feeding into a 50 Ω preamp.

Fig. 4 — The equivalent circuit for the fast passive integrator designated with consistent units to illustrate the calculation of the transfer function.
Fig. 5 — The circuit diagram of the fast passive integrator as it would look without parasitic and compensating capacitors.

Fig. 6 — The integrating capacitor of the fast passive integrator (a) and its representation as a transmission line (b)
Fig. 7 — Construction detail of the fast passive integrator

Fig. 8 — (a) A square wave input signal, vertical scale 2 volt/cm, time base 10 ns/cm, (b) the output signal from the fast passive integrator. Vertical scale 10 mV/cm, time base 10 ns/cm