A General Algorithm for the Calculation of Laser Beam Spreading

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By

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Atmospheric Sciences Laboratory
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A GENERAL ALGORITHM FOR THE CALCULATION OF LASER BEAM SPREADING

Please change subject document as follows.

Page 10  Second line from top of page, change "... [(equations (1) and (2a)]" to "... [equations (1) and (2a)]"

Page 10  Third line from top of page, change "... equations (3), (4), (5), and (6);" to "... [equations (3), (4), (5), and (6)]"

Page 11  First line on page, change "... [(equations (2a) and (2b)]" to "... [equations (2a) and (2b)]"

Page 14  Last line in figure 3 caption, change "... \( \Omega = \text{to } 20 \)" to "... \( \Omega = 2.0 \)"
This paper compares two different beam spreading formalisms in the limits of near- and far-field, short exposure and long exposure. A description is given of an interpolation algorithm bridging the limits, giving entirely reasonable results in the intermediate regimes while having the proper limiting values.
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Laser beam spreading is an unavoidable consequence of finite aperture size (resulting in diffraction) and atmospheric turbulence (resulting in temporal variations in the index of refraction, which in turn induces beam spread). Although the quantification of beam spreading is important in the design and subsequent performance of electro-optical systems, (e.g., laser designators and homing seekers), previous calculations of turbulence effects have been limited to the cases of very short exposure in the near-field and far-field and to the case of very long exposure. 1 2 The difficulty in quantifying turbulence effects is in the accurate representation of the stochastic nature of turbulence.

In the near-field limit, the beam profile maintains its initial shape. As the beam propagates, however, in the absence of turbulence the beam profile gradually changes to a diffraction-induced profile in the far-field limit (e.g., an "Airy pattern" for a uniform circular beam). An indication of whether we are in the near-field or far-field limit is determined by the value of the Fresnel number $\alpha = \frac{kD^2}{\lambda L}$. The variables are the wavenumber $k = \frac{2\pi}{\lambda}$, $\lambda$ = wavelength, $D$ = aperture diameter, and $L$ = path length. When $\alpha >> 1$, we are in the near-field limit; when $\alpha << 1$, we are in the far-field limit.

It is readily apparent that for typical laser designator parameters (e.g., $\lambda = 10.6 \mu$m, $D = 0.1$ m, and $L = 3$ km), $\alpha = 2$; thus, neither the near-field or far-field limit is applicable. The condition of short or long exposure is determined by a comparison of the pulse duration time $t$ with the time $t_c$ it takes a turbulent eddy whose size is on the order of the beam size to cross the beam. An indication of whether we are in the short or long exposure limit is given by the parameter $\tau = \frac{t_c}{t + t_c}$, where $\tau \approx 1$ in the short exposure limit and $t \approx 0$ in the long exposure limit. For an eddy $\approx 0.1$ m and an eddy velocity $\approx 10$ m/s, $t_c = \frac{0.1}{10} = 0.01$ ms. Since pulse duration times range from nanoseconds ($\tau \approx 1$) to seconds ($\tau \approx 0$), neither the short nor long exposure limits are, in general, applicable.


Since, as shown above, the limiting cases do not apply to typical scenarios, a
general expression for beam spreading, valid for arbitrary values of \( \sigma \) and \( t \),
is needed. Using an interpolation scheme, this paper describes an algorithm
that bridges the near- and far-field, short exposure and long exposure
limits. Also, comparisons are made of laser beam spreading due to turbulence
as calculated by Fried\(^1\) in the near-field, short exposure and long exposure
limits, with that calculated by Breaux.\(^2\)

**Short and Long, Exposure Limit Expressions**

Fried\(^1\) derived expressions for the average resolution of very long and very
short exposure images in terms of the modulation transfer function (MTF). In
the short exposure case, the image sharpness and the MTF are insensitive to
the tilt of the wavefront. In the long exposure case, the random variations
in the tilt substantially add to the image spreading and, therefore, result in
poorer resolution.

Beam spreading, using Fried's\(^1\) resolution expressions, is calculated from the
following equation.

\[
\frac{r_T}{r_D} = \left( \frac{D}{r_0} \right) R
\]

where \( r_T \) = turbulence- and diffraction-induced beam spread,

\[
r_D = \text{diffraction-induced beam spread},
\]

\[
\frac{D}{r_0} = \text{strength of turbulence},
\]

\[
D = \text{aperture diameter},
\]

\[
r_0 = \text{coherence length} = 1.67 \left( \int_0^L C_n^2(z) W(z) dz \right)^{-3/5},
\]

\[
C_n^2 = \text{refractive index structure constant},
\]

\[
W = \text{geometry-dependent weighting function}, \text{ and}
\]

\[
L = \text{total propagation distance}.
\]

The function \( R \), using the notation of Fried,\(^1\) is given by

\(1\) Fried, D. L., "Optical Resolution Through a Randomly Inhomogeneous Medium for

\(2\) Breaux, Harold J., "Correlation of Extended Huygens-Fresnel Turbulence
Calculations for a General Class of Tilt Corrected and Uncorrected Laser
Apertures," BRL IMR 600, Ballistic Research Laboratory, Aberdeen Proving
\[
R = \begin{cases} 
\left( \frac{n R_0}{R_{\text{max}}} \right)^{-1/2} & \text{near-field, short exposure limit (} a \gg 1, \tau + 0), \\
\left( \frac{f R_0}{R_{\text{max}}} \right)^{-1/2} & \text{far-field, short exposure limit (} a \ll 1, \tau + 0), \\
\left( \frac{R}{R_{\text{max}}} \right)^{-1/2} & \text{long exposure limit (} \tau + 1),
\end{cases}
\]

where the expressions for the resolution, normalized by the limiting resolution \( R_{\text{max}} \), are given by

\[
\frac{n R_0}{R_{\text{max}}} = \left\{ \frac{16}{x} (\frac{D}{R_0})^2 \int_0^1 u \left[ \cos^{-1}(u) - u(1-u)^{1/2} \right] \\exp \left[ -3.44 (\frac{D}{R_0})^{5/3} u^{5/3} (1-u^{1/3}) \right] \right\}, \tag{2a}
\]

\[
\frac{f R_0}{R_{\text{max}}} = \left\{ \cdots \right\} x \exp \left[ -3.44 (\frac{D}{R_0})^{5/3} u^{5/3} (1-0.5 u^{1/3}) \right], \tag{2b}
\]

and

\[
\frac{R}{R_{\text{max}}} = \left\{ \cdots \right\} x \exp \left[ -3.44 (\frac{D}{R_0})^{5/3} u^{5/3} \right]. \tag{2c}
\]

The quantity in braces corresponds to the optical-system MTF and the exponential factor corresponds to the atmospheric MTF.

In figure 1 the normalized resolutions given in equations (2a), (2b), and (2c) are plotted as a function of strength of turbulence. Note that in the limits of very strong \((\frac{D}{R_0} \gg 1)\) and very weak \((\frac{D}{R_0} \ll 1)\) turbulence, the results approach a single curve. In the limit of strong turbulence, the exponential factors in (2a), (2b), and (2c) go to zero. In this case all the resolutions approach the value of the limiting resolution in the long exposure limit, \( R_{\text{max}} \); thus, the normalized resolutions all approach unity. In the limit of weak turbulence, the exponential factors go to unity. In this case, all the
normalized resolutions are proportional to the factor \( \left( \frac{D}{r_0} \right)^2 \) which shows up as the linear section of the curve seen in figure 1. However, in the intermediate regime \( \left( \frac{D}{r_0} \sim 3 \right) \), all three expressions differ by as much as a factor of 5. Because turbulence is often encountered in this intermediate regime, the need exists for an expression for beam spreading that gives values intermediate to the extremes.

Breaux's approach to the calculation of beam spreading was to develop beam spreading algorithms in terms of a scaling variable in the near-field limit. The scaling variable chosen was the strength of turbulence \( \frac{D}{r_0} \) for various beam shapes, where \( D \) is the effective aperture diameter and \( r_0 \) is the coherence length.

Figure 1. Normalized resolution \( R \) as a function of strength of turbulence \( \frac{D}{r_0} \) in the near-field, short exposure limit; far-field, short exposure limit; and long exposure limit.

---


Breaux's expression for beam spreading,

\[ \frac{r_T}{r_D} = \text{SR}^{-1/2} \]  

is calculated as a function of the Strehl ratio,

\[ \text{SR} = \frac{I_T}{I_D} \]  

where \( I_T \) = on-axis irradiance with turbulence and diffraction and \( I_D \) = on-axis irradiance with only diffraction. The quantities \( r_T \) and \( r_D \) have the same meanings as in equation (1).

For a uniform beam, the long-term Strehl ratio is

\[ \text{SR}_{LT} = \left[ 1 + \left( \frac{D}{r_0} \right)^2 \right]^{-1} \]  

For a uniform beam, the near-field, short exposure Strehl ratio is

\[ \text{SR}_{ST} = \begin{cases} \left[ 1 + 0.182 \left( \frac{D}{r_0} \right)^2 \right]^{-1} & \frac{D}{r_0} \leq 3 \\ \left[ 1 + \left( \frac{D}{r_0} \right)^2 - 1.18 \left( \frac{D}{r_0} \right)^{5/3} \right]^{-1} & \frac{D}{r_0} > 3 \end{cases} \]  

Short Exposure Comparison: Near-Field, Far-Field and Intermediate Cases

Fried's\textsuperscript{1} integral expression [(equations (1) and (2a))] are compared with Breaux's\textsuperscript{2} analytical expression equations (3), (4), (5), and (6) in the near-field, short exposure limit in table 1. Equation (2a) was evaluated numerically using a 1/2 Simpson rule integration. The agreement between the two sets of results is excellent. For \( \frac{D}{r_0} \approx 2 \), the results are within 1 percent of each other; for \( \frac{D}{r_0} \approx 4 \), the results are within 6 percent of each other.

\begin{table}[h]
\centering
\caption{Comparison of Breaux's Analytical Expression (6) With Fried's Integral Expression (2a) in the Near-Field, Short Term Limit.}
\begin{tabular}{ccccccc}
\hline
D(m) & \( C_n^2 \) (m\(^{-2/3}\)) & \( r_0 \) (m) & \( \frac{D}{r_0} \) & \( \alpha \) & Fried(2a) & Breaux(6a,b) \\
\hline
0.1 & \( 1 \times 10^{-13} \) & 0.2363 & 0.4233 & 2.964 & 0.2441 & 0.2430 \\
0.1 & \( 5 \times 10^{-13} \) & 0.0900 & 1.112 & 2.964 & 0.2646 & 0.2647 \\
0.1 & \( 1 \times 10^{-12} \) & 0.0593 & 1.685 & 2.964 & 0.2921 & 0.2945 \\
0.4 & \( 1 \times 10^{-13} \) & 0.2363 & 1.693 & 47.42 & 0.0731 & 0.0737 \\
0.4 & \( 5 \times 10^{-13} \) & 0.0900 & 4.447 & 47.42 & 0.1445 & 0.1534 \\
0.4 & \( 1 \times 10^{-12} \) & 0.0593 & 6.740 & 47.42 & 0.2505 & 0.2540 \\
\hline
\end{tabular}
\end{table}

For the far-field, short exposure limit, a similar comparison cannot be made owing to the lack of an analytical expression analogous to (6) in the far-field limit. However, for values of \( \frac{D}{r_0} \approx 1 \), the near-field and far-field, short exposure values can differ by more than 50 percent. In order to bridge the near-field and far-field limits, the short exposure limit resolutions


\textsuperscript{2}Breaux, Harold J., "Correlation of Extended Huygens-Fresnel Turbulence Calculations for a General Class of Tilt Corrected and Uncorrected Laser Apertures," BRL IMR 600, Ballistic Research Laboratory, Aberdeen Proving Ground, MD, 1978.
[(equations (2a) and (2b)) were combined in the following interpolation expression:

\[
\frac{\ln R_0}{R_{\text{max}}} = \left\{ \ldots \right\} \times \exp \left[ -3.44 \left( \frac{D}{r_0} \right)^{5/3} u^{5/3} (1-F \cdot u^{1/3}) \right]
\]

(7)

where \(\left\{ \ldots \right\}\) is the MTF of the optical system listed in (2a), the near-field/far-field interpolation factor \(F = \frac{a+1}{a+2}\), and \(a\) is the Fresnel number.

The interpolated, normalized resolution given in equation (7) was used in conjunction with equation (1) to get the beam spread.

In table 2 we compare Fried's\(^1\) near-field and far-field, short exposure results for beam spreading with the interpolation expression. Consistent with the resolutions shown in figure 1, the beam spread values approach one another for \(\frac{D}{r_0} = 0.4233\), diverging for \(\frac{D}{r_0} > 1\). The interpolation formula in the present study correctly goes to the proper limits in the near-field \((a >> 1)\) cases. For intermediate cases \((a \sim 1)\), the results fall between the near- and far-field limits.

**TABLE 2. BEAM SPREAD IN SHORT EXPOSURE LIMIT.**

(path length = 2 km, wavelength = 10.6μm)

<table>
<thead>
<tr>
<th>(D) (m)</th>
<th>(C_n^2)</th>
<th>(r_0) (m)</th>
<th>(\frac{D}{r_0})</th>
<th>(\alpha)</th>
<th>(r_T) (m)</th>
<th>(\text{near-field})</th>
<th>(\text{far-field})</th>
<th>(\text{in} \rightarrow)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>(1 \times 10^{-12})</td>
<td>0.2363</td>
<td>0.4233</td>
<td>2.964</td>
<td>2.964</td>
<td>0.2441</td>
<td>0.2565</td>
<td>0.2491</td>
</tr>
<tr>
<td>0.1</td>
<td>(5 \times 10^{-13})</td>
<td>0.0900</td>
<td>1.112</td>
<td>2.964</td>
<td>2.964</td>
<td>0.2646</td>
<td>0.3266</td>
<td>0.2906</td>
</tr>
<tr>
<td>0.1</td>
<td>(1 \times 10^{-12})</td>
<td>0.0593</td>
<td>1.685</td>
<td>2.964</td>
<td>2.964</td>
<td>0.2921</td>
<td>0.4128</td>
<td>0.3457</td>
</tr>
<tr>
<td>0.4</td>
<td>(1 \times 10^{-12})</td>
<td>0.2363</td>
<td>1.693</td>
<td>47.42</td>
<td>47.42</td>
<td>0.0731</td>
<td>0.1035</td>
<td>0.0746</td>
</tr>
<tr>
<td>0.4</td>
<td>(5 \times 10^{-13})</td>
<td>0.0900</td>
<td>4.447</td>
<td>47.42</td>
<td>47.42</td>
<td>0.1445</td>
<td>0.2386</td>
<td>0.1539</td>
</tr>
<tr>
<td>0.4</td>
<td>(1 \times 10^{-12})</td>
<td>0.0593</td>
<td>6.740</td>
<td>47.42</td>
<td>47.42</td>
<td>0.2505</td>
<td>0.3604</td>
<td>0.2651</td>
</tr>
</tbody>
</table>

*Includes diffraction and turbulence

\(\text{in} \rightarrow\) interpolation

Though the interpolation scheme is ad hoc, the results are reasonable since one expects a smooth, continuous shift from the near-field to far-field results, as a function of decreasing Fresnel number.

Long Exposure Comparison

In the limit of long exposures, the random tilt variations are responsible for most of the distortion and beam spread. Thus, no distinction between near-field and far-field is necessary.

Fried's\(^1\) integral expression is compared with Breaux's\(^2\) analytical expression in the long exposure limit in table 3. A reasonably good agreement exists between the two sets of calculations, always within at least 7 percent of each other. The results, using Breaux's formalism, yield slightly smaller values for the beam spread; the differences are not considered significant.

Beam spread in the long exposure limit has larger values compared with beam spread for the same parameters in the short exposure limit (compare tables 3 and 2). This difference occurs because the long exposure beam spread consists of random tilt variations along with the short exposure beam spreading; therefore, beam spread in the long exposure limit will always be larger than in the short exposure limit.

Short, Long, and Intermediate Exposure Comparison

Analogous to the interpolation scheme used to bridge the near-field/far-field results, an interpolation factor for short and long exposures was incorporated into an expression for the normalized resolution. The normalized resolution now becomes:

\[
\frac{R_{SE-LE}}{R_{\text{max}}} = \left\{ \frac{1}{\tau} \right\} \exp \left(-3.44 \frac{D}{r_0} \right)^{5/3} u^{5/3} \cdot (1 - \tau \cdot F \cdot u^{1/3})
\]  

(8)

where \(\tau\) is the short exposure/long exposure interpolation factor.

---


TABLE 3. COMPARISON OF BREAUX'S ANALYTICAL EXPRESSION (5) WITH FRIED'S INTEGRAL EXPRESSION (2c) IN THE LONG EXPOSURE LIMIT.

(path length = 2 km, wavelength = 10.6 μm)

<table>
<thead>
<tr>
<th>D(m)</th>
<th>C^2_n (m^-2/3)</th>
<th>r_0 (m)</th>
<th>( \frac{D}{r_0} )</th>
<th>a</th>
<th>Beam Spread ( r_T ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1 x 10^-11</td>
<td>0.2363</td>
<td>0.4233</td>
<td>2.964</td>
<td>0.2685</td>
</tr>
<tr>
<td>0.1</td>
<td>5 x 10^-11</td>
<td>0.0900</td>
<td>1.112</td>
<td>2.964</td>
<td>0.3802</td>
</tr>
<tr>
<td>0.1</td>
<td>1 x 10^-12</td>
<td>0.0593</td>
<td>1.685</td>
<td>2.964</td>
<td>0.5013</td>
</tr>
<tr>
<td>0.4</td>
<td>1 x 10^-11</td>
<td>0.2363</td>
<td>1.693</td>
<td>47.42</td>
<td>0.1258</td>
</tr>
<tr>
<td>0.4</td>
<td>5 x 10^-11</td>
<td>0.0900</td>
<td>4.447</td>
<td>47.42</td>
<td>0.2864</td>
</tr>
<tr>
<td>0.4</td>
<td>1 x 10^-12</td>
<td>0.0593</td>
<td>6.740</td>
<td>47.42</td>
<td>0.4232</td>
</tr>
</tbody>
</table>

In figure 2 the normalized resolutions given in the short exposure limit using the near-field/far-field interpolated resolution given in equation (7), the long exposure limit using equation (2c), and the interpolated case using the short exposure/long exposure resolution given in equation (8) are presented as a function of strength of turbulence. For the interpolated resolution, the Fresnel number \( \alpha \) was set to 2 and the pulse time over eddy cross-time \( t/t_c \) was set to 0.11. Note that the interpolated results always lie between the two limiting results and that, as in figure 1, the results go to the same values in the limit of weak or strong turbulence.

Beam spreading, using the normalized resolutions plotted in figure 2, is tabulated in table 4 for a pathlength of 2 km, wavelength of 10.6 μm, and a short/long exposure interpolation factor \( \tau \) of 0.5. Note that the long exposure beam spread is always larger than the short exposure value—as it should be. The intermediate cases all fall between the long and short exposure values. For small values of \( \frac{D}{r_0} \) (e.g., the first entry in table 4, \( \frac{D}{r_0} = 0.4233 \)), the differences between the three formulations are small. For values of \( \frac{D}{r_0} > 1 \), the differences become appreciable. For the 0.4 m aperture diameter, \( \frac{D}{r_0} = 4.447 \) entry, the long exposure value is 86 percent greater than the short exposure value. The intermediate value is 56 percent larger than the short exposure value and 19 percent smaller than the long exposure value.

Beam spread in the short, long, and intermediate exposure cases is plotted as a function of strength of turbulence in figure 3. In calculating the values for the intermediate cases, \( \alpha \) was set to 2 and \( t/t_c = 0.11 \) (equivalent to \( \tau = \) ...
Figure 2. Normalized resolution $R$ as a function of strength of turbulence $D_0$ in the short exposure limit (using the near-field/far-field interpolated resolution given by equation (7)); long exposure limit, and interpolated case (using the short exposure/long exposure resolution given by equation (8)). For the interpolated case, $\alpha = 2.0$ and $t/t_c = 0.11$.

Figure 3. Beam spread (in meters) as a function of strength of turbulence $D_0$ in the short exposure, long exposure and interpolated cases. For the interpolated case, $\alpha = 2.0$ and $t/t_c = 0.11$. 
Again note the convergence to the same beam spread value for weak \( \frac{D}{\rho_0} \ll 1 \) and strong \( \frac{D}{\rho_0} \gg 1 \) turbulence. The intermediate exposure curve \( r_0 \) falls between the two extremes with the differences largest around \( \frac{D}{\rho_0} \approx 1 \).

**TABLE 4. BEAM SPREAD IN THE SHORT EXPOSURE, LONG EXPOSURE, AND INTERMEDIATE CASE.**

(path length = 2 km, wavelength = 10.6\( \mu \)m and \( \tau = 0.5 \))

<table>
<thead>
<tr>
<th>( D(\text{m}) )</th>
<th>( C_n^2(\text{m}^{-2/3}) )</th>
<th>( r_0(\text{m}) )</th>
<th>( \frac{D}{\rho_0} )</th>
<th>( \alpha )</th>
<th>Beam Spread ( r_T(\text{m}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>( 1 \times 10^{-13} )</td>
<td>0.2363</td>
<td>0.4233</td>
<td>2.964</td>
<td>0.2491</td>
</tr>
<tr>
<td>0.1</td>
<td>( 5 \times 10^{-13} )</td>
<td>0.0900</td>
<td>1.112</td>
<td>2.964</td>
<td>0.2906</td>
</tr>
<tr>
<td>0.1</td>
<td>( 1 \times 10^{-12} )</td>
<td>0.0593</td>
<td>1.685</td>
<td>2.964</td>
<td>0.3457</td>
</tr>
<tr>
<td>0.4</td>
<td>( 1 \times 10^{-13} )</td>
<td>0.2363</td>
<td>1.693</td>
<td>47.42</td>
<td>0.0746</td>
</tr>
<tr>
<td>0.4</td>
<td>( 5 \times 10^{-13} )</td>
<td>0.0900</td>
<td>4.447</td>
<td>47.42</td>
<td>0.1539</td>
</tr>
<tr>
<td>0.4</td>
<td>( 1 \times 10^{-12} )</td>
<td>0.0593</td>
<td>6.740</td>
<td>47.42</td>
<td>0.2651</td>
</tr>
</tbody>
</table>

*SE = Short exposure  
†LE = Long exposure  
‡IN = Intermediate
SUMMARY OF BEAM SPREADING RESULTS

Comparison of Fried's near-field, short exposure expressions and long exposure expressions with those of Breaux showed excellent overall agreement. The interpolation expression of the present study shows the proper limiting results in the near- and far-field, short exposure cases and in the long exposure case. It also gives entirely reasonable results in the intermediate cases. It would be preferable—for ease of computation—to have an analytical interpolation expression similar to the limiting expressions developed by Breaux. The lack of an analytical, far-field, short exposure expression, however, prevents formulating such an expression.

The complete beam spreading expression is given by

\[ r_T = r_T(\frac{D}{r_0}) \cdot \left\{ \frac{16(D/r_0)^2}{\pi} \int_0^1 u \cos^{-1}(u) - u(1-u^2)^{1/2} \right\} \]
\[ \times \exp \left[ -3.44(D/r_0)^{5/3} u^{5/3} \cdot (1-\tau F u^{1/3}) \right] \]
\[ \cdot (-1/2) \]

where \( \tau = \frac{t}{t + t_c} \), and \( F = \frac{a + 1}{a + 2} \), and

where \( \tau \) interpolates between the short and long exposure limits and \( F \) interpolates between the near- and far-field limits.


REFERENCES


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