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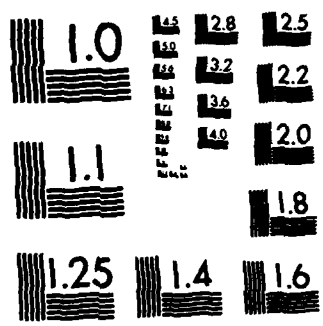

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A Stable Hybrid Adaptive Algorithm with Periodic Sampling and Gain Adjustment

by

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In this report a hybrid algorithm for model reference adaptive control of single-input single-output systems is presented. The control structure involves a continuous time as well as a discrete time part, instead of being all discrete or all continuous as in previous approaches. The system is sampled periodically at a frequency F, and a bound F* is determined such that the closed loop system is stable whenever $F > F^*$.

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Abstract

In this report a hybrid algorithm for model reference adaptive control of single-input single output systems is presented. The control structure involves a continuous time as well as a discrete time part, instead of being all discrete or all continuous as in previous approaches. The system is sampled periodically at a frequency F, and a bound F* is determined such that the closed loop system is stable whenever F > F*.

Introduction

The theory and application of Adaptive Control Systems have been a center of discussion in the last few years. Continuous-time [1], [6], [7], [8], as well as discrete-time [2], [5], [9], [10] schemes have been devised, and stability has been proved.

In spite of the continuous-time nature of real systems, from a point of view of applications, discrete-time algorithms are preferred to continuous-time, due to recent advances in digital technology.

However, the discrete approach is not closely coupled to the continuous-time behavior of real plants, making a "hybrid" approach (partly discrete, partly continuous) desirable. It is a well known result [1], [6], that, for a given plant, poles and zeroes can be arbitrarily placed with appropriate compensators as in Fig. 1. If the plant parameters are known exactly, then the control input which gives the desired behavior is on the form

$$u(t) = \underline{K}^* \; \underline{\psi}(t),$$

$\underline{\psi}(t)$ being filtered versions of the plant input and output, and $\underline{K}^*$ an array of constants. In case of plant unknown, or partially known, the input assumes the form

$$u(t) = \underline{K}(t) \; \underline{\psi}(t),$$

where $\underline{K}(t)$ are adapted in order to have $\underline{K}(t) \to \underline{K}^*$.

In the hybrid scheme which will be the subject of this paper, the set of parameters $\underline{K}(t)$ are updated by a digital computer at discrete intervals of time $\{t_k\}$, and the continuous-time nature of $u(t)$ is preserved.

The overall scheme of the control system is shown in Fig. 2.

Recently, hybrid algorithms for adaptive control [4] as well as self-tuning regulators [11], have been devised. In [4] the adaptive gains $\underline{K}(t)$

are discretely updated at a fixed rate, in base of samples taken from the plant in a random fashion.

It turns out that the sampling scheme is crucial in order to establish stability of the closed loop system.

In many practical applications bounds on the parameters of the plant are known, what enables us to determine a suitable sampling frequency which guarantees stability.

The problem is stated in Section 1, with the error model given in Section 2. The adaptive law is as in Section 3, and the variable and fixed rate sampling schemes are discussed in Sections 4, 5, and 6.

Notation

The following notation will be used:

- vectors: $\underline{a} = [a_1, a_2, \ldots, a_n]^T$;
- time delay operator: z;
- differential operator: $p = \frac{d}{dt}$
- $x(t) = O[y(t)]$ iff there exists a positive constant M such that $|x(t)| \leq M|y(t)|$, for any t;
- $x(t) = o[y(t)]$ iff $|x(t)| \leq \beta(t)|y(t)|$ for some function $\beta(t)$ such that $\beta(t) \to 0$;
- $x(t) \sim y(t)$ iff $x(t) = O[y(t)]$ and $y(t) = O[x(t)]$;
- $\mathcal{L}$ denotes Laplace Transform operation.

1. Statement of the Problem

A continuous time dynamic system (plant) can be described by the linear line invariant, non-autonomous differential equation

$$(1.1) \quad D_p(p)\, x(t) = D_u(p)\, u(t)$$

with
$$D_p(p) = p^n + a_1 p^{n-1} + \ldots + a_n$$
$$D_u(p) = b_o p^m + b_1 p^{m-1} + \ldots + b_m$$

The following assumptions are made on the plant parameters:

(i) the values of a_i, $i = 1, \ldots, n$ and b_i, $i=0, m$, are unknown;

(ii) $m \leq n-1$ is known;

(iii) the plant is minimum phase; i.e., the polynomial $D_u(p)$ is Hurwitz;

(iv) the sign of b_o is known, as are bounds b_{om} and b_{oM}, where $b_{oM} \geq b_o \geq b_{om}$.

Without loss of generality, $b_{om} > 0$ will be assumed.

Given a model

(1.2) $D_m(p)\, x_m(t) = K_o r(t)$

with $D_m(n) = p^n + a_{m1}p^{n-1} + \ldots + a_{mn}$, Hurwitz.

The design objective is to determine an input to the plant u(t) such that, for some $E_o > 0$, $t_F > 0$

(1.3) $|e(t)| \leq E_o$, for every $t \geq t_F$,

where

(1.3) $e(t) \triangleq x_m(t) - x(t)$

In particular we restrict the input u(t) to be on the form

(1.4) $u(t) = \sum_1^n i\ K_i(k)\psi_i(t)$, for $t\varepsilon[t_k, t_{k+1})$

where $K_i(k)$, i=1,n, is a set of gains updated only at discrete instants $\{t_k\}$, and $\psi_i(t)$ are continuous time, observable state variables of the system.

2. The Error Model

It has been shown in [1] that constant vectors β_u and β_x exist such that

(2.1) $D_m(p)e(t) = D_w(p)[-b_o u_f(t) + \underline{\beta_u}^T \underline{\phi_u}(t) + \underline{\beta_x}^T \underline{\phi_x}(t) + K_o\phi_o(t)]$

where the following definitions pertain:

- $D_w(p) \triangleq p^{n-1} + c_1 p^{n-2} + \ldots + c_{n-1}$ is a Hurwitz polynomial such that $\dfrac{D_w(p)}{D_m(p)}$ is Strictly Positive Real (S.P.R.);
- $u_f(t)$ is such that $D_f(p)u_f(t) = u(t)$ where $D_f(p) = p^{n-m-1} + F_1 p^{n-m-2} + \ldots + F_{n-m-1}$ is any Hurwitz polynomial of degree n-m-1;
- $\phi_u^i(t)$, i = 0, ..., n-2 are solutions of $D_w(p)D_f(p)\phi_u^i(t) = p^i u(t)$;
- $\phi_x^i(t)$, i = 0, ..., n-1 are solutions of $D_w(p)D_f(p)\phi_x^i(t) = p^i x(t)$;
- $\phi_o(t)$ is solution of $D_w(p)\phi_o(t) = r(t)$.

If we choose $D_m(p) = (p+\alpha)D_w(p)$, with $\alpha > 0$, a sequence $\{t_k\}$, and

(2.2) $u_f(t) = \underline{K}_u^T(k)\underline{\phi}_u(t) + \underline{K}_x^T(k)\underline{\phi}_x(t) + K_o(k)\phi_o(t) + w_1(t),$

for $t\epsilon[t_k, t_{k+1})$,

we can write (2.1) as

(2.3) $(p+\alpha)e(t) = \underline{\delta}_u^T(k)\underline{\phi}_u(t) + \underline{\delta}_x^T(k)\underline{\phi}_x(t) + \delta_o(k)\phi_o(t) - b_o w_1(t),$

for $t\epsilon[t_k, t_{k+1})$

where $\underline{\delta}_j(k) \triangleq \underline{K}_j(k) - b_o\underline{\delta}_j$, $j = u,x,o$.

In what follows the sequences $\underline{K}_j(k)$ will be called the Adaptive Gains, and will be updated at the sampling instants $\{t_k\}$ only. Furthermore the input u(t) has to be determined such that (1.3) is satisfied.

If (2.3) is sampled at instants $\{t_k\}$, the samples of the error are related by the linear, time variant difference equation

(2.4) $e(t_k) = A_k e(t_{k-1}) + \underline{\delta}_u^T(k-1)\underline{\tilde{\phi}}_u(k) + \underline{\delta}_x^T(k-1)\underline{\tilde{\phi}}_x(k) +$

$+ \delta_o(k-1)\tilde{\phi}_o(k) - b_o\tilde{w}_1(k)$

where we define

$T_k = t_k - t_{k-1};$

$A_k = \exp -\alpha T_k;$

(2.5) $\underline{\tilde{\Phi}}_j(k) = \underline{\xi}_j(t_k) - A_{k-1}\,\underline{\xi}_j(t_{k-1})$, $j = 0,u,x;$

$(p+\alpha)\,\underline{\xi}_j(t) = \underline{\phi}_j(t)$, $j = 0,u,x;$

$$\tilde{w}_1(k) = \int_{t_{k-1}}^{t_k} \exp -\alpha(t_k-\tau)w_1(\tau)d\tau$$

Introducing the auxiliary network

$$(2.6)\quad y(k) = A_k y(k-1) + q(k) + w(k)$$

with $\eta(k) \triangleq e(t_k) + y(k)$, equations (2.4) and (2.6) yield

$$(2.7)\quad \eta(k) = A_k \eta(k-1) + \underline{\xi}^T(k-1)\,\underline{\tilde{\phi}}(k) + w(k) - b_o \tilde{w}_1(k) + q(k)$$

where

$$(2.8)\quad \begin{aligned} \underline{\delta}^T(k) &\triangleq [\underline{\delta}_u^T(k) \;\vdots\; \underline{\delta}_x^T(k) \;\vdots\; \delta_o(k)] \\ \underline{\tilde{\phi}}(k) &\triangleq [\underline{\tilde{\phi}}_u^T(k) \;\vdots\; \underline{\tilde{\phi}}_x(k) \;\vdots\; \tilde{\phi}_o(k)] \end{aligned}$$

Let us choose

$$(2.9)\quad w(k) = K_w(k-1)\tilde{w}_1(k),$$

then (2.7) becomes

$$(2.10)\quad \eta(k) = A_k(k-1) + \underline{\delta}^T(k-1)\,\underline{\tilde{\phi}}(k) + \delta_w(k-1)\tilde{w}_1(k) + q(k),$$

which is the augmented error equation.

3. Adaptive Law

The equations in the previous section hold for any sampling sequence $\{t_k\}$, on which no hypothesis has been made so far.

If we suppose $\{t_k\}$ be a sequence with an infinite number of elements, then it is a well known result--[2], [3]--that equation (2.10) and the following adaptive law

$$(3.1)\quad \begin{aligned} \underline{\delta}(k) &= \underline{\delta}(k-1) - F\,\underline{\tilde{\phi}}(k)\,\eta(k) \\ q(k) &= -\gamma_1 \;||\underline{\tilde{\phi}}(k)||^2\,\eta(k) \\ \delta_w(k) &= \delta_w(k-1) + \frac{1}{\lambda_w}\tilde{w}_1(k)\,\eta(k) \end{aligned}$$

with $F = \text{diag}\,\{\frac{1}{\lambda_i},\ i = 1,N\}$, $\gamma > 1/2 \min(\lambda_i, \lambda_w)$, $\lambda_i, \lambda_w > 0$, yield $\{\underline{\delta}(k)\}$ be a uniformly bounded sequence, and moreover

(3.2) $\lim_{k\to\infty} \eta(k) = 0$

(3.3) $\lim_{k\to\infty} \tilde{\underline{\phi}}(k)\ \eta(k) = 0$

Let us define the control input as

(3.4) $u(t) = \underline{K}^T(k)\ \underline{\psi}(t), \quad t \in [t_k, t_{k+1})$

where

(3.5) $\underline{\psi}(t) \triangleq D_f(n\text{-}m\text{-}1)\ \ \underline{\phi}(t);$

equations (3.4) and (2.2) then yield

(3.6) $w_1(t) = u_f(t) - \underline{K}^T(k)\ \ \underline{\phi}(t), \ t \in [t_k, t_{k+1}),$

which, together with (2.5), gives the remaining input to the auxiliary network

(3.7) $\tilde{w}_1(k) = \tilde{u}_f(k) - \underline{K}^T(k\text{-}1)\ \tilde{\underline{\phi}}(k)$

(3.8) $\tilde{u}_f(k) \triangleq \int_{t_{k-1}}^{t_k} \exp -\alpha(t_k-\tau)\ u_f(\tau)\ d\tau.$

4. Stability and Sampling Scheme

A suitable choice of the sampling sequence $\{t_k\}$ is crucial to prove stability of the closed loop system. It is evident, in fact, from (3.1) that if the output of the plant grows without bound in an oscillating fashion, we might choose $\{t_k\}$ such that $\eta(k) = 0$ for every k, and the gains never be updated.

A sufficient requirement on the sampling sequence can be stated as follows:

Theorem 4.1. Let the sampling sequence $\{t_k\}$ have an infinite number of terms, and be such that

(4.1) $\sup_{s \le t_k} |\ e(s)\ | \le M_0 \sup_{n \le k} |\ e(t_n)\ | + M_1$

for some constants $M_0 > 0$, and $M_1 \ge 0$. Then the hybrid system described in the previous sections is uniformly stable and

(4.2) $\lim_{k\to\infty} e(t_k) = 0.$

In fact, equations (1.3), (2.4), (2.7) yield

$$(4.3)\quad x(t_k) = A_k\, x(t_{k-1}) + K_o\, \tilde{\phi}_o(k) - \eta(k) + A_k\eta(k-1) + K_w(k-1)\, \tilde{w}_1(k) - \gamma\, \tilde{\underline{\phi}}^T(k)\, \tilde{\underline{\phi}}(k)\, \eta(k)$$

Since the model is stable, driven by a bounded input, condition (4.1) on $\{t_k\}$ implies

$$(4.4)\quad \sup_{s \le t_k} |x(s)| \le \sup_{s \le t_k} |e(s)| + M_2 \le M_0 \sup_{n \le k} |e(t_n)| + M_3 \le \le M_0 \sup_{n \le k} |x(t_n)| + M_4$$

for some $M_4 \ge 0$.

Combining (4.4) with the results obtained in Appendix A, which yield

$$\tilde{w}_1(k) = o[\sup_{s \le t_k} |x(s)|]$$

$$||\tilde{\underline{\phi}}(k)|| = O[\sup_{s \le t_k} |x(s)|],$$

we obtain

$$(4.5)\quad \tilde{w}_1(k) = o[\sup_{n \le k} |x(t_n)|]$$

$$||\tilde{\underline{\phi}}(k)|| = O[\sup_{n \le k} |x(t_n)|]$$

If we take equations (3.2), (3.3), (4.5) into account, we can write (4.3) in the form

$$(4.6)\quad x(t_k) = A_k x(t_{k-1}) + \beta_0(k) \sup_{n \le k} |x(t_n)| + \beta_1(k)$$

for some sequence β_0, β_1 such that $\lim_{k\to\infty} \beta_0(k) = 0$ and $\beta_1(k)$ uniformly bounded.

It is easy to see that (4.6) implies uniform boundedness of the sequence $\{x(t_k)\}$. Using this result in (4.4) we conclude that the plant output $x(\cdot)$ is uniformly bounded, which proves the first part of the Theorem.

In order to prove (4.2) notice that, by equations (4.5), $\{\underline{\tilde{\phi}}(k)\}$ and $\{\tilde{w}(k)\}$ are uniformly bounded sequences. This fact together with equations (3.1), (3.2), (2.6) implies that for the augmenting network

$$\lim_{k\to\infty} y(k) = 0$$

and (4.2) follows from being $e(t_k) = \eta(k) - y(k)$.

QED

The central idea contained in Theorem 4.1 is that stability of the overall system is guaranteed if the sampled error $\{e(t_k)\}$ grows at the same rate as the continuous time error itself.

In particular, for a fixed rate sampling scheme we define

$$(4.7) \quad t_k \triangleq kT, \text{ for } k = 0, 1, 2, \ldots$$

where T has to be determined in order to guarantee stability of the closed loop system.

In Section 5 it is shown that a suitable sampling frequency F^* can be computed from the knowledge available on the bounds of the plant parameters, and the adaptive control system is stable if t_k is as in (4.7) with $T < \frac{1}{F^*}$.

Before going into the details of the two schemes mentioned above, some preliminary results, which will be used throughout the paper, need to be proved.

From the definition of the problem in Section 2, it turns out that the (2n-1) order polynomial

$$(4.7) \quad b_0(s+\alpha_1) \ldots (s+\alpha_{2n-1}) \triangleq D_m(s)D_f(s)D_n(s)$$

is Hurwitz. Define λ and ρ to be such that

$$(4.8) \quad \rho > |\alpha_i|, \text{ for } i = 1, \ldots, 2n-1$$

$$0 < \lambda < \text{Re}\,[\alpha_i].$$

Then the following can be proved:

Lemma 4.1 For λ and ρ as in (4.8) the following inequalities hold

(4.9) $|\phi_x^i(t)| \le \dfrac{(\lambda+\rho)^i}{\lambda^{2n-m-2}} \sup |x(\tau)|$, for $i = 0, \ldots, n-1$.

(4.10) $|\phi_n^j(t)| \le \dfrac{(\lambda+\rho)^j}{|b_o|\,\lambda^{2n-2}} \sum_o^n k|a_{n-k}|(\lambda+\delta)^k \sup_{\tau \le t} |x(\tau)|$

for $j = 0, \ldots, n-2$,

where a_i, $i = 0, 1, \ldots, n-1$ and b_o are coefficients of the plant transfer function as in (1.1).

Proof. By the definitions in Section 2 we can write $\phi_x{}^i$ as

(4.11) $\phi_x{}^i(t) = \int_o^t h_x{}^i\,(t-\tau)\,x(\tau)d\tau$, $i = 0, \ldots, n-1$

where

$$h_x{}^i(\cdot) = \mathcal{L}^{-1}\left[\frac{s^i}{D_w(s)D_f(s)}\right]$$

Application of the results in Appendix B yields (4.9).

To prove (4.10) notice that $\phi_u{}^i$ can be written as

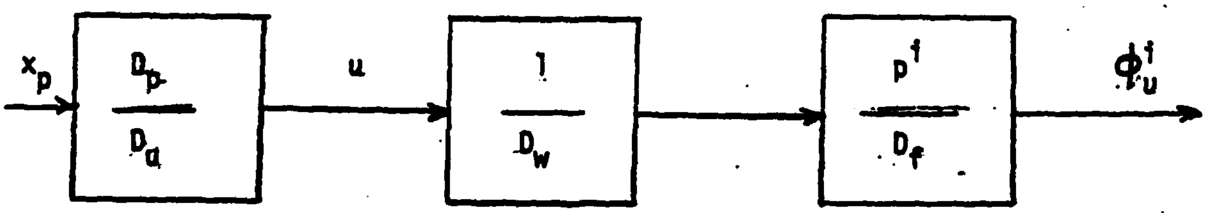

Fig 4.1

$$(4.12) \qquad \phi_u^{\,i}(t) = \frac{1}{b_0} \sum_{0}^{n}{}_{k} \; a_{n-k} \quad \zeta^{i+k}(t), \; i = 0, \ldots, n-2$$

where we define

$$(4.13) \qquad \zeta^k(t) \triangleq \int_0^t h_u^{\,k}(t-\tau)\, x(\tau)\, d\tau$$

$$h_u^{\,k}(\cdot) \triangleq \mathcal{L}^{-1}\left[\frac{s^k}{\frac{1}{b_0} D_u(s) D_w(s) D_f(s)} \right]$$

Appendix B, (4.12) and (4.13) yield (4.10).

QED

<u>Lemma 4.2</u> For the Hybrid Control System discussed in Sections 2 and 3, constants $F^* > 0$ and $M_1 \geq 0$ exist such that, for some $\beta(\cdot)$ with $\beta(t) \to 0$,

$$(4.14) \qquad |\dot{e}(t)| \leq [F^* + \beta(t)] \sup_{\tau \leq t} |e(\tau)| + M_1$$

where e is the output error between the model and the plant.

<u>Proof</u>. Equation (2.3), lemma 4.1 and Appendix A imply that

$$(4.15) \qquad |\dot{e}(t)| \leq \alpha |e(t)| + M_0 \, ||\underline{\delta}(k)|| \sup_{\tau \leq t} |x(\tau)| +$$

$$+ \beta(t) \sup_{\tau \leq t} |x(\tau)|, \quad t \in [t_k, t_{k+1})$$

for some $M_0 > 0$, and $\beta(\cdot)$ such that $\lim_{t\to\infty} \beta(t) = 0$.

Boundedness of the sequence $\{\underline{\delta}(k)\}$, as seen in Section 3, and of the model output, make (4.14) to follow from (4.15).

QED

5. Existence and Determination of a Minimum Sampling Frequency

Throughout this section, bounds on the values of the constants $\underline{\beta} \triangleq [\underline{\beta}_u, \underline{\beta}_x]$ introduced in equation (2.1) are supposed to be available. In particular, we know constants $\hat{\beta}_j^m$, $\hat{\beta}_j^M$ such that

$$(5.1) \qquad \hat{\beta}_j^m \leq \frac{\beta_j}{b_0} \leq \hat{\beta}_j^M, \text{ for } j = 0, 1, \ldots 2n\text{-}2.$$

This enables us to determine an updating law which takes (5.1) into account, and to compute a minimum sampling frequency F* which makes uniformly stable the Hybrid Adaptive Control System defined in Sections 2 and 3.

Lemma 5.1. Let $\{t_k\}$ be an infinite sequence. Then equation (2.10) and the adaptive law given by

$$(5.2) \quad K_j(k) = \begin{cases} \hat{\beta}_j^m & \text{, if } K_j(k-1) + \Delta_j(k) \geq \hat{\beta}_j^m; \\ \hat{\beta}_j^m & \text{, if } K_j(k-1) + \Delta_j(k) \leq \hat{\beta}_j^m; \\ K_j(k-1) + \Delta_j(k) & \text{, otherwise} \end{cases}$$

$$j = 0, 1, \ldots, 2n-2,$$

$$q(k) = -\gamma_1[||\underline{\tilde{\phi}}(k)||^2 + |\tilde{w}_1(k)|^2]\eta(k)$$

$$\delta_w(k) = \delta_w(k-1) + \frac{1}{\lambda_w} \tilde{w}_1(k)\eta(k),$$

with

$$\Delta_j(k) \triangleq \frac{1}{b_0\lambda} \tilde{\phi}_j(k)\eta(k)$$

$$\underline{\tilde{\phi}}(k) = [\tilde{\phi}_0(k), \ldots, \tilde{\phi}_{2n-2}(k)]$$

yields (3.2) and (3.3).

Proof. By (5.2) we can express $K_j(k)$ as

$$(5.3)\qquad K_j(k) = K_j(k-1) + \Delta_j(k) + F_j(k),$$

for some $F_j(k)$ such that

$$(5.4)\qquad \underline{\delta}^T(k)\,\underline{F}(k) \geq 0 \quad \text{for every } k,$$

$$\underline{F}^T(k) \triangleq [F_0(k), \ldots, F_{2n-2}(k)].$$

Let us choose as a candidate Lyapunov function

$$(5.5)\qquad V(k) = \lambda||\underline{\delta}(k)||^2 + \lambda_w|\delta_w(k)|^2 + \eta^2(k).$$

Then (5.3), (2.10), and $\underline{\delta}(k) \triangleq \underline{\beta} - b_0\underline{K}(k)$ yield

$$(5.6)\qquad V(k) - V(k-1) = -(2\gamma_1 - \frac{1}{\lambda})||\tilde{\underline{\phi}}(k)||^2\eta^2(k)$$

$$- (2\gamma_1 - \frac{1}{\lambda_W})|\tilde{w}_1(k)|^2\eta^2(k)$$

$$- [\eta^2(k) - A_k\eta(k)\eta(k-1) + \eta^2(k-1)]$$

$$- b_0\lambda\,||\underline{F}(k)||^2 - 2b_0\lambda[\underline{\delta}(k-1) - \frac{1}{\lambda}\tilde{\underline{\phi}}(k)\eta(k)$$

$$- b_0\underline{F}(k)]^T\underline{F}(k)$$

The last term in square brackets is $\underline{\delta}(k)$; then from (5.4) we conclude that $V(k) - V(k-1) \leq 0$ for every k. This yields $V(\infty) < \infty$, and the lemma is proved.

QED

The existence of a minimum sampling rate F* which guarantees stability of the closed loop system for the Hybrid MRAC discussed in the previous sections, is stated by the following:

Lemma 5.2. Let the sampling sequence be on the form

$$(5.7)\qquad t_k = kT,\ k = 0, 1, \ldots$$

for some constant $T > 0$.

Then a value T* exists such that the Hybrid MRAC described in Sections 2 and 3 is uniformly stable for $T < T^*$, and

$$(5.8)\qquad \lim_{k\to\infty} e(t_k) = 0$$

Proof. Suppose that the continuous time error $e(\cdot)$ grows without bounds.

Then an infinite sequence of time $\{\xi_j\}$ exists such that

(5.9) $$|e(\xi_j)| = \sup_{\tau \le \xi_j} |e(\tau)|$$

$$0 < |e(\xi_j)| < |e(\xi_{j+1})|, \ j = 0, 1, \ldots$$

Let us define a sequence $\{k_j\}$ of integer values such that

(5.10) $$(k_{j-1})T \le \xi_j < k_j T, \ j = 0, 1, \ldots$$

First we can prove that positive constants M_5 and M_6 exist such that

(5.11) $$|e(\xi_j)| < M_5 \ |e(k_j T)| + M_6, \ j = 0, 1, 2, \ldots$$

when $T < \frac{1}{F^*}$, with F^* as in Lemma 4.4.

In fact, suppose (5.11) does not hold; then a sequence $\beta(j)$, such that $\lim_{j \to \infty} \beta(j) = 0$, exists for which

(5.12) $$|e(k_j T)| \le \beta(j) \ |e(\xi_j)|.$$

Since the error $e(\cdot)$ is a continuous function of time--the plant and the model being strictly proper and the adaptive gains uniformly bounded--by Lagrange theorem instants $\tau_j \in (\xi_j, k_j T)$ exist such that

(5.13) $$|\dot{e}(\tau_j)| = \frac{|e(\xi_j) - e(k_j T)|}{|\xi_j - k_j T|}, \ j = 0, 1, \ldots$$

Substituting (5.12) into (5.13) we obtain

(5.14) $$|\dot{e}(\tau_j)| \ge \frac{[1 - \beta(j)]}{T} \ | e(\xi_j)|$$

By the fact that $\beta(j) \to 0$ and $\frac{1}{T} > F^*$, an index N exists such that, for $j > N$,

$$|\dot{e}(\tau_j)| > F^* |e(\xi_j)| = F^* \sup_{\tau \le \tau_j} |e(\tau)|.$$

But this contradicts Lemma 4.2, and then proves that (5.11) is true. Finally, equation (5.11) and Theorem 4.1 prove the Lemma.

QED

The knowledge available on the plant parameter enables us to determine a suitable value for the sampling frequency F^*, as shown in the following

Theorem 5.1. Under the conditions of Lemma 5.2, the overall system is uniformly stable and (5.8) holds, if $T < \frac{1}{F^*}$, with

$$(5.15)\qquad F^* = \max\left\{ \frac{||\underline{\delta}_u||_\infty}{|b_0|\lambda^{2n-2}} \bullet \sum_{0}^{n}{}_k |a_{n-k}|(\lambda+\rho)^k \sum_{0}^{n-2}{}_j (\lambda+\rho)^j,\ \frac{||\underline{\delta}_x||_\infty}{\lambda^{2n-m-2}} \bullet \sum_{0}^{n-1}{}_j (\lambda+\rho)^j,\ \alpha \right\}$$

where $||\underline{\delta}||_\infty \overset{\Delta}{=} \max |\delta_i|$, and λ, ρ as in (4.8).

Proof. Using the result of the previous lemma, we have to show that F^* given by (5.15) satisfies inequality (4.14) of Lemma 4.2.

From equation (2.3) and Appendix A we can write

$$(5.16)\qquad |\dot{e}(t)| \leq \alpha|e(t)| + ||\underline{\delta}_u||_\infty ||\underline{\phi}_u(t)||_\infty + ||\underline{\delta}_x||_\infty ||\underline{\phi}_x(t)||_\infty + |\delta_0||\phi_0(t)| + \beta(t) \sup_{\tau \leq t} |x(\tau)|,$$

where $\lim_{t\to\infty} \beta(t) = 0$.

Application of the results in lemma 4.1 to inequality (5.16), yields

$$(5.17)\qquad |e(t)| \leq [F^* + \beta(t)] \sup_{\tau \leq t} |x(\tau)| + |\delta_0| \sup_{\tau < \infty} |\phi_0(\tau)| + \nu(t)$$

with $\lim_{t\to\infty} \nu(t) = 0$.

Finally, being $|x(t)| \leq |e(t)| + X_M$, where $X_M = \sup_{\tau < \infty} |X_m(\tau)|$, (5.17) yields

$$(5.18) \qquad |e(t)| \leq [F^* + \beta(t)] \sup_{\tau \leq t} |e(\tau)| + F^* X_M + |\delta_0| \sup_{\tau \leq \infty} |\phi_0(\tau)|$$

$$+ \nu(t);$$

which implies that F^* as in (5.15) satisfies inequality (4.14).

QED

Conclusions

An algorithm for hybrid adaptive control of single-input single-output systems has been presented. The parameters of the controller are updated periodically by a digital computer, and the continuous time nature of the closed loop system is preserved.

A bound F^* on the sampling frequency F has been computed in base of the information available on the plant, and uniform stability is shown for $F > F^*$.

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Appendix A

<u>Lemma</u>. $\tilde{w}_1(k) = O[e^{-\lambda Tk} \sup_{s \leq t_k} |x(s)|]$

with λ as in (4.8).

<u>In fact</u>, equations (3.4), (3.6) yield

(A.1) $$D_f(p)w_1(t) = \underline{K}^T(k)D_f(n-m-1)\underline{\phi}(t)-D_f(n-m-1)\underline{K}^T(k)\underline{\phi}(t),\ t \in [t_k, t_{k+1})$$

Using the identity, [1],

(A.2) $$D_f(p)\ \underline{K}^T(k)\ \underline{\phi}(t) = \underline{K}^T(k)\ D_f(p)\ \underline{\phi}(t) + + \sum_{0}^{n-m-2}{}_i\ D_f(i)(p\underline{K}^T(k)(p^{n-m-2-i}\underline{\phi}(t))$$

where we define

(A.3) $$D_f(o) = 1;\ D_f(1) = p + F_1;\ \ldots$$
$$\ldots;\ D_f(n-m-2) = p^{n-m-2} + F_1p^{n-m-3} + \ldots + F_{n-m-2},$$

and considering that, for $t \in [t_k, t_{k+1})$

(A.4) $$p\underline{K}(k) = [\underline{K}(k) - \underline{K}(k-1)]\ \delta(t-t_k)$$

with $\delta(t)$ the Dyrac function, equation (A.1) can be written as

(A.5) $$D_f(p)w_1(t) = \sum_{0}^{n-m-2}{}_i\ D_f(i)[\Delta\underline{K}^T(k)\delta(t-t_k)][p^{n-m-2-i}\underline{\phi}(t)]$$

The polynomial $D_f(n-m-1)$ being arbitrary, it is not restrictive to choose with real zeroes, i.e.

(A.6) $$D_f(p) = (p + \beta_1)\ \ldots\ (p + \beta_{n-m-1}).$$

Partial fraction expansion yields

(A.7) $$\frac{D_f(i)}{D_f(n-m-1)} = \sum_{1}^{n-m-1}{}_j \frac{B_j^i}{p+\beta_j}$$

and (A.5) becomes

(A.8) $$w_1(t) = \sum_{1}^{n-m-1}{}_j w_1{}^j(t),$$

(A.9) $$(p+\beta_j)\, w_1{}^j(t) = \Delta\underline{K}^T(k)[\sum_{0}^{n-m-2}{}_i \beta_j{}^i p^{n-m-2-i}\underline{\phi}(t)]\delta(t-t_k)$$

for $t \in [t_k, t_{k+1})$.

Solution of (A.9) leads to

(A.10) $$w_1{}^j(t) = \sum_{0}^{K}{}_h \Delta\underline{K}^T(h)[Q_j(n-m-2)\underline{\phi}(t)]_{\sigma = t_h} e^{-\beta_j(t-t_n)}$$

where $t \in [t_k, t_{k+1})$, and

(A.11) $$Q_j(n-m-2) \triangleq \sum_{0}^{n-m-2}{}_i \beta_j^i p^{n-m-2-i}$$

Definition (2.5) and equation (A.8) imply that

(A.12) $$\tilde{w}_1(k) = \sum_{1}^{n-m-1}{}_j \tilde{w}_1{}^j(k)$$

where

(A.13) $$\tilde{w}_1{}^j(k) \triangleq \int_{t_{k-1}}^{t_k} e^{-\alpha(t_{k-\tau})} w_1{}^j(\tau)\, d(\tau) =$$

$$= \int_{t_{k-1}}^{t_k} e^{-\alpha(t_k-\tau)} e^{-\beta_j(\tau-t_{k-1})} d\tau \sum_{0}^{k-1}{}_h \Delta K^T(h)[Q_j(n-m-2)\underline{\phi}]_{\sigma=t_h} \cdot$$

$$\cdot e^{-\beta_j(t_{k-1}-t_h)}$$

The following facts hold:

a) $\Delta \underline{K}(k) \to 0$,

b) the elements of the vector Q (n-m-2) $\underline{\phi}$ are strictly proper, linear transformation of x, and u,

c) the plant is minimum phase then, as shown in [7];

$$|u(t)| = 0\,[\sup_{\tau \leq t} |x(\tau)|].$$

Facts a), b), c) imply that

$$|\tilde{w}_1{}^j(k)| \leq \frac{e^{-\beta_j Tk}}{|\beta_j - \alpha|} \quad e^{(\beta_j - \alpha)Tk} - 1 \quad \beta(k) \sup_{\tau \leq t_k} |x(\tau)| =$$

$$= o[e^{-\lambda T_k} \sup_{\tau \leq t_k} |x(\tau)|], \forall_j$$

with λ as in (4.8).

QED

Appendix B

Let $D(s) \triangleq (s + \alpha_1)(s + \alpha_2) \ldots (s + \alpha_n)$ be a Hurwitz polynomial, and let $\lambda, \rho \in R$ be such that

(1) $\quad \mathrm{Re}\,[\alpha_i] > \lambda > 0$, for $i = 1, 2, \ldots, n$

$\qquad \rho > |\alpha_i| \quad ;$

the following can be proved:

Lemma. For every pair of functions $x,y: R \to R$ related by the linear transformation

(2) $$y(t) = \int_0^t h^k(t-\tau)\, x(\tau)d\tau$$

where

(3) $$h^k(\cdot) \triangleq \mathcal{L}^{-1}\left[\frac{s^k}{D(s)}\right],\ 0 \le k \le n,$$

the inequality

(4) $$|y(t)| \le \frac{(\lambda + \rho)^k}{\lambda^n} \sup_{\tau \le t} |x(\tau)|$$

holds for every $t \ge 0$.

Proof. Let us define

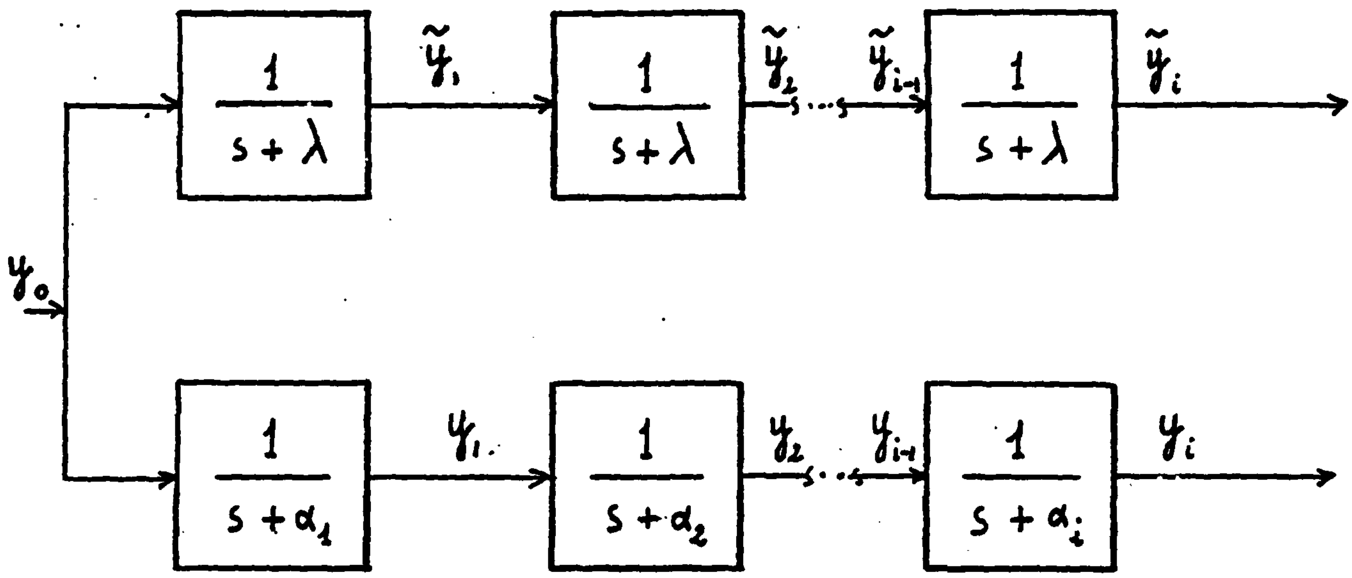
(5) $$Z_i^j(\cdot) \triangleq \mathcal{L}^{-1}\left[\frac{s^j}{(s+\alpha_1) \ldots (s+\alpha_i)}\right]$$

$$\tilde{Z}_i(\cdot) \triangleq \mathcal{L}^{-1}\left[\frac{1}{(s + \lambda)^i}\right]$$

for $i = 1, 2, \ldots, n$ and $0 \le j < i$.

First notice that

(6) $|Z_i^{\,o}(t)| \leq \tilde{Z}_i(t) = \dfrac{t^{i-1}}{(i-1)!}\; e^{-\lambda t}.$



This can be seen from figure B.1 according to the following considerations. Let Y_o: $R \to R+$ be any non-negative valued function. Proceeding by induction we can show that

(7) $\tilde{Y}_i(t) \geq |Y_i(t)|$, for every $t \in \Pi^+$.

In fact

(8) $|Y_1(t)| \leq \displaystyle\int_0^t e^{-\alpha_1(t-\tau)} Y_o(\tau)d\tau \leq \int_0^t e^{-\lambda(t-\tau)} Y_o(\tau)d\tau = \tilde{Y}_1(t)$

which proves (7) for $i = 1$. Suppose (7) is true for $i \leq j - 1$; then if α_j is real we obtain

(9) $|Y_j(t)| \leq \displaystyle\int_0^t e^{-\alpha_j(t-\tau)} \left| Y_{j-1}(\tau) \right| d\tau \leq \int_0^t e^{-\lambda(t-\tau)} \tilde{Y}_{j-1}(\tau)d\tau \leq \tilde{Y}_j(t)$

and if $\alpha_j = \alpha^*_{j-1}$ is complex we obtain

$$(9') \quad |Y_j(t)| \le \int_0^t (t-\tau)e^{-\mathrm{Re}[\alpha_j](t-\tau)} \left|Y_{j-2}(\tau)\right| d\tau \le \tilde{Y}_j(t)$$

which proves (7). Finally, using definitions (5), inequality (7) implies

$$(10) \quad \int_0^t |Z_i^{\,o}(t-\tau)|Y_o(\tau)\, d\tau \le \int_0^t \tilde{Z}_i(t-\tau)\, Y_o(\tau)\, d\tau$$

for every non-negative valued function Y_o, and then (6) follows directly.

In order to prove (4) notice that the following recursion

$$(11) \quad Z_i^{\,k} = Z^{k+1}_{i+1} + \alpha_{i+1}\, Z^{k}_{i+1} \qquad , \quad 0 \le k < i$$

holds from the fact that

$$\frac{s^j}{(s+\alpha_1)\ldots(s+\alpha_i)} = \frac{s^j(s+\alpha_{i+1})}{(s+\alpha_1)\ \ldots\ (s+\alpha_i)(s+\alpha_{i+1})}$$

This enables us to write

$$(12) \quad \left|Z^{j+1}_{i+1}\right| \le \left|Z^{j}_{i}\right| + \rho\left|Z^{j}_{i+1}\right|$$

where ρ is as in (1).

Using definitions (2), (3) and (5) we can write

$$(13) \quad |Y(t)| \le q^k_n \sup_{\tau \le t} |x(\tau)|$$

where we define

$$(14) \quad q^j_i \triangleq \int_0^\infty \left|Z^j_i(\tau)\right| d\tau \ , \text{ for } o \le j < i, \quad i = 1, 2, \ldots, n.$$

In particular, by inequalities (6) and (12) the following recursion holds

(15) $q_{i+1}^{j+1} \leq q_i^j + \rho\, q_{i+1}^j$

(16) $0 < q_i^0 \leq \int_0^\infty \tilde{Z}_i(\tau)\, d\tau = \dfrac{1}{\lambda^i}$

By induction we can prove that inequalities (15) and (16) yield

(17) $q_i^j \leq \dfrac{(\lambda + \rho)^j}{\lambda^i}$

In fact, for $j = 0$ and every $i = 1, 2, \ldots, n$ (17) is proved by (16). Furthermore, by (15) and (17) we obtain

$$q_{i+1}^{j+1} \leq \frac{(\lambda + \rho)^j}{\lambda^i} + \rho \frac{(\lambda + \rho)^j}{\lambda^{i+1}} = \frac{(\lambda + \rho)^{j+1}}{\lambda^{i+1}}$$

which proves (17).

Finally by (14), (13) and (17) the Lemma is proved.

<u>QED</u>

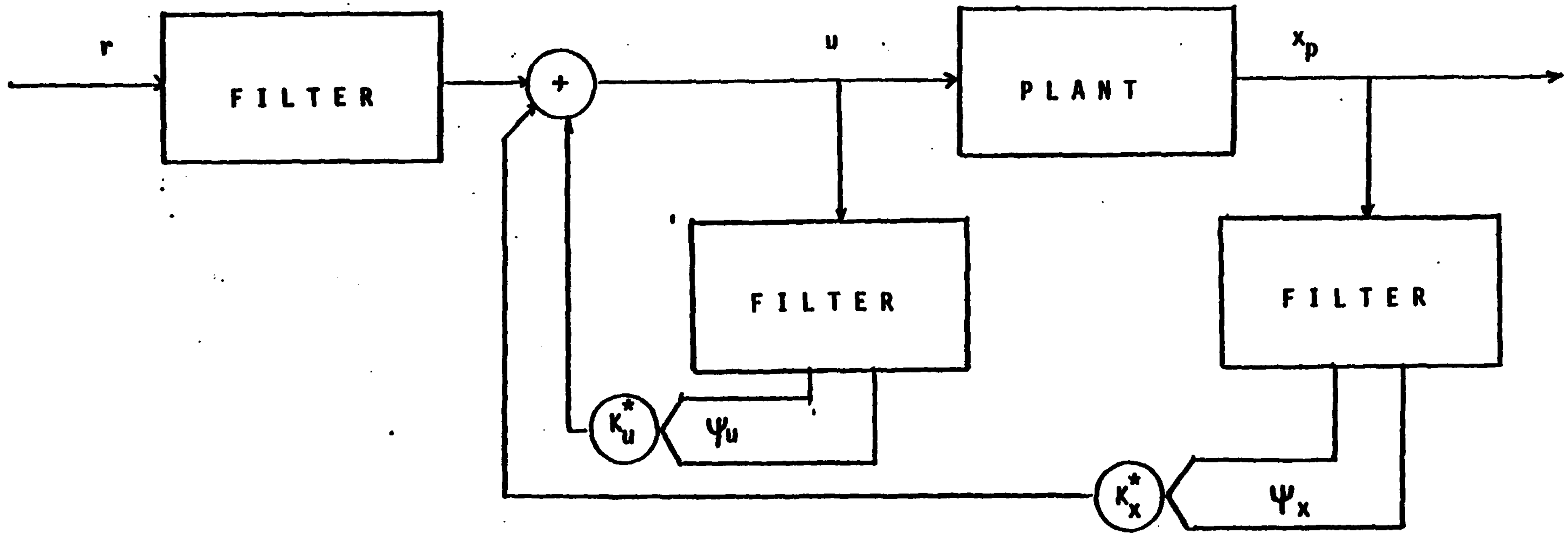


Fig.1 Algebraic Problem.

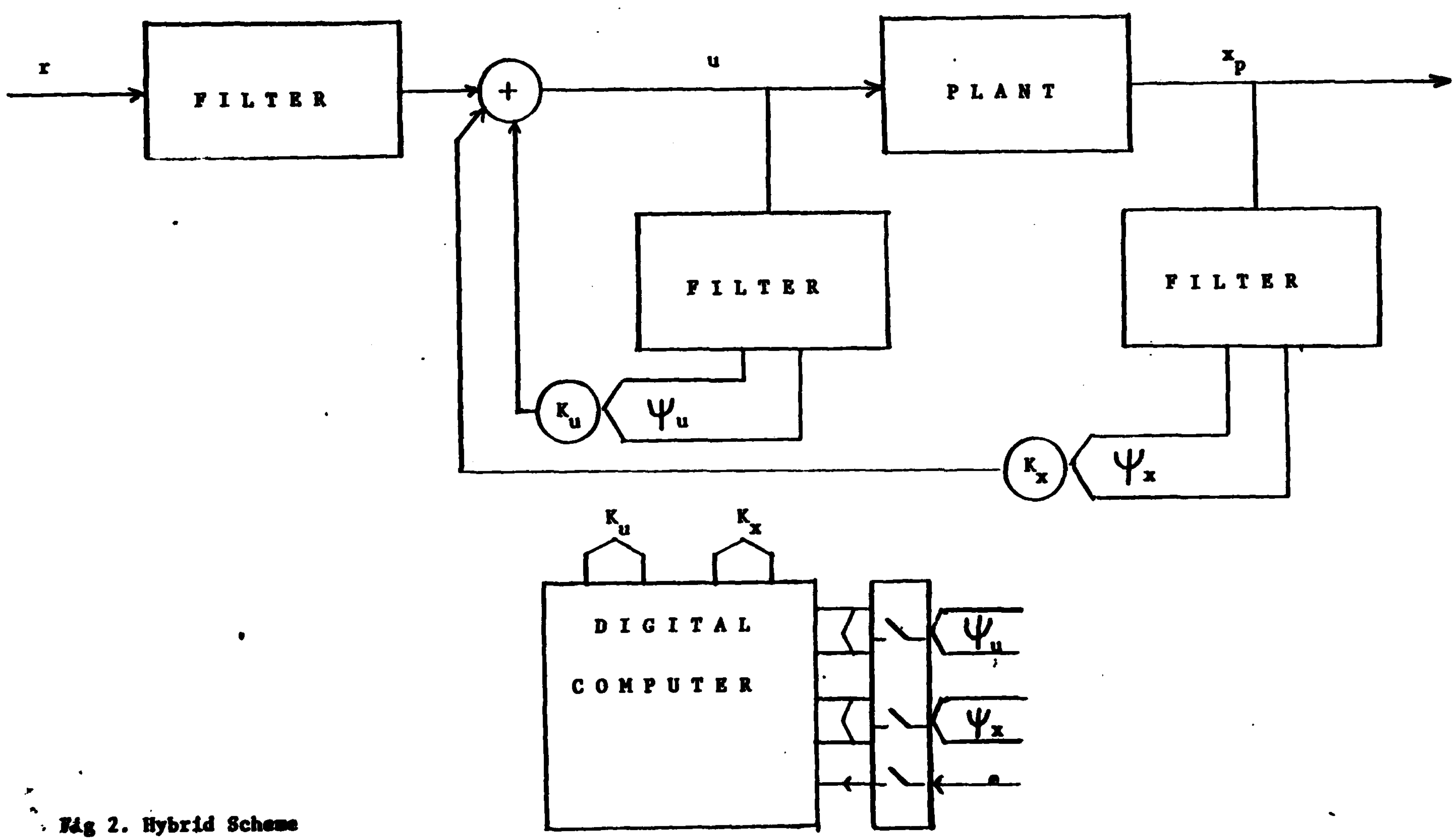
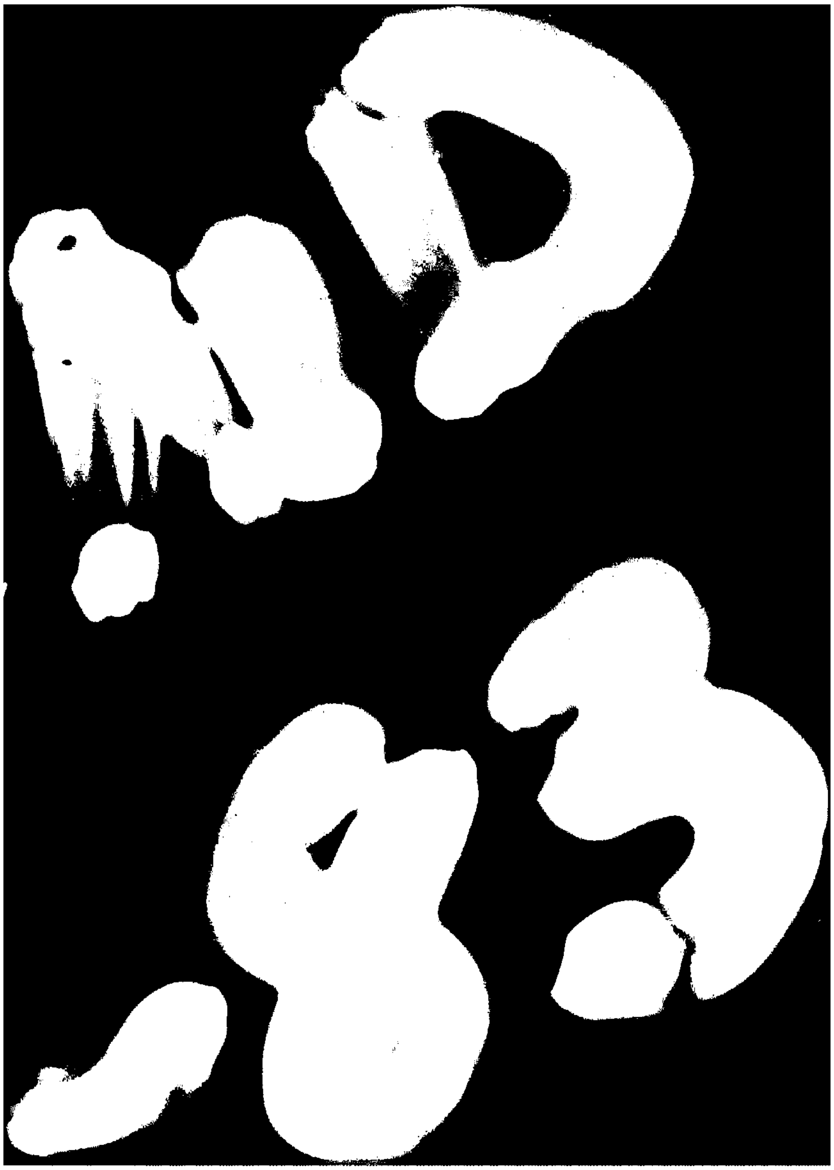


Fig 2. Hybrid Scheme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