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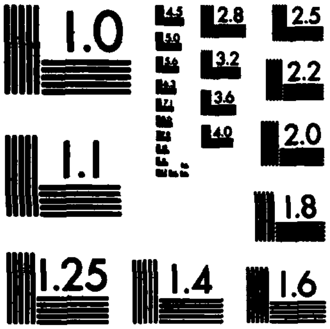

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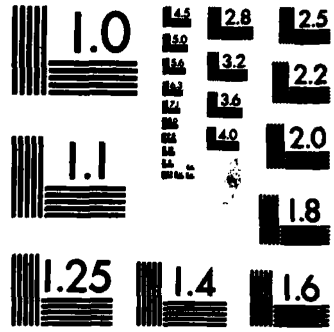

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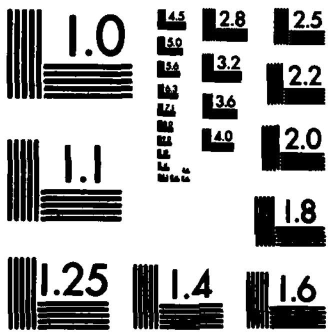

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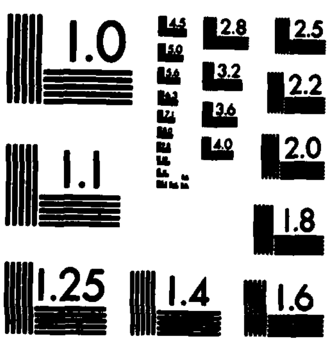

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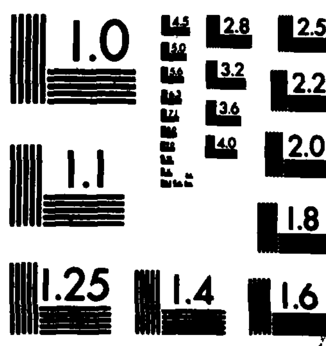

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After a terse discussion of the initial stress method, the procedure of computation is extended in the analysis by using an incremental load process. The size and the shape of the plastic zone of a center crack specimen is investigated

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It was found that the moving finite element technique can be modified to solve the crack branching problems.

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FINAL REPORT

Finite Element Modeling of Elastic-Plastic Crack Growth

by

Francis T.C. Loo

Department of Mechanical and Industrial Engineering

Clarkson College of Technology

Potsdam, New York 13676

Grant/Contract No. AFOSR-81-0174

FOREWORD

This report was prepared by the Research Division, Clarkson College of Technology, Potsdam, New York. This work was supported by the Air Force Office of Scientific Research under the grant No. AFOSR-81-0174. The work was performed by Dr. Francis T.C. Loo of the department of Mechanical and Industrial Engineering, Clarkson College of Technology, Potsdam, New York.

This report describes work performed from June 1, 1981 through May 31, 1982.

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It was found that the moving finite element technique can be modified to solve the crack branching problems.

LIST OF SYMBOLS

A	hardening parameter
a	crack length
Δa	crack increment
$\bar{a}$	new crack length
$[\bar{B}]$	partial differential operator matrix
[B]	strain-displacement relationship matrix
[D]	stress-strain relationship matrix
$[D]_{ep}$	elasto-plastic matrix
E	Young's modulus
F	function of stress
G	shear modulus
J	the J-integral
[J]	Jacobian matrix
k	constant
[K]	stiffness matrix
K_I	mode I stress intensity factor
ℓ	crack-tip element size
L	length of the strip
$\vec{n}$	normal vector
N	shape function
{P}	Plastic load vector
{q}	generalized displacement vector
r_p	radius of plastic zone
R,S	local coordinates of isoparametric element

s	distance along integration path
T_i	surface traction vector
u	displacement in x-direction
$\{u\}$	displacement vector
U	strain energy
v	displacement in y-direction
W	width of strip
w	elastic strain energy density
x,y	global Cartesian coordinates
$\{\sigma\}$	stress vector
$\{\Delta\sigma'\}$	increment of stress vector
$\{\sigma_o\}$	initial stress vector
$\bar{\sigma}$	effective equivalent stress
σ_e	effective stress
σ_{yp}	yield strength of material
σ_o^*	post yield applied load
$\sigma_1,\sigma_2,\sigma_3$	principal stresses
τ_{ij}	shear stress component
$\{\varepsilon\}$	total strain vector
$\{\varepsilon\}_e$	elastic strain vector
$\{\varepsilon\}_p$	plastic strain vector
$\varepsilon_1,\varepsilon_2,\varepsilon_3$	principal strains

$\bar{\varepsilon}$ effective strain

ν Poisson's ratio

λ positive scalar constant

Γ contour of J-integral

SECTION 1

Introduction

Fracture mechanics has in recent years become an independent discipline that deals with the determination of the conditions under which machine or structural elements attain uncontrollable failure by crack propagation. This topic has been a challenging subject in the area of solid mechanics; its historical development can be traced back to the early part of the century. A knowledge of this subject can assist the engineer or designer to safeguard machine elements against catastrophic failure.

Two fracture criterions of failure which have played the major and fundamental roles in the study of fracture mechanics are based on the concepts of energy balance and stress intensity factors. In 1921 and 1924, Griffith [1,2] introduced a fracture criterion of failure which would take into account the effect of minute flaws and small cracks in material. His theory basically involves the energy balance, which states that the energy consumed in creating a new fracture surface within solid body must be supplied from the release of the elastic strain energy and from the work done by the external load. Although this theory has since been proven inadequate, it still provided the basis for newer theories developed in the field of fracture mechanics.

An alternative approach was later proposed by Irwin [3,4]. Rather than follow Griffith's theory, he concentrated on a crack tip region which is small in comparison with the body as a whole but sufficiently large with respect to atomic dimensions for him to be reasonably comfortable with the application of linear elasticity theories. He stated that fracture initiation occurs when the intensity of stress in the near crack tip region reaches a critical value. He subsequently introduced the three modes of crack extension and the corresponding stress intensity factors K_I, K_{II}, and K_{III}, which provide a measure of the amplitude of the stress field in the near crack tip region.

The study of the elastic-plastic fracture problems are the main concern of this report. The discussion which follows will be restricted to Mode I (in-plane tensile mode) crack propagation. The problem from a designer's viewpoint is to prevent brittle fracture. For this purpose, the state of stress and strain close to the crack tip should be accurately known. The common approach, known as Linear Elastic Fracture Mechanics (LEFM), is to assume linearly elastic material behavior around the crack tip. Since it is known that there is always some amount of plastic deformation at the crack tip, the LEFM approach can provide satisfactory answers only for highly brittle materials and under such loading conditions which cause a negligibly small plastic zone at the crack tip as compared to the crack length and the thickness of the specimen. Since a considerable number of engineering materials are relatively ductile, and since the thickness of the specimen cannot always be considered large, the plastic deformation at the crack tip cannot always be ignored. The significance of the size of the plastic zone ahead of the crack tip as one of the parameters which govern the growth of a crack was first emphasized by Irwin [3,4,9]. Later researchers, such as Rice [5,8,10], Hutchinson [6], Cherepanov [7], and others, also incorporated the effect of the shape and size of plastic zone in their proposed fracture criteria.

One of the challenging problems is to investigate the shape and the size of the plastic zone, which would depend on material properties, specimen geometry, loading and boundary conditions. A considerable amount of research in this direction had been reported in the literature. A good bibliography on the subject of elasto-plastic stress analysis can be found in references [10] and [36].

In recent developments of fracture mechanics, the use of the finite element method has been quite extensive in both elastic and elasto-plastic analyses. A number of special finite element techniques have been developed [16,17,18], by using the displacement finite element method. These special elements contain a singularity. The disadvantages of these elements are that the solutions do not always converge and that they take too much computing time, especially for elastic-plastic fracture problems.

From the above considerations and practical viewpoint, Barsoum [11] formulated the eight-noded quadrilateral isoparametric element. The singularity in the element can be achieved by placing the mid-sided node near the crack tip at the quarter point. It has been shown that such elements in their non-singular formulation satisfy the essential convergence criteria, namely, continuity of displacements, compatibility, constant strain modes and rigid body motion modes. Therefore, their convergence characteristics and the high accuracy of the results in a reasonably-sized finite element mesh make their application in the modeling of the elastic-plastic fracture problem very docile.

SECTION II

Finite Element Analysis

c Concepts

fundamental concept of the finite element method is to construct a model composed of a set of piece-wise continuous functions defined inite number of closed sub-regions. The sub-regions are called ts'' which are connected to each other at their common nodes, and rely approximate the shape of the domain. The generalized nent components of these nodes are the basic unknowns in the n.

the column vector {q} be the generalized displacement components at es; and column vector {U} be the state of displacement for the The two parameters can be related and expressed in matrix form as:

$$= [N]\{q\} \tag{1}$$

] is the shape function, which is a function of special coordinates to be evaluated at each of the nodal points. The notations { } and te a column vector and a matrix, respectively.

oting the generalized strain vector as $\{\varepsilon\}$, the strain components can expressed in terms of the displacements through the compatibility ns:

$$= [\overline{B}]\{U\} \tag{2}$$

where $[\bar{B}]$ is a partial differential operator matrix which can be written as:

$$[\bar{B}] = \begin{bmatrix} \frac{\partial}{\partial x} & o \\ o & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \tag{3}$$

Through the constitutive relations of stress and strain, the generalized stress vector $\{\sigma\}$ can be written, in terms of the strain vector $\{\varepsilon\}$, as:

$$\{\sigma\} = [D]\{\varepsilon\} \tag{4}$$

where [D] is the material matrix which is given below

a) for plane stress

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & o \\ \nu & 1 & o \\ o & o & \frac{1-\nu^2}{2} \end{bmatrix} \tag{5}$$

b) for plane strain

$$[D] = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \nu & o \\ \nu & 1 & o \\ o & o & \frac{1-\nu}{2} \end{bmatrix} \tag{6}$$

where E, ν are the modulus of elasticity and Poisson's ratio, respectively.

2. Quadrilateral Isoparametric Element

The formulation of the stiffness matrix of the eight-noded isoparametric element is well documented [28]. Through the following transformation, the stiffness matrix is found as follows:

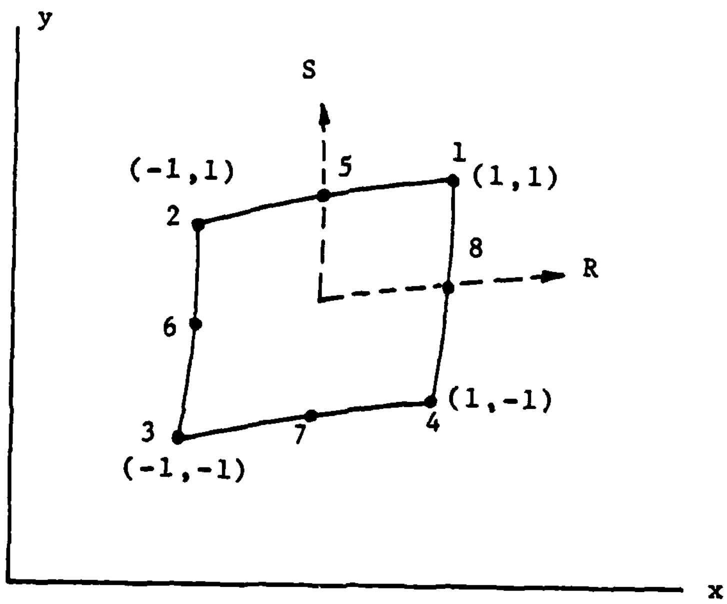


Figure 1. Quadrilateral Isoparametric Element

$$x = \sum_{i=1}^{8} N_i(R,S)x_i$$

$$y = \sum_{i=1}^{8} N_i(R,S)y_i \tag{7}$$

where N_i are the shape functions corresponding to the node i, whose coordinates are (x_i, y_i). In the transformed R-S coordinates system, R_i, S_i $= \pm 1$ for the corner nodes and zero for the mid-side nodes.

$$N_i(R,S) = [(1+RR_i)(1+SS_i) - (1-R^2)(1+SS_i) - (1-S^2)(1+RR_i)]\,\frac{R_i^2 S_i^2}{4} + (1-R^2)(1+SS_i)(1-R_i^2)\,\frac{S_i^2}{2} + (1-S^2)(1+RR_i)(1-S_i^2)\frac{R_i^2}{2} \tag{8}$$

It is quite easy to understand that if the boundaries of each element vary from -1 to +1 in the R-S coordinate system, the terms $(1-R^2)$ and $(1-S^2)$ cannot coexist on any boundary. The interpolation function for an eight-node isoparametric element can thus be written in the following form:

$$\begin{aligned}
N_1(R,S) &= -\tfrac{1}{4}(1+R)(1+S)(1-R-S)\\
N_2(R,S) &= -\tfrac{1}{4}(1-R)(1+S)(1+R-S)\\
N_3(R,S) &= -\tfrac{1}{4}(1-R)(1-S)(1+R+S)\\
N_4(R,S) &= -\tfrac{1}{4}(1+R)(1-S)(1-R+S)\\
N_5(R,S) &= \tfrac{1}{2}(1+S)(1-R^2)\\
N_6(R,S) &= \tfrac{1}{2}(1-R)(1-S^2)\\
N_7(R,S) &= \tfrac{1}{2}(1-S)(1-R^2)\\
N_8(R,S) &= \tfrac{1}{2}(1+R)(1-S^2)
\end{aligned} \tag{9}$$

The displacements are interpolated by:

$$u = \sum_{i=1}^{8} N_i(R,S)u_i \tag{10}$$

$$v = \sum_{i=1}^{8} N_i(R,S)v_i$$

The strain-displacement relations can be written as:

$$\{\varepsilon\} = [B]\begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \tag{11}$$

where [B] is

$$B = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix} \tag{12}$$

where

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial R} \\ \frac{\partial N_i}{\partial S} \end{Bmatrix} \tag{13}$$

where [J] is the Jacobian matrix and is in the following form:

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial R} & \frac{\partial y}{\partial R} \\ \frac{\partial x}{\partial S} & \frac{\partial y}{\partial S} \end{bmatrix} \tag{14}$$

$$= \begin{bmatrix} \sum_{i=1}^{8} \frac{\partial N_i}{\partial R} x_i & \sum_{i=1}^{8} \frac{\partial N_i}{\partial R} y_i \\ \sum_{i=1}^{8} \frac{\partial N_i}{\partial S} x_i & \sum_{i=1}^{8} \frac{\partial N_i}{\partial S} y_i \end{bmatrix} \tag{14}$$

The variation of the displacement field can now be obtained through the Jacobian operator:

$$\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{Bmatrix} = J^{-1} \begin{Bmatrix} \frac{\partial u}{\partial R} \\ \frac{\partial u}{\partial S} \end{Bmatrix} \tag{15}$$

$$\begin{Bmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \end{Bmatrix} = J^{-1} \begin{Bmatrix} \frac{\partial v}{\partial R} \\ \frac{\partial v}{\partial S} \end{Bmatrix} \tag{16}$$

The inverse of Jacobian matrix is defined as:

$$[J]^{-1} = \frac{1}{\det\ [J]} \begin{bmatrix} \frac{\partial y}{\partial S} & -\frac{\partial x}{\partial S} \\ -\frac{\partial y}{\partial R} & \frac{\partial x}{\partial R} \end{bmatrix} \tag{17}$$

$$= \frac{1}{\frac{\partial x}{\partial R}\frac{\partial y}{\partial S} - \frac{\partial y}{\partial R}\frac{\partial x}{\partial S}} \begin{bmatrix} \frac{\partial y}{\partial S} & -\frac{\partial y}{\partial R} \\ -\frac{\partial x}{\partial S} & \frac{\partial x}{\partial R} \end{bmatrix}$$

The stress can be found from the following equation:

$$\{\sigma\} = [D]\{\varepsilon\} \tag{18}$$

where [D] is the material matrix which is a function of mechanical properties of the material. The element stiffness matrix [K] is given as

$$[K] = \int_{-1}^{1} \int_{-1}^{1} [B]^T [D][B] \det|J| \; dR \; dS \tag{19}$$

Since the form of the shape function $N_i(R,S)$ in all elements are polynomials, the derivatives, $\frac{\partial N_i}{\partial R}$, $\frac{\partial N_i}{\partial S}$, are non-singular. From equations (11), (12), and (13), the strain can be written in the following form:

$$\{\epsilon\} = [J]^{-1}[B'(R,S)] \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \tag{20}$$

Therefcre, from the above equation, the singularity can be obtained by requiring that the Jacobian matrix [J] be singular at the crack tip. In other words, the determinant of the Jacobian det$|J|$ vanishes at the crack tip. This can be achieved by placing the mid-side mode at the quarter points of the sides.

3. Quarter Point Elements Around the Crack Tip

In order to obtain the singular element matrices to be used around the crack tip, [ϵ] and [σ] must be singular. The singularity can be achieved by placing the mid-side node of any side at the quarter point. Figure 2a shows the two-dimensional, eight-noded quadrilateral quarter point element and Figure 2b, the six-noded triangular isoparametric elements with the mid-side nodes near the crack tip at the quarter point.

One of the most important features of these elements is that they satisfy the necessary requirements for convergence [11] in their singular form as well as in their non-singular form. They possess rigid body motion, constant strain modes, interelement compatibility, and continuity of displacements, in contrast with many other special crack tip elements.

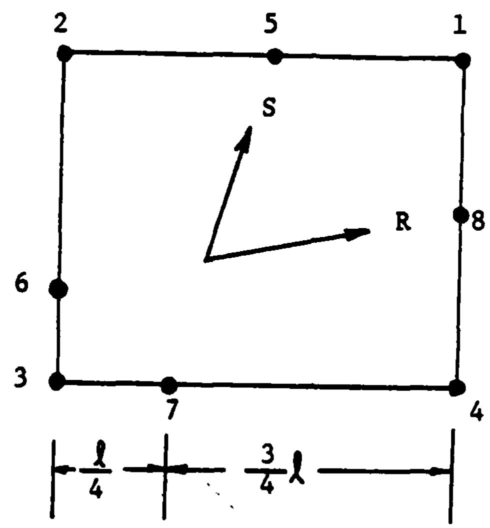


Figure 2a. Eight-Noded Quadrilateral Quarter Point Element

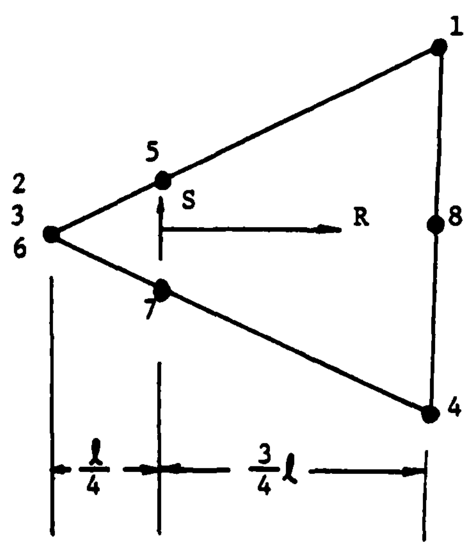


Figure 2b. Six-Noded Triangular Quarter Point Element

4. Initial Stress Finite Element Method

It is probably true in saying that the elastic-plastic behavior of the material played a dominating role in recent developments of fracture mechanics. One of the major objectives of this report is to provide a simplified treatment of a very complex problem so that with a minimum of programming effort, a reasonable solution can be obtained.

The practical interest in solving the elastic-plastic fracture problem may be in the prediction of displacements stresses and strains at various stages of loading, the size of the plastic zone, and the growth of crack.

The method, which serves the purpose, called the initial stress finite element method [13], is outlined in the following.

It is assumed that the von Mises yield criterion [45,46] and flow rules (See Appendix 1) are valid. Therefore, the following hypothesis appears to be generally accepted. If $\delta\{\varepsilon\}_p$ denotes the increment of plastic strain, then,

$$\delta\{\varepsilon\}_p = \lambda \frac{\partial F}{\partial\{\sigma\}} \tag{21}$$

In this relation, λ is a proportionality constant and F is a function of stresses. For the well-known von Mises yield surface, F is given in the following form:

$$F = [\tfrac{1}{2}(\sigma_x - \sigma_y)^2 + \tfrac{1}{2}(\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 3\tau_{xy}^2 + 3\tau_{yz}^2 + 3\tau_{zx}^2]^{1/2} - \bar{\sigma} \tag{22}$$

where $\bar{\sigma}$ is the effective equivalent stress.

During an infinitesimal increment of stress, the change of strain can be assumed that it is the sum of the elastic and plastic parts. Thus,

$$\delta\{\varepsilon\} = \delta\{\varepsilon\}_e + \delta\{\varepsilon\}_p \tag{23}$$

The elastic strain increments are related to stress increments by

$$\delta\{\varepsilon\}_e = [D]^{-1}\delta\{\sigma\} \tag{24}$$

Therefore, the strain increment can be written as

$$\delta\{\varepsilon\} = [D]^{-1}\delta\{\sigma\} + \frac{\partial F}{\delta\{\sigma\}} \cdot \lambda \tag{25}$$

From the above equation, it is convenient to solve for the stress changes in terms of the imposed strain changes [13].

$$\delta\{\sigma\} = [D]_{ep}\delta\{\varepsilon\} \tag{26}$$

where $[D]_{ep}$ is the elasto-plastic matrix. It is symmetric, and positive.

$$[D]_{ep} = [D] - [D] \left\{\frac{\partial F}{\delta\{\sigma\}}\right\}\left\{\frac{\partial F}{\delta\{\sigma\}}\right\}^T [D] \left[A + \left\{\frac{\partial F}{\delta\{\sigma\}}\right\}^T [D] \left\{\frac{\partial F}{\delta\{\sigma\}}\right\}\right]^{-1} \tag{27}$$

where A is the hardening parameter. For ideal plasticity with no hardening, A will be zero. If hardening is considered, in general, the value of A can be obtained from the slope of the uniaxial stress-plastic strain curve at the particular value of plastic strain.

For an elasto-plastic situation, the computation procedures during a series of load increment can be outlined as follows:

a) Apply the load increment and calculate elastic increments of stress $\{\Delta\sigma'\}_1$, and the corrsponding strain $\{\Delta\varepsilon'\}_1$.

b) Add $\{\Delta\sigma'\}_1$ to the existing stresses at the start of increment $\{\sigma_o\}$ to obtain $\{\sigma'\}$.

c) Check whether $F\{\sigma'\} < 0$. If this condition is satisfied, the pure elastic case is continued.

d) If $F\{\sigma'\} \geq 0$ and $F\{\sigma_o\} = 0$, this means that the element was yield at the start of load increment. Find $\{\Delta\sigma\}_1$ by following the equation

$$\{\Delta\sigma\}_1 = [D]_{ep}\{\Delta\varepsilon'\}_1$$

e) Evaluate stress which has to be supported by body forces

$$\{\Delta\sigma''\}_1 = \{\Delta\sigma'\}_1 - \{\Delta\sigma\}_1$$

f) Store the current stresses and strains

$$\{\sigma\} = \{\sigma'\} - \{\Delta\sigma''\}_1$$

$$\{\varepsilon\} = \{\varepsilon'\} + \Delta\{\varepsilon'\}_1$$

g) If $F\{\sigma\} > 0$, but $F\{\sigma_o\} < 0$, find the intermediate stress at which yield begins and calculate the stress increment $\{\Delta\sigma\}$. Then proceed as in (d).

h) Calculate nodal forces, or the plastic load vector corresponding to the equilibrating body forces for any element by following relation

$$\{P\}_1^e = \int [B]^T\{\Delta\sigma''\}_1 d(\mathrm{vol})$$

i) Find $\{\Delta\sigma'\}_2$ and $\{\Delta\sigma'\}_2$ using original elastic properties and the load system.

j) Find the current value of $\bar{\sigma}$ and repeat steps b) to i) etc...

The cycling will be terminated if the nodal forces in step h) reach sufficiently small values.

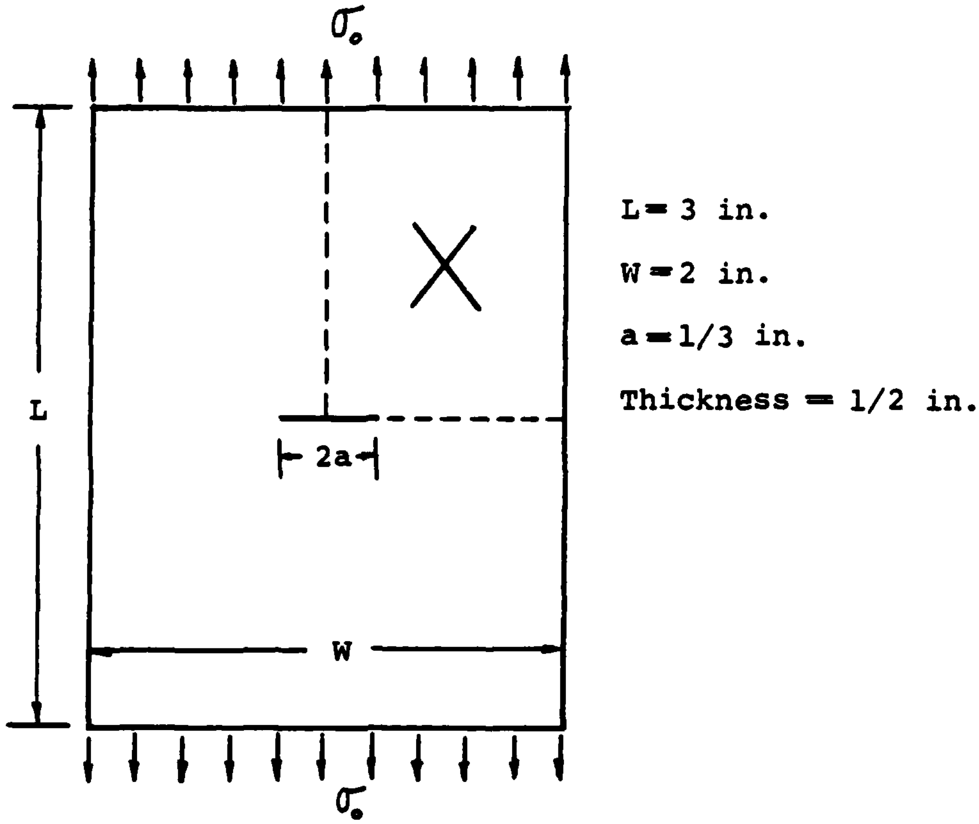


Figure 3a. Plate With Center Crack Under Tension

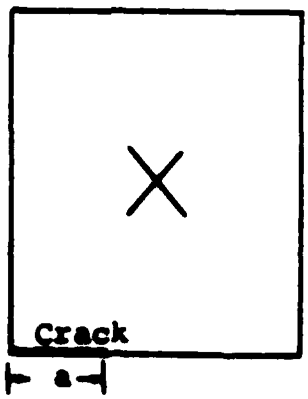


Figure 3b. Region ~ Be Analyzed (Plate With Center Crack)

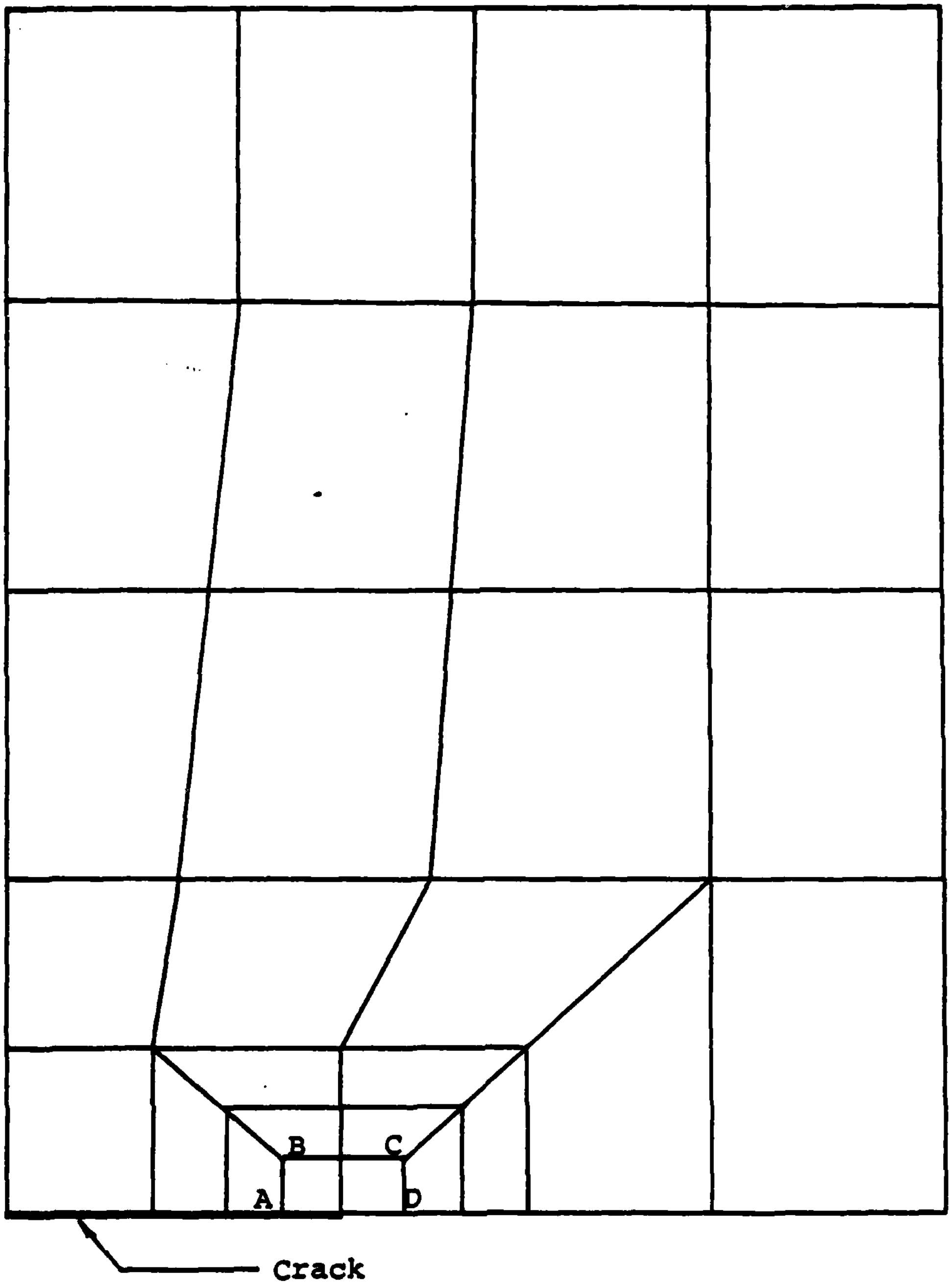


Figure 4. Finite Element Mesh of the Region to be Analyzed

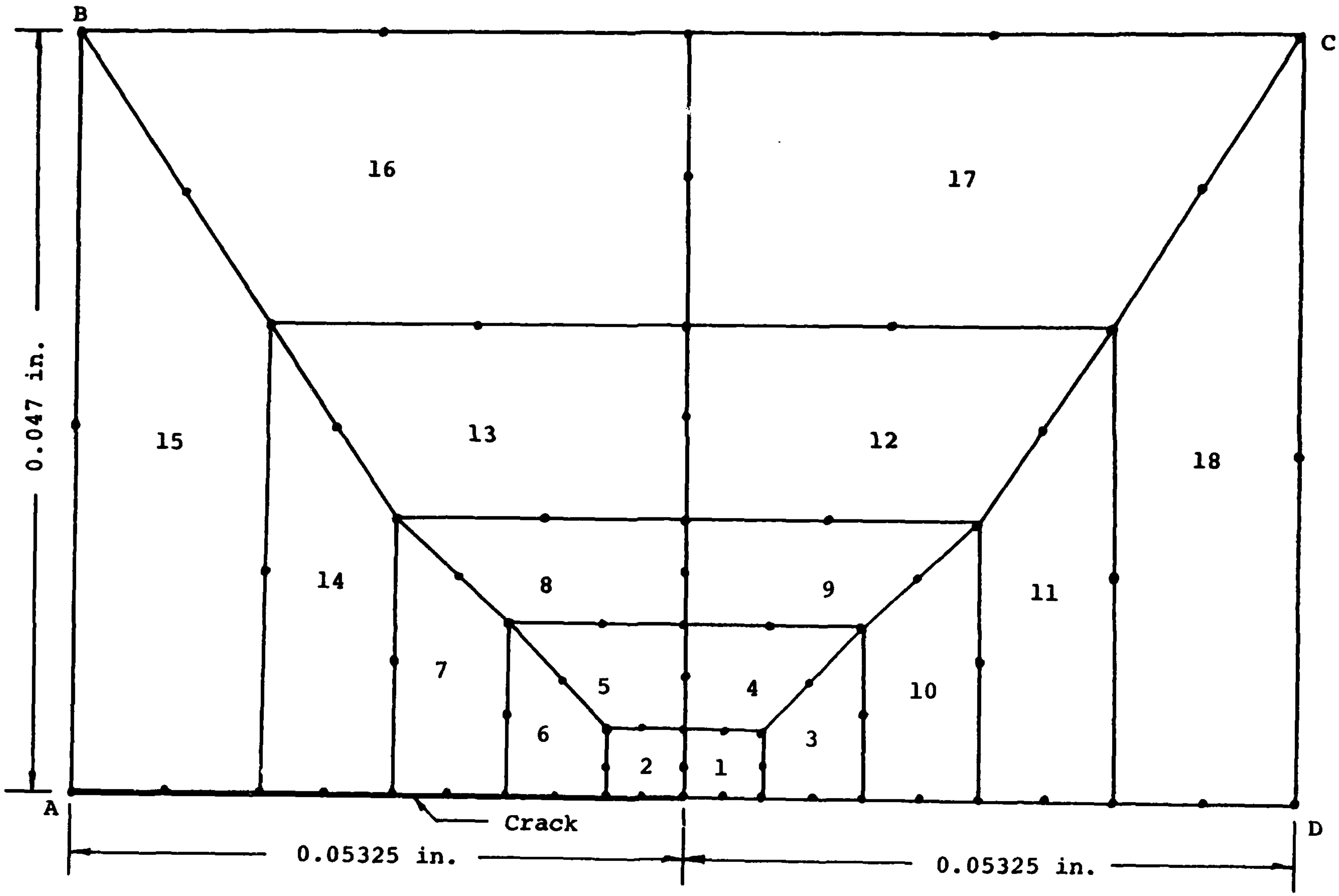
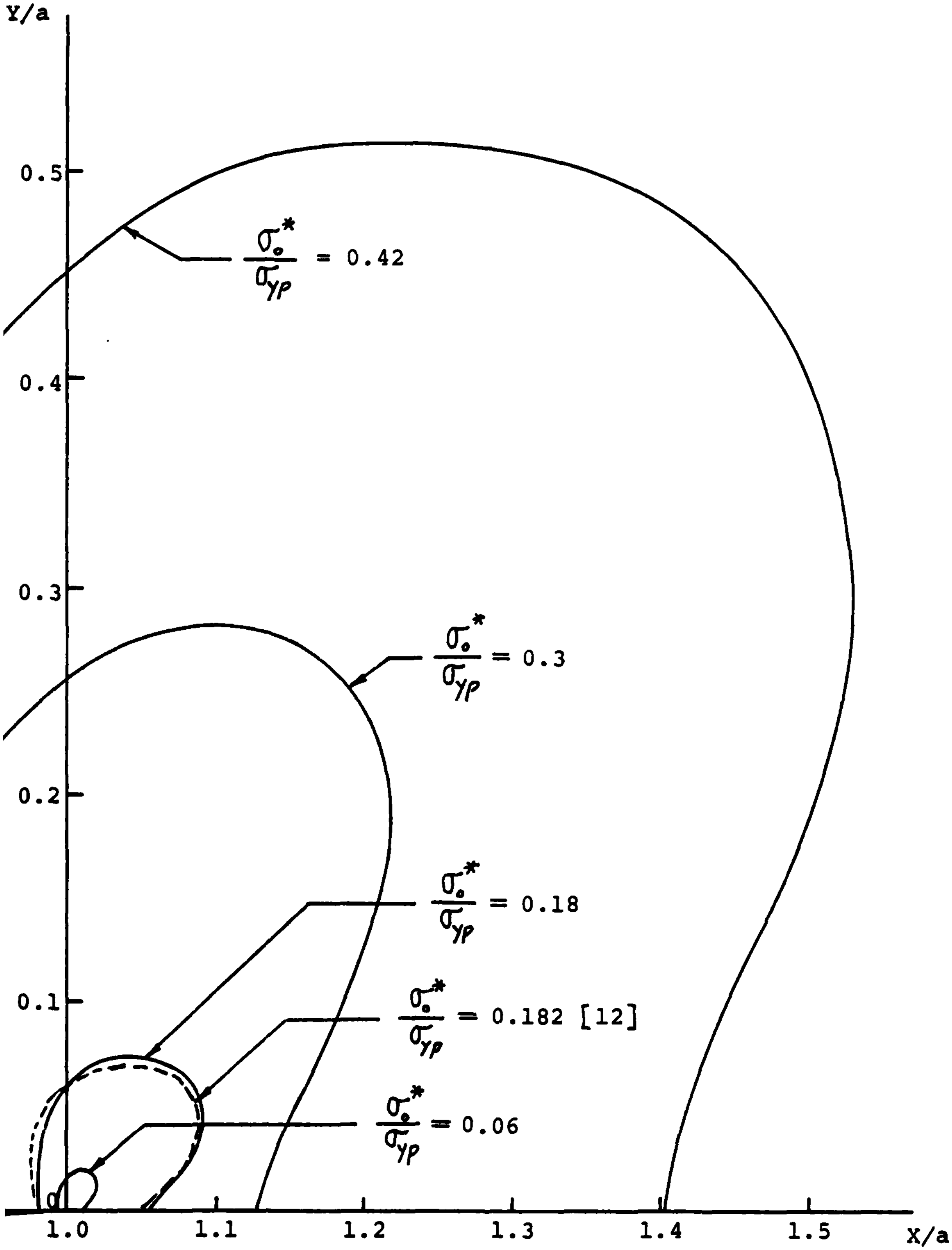


Figure 5. Details of the Near Crack-Tip Region



igure 6. Growth of Plastic Zone for a Center Crack Plate
(σ_o^* is the post yield applied load)

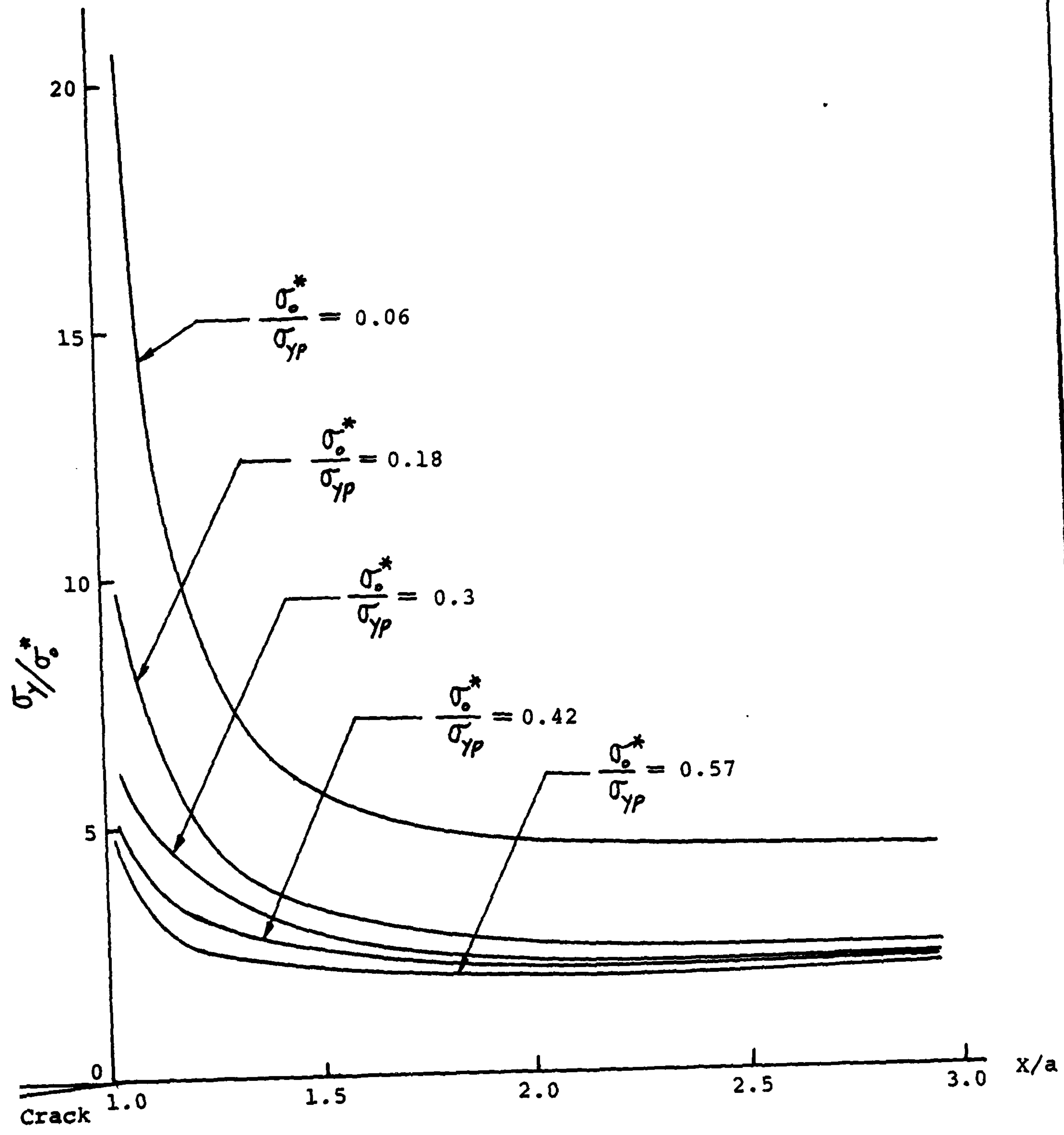


Figure 7. Post Yield σ_y Stress on Crack Plane, y = 0

σ_o^* is the Post Yield Applied Load

SECTION III

The J-Integral

The J-Integral [8] is basically a generalization of Griffith's theory to elasto-plastic rather than purely elastic crack tip behavior. J is a path-independent integral which can be interpreted as the energy release rate when a crack starts to propagate. In the finite element analysis of a static crack problem, the stress intensity factor K_I is commonly evaluated by the path-independent J-integral as follows:

$$J = \int_{\Gamma} [Wdy - T_i \frac{\partial U_i}{\partial x} ds] \tag{28}$$

where w is the elastic strain energy density and Γ is a path of arbitrarily chosen contour which is followed by the integration and which begins at any point on the lower crack surface, encircles the crack tip, and ends at any point on the upper crack surface. T_i is a traction vector acting along the outward normal to the contour.

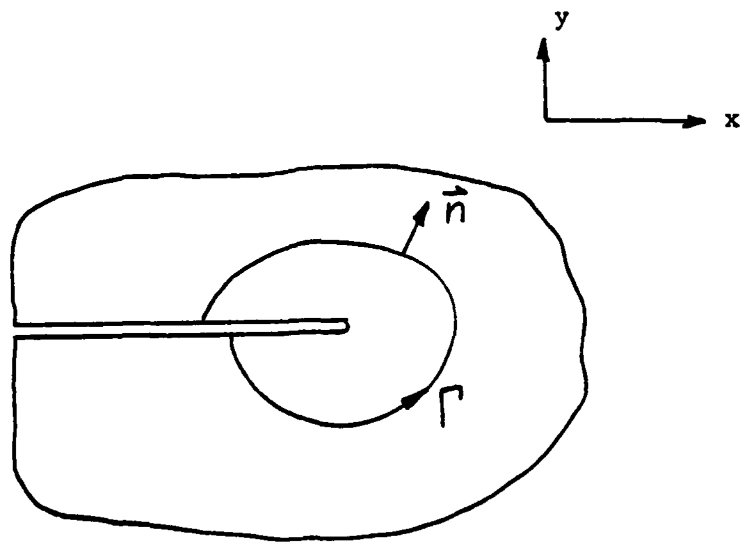


Figure 8. J-Integral Representation

SECTION IV

Movement of the Singular Elements

One of the latest developments to numerically model a propagating crack is the use of moving singular elements. We treat the singular element mesh as a function of stresses. When the Von Mises stress around the crack tip reaches a level greater than the yield strength of the material, the crack will be propagating at a distance of

$$r_p = \frac{1}{2\pi} \left(\frac{K_I}{\sigma_{yp}} \right)^2 \qquad (29)$$

and

$$J = \frac{1-\nu^2}{E} K_I^2 \qquad (30)$$

The schematic representation of the movement of the singular elements is in the following figure.

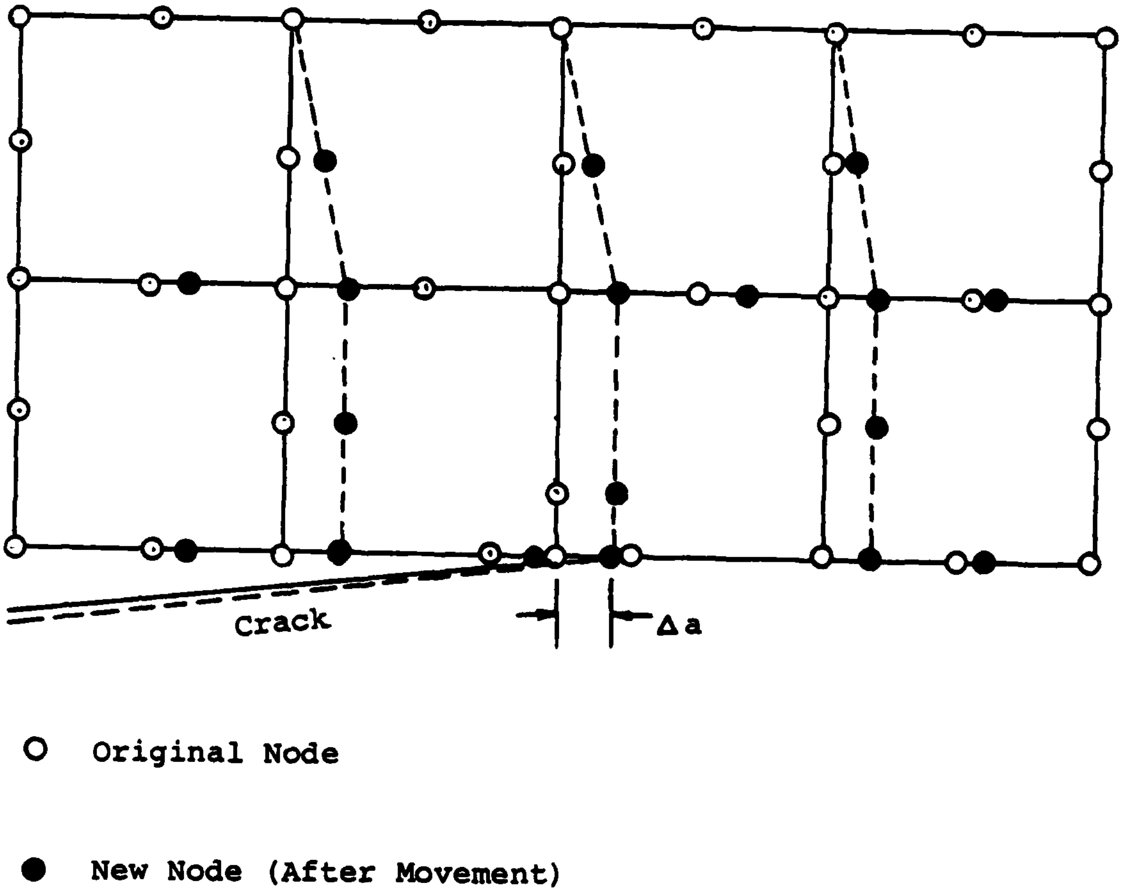


Figure 9. Schmatic Representation of the Movement of Crack-Tip Elements

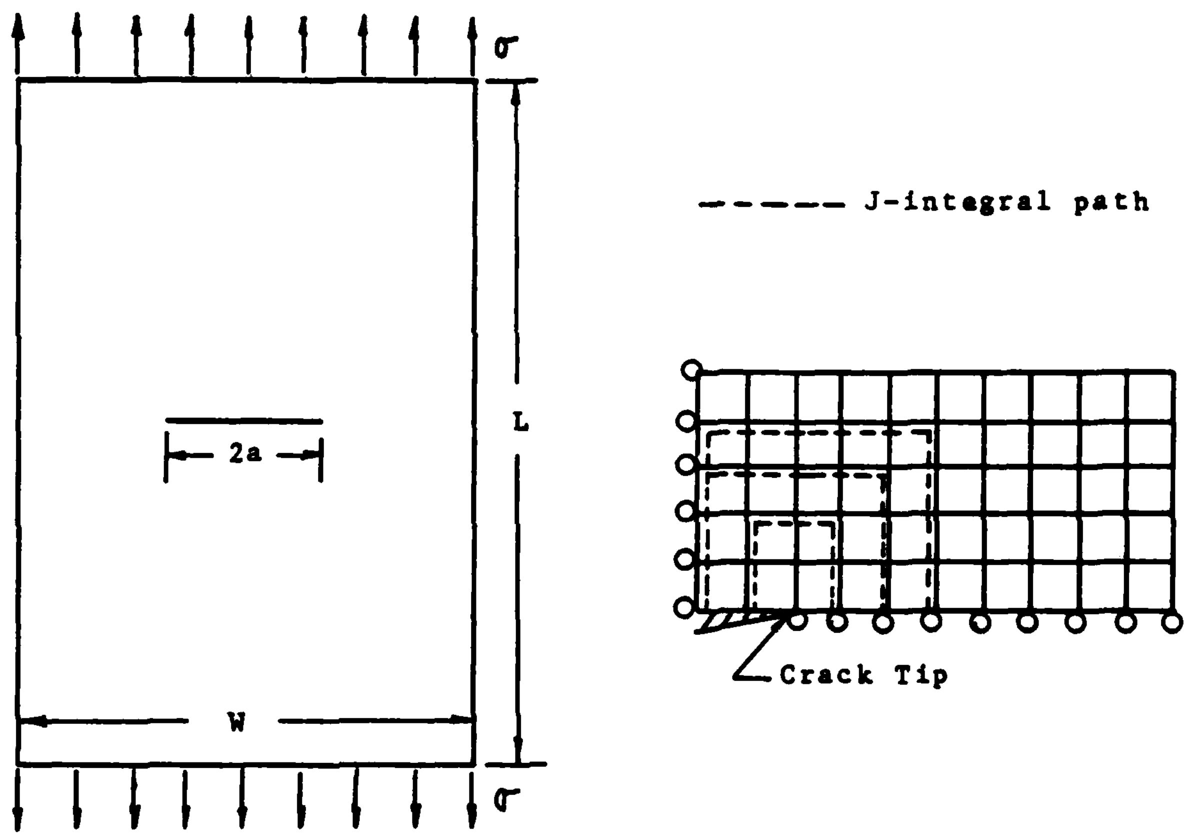


Figure 10. Finite Element Mesh with Quadrilateral Quarter Point Element for a Center Crack Plate

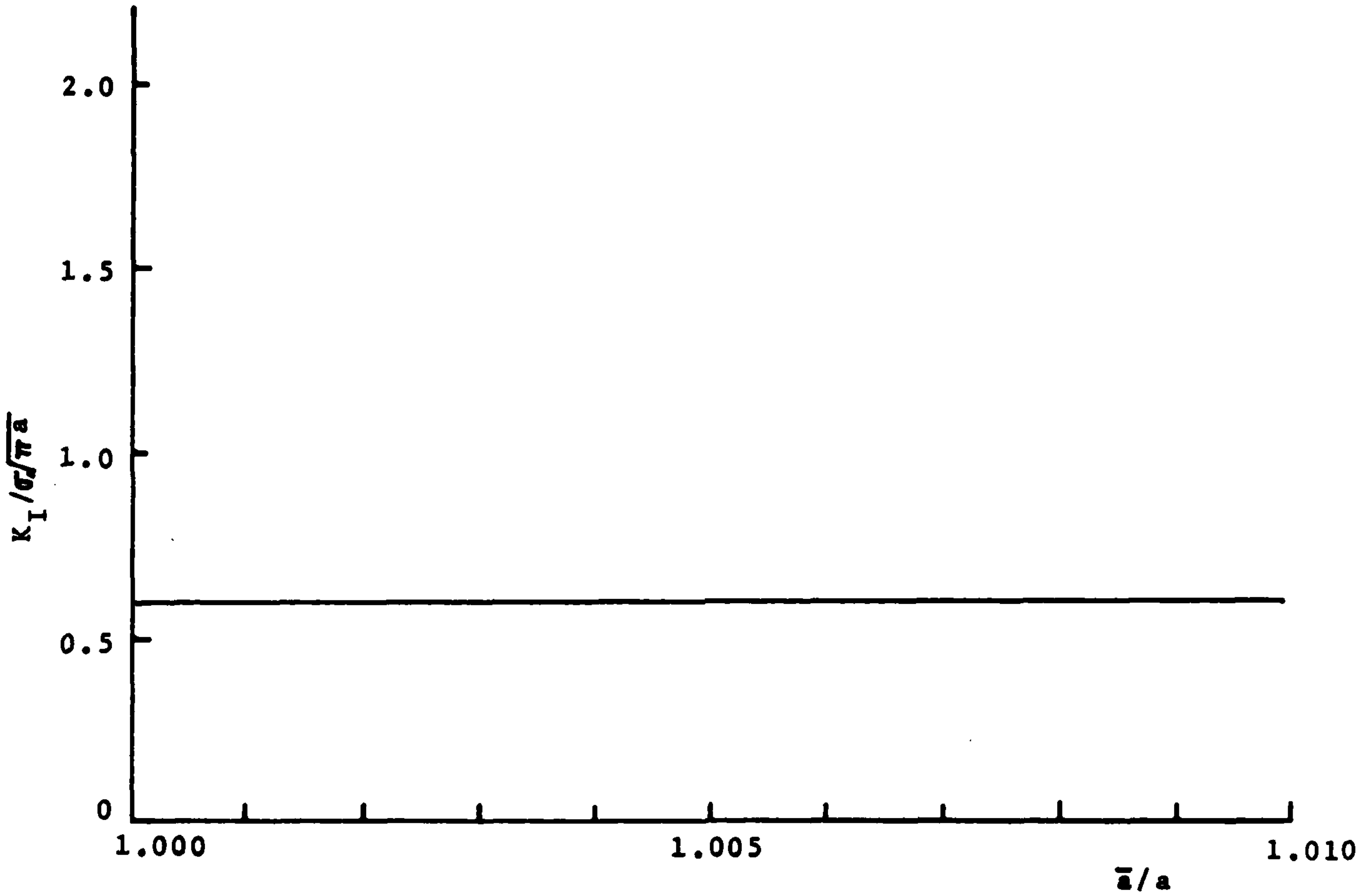


Figure 11. Stress Intensity Factor for a Crack Propagation with $a = 1/3$ in. and $W = 2$ in. ($\bar{a}$--new crack, after crack propagating)

SECTION V

Results of Numerical Example

A rectangular plate, L = 3 in. and W = 2 in., with a center crack, Fig. 3a, was considered. The thickness of the plate was chosen to be 1/2 inch and plane stress conditions were assumed in this case. The material properties were chosen to be the following:

$$E = 10 \times 10^6 \text{ psi}$$

$$\nu = 0.33$$

$$\sigma_{yp} = 60{,}000 \text{ psi}$$

Due to the symmetry of the problem, only one-quarter of the plate needs to be analyzed. A total of 163 nodes and 44 elements were used in the analysis of growth of the size of the plastic zone with increasing external loads. The plastic zones were plotted by connecting the centroidal points of the yielded elements around the crack tip.

For the analysis of the propagation of the crack, moving singular elements were used. The criterion of moving condition was that when the stress of the crack tip element was greater than the von Mises effective stress, the crack will propagate. The normalized stress intensity factors are determined from averaging the three different J-integral paths. The variation among these paths is less than 3%.

SECTION VI

Summary and Conclusions

Although considerable progress has been made in the field of elasto-plastic fracture mechanics, it is the common consensus of most of the foremost researchers in the field that the actual size and shape of the plastic zone near the crack tip are not accurately established yet. It would, therefore, be improper to claim that the results presented in this report are the most reliable and accurate; nevertheless, they provide a basic knowledge that can be used to better understand more complex problems in the near future. The analysis and procedure presented in this report for elasto-plastic analysis do provide a better and more reliable method in comparison with the other similar procedures available in the literature [12, 17, 19, 20, 36]. For physical engineering problems, the eight-noded quadrilateral isoparametric elements and the eight-noded quarter-point elements yield a good result with proper calibration. The main improvements in using these elements instead of the constant strain triangular elements are the convergence and the stability of the solution warranted.

It has been demonstrated that the technique can be applied where crack propagation is very small and may not otherwise be measured by conventional experimental techniques.

The following conclusions are based on the elastic-plastic crack growth analysis.

1) A method was developed to calculate the size and shape of the plastic zone around the crack tip. The problem was solved by using the initial stress finite element method. Eight-noded quadrilateral isoparametric

elements were used in the analysis. It was proved that this method is quite efficient and provided reasonable, accurate results.

2) The eight-noded quarter-point element yielded a good result with proper calibration. This singular element, when coupled with the moving element technique, was preferable for studying the fast fracture problem.

3) It was found that the procedure of the moving singular element was much easier to use than the so-called crack tip node release method. Furthermore, the moving singular element provided realistic crack growth values.

4) The moving finite element technique can be modified to solve the crack branching problem.

The above advancements in the understanding of elastic-plastic crack growth are specially suited for aiding the future study of the fast or dynamic fracture problems. In addition, the crack growth criteria investigation and the calculation of the plastic zone size provide significant progress towards life prediction of machine elements, such as turbine blades and turbine disks.

The following additional research work is recommended to further the machine element life prediction capability.

1) Future work needs to include impact loading and cyclic loading conditions.

2) Use present approach to analyze other test specimen geometries to determine the dependence/independence of results on speciment geometry. Also, investigate the repeatability.

3) Use present approach to analyze the crack growth of the composite material structures.

4) Environmental effects should be further researched by conducting fatigue and creep crack growth tests in different environmental conditions, such as high sulfur content, vacuum, etc.

In summary, the technique developed here is worth exploring further to provide a better understanding of crack growth rate behavior in machine elements under normal operating conditions.

SECTION VII.

Appendix I

1. Von Mises Criteron

A criterion that involves a smooth function was proposed by Von Mises. In its most widely used form, the von Mises criterion, in terms of principal stresses, predicts that yielding occurs when

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = \text{constant} \qquad (1A)$$

In a more general form,

$$(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) = \text{constant} \qquad (2A)$$

To determine the constant, the following procedure is used:

a) For uniaxial tension, yielding occurs when $\sigma_1 = \sigma_{yp}$, $\sigma_2 = \sigma_3 = 0$. Therefore equation (1A) becomes

$$2\sigma_1^2 = \text{constant} = 2\sigma_{yp}^2$$

b) For pure shear, yielding occurs when $\sigma_1 = -\sigma_3 = k$, $\sigma_2 = 0$ and using equation (1A):

$$\sigma_1^{\,2} + \sigma_1^{\,2} + 4\sigma_1^{\,2} = 6\sigma_1^{\,2} = \text{constant} = 6k^2$$

Thus the von Mises criterion may be written as :

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_{yp}^{\,2} = 6k^2 \qquad (3A)$$

According to this criterion the tensile and shear yield stresses are related as $\sigma_{yp} = \sqrt{3}k.$

It is convenient to consider this criterion as a function of an effective stress, denoted as $\bar{\sigma}$, where $\bar{\sigma}$ is a function of the applied stresses. Whenever its magnitude reaches the yield strength in uniaxial tension, then that applied stress should cause yielding to occur. Thus

$$\bar{\sigma} = \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + \sigma_3 - \sigma_1)^2]^{1/2} \qquad (4A)$$

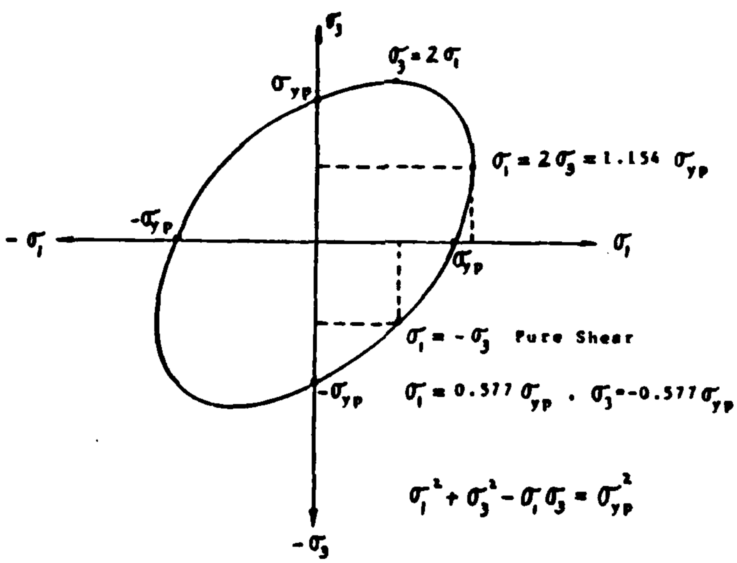


Figure 1A. Von Mises Yield Locus: $\sigma_2 = 0$

2. Distortion Energy

An interpretation of the Mises criterion is that yielding occurs when the elastic energy causing distortion reaches a critical value. This strain energy is found in a general way by subtracting the dilatational strain energy from the total elastic strain energy. The total strain energy per unit volume is:

$$U_v = 1/2\ (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx}) \tag{5A}$$

or for the case of principal stresses:

$$U_v = 1/2\ (\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3) \tag{6A}$$

To express the above equation as a function of stresses, the generalized form of Hooke's law, gives

$$U_v = \frac{1}{2E}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - \frac{\nu}{E}(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \tag{7A}$$

Since only normal stresses cause a volume change, the dilatation is:

$$\Delta = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \frac{1-2\nu}{E}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{3}{E}(1-2\nu)\ \sigma_m \tag{8A}$$

Let $\Delta = 3\varepsilon_m$

Therefore $\varepsilon_m = \dfrac{1-2\nu}{E}\ \sigma_m$

From the above discussion, it can be seen that the work due to dilatation is

$$U_d = 3(1/2\ \sigma_m \varepsilon_m).$$

then $U_d = [3(1-2\nu)\sigma_m^2]/2E$

$$U_d = \frac{1-2\nu}{6E}(\sigma_1 + \sigma_2 + \sigma_3)^2 \tag{9A}$$

By subtracting eq. (9A) from (7A) to give the shear strain energy, U_s, the result after rearrangement is:

$$U_S = \frac{1}{12G}\ [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \tag{10A}$$

Now, the shear strain energy induced during uniaxial tension where $\sigma_2 = \sigma_3 = 0$ is:

$$U_s = \frac{\sigma_1^{\;2}}{6G}$$

The critical value, U_{sc}, that may be developed to cause yielding will result when $\sigma_1 = \sigma_{yp}$. Setting eq. (10A) equal to this critical value leads to:

$$\frac{1}{12G} \quad [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = \frac{\sigma_{4p}^{\;2}}{6G} \tag{11A}$$

3. <u>Flow Rules or Plastic Stress-Strain Relationships</u>

Consider plastic flow under uniaxial tension as indicated in following figure:

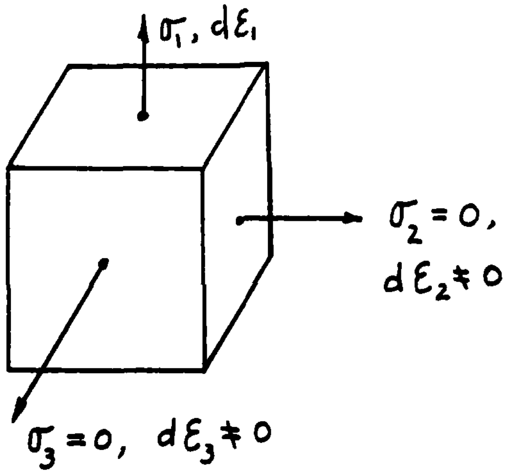


Figure 2A. Stresses and Incremental Plastic Strains for Uniaxial Tension

Now the deviatoric stress in the 1 direction is $\sigma_1{}' = \sigma_1 - \sigma_m$ and at the particular instant represented in the above figure,

$$\sigma'_1 = 2/3\ \sigma_1 \text{ and } \sigma_2{}' = \sigma_3{}' = 0 - 1/3\ \sigma_1 = -\ 1/3\ \sigma_1$$

Therefore,

$$\sigma_1{}' = -\ 2\sigma_2{}' = -\ 2\sigma_3{}'$$

volume constancy, the sume of the plastic strain increments must be

erefore,

$+ d\varepsilon_2 + d\varepsilon_3 = 0$

ere is symmetry in this instance, $d\varepsilon_2 = d\varepsilon_3$, therefore, $d\varepsilon_1 = - 2d\varepsilon_2$

. This leads to:

$$\frac{d\varepsilon_2}{\sigma_2} = \frac{d\varepsilon_3}{\sigma_3} = \text{constant} = d\lambda \tag{12a}$$

ication is that the ratio of the current incremental plastic strain

s to the current deviatoric stresses is a constant. This expresses

dtl-Reuss flow rules, where the elastic strain increments have been

convenience, the flow rules may be expressed in other forms

$$\frac{{}_1 - d\varepsilon_2}{{}_1 - \sigma_2} = d\lambda$$

$$= \frac{2}{3} d\lambda[\sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3)] \tag{13a}$$

$$= \frac{d\bar{\varepsilon}}{\bar{\sigma}}[\sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3)]$$

is the incremental effective strain which is defined as

$$[(d\varepsilon_1 - d\varepsilon_2)^2 + (d\varepsilon_2 - d\varepsilon_3)^2 + (d\varepsilon_3 - d\varepsilon_1)^2]^{1/2} \tag{14a}$$

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