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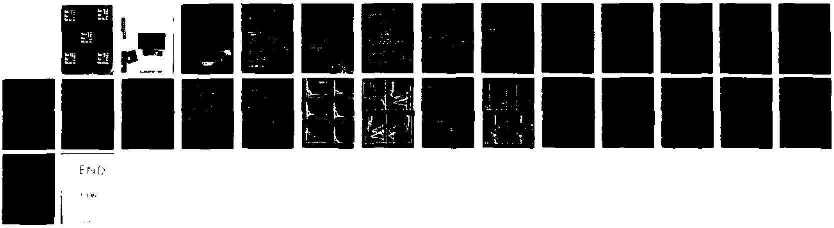

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STABLE HYBRID ADAPTIVE CONTROL

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Stable Hybrid Adaptive Control

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Abstract: The paper deals with hybrid adaptive control of single-input single-output linear dynamical systems with unknown parameters. The system operates in continuous time while control parameters are updated only at discrete instants. Using a hybrid error model it is shown that adaptive algorithms used in discrete and continuous systems can be directly extended to hybrid systems. The resulting nonlinear time-varying systems are globally stable and independent of the frequency with which the parameters are adjusted.

Key Words. Hybrid Systems, Error Models, Global Stability.

1. **INTRODUCTION:** Stable adaptive algorithms for continuous systems (Narendra et al, 1980, Morse 1980) and discrete systems (Narendra and Lin, 1980, Goodwin et al, 1980) are currently well-known. This paper deals with hybrid systems in which the plant to be controlled operates in continuous time while the control parameters are updated at discrete instants. The main contribution of the paper is that by suitably modifying well-known discrete and continuous algorithms to suit corresponding hybrid error models the global stability of the overall system can be established. The method can also be extended to purely discrete systems in which the input and output are observed at a certain rate but the control parameters are adjusted at slower rates.

The concept of hybrid self-tuning control was introduced by Gawthrop (1980) as one well suited to current digital technology. In 1982, Elliott described a stable discrete adaptive algorithm which randomly samples filtered versions of the plant input and output signals and periodically updates the controller param-

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eters. Recently Cristi and Monopoli (1982) also attempted to obtain similar results for direct control systems by discretising the differential equation over intervals on which the control parameters remain constant. This procedure leads to questions of stability which depend on the rate at which parameters are adjusted. In this paper a simple hybrid adaptive algorithm for adjusting the control parameter vector is described which assures the global stability of the overall system independent of the frequency with which the control parameters are adjusted.

For a clear and concise statement of the desirability of hybrid control the reader is referred to the paper by Elliott (1982). The hybrid adaptive control problem is described in section 2 and differs from that considered in Narendra et al (1980) only in that the control parameter vector is updated at discrete instants rather than continuously. The principal results of the paper are contained in sections 3 and 4. In section 3, an error model is analysed in detail. These results together with Lemma 1 for hybrid systems in the appendix are applied in section 4 to demonstrate the global stability of the overall adaptive system. Simulation studies are included towards the end of the paper to indicate the effect various adaptive parameters have on the performance of the overall system.

2. STATEMENT OF THE PROBLEM

A continuous time plant P to be controlled is completely represented by the input-output pair $\{u(t), y_p(t)\}$ and can be modeled by a time-invariant system

$$\begin{aligned} \dot{x}_p &= A_p x_p + b_p u \\ y_p &= h_p^T x_p \end{aligned} \qquad (1)$$

where A_p is an $(n \times n)$ matrix, and h_p and b_p are n-vectors. The transfer function of the plant is $W_p(s)$ where

$$W_p(s) = h_p^T(sI-A_p)^{-1}b_p \triangleq \frac{k_p Z_p(s)}{R_p(s)} \tag{2}$$

where $W_p(s)$ strictly proper, $Z_p(s)$ a monic polynomial of degree $m(\leq n-1)$, $R_p(s)$ a monic polynomial of degree n and k_p a constant gain parameter. It is further assumed that

(i) the sign of k_p

(ii) the order n of the plant

(iii) the relative degree $n^* = n-m$ of the plant (3)

are known and that

(iv) the monic polynomial $Z_p(s)$ is Hurwitz.

A model M which represents the behavior desired from the plant has a uniformly bounded input $r(\cdot)$, an output $y_M(\cdot)$ and a transfer function $W_M(s)$ where

$$W_M(s) \triangleq k_M \frac{Z_M(s)}{R_M(s)} \tag{4}$$

where $Z_M(s)$ is a monic polynomial of degree $m \leq n$ and $R_M(s)$ is a monic Hurwitz polynomial of degree n and k_M is a constant.

The aim of the design (whether a continuous or a hybrid controller is used) is to generate a suitably bounded input $u(\cdot)$ to the plant so that the deviation $e_1(\cdot)$ of the plant output from the desired behavior satisfies

$$|e_1(t)| \triangleq |y_p(t) - y_M(t)| \to 0 \quad \text{as } t \to \infty \tag{5}$$

The solution of this problem when the parameters are adjusted continuously is now well-known and described in the papers by Narendra et al (1980) and Morse (1980). The corresponding discrete adaptive control problem has also been resolved by Narendra and Lin (1980) and Goodwin et al (1980). For the hybrid adaptive control problem we use an identical structure for the controller as in Narendra et al (1980) but merely adjust the controller parameters at discrete instants. The controller structure may be described as follows:

$$\dot{v}^{(1)} = Av^{(1)} + bu \qquad \dot{v}^{(2)} = Av^{(2)} + by_p$$
$$w^{(1)} = c^T v^{(1)} \qquad w^{(2)} = d^T v^{(2)} + d_0 y_p \tag{6}$$

where $c^T = \{c_1, c_2, \ldots c_{n-1}\}$, $d^T = \{d_1, d_2, \ldots d_{n-1}\}$, $v^{(1)}, v^{(2)}: R^+ \to R^{n-1}$ and A is an $(n-1)x(n-1)$ stable matrix.

Defining

$$\bar{\omega}^T(t) \triangleq [r(t), v^{(1)^T}(t), y_p(t), v^{(2)^T}(t)]; \; \bar{\theta}^T(t) \triangleq [c_0(t), c^T(t), d_0(t), d^T(t)] \tag{7}$$

The control input to the plant may be expressed as

$$u(t) = \bar{\theta}^T(t)\bar{\omega}(t) \;. \tag{8}$$

We shall refer to $\bar{\omega}$ as the vector of sensitivity functions. It is well-known that under conditions specified in (2) a unique constant vector $\bar{\theta}^*$ exists such that the transfer function of the plant together with the controller matches exactly the transfer function of the model when $\bar{\theta}(t) \equiv \bar{\theta}^*$. The objective of the adaptive control problem is to determine the adaptive laws for updating the parameter vector $\bar{\theta}(t)$ using all available data such that all signals in the system remain bounded while $e_1(t) \to 0$ as $t \to \infty$. In the continuous control problem $\bar{\theta}(t)$ is adjusted continuously. In the hybrid problem under consideration $\bar{\theta}(t)$ is updated at discrete instants t_i ($i \in N$) and remains constant over each interval $[t_i, t_{i+1})$. The questions to be resolved are how such a discrete adjustment of the parameter vector $\bar{\theta}$ affects the stability of the overall system and to what extent the latter depends upon the sequence $\{t_i\}$. The principal contribution of this paper is that discrete and continuous adaptive laws can be modified in a straightforward fashion to obtain discrete adaptive laws for adjusting $\bar{\theta}$ and that for any infinite unbounded sequence $\{t_i\}$ with $|t_i - t_{i-1}|$ bounded for all $i \in N$, the overall system will be globally stable and $e_1(t)$ will tend to zero as $t \to \infty$.

As in the continuous case, central to the stability analysis of the hybrid adaptive control problem are the error models. If $\bar{\phi} \triangleq \bar{\theta}-\bar{\theta}^*$, the error models relate the sensitivity vector $\bar{\omega}$, the parameter error vector $\bar{\phi}$ and the output error e_1. In Narendra and Khalifa(1982)several error models have been analyzed and stable hybrid algorithms have been derived. One of these algorithms is analysed in some detail in the following section. It is used in Section 4 to adjust the control parameter vector $\bar{\theta}$ and the global stability of the resulting system is established. The other algorithms in Narendra and Khalifa(1982)can also be used in a similar manner to design stable adaptive systems.

3. ERROR MODEL:

The error model described in this section is a continuous time system in which $t\epsilon R^+$, the set of positive real numbers. $\bar{\omega}:R^+\to R^{2n}$ and $e_1:R^+\to R$ are piecewise continuous functions and will be referred to as the input and output functions respectively of the error model. They correspond to the sensitivity function defined in equation (7) and the output error function of the control problem defined in Section 4.

Let $\{t_i\}$ be an unbounded monotonically increasing sequence with $0<T_{min} \le T_i \le T_{max} < \infty$, where $T_i \triangleq t_{i+1}-t_i$. In the following sections $\{t_i\}$ will be referred to as the sampling sequence. $\bar{\phi}:R^+\to R^{2n}$ is a piecewise constant function and assumes the values

$$\bar{\phi}(t) = \bar{\phi}_k \qquad t\epsilon[t_k,t_{k+1}) \quad k\epsilon N \tag{9}$$

where $\bar{\phi}_k$ is a constant vector. The error model of interest in this paper is then described by the equation

$$\bar{\phi}_k^T\bar{\omega}(t) = e_1(t) \qquad t\epsilon[t_k,t_{k+1}) \quad k\epsilon N \tag{10}$$

It is assumed that $\bar{\phi}_0$ is unknown while $e_1(t)$ and $\bar{\omega}(t)$ can be measured for all $t \in R^+$. The objective is to determine the adaptive law for choosing the sequence $\{\Delta\bar{\phi}_i\}$, where $\Delta\bar{\phi}_i \triangleq \bar{\phi}_{i+1} - \bar{\phi}_i$ so that $\lim_{t\to\infty} e_1(t) = 0$.

Consider the Lyapunov function candidate.

$$V(k) = \frac{1}{2} \bar{\phi}_k^T \bar{\phi}_k . \tag{11}$$

Then

$$\Delta V(k) \triangleq V(k+1) - V(k) = [\bar{\phi}_k + \Delta\bar{\phi}_k/2]^T \Delta\bar{\phi}_k \tag{12}$$

where

$$\Delta\bar{\phi}_k \triangleq \bar{\phi}_{k+1} - \bar{\phi}_k \tag{13}$$

Choosing the adaptive law as

$$\Delta\bar{\phi}_k = -\frac{1}{T_k} \int_{t_k}^{t_{k+1}} \frac{e_1(\tau)\bar{\omega}(\tau)}{1+\bar{\omega}^T(\tau)\bar{\omega}(\tau)} d\tau \tag{14}$$

where $T_k \triangleq (t_{k+1} - t_k)$, yields

$$\Delta V(k) = -\frac{1}{2} \bar{\phi}_k^T [2I - R_{k,k+1}] R_{k,k+1} \bar{\phi}_k \tag{15}$$

where

$$R_{k,k+1} = \frac{1}{T_k} \int_{t_k}^{t_{k+1}} \frac{\bar{\omega}(\tau)\bar{\omega}^T(\tau)}{1+\bar{\omega}^T(\tau)\bar{\omega}(\tau)} d\tau \tag{16}$$

Since $R_{k,k+1}$ is a positive semi-definite matrix with eigenvalues less than unity, it follows that $[2I - R_{k,k+1}] > \beta I$ for some constant $\beta>0$.

Hence

$$\Delta V(k) < - \beta\tilde{\phi}_k^T R_{k,k+1}\tilde{\phi}_k \leq 0 \tag{17a}$$

and $V(k)$ is a Lyapunov function and assures the boundedness of $||\tilde{\phi}_k||$ if $||\tilde{\phi}_0||$ is bounded. From (17a) it follows that

$$\Delta V(k) \to 0 \quad \text{as} \quad k \to \infty$$

$$\Delta\tilde{\phi}_k \to 0 \tag{17b}$$

and from (16), (17a) and (17b) we have

$$\frac{\beta}{T_k}\int_{t_k}^{t_{k+1}} \frac{e_1^2(\tau)}{1 + \bar{\omega}^T(\tau)\bar{\omega}(\tau)}\, d\tau \to 0 \tag{18}$$

$$\text{as } k \to \infty$$

Case i: If $\bar{\omega}$ is uniformly bounded in $[0,\infty)$ it follows that e_1 is also uniformly bounded. If $\bar{\omega}$ is differentiable and $||\dot{\bar{\omega}}||$ is also bounded it follows from (18) that $\lim_{t\to\infty} e_1(t) = 0$. Hence for a uniformly bounded input with a bounded derivative, $|e_1|$ tends to zero and $\Delta\tilde{\phi}_k \to 0$ as $k \to \infty$.

Case ii: If in addition to being uniformly bounded, $\bar{\omega}$ is "sufficiently rich" (Morgan and Narendra, 1977) over any interval of length T_{min} so that $R_{k,k+1}$ is positive definite for all $k\in N$, $\Delta V(k)<0$ and hence the parameter error vector $\tilde{\phi}_k \to 0$ as $k \to \infty$.

adaptive law can be modified to

$$\Delta\bar{\phi}_k = -\frac{\Gamma}{T_k}\int_{t_k}^{t_{k+1}} \frac{e_1(\tau)\bar{\omega}(\tau)}{1+\bar{\omega}^T(\tau)\Gamma\bar{\omega}(\tau)}\, d\tau \tag{20}$$

where Γ is a positive definite diagonal matrix with eigenvalues within the unit circle.

4. GLOBAL STABILITY OF THE HYBRID ADAPTIVE SYSTEM

The proof of stability of the hybrid adaptive control problem follows generally along the same lines as those for the continuous case discussed in Narendra et al (1980). However, since the adjustment of the parameters is done at discrete instants, $\bar{\phi}(t)$ is discontinuous at these instants and hence the arguments have to be suitably modified. Only those features which are pertinent to the hybrid control problem are discussed here and the reader is referred to the earlier paper (Narendra et al, 1980) for some of the details. The basic mathematical concepts as well as Lemma 1 essential for the proof of stability, are developed briefly in the appendix.

For ease of exposition the stability problem is discussed in two stages. In the fir case the high frequency gain K_p of the plant is assumed to be known while the more general problem with K_p unknown is discussed in Case (ii).

a) Case i (K_p known)

With no loss of generality we assume that $K_M = K_p = 1$ so that $c_o = 1$ and only (2n-1) parameters have to be adjusted. Defining

$\bar{\theta}^T(t)=[c_o,\theta^T(t)]$, $\bar{\omega}^T(t)=[r(t),\omega^T(t)]$ and $\bar{\phi}^T(t)=[o,\phi^T(t)]$

the input and the output of the plant can be expressed as

$$u(t) = r(t) + \theta^T(t)\omega(t)$$

$$y_p(t) = W_m(s)[r(t) + \phi^T(t)\omega(t)] \tag{21}$$

As in the continuous case an augmented error $\bar{e}_1(t)$ is generated by adding an auxiliary signal y_a to the plant output, where

$$y_a(t) \triangleq [\theta^T(t)W_m(s)I - W_m(s)\theta^T(t)]\omega(t). \tag{22}$$

The augmented error is then given by

$$\bar{e}_1(t) \triangleq e_1(t) + y_a(t)$$
$$= \phi^T(t)\ \zeta(t) \tag{23}$$

where $W_m(s)\omega(t) = \zeta(t)$. The adaptive law for adjusting $\phi(t)$ is generated using $\bar{e}_1(t)$ and $\zeta(t)$ in equation (23) but it remains to be shown that equation (21) will be globally stable with such an adaptive law.

In the hybrid control problem $\theta(t)=\theta_k$ and $\phi(t)=\phi_k$ over the interval $[t_k,t_{k+1})$, $k\epsilon N$ where θ_k and ϕ_k are constant vectors. Hence equation (23) can be expressed in the form of the error model described in Section 3 as:

$$\phi_k^T\zeta(t) = \bar{e}_1(t) \qquad t\epsilon[t_k,t_{k+1}),\ k\epsilon N \tag{24}$$

The corresponding adaptive law is given by (14) as

$$\Delta\phi_k = -\frac{1}{T_k}\int_{t_k}^{t_{k+1}} \frac{\bar{e}_1(\tau)\zeta(\tau)}{1 + \zeta^T(\tau)\zeta(\tau)}\, d\tau \quad * \tag{25}$$

*Again, to avoid obscuring the principal results the adaptive gain matrix Γ is not included here.

It follows from the discussions in Section 3 that ϕ_k will be bounded and hence the plant output y_p as well as the state variables of the entire system can grow at most exponentially. Again from Section 3 we have

$$|\bar{e}_1(t)| = o\left[\sup_{t\geq\tau}||\zeta(\tau)||\right] \tag{26}$$

or the augmented error $|\bar{e}_1(t)|$ will grow more slowly than the norm of the vector $\zeta(t)$.

b) <u>Case ii</u> <u>(K_p Unknown)</u>

The error equations in this case appear, at first sight, to be considerably more involved than in case (i). However they can be reduced to the form of the first error model by a change of variables. The input to the plant is

$$u(t) = \bar{\theta}^T(t)\bar{\omega}(t)$$

The main difficulty arises since $K_p \neq K_M$ in general, resulting in a plant output which can be described by

$$y_p(t) = W_M(s)r(t) + \frac{K_p}{K_M} W_M(s)\bar{\phi}^T(t)\bar{\omega}(t) \tag{27}$$

and an error equation

$$e_1(t) = \frac{K_p}{K_M} W_M(s)\bar{\phi}^T(t)\bar{\omega}(t) \; . \tag{28}$$

In generating the augmented error, an additional parameter $\psi_1(t)$ has to be used making a total of $(2n+1)$ adjustable parameters. Defining the augmented error once again as

$$\bar{e}_1(t) = e_1(t) + y_a(t)$$

where

$$y_a(t) \triangleq \psi_1(t)\left[\bar{\theta}^T(t)W_M(s)I - W_M(s)\bar{\theta}^T(t)\right]\bar{\omega}(t) \tag{29}$$

we obtain

$$\bar{e}_1(t) = \frac{K_p}{K_M}\left[\bar{\phi}^T(t)W_M(s)\bar{\omega}(t) + \psi(t)\xi(t)\right] \tag{30}$$

where

$$\psi_1(t) \triangleq \frac{K_p}{K_M}\left[1 + \psi(t)\right] \quad \text{and}$$

$$\xi(t) \triangleq \left[\bar{\theta}^T(t)W_M(s)I - W_M(s)\bar{\theta}^T(t)\right]\bar{\omega}(t) \tag{31}$$

Defining

$$\bar{\bar{\phi}}^T \triangleq \left[\bar{\phi}^T, \psi\right] \quad \bar{\bar{\zeta}}^T = \left[\bar{\zeta}^T, \xi\right] \tag{32}$$

The augmented error equation may be expressed as

$$\bar{e}_1(t) = \frac{K_p}{K_M}\,\bar{\bar{\phi}}^T(t)\bar{\bar{\zeta}}(t) \tag{33}$$

which again corresponds to the error model described in Section 3. In the hybrid control problem the $(2n+1)$ elements of the control vector $\bar{\bar{\theta}}$ and hence of the parameter vector $\bar{\bar{\phi}}$ are adjusted at discrete instants of time and the adaptive law can be expressed as

$$\Delta\bar{\bar{\theta}}_k = \Delta\bar{\bar{\phi}}_k = -\frac{1}{T_k}\int_{t_k}^{t_{k+1}} \frac{\bar{e}_1(\tau)\bar{\bar{\zeta}}(\tau)}{1+\bar{\bar{\zeta}}^T(\tau)\bar{\bar{\zeta}}(\tau)}\,d\tau \tag{34}$$

From the discussions in Section 3 it follows once again that $\bar{\bar{\phi}}_k$ is bounded and hence the plant output as well as all variables of the system grow at most exponentially. From the results of Section 3 this implies that

$$|\bar{e}_1(t)| = o\left[\sup_{t\geq\tau}\left|\left|\bar{\bar{\zeta}}(\tau)\right|\right|\right] \text{ or equivalently } o\left[\sup_{t\geq\tau}\left|\left|\bar{\zeta}(\tau)\right|\right|\right]$$

$$\text{since } \phi_k \text{ is bounded.} \tag{35}$$

c) **Proof of Global Stability:**

The main results of the analysis carried out so far, which are central to the proof of global stability, may be summarized as follows:

(i) the hybrid adaptive equations assure the boundedness of all the parameters so that the signals in the system can grow at most exponentially.

(ii) the augmented error $|\bar{\varepsilon}_1(t)|$ can grow only at a rate slower than that of the sensitivity functions (equations (26) and (35). Using these results it is shown in this section that the output of the plant as well as all the relevant signals of the adaptive system will remain bounded for all $t \epsilon R$.

Let the plant output $y_p \epsilon L_e^\infty$ and grow in an unbounded fashion. Since the parameter error vector $\bar{\phi}$ is bounded, y_p can grow at most exponentially (Narendra et al,1980). The output error $e_1(t)$ is given by equation (28) as

$$e_1(t) = \frac{K_p}{K_M} W_M(s)\, \bar{\phi}_k^T \bar{\omega}(t) \qquad t \epsilon [t_k, t_{k+1}) \quad k \epsilon N$$

Since $W_M(s)\bar{\omega}(t) = \bar{\zeta}(t)$ and by (17b) $\Delta\bar{\phi}_k \to 0$ as $k \to \infty$, we have by Lemma 1 in the appendix:

$$\left[\bar{\phi}_k^T W_M(s) I - W_M(s)\bar{\phi}_k^T\right] \bar{\omega}(t) = o\left[\sup_{t \geq \tau} ||\bar{\omega}(\tau)||\right] \tag{36}$$

or

$$e_1(t) = \frac{K_p}{K_M}\left\{\bar{\phi}_k^T \bar{\zeta}(t) + o\left[\sup_{t \geq \tau} ||\bar{\omega}(\tau)||\right]\right\} \tag{37}$$

From equations (26) and (35) we have

$$\tilde{\phi}_k^T\bar{\zeta}(\tau) = o\left[\sup_{t\geq\tau}||\bar{\zeta}(\tau)||\right] = o\left[\sup_{t\geq\tau}||\bar{\omega}(\tau)||\right] \tag{38}$$

Since $\sup_{t\geq\tau}||\bar{\zeta}(\tau)|| \sim \sup_{t\geq\tau}||\bar{\omega}(\tau)||$. Since the output of the model $y_M(t)$ is uniformly bounded, $|y_p(t)| = O\left[\sup_{t\geq\tau}|e_1(\tau)|\right]$ and hence from equations (26), (35) and (38) we have

$$|y_p(t)| = o\left[\sup_{t\geq\tau}||\bar{\omega}(\tau)||\right] \tag{39}$$

From (27) and (39) it follows that $y_p(t)$ and $\bar{\omega}(t)$ are uniformly bounded contradicting the original assumption that y_p grows in an unbounded fashion. This in turn implies that $\bar{\zeta}(t)$ is bounded and hence

$$\lim_{t\to\infty} e_1(t) = 0, \qquad \lim_{t\to\infty} \varepsilon_1(t) = 0 \qquad \text{and} \qquad \lim_{t\to\infty} y_a(t) = 0.$$

Hence the adaptive system is globally stable.

Comments:

(i) Lemma 1 in the appendix assures that when $\Delta\tilde{\phi}_k \to 0$ as $k\to\infty$ the auxiliary error $y_a(t)$ is $o||\bar{\omega}(t)||$. Since, from the analysis of the error model it is known by equations (26) and (35) that the augmented error $|\varepsilon_1(t)| = o||\bar{\zeta}(t)||$, it follows that the true error $|e_1(t)| = o||\bar{\omega}(t)||$ which contradicts the assumption that $y_p(t)$ is unbounded.

(ii) As in the error model, the introduction of a diagonal gain matrix $\Gamma=\Gamma^T>0$ in the adaptive law does not affect the arguments of this section.

(iii) The same stability arguments could also be used with all the hybrid adaptive algorithms given in Narendra and Khalifa (1982).

5. SIMULATION RESULTS:

In this section computer simulations of two typical examples of the error model discussed in Section 3 and the adaptive control problem discussed in Section 4 are presented. In all cases the parameters are adjusted periodically so that $t_k = kT$ and $T_k = T$ where T denotes the period. The main interest in these simulations is in the effect of T on the adaptive process. In both examples the results for an approximation to the continuous case (using a sufficiently small value of T) are included for purposes of comparison.

Example 1: The hybrid error model described by equation (10) contains 4 parameters so that $\tilde{u}$ and $\tilde{\theta} \in \mathbb{R}^4$. The elements of the input vector $\tilde{u}(t)$ are

$$\tilde{u}_1(t) = 8 \sin .261\, t \qquad \tilde{u}_2(t) = 8 \sin .522\, t$$

$$\tilde{u}_3(t) = 8 \sin 1.044 t \qquad \tilde{u}_4(t) = 8 \sin 2.088 t$$

The adaptive law $\Delta\tilde{\theta}_k = -\frac{\gamma}{T}\int_{kT}^{(k+1)T} \frac{e_1(\tau)\tilde{u}(\tau)}{1+\tilde{u}^T(\tau)\tilde{u}(\tau)}\, d\tau$ was used to adjust the discrete parameters, where γ is the adaptive gain. An approximation to the continuous algorithm was obtained using a sampling period $T = .3$ and the hybrid algorithm was also simulated for periods $T = 1.8$ and 3.3, respectively.

Figure 1 shows the variation of the four parameters with respect to time for all three cases. Figure 2 indicates the effect of changing the adaptive gain on the speed of convergence. As might be expected, an increased adaptive gain is needed as T is increased to obtain a response comparable to that of the continuous case.

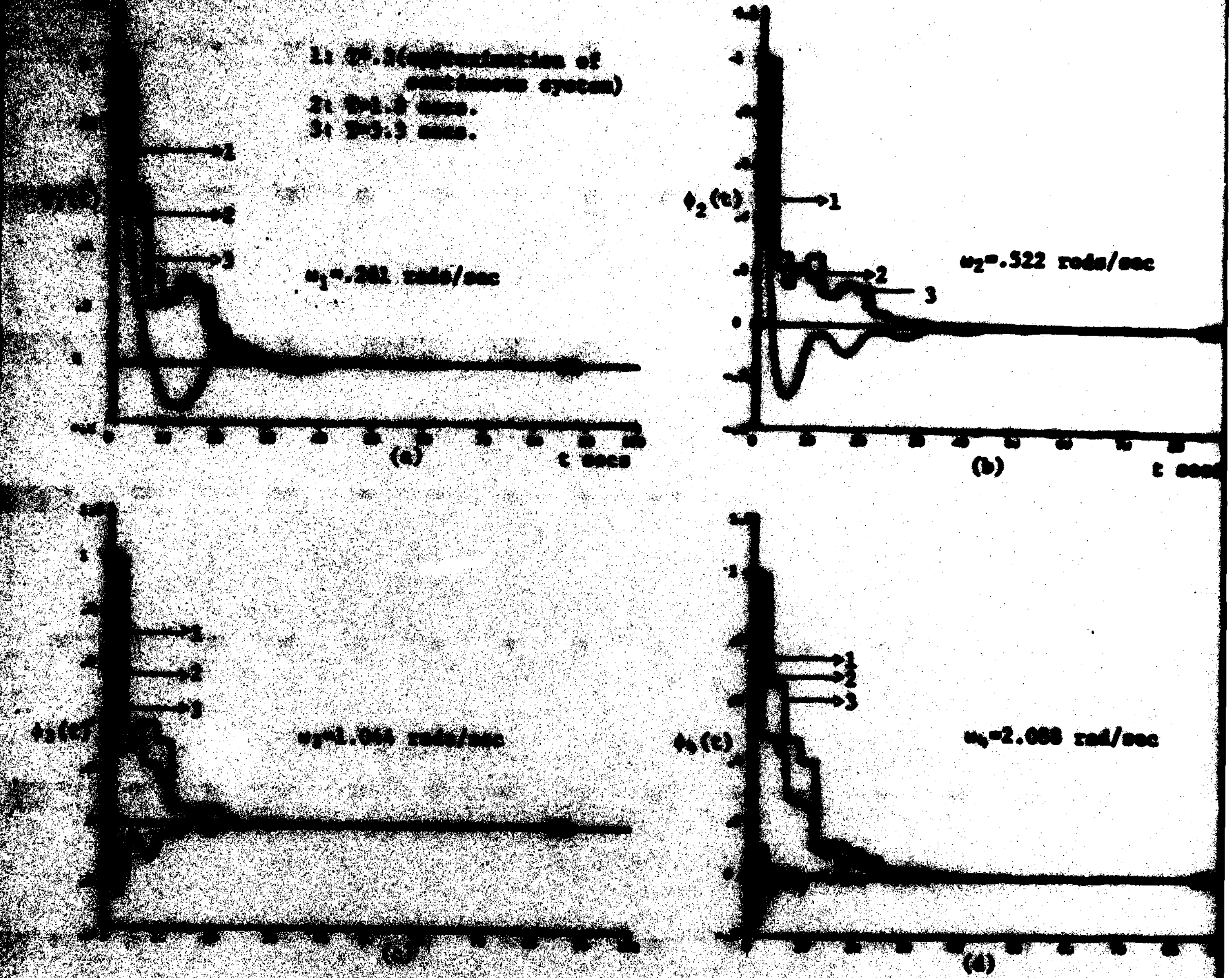


Figure 1: Evolution of parameter errors ϕ_i with respect to time.

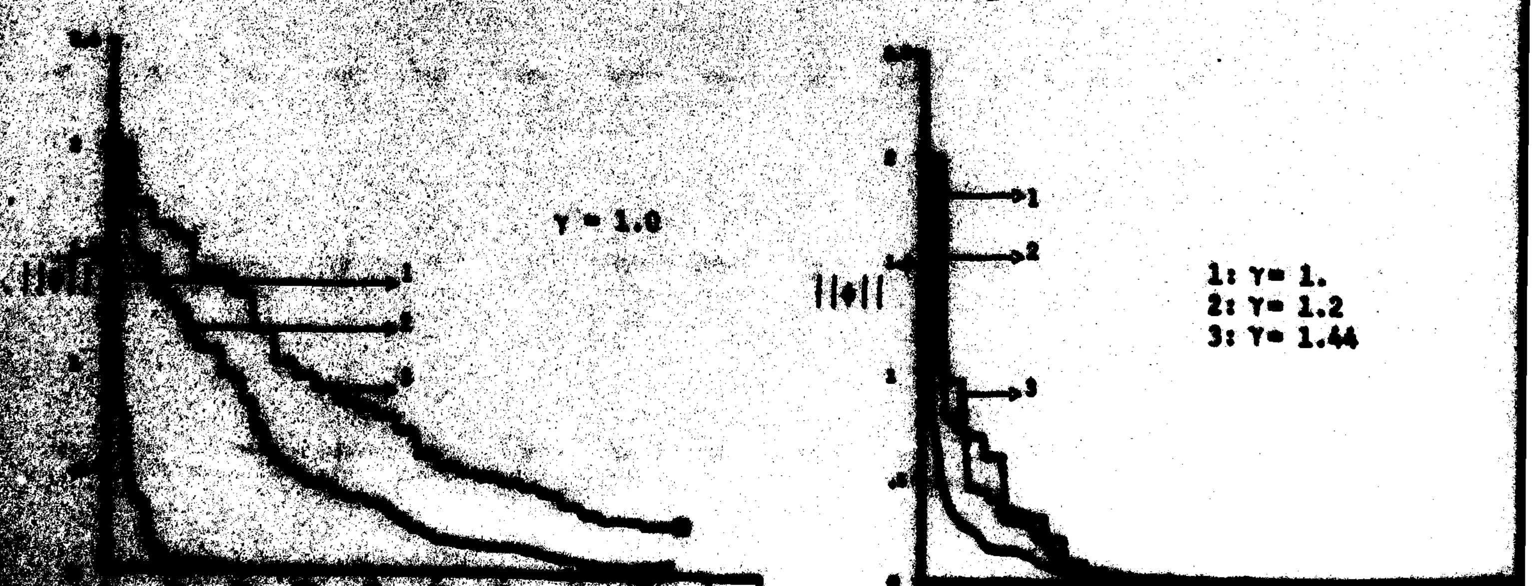


Figure 2: Effect of changing the adaptive gain on speed of convergence

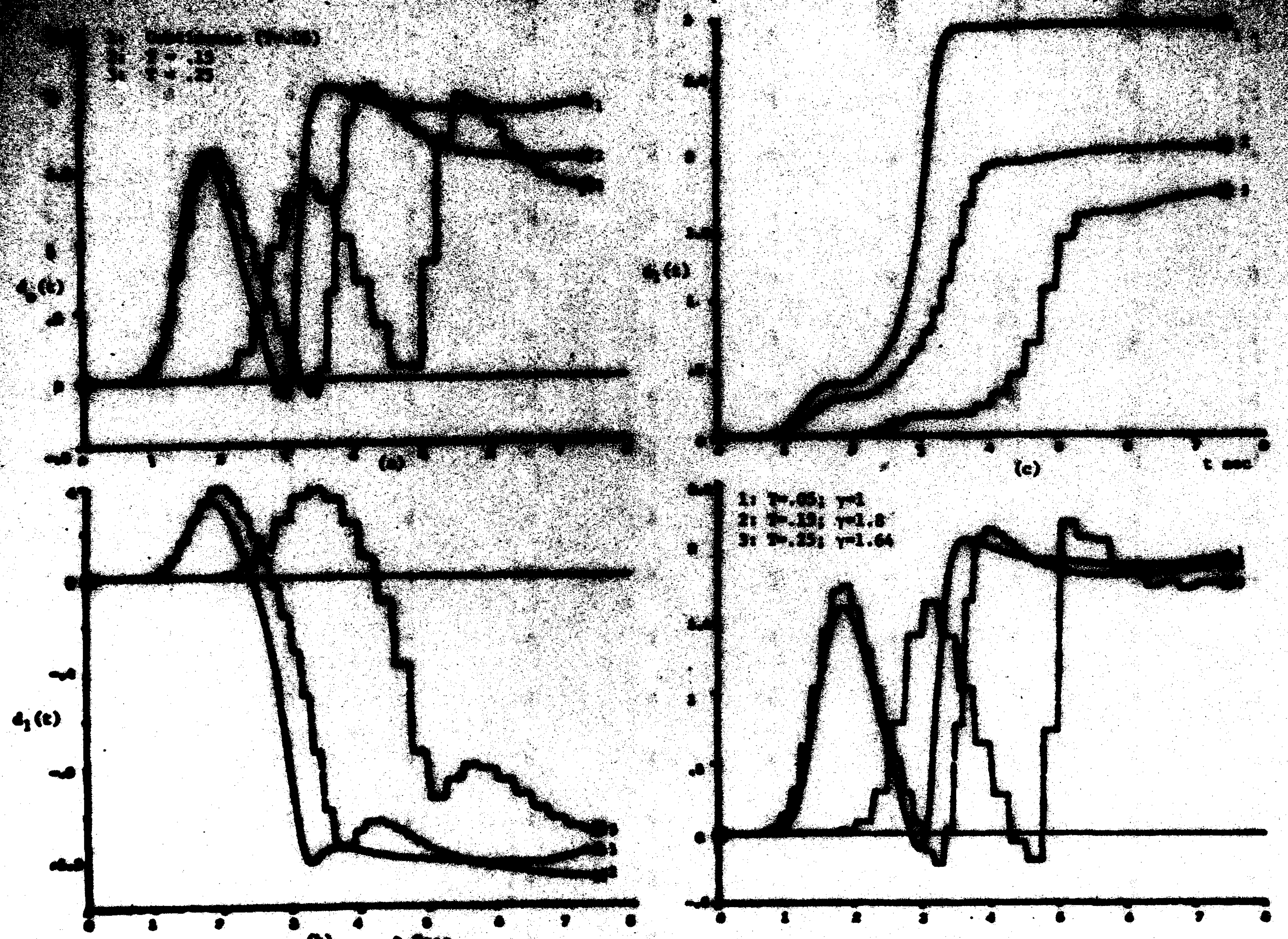


Figure 4: Evolution of the parameter $d_n(t)$.

Example 2: In the second example, the hybrid adaptive control of an unstable second order plant is considered. The plant transfer function is

$$W_p(s) = \frac{5}{(s^2+.5s - 1.5)}$$

where only the high frequency gain $K_p = 5$ is assumed to be known. The model whose output the plant output has to follow has the stable transfer function

$$W_m(s) = \frac{5}{s^2+4s+3}$$

and the filter used in the controller has a transfer function

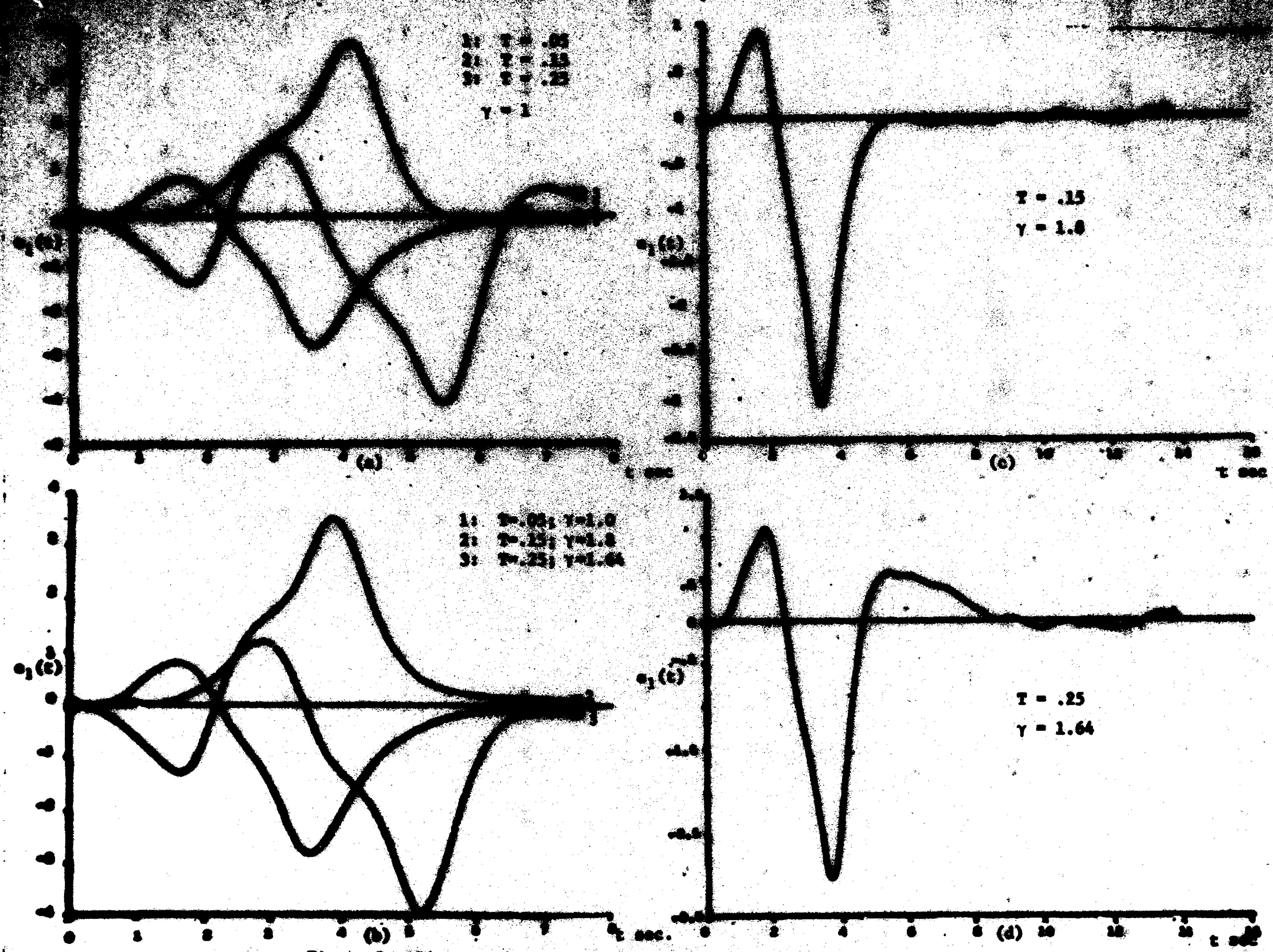
$$\Lambda(s) = \frac{1}{s+2}$$

The elements of the control parameter vector $\theta(t)$ are $c_1(t), d_o(t)$ and $d_2(t)$, and their desired values c_1^*, d_o^* and d_2^* are

$$c_1^* = 3.5, \qquad d_o^* = 1.95 \qquad \text{and} \qquad d_2^* = -1.05, \text{ respectively.}$$

The input to the system is zero mean white noise with unit variance. Figure 3 shows the evolution of the three parameters for the continuous case (approximated by $T=.05$) as well as for the cases $T=.15$ and $T=.25$, respectively. For the same adaptive gain the speed of response is seen to decrease with the period T.

In Figure 4, the improvement in performance achieved by adjusting the adaptive gain γ is shown. The adaptive gain for the three cases are $\gamma=1$ $(T=.05)$, $\gamma=1.8$ $(T=.15)$ and $\gamma=1.64$ $(T=.25)$, respectively and the parameter observed is $d_o(t)$.
Even though the final value of $d_o(t)$ is approximately the same in all three cases, the response with $T=.25$ is seen to be substantially different from that of the continuous case. The output error e_1 for all three cases is shown in Figure 5. Different adaptive gains were used in each case and are indicated



1: T = .05
2: T = .15
3: T = .25
γ = 1
(a)
t sec
1: T=.05; γ=1.0
2: T=.15; γ=1.8
3: T=.25; γ=1.64
(b)
t sec.
T = .15
γ = 1.8
(c)
t sec
T = .25
γ = 1.64
(d)
t sec

on the figure. While the error e_1 is small for all three responses at $t=7.5$ secs, the amplitude of oscillations when $T=.25$ secs is relatively large. This may be attributed to the fact that the plant is unstable. Hence, though the overall system is theoretically globally stable for all finite values of the parameter T, the transient response may deteriorate with increasing T, particularly if the plant is unstable. Hence, in such cases the desired transient response will dictate the choice of T.

6. COMMENTS AND CONCLUSIONS

An adaptive algorithm is presented in this paper which assures the global stability of hybrid systems in which the signals are continuous but the parameters are adjusted at discrete instants. The global stability of the overall system is independent of T, the period between parameter adjustments provided T is bounded. Simulation results indicate that the choice of T will be dictated by the desired transient response, particularly when the plant is unstable.

The adaptive law depends upon the specific error model analyzed in Section 3. Several other hybrid error models have also been analyzed by Narendra and Khalifa (1982) and similar results can be obtained using the adaptive laws corresponding to these error models. The techniques developed, though applied only to single input-single output systems (SISO) here, carry over to multivariable systems as well. Further the same approach can also be used for the adaptive control of discrete linear systems in which the plant output is measured at a certain rate but the control parameters are adjusted at slower rates. The latter is obviously significant in the digital control of complex processes.

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Result 1: If the input to a linear time-invariant exponentially stable system is $x(\cdot)\varepsilon L_e^\infty$ and the corresponding output is $y(\cdot)$, then

$$|y(t)| = o\left[\sup_{t\geq\tau}|x(\tau)|\right]$$

Result 2: If $W_L(s)$ is a rational transfer function of a linear time-invariant discrete system with all its poles and zeros within the unit circle and with input and output $x(\cdot)$ and $y(\cdot)$, respectively, then

$$\sup_{k\geq\nu}|x(\nu)| \sim \sup_{k\geq\nu}|y(\nu)|$$

Lemma 1: Let $\omega(\cdot)$, $\zeta(\cdot)$: $R^+ \to R^n$ be the input and output, respectively of a transfer matrix $H(s)I$ where $H(s)$ is a rational transfer function and I is the $(n\times n)$ unit matrix. Let $H(s)$ have all its poles and zeros in the open left half plane. Further suppose that there is a vector $\phi(t)\varepsilon R^n$ and $||\phi||$ is uniformly bounded and

$$\phi(t) = \phi_k \qquad t\ \varepsilon\ [t_k, t_{k+1}) \quad k\ \varepsilon\ N$$

where ϕ_k is a constant vector and

$$\Delta\phi_k \triangleq \phi_{k+1} - \phi_k \longrightarrow 0 \qquad \text{as } k \longrightarrow \infty$$

Then

$$[\phi^T(t)\ H(s)I - H(s)\phi^T(t)]\omega(t) = o\left|\sup_{t\geq\tau}||\omega(\tau)||\right| \tag{A-1}$$

According to Lemma 1 if the input is $\omega(t)$, the outputs of the two systems $\phi^T(t)\ H(s)$ and $H(s)\phi^T(t)$ differ by $o\left|\sup_{t\geq\tau}||\omega(\tau)||\right|$ if $\Delta\phi_k \longrightarrow 0$ as $k \longrightarrow \infty$.

Proof: At time $t = (n+1)T$

$$\phi^T(t)\ H(s)\ I\omega(t) = \left[\phi_o + \sum_{i=0}^{n-1} \Delta\phi_i\right]^T H(s) I\omega(t)$$

$$= \zeta(t) \tag{A-2}$$

If the impulse response of $H(s)$ is $h(t)$ where $|h(t)| \leq \beta e^{-rt}$ for some positive constants β and r.

$$H(s)\phi^T(t)\omega(t)\Big|_{t=(n+1)T} = \phi_o^T H(s) I\omega(t) + \sum_{i=0}^{n-1} \Delta\phi_i^T \int_{(i+1)T}^{(i+2)T} h[(n+1)T-\tau]\omega(\tau)d\tau$$

$$= \left[\phi_o + \sum_{i=0}^{n-1} \Delta\phi_i\right]^T H(s) I\omega(t)\Big|_{t=(n+1)T}$$

$$- \sum_{i=0}^{n-1} C_i \int_{iT}^{(i+1)T} h[(n+1)T-\tau]\omega(\tau)d\tau \tag{A-3}$$

where $C_o = \left\{\phi_n - \phi_o\right\}$ and $C_i = \sum_{j=i}^{n} \Delta\phi_j$

Since the vector ϕ is bounded $C_i (i=0,\ldots,n)$ are bounded. Further since $\Delta\phi_n \longrightarrow 0$, $C_n \longrightarrow 0$ as $n \longrightarrow \infty$. From (A-2) and (A-3) it follows that

$$[\phi^T(t)H(s)I - H(s)\phi^T(t)]\omega(t)\Big|_{t=(n+1)T} = \sum_{i=0}^{n-1} C_i \int_{iT}^{(i+1)T} h[(n+1)T-\tau]\omega(\tau)d\tau$$

$$= \nu[(n+1)T] \tag{A-4}$$

$$|\nu[(n+1)T]| \le \sum_{i=0}^{n-1} C_i \quad \beta \bar{e}^{r(n+1)T} \int_{iT}^{(i+1)T} \bar{e}^{r\tau} ||\omega(\tau)|| d\tau$$

$$\le \gamma_1 \beta \sup_{t \ge \tau} ||\omega(\tau)|| \left[\sum_{i=0}^{n-1} |C_i| \; \bar{e}^{r(n+1)T} \right] \tag{A-4}$$

for some constant γ_1.

Since $|C_n| \to 0$ as $n \longrightarrow \infty$, the term in the brackets as $n \longrightarrow \infty$ tends to zero by result 2. Hence

$$|\nu(t)| = o\left[\sup_{t > \tau} ||\omega(\tau)|| \right]$$

proving Lemma 1.