ON THE OPTIMAL LOCATION OF VIBRATION SUPPORTS (U)
1982 B P Wang, W D Pilkey
N00014-75-C-0374

UNCLASSIFIED
A-82-3

END
ON THE OPTIMAL LOCATION OF VIBRATION SUPPORTS

B. P. Wang
W. D. Pilkey

Department of Mechanical & Aerospace Engineering
University of Virginia
Charlottesville, VA 22901

Office of Naval Research
Arlington, VA 22217

1982

This document has been approved for public release and sale; its distribution is unlimited.

The problem of optimal positioning of vibration supports to raise the fundamental natural frequency of a system is studied. It is established that for the optimal locations criterion the corresponding lowest antiresonant frequency is a maximum. A numerical example illustrates this criterion.
On the Optimal Location of Vibration Supports

B. P. Wang, W. D. Pilkey
University of Virginia

ABSTRACT

The problem of optimal positioning of vibration supports to raise the fundamental natural frequency of a system is studied. It is established that for the optimal locations criterion the corresponding lowest antiresonant frequency is a maximum. A numerical example illustrates this criterion.

1. Introduction

Intermediate supports are often introduced in engineering structures to increase the resonant frequencies of the system as well as to support weights. These supports, when realized by actual structural components, are elastic supports. Thus, the problem of designing vibration supports to raise the fundamental frequency involves both finding the location and the required stiffness of the supports.

In an earlier paper, Bezler and Curreri [1] studied the design of vibration supports for piping systems. They used the transfer matrix method to study a spring supported cantilever beam and a spring supported "L" bend. They found the optimum spring location, i.e., the most effective location to put a spring to increase the fundamental frequency, from numerical experimentation. They concluded that a "near optimal" position for a flexible spring is at a node of the second mode. For a rigid support this would be the optimal location.

In the present paper, a criterion for selecting the optimal spring locations will be derived. This criterion can also be used to compare the relative effectiveness of sets of proposed support locations.

2. Problem Formulation

For a multiple-degree-of-freedom, undamped system with a spring introduced at dof J, the frequency equation is

\[ \frac{1}{k} + R_{JJ}(\omega) = 0 \]  

where \( R_{JJ}(\omega) \) is the receptance of dof J. Equation (1) can be derived using the receptance method [2]. Alternatively, it can be found by considering the addition of a spring to a system as a local modification [3]. The receptance \( R_{JJ}(\omega) \) can be expressed in modal summation form as
where \( \omega_t \) is the natural frequency of the \( t \)th mode of the unsupported system. \( \rho_{jt} \) is the corresponding eigenvector. \( \rho_{jt} \) is the \( j \)th component of \( \rho_t \). and \( G_t = (\rho_t)^T[m](\rho_t) \) is the generalized mass of the \( t \)th mode. Thus, for any given spring rate \( k \), Eq. (1) along with Eq. (2) can be used to solve for the new frequencies \( \omega \). The natural frequencies of the supported system increase as the spring rate increases. In the limit, as \( k \) approaches infinity, i.e., as the support becomes ideally rigid, the frequency equation becomes

\[
R_{JJ}(\omega) = 0
\]  

Denote the lowest \( \omega \) that satisfies Eq. (3) as \( \omega^{(J)} \). Then \( \omega^{(J)} \) is the lowest antiresonant frequency of dof \( J \). That is, \( \omega^{(J)} \) is the highest fundamental frequency achievable when the support at dof \( J \) becomes rigid. It follows from the eigenvalue separation property [6], that \( \omega^{(J)} < \omega_2 \), where \( \omega_2 \) is the second natural frequency of the unsupported system. Thus, by choosing dof \( J \) for a rigid support as a node in the second mode of the unsupported system, we have \( \omega^{(J)} = \omega_2 \) which is the maximum obtainable fundamental frequency. This result has been known for some time [1].

Now consider the case of introducing \( s \) springs at dof \( J_1, J_2, \ldots, J_s \). Following the procedure of Ref. 5, the frequency equation of the supported system is given by

\[
\det([I] + [\hat{R}][\Delta K]) = 0
\]  

where

- \([I]\) is an \( s \times s \) identity matrix
- \([\hat{R}]\) is the receptance matrix associated with the dof \( J_1, J_2, \ldots, J_s \), i.e.,

\[
\hat{R}_{ij} = R_{ij}^{(J)}
\]

\[
[\Delta K] = \begin{bmatrix}
\Delta k_1 \\
\Delta k_2 \\
\vdots \\
\Delta k_s
\end{bmatrix}
\]  

\( \Delta k_j \) is the spring rate of the support at dof \( j \).
In the limiting case when all $\Delta k_j \to \infty$, Eq. (4) becomes

$$\det[R] = 0$$

Let $\lambda$ be the lowest root of Eq. (6). Then the optimal support locations will be where $\lambda$ is a maximum. We are now in a position to establish a criterion for optimal support locations.

3. **Maximum Antiresonant Frequency Criterion**

For given sets of support locations, the best set of locations is where the corresponding lowest antiresonant frequency is a maximum.

We will call this criterion the **Maximum Antiresonant Frequency Criterion** (MAFC). To find the antiresonant frequency, one can either solve an eigenvalue problem of order $(n-s)$ or solve the nonlinear Eq. (6).

4. **Numerical Example**

To illustrate the basic contention of the MAFC criterion, consider the simply supported beam of Fig. 1. The fundamental frequency of this beam is $15.71$ Hz. It is desired to introduce two intermediate supports to increase the fundamental natural frequency to above $25$ Hz. For this example it is practical to restrict the support locations to two possible sets of positions, say A $(x_1 = 0.1L, x_2 = 0.5L)$ and B $(x_1 = 0.34L, x_2 = 0.67L)$.

For this case with two supports, we have

$$\Lambda = \begin{bmatrix} \Delta K_1 & 0 \\ 0 & \Delta K_2 \end{bmatrix}$$

and

$$[\hat{R}] = \begin{bmatrix} \hat{R}_{11} & \hat{R}_{12} \\ \hat{R}_{21} & \hat{R}_{22} \end{bmatrix}$$

It is convenient to calculate the elements $R_{i,j}$ with a modal summation. Thus,

$$\hat{R}_{i,j} = R(x_i, x_j)$$

$$= \sum_{l=1}^{n} \frac{\psi_j(x_i) \psi_j(x_j)}{G_l(\omega_l^2 - \omega^2)}$$

(8)
L = 2.54m (100 in.)
E = 69 GPa (10^7 psi)
ρ = 8748.73 kg/m³ (0.01 lb·sec²/in²)
I = 4.1623x10⁻⁶ m⁴ (10 in.⁴)

Figure 1 A simply supported beam
where, for a simply supported beam,

\[ \rho_e(x) = \sin \frac{i\pi x}{L} \]

\[ G_e = \frac{1}{2} \rho L \]

\[ \omega_e = \frac{(i\pi)^2}{L^2} \sqrt{\frac{EI}{\rho}} \]

In the numerical calculation \( n = 20 \) is used, or, in other words, 20 modes are used to evaluate the receptances in Eq. (8). The frequency determinant of Eq. (6) gives

\[ f_1^{(A)} = \text{fundamental natural frequency for rigid supports at} \]
\[ x_1 = 0.1L, x_2 = 0.5L \]
\[ = 70.9 \text{ Hz} \]

\[ f_1^{(B)} = \text{fundamental natural frequency for rigid supports at} \]
\[ x_1 = 0.34L, x_2 = 0.67L \]
\[ = 180.1 \text{ Hz} \]

Since \( f_1^{(B)} > f_1^{(A)} \), we conclude that the location pair B is more effective than location pair A in raising the fundamental frequency of the system.

To check the above proposition, we will compute the fundamental frequencies of the spring supported beam for the special case of equal spring rates. The results are summarized in Table 1. Alternatively, we can compute the required (equal) spring rates for both springs for given fundamental frequencies. The results are summarized in Table 2. We observe that to raise the fundamental frequency above 25 Hz, springs with rates of about \( 1.23 \times 10^6 \text{ N/m} \) (7000 lb/in) are needed at location \( x_1 = 0.1L \) and \( x_2 = 0.5L \), while less stiff springs with rates of \( 0.88 \times 10^6 \text{ N/m} \) (5000 lb/in) are needed if they are located at \( x_1 = 0.34L, x_2 = 0.67L \).
Table 1
Natural Frequencies for Simply Supported Beam with Two Equal Intermediate Springs

<table>
<thead>
<tr>
<th>Spring Stiffness (N/m)</th>
<th>Fundamental Natural Frequency of the Supported System (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_1 = 0.1L$, $x_2 = 0.5L$</td>
</tr>
<tr>
<td></td>
<td>(lb/in.)</td>
</tr>
<tr>
<td>17513 (100)</td>
<td>15.88</td>
</tr>
<tr>
<td>87565 (500)</td>
<td>16.67</td>
</tr>
<tr>
<td>175130 (1000)</td>
<td>17.38</td>
</tr>
<tr>
<td>350268 (2000)</td>
<td>18.90</td>
</tr>
<tr>
<td>525390 (3000)</td>
<td>20.30</td>
</tr>
<tr>
<td>700520 (4000)</td>
<td>21.61</td>
</tr>
<tr>
<td>875650 (5000)</td>
<td>22.82</td>
</tr>
<tr>
<td>1751300 (10000)</td>
<td>28.07</td>
</tr>
<tr>
<td>$\infty$</td>
<td>76.9</td>
</tr>
</tbody>
</table>

Table 2
Required Spring Rate to Achieve Prescribed Natural Frequency

<table>
<thead>
<tr>
<th>Fundamental Natural Frequency (Hz)</th>
<th>Required Spring Stiffness (lb/in) for $x_1 = 0.1L$, $x_2 = 0.5L$</th>
<th>Required Spring Stiffness (lb/in) for $x_1 = 0.34L$, $x_2 = 0.67L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>29238 (166.95)</td>
<td>21220.5 (121.17)</td>
</tr>
<tr>
<td>17</td>
<td>133608 (762.91)</td>
<td>968962.7 (553.09)</td>
</tr>
<tr>
<td>18</td>
<td>244432 (1396.86)</td>
<td>177135 (1011.45)</td>
</tr>
<tr>
<td>19</td>
<td>362386 (2069.24)</td>
<td>262047 (1496.3)</td>
</tr>
<tr>
<td>20</td>
<td>486935 (2780.42)</td>
<td>351610 (2007.71)</td>
</tr>
<tr>
<td>25</td>
<td>1214360 (6934.07)</td>
<td>869482 (4964.78)</td>
</tr>
<tr>
<td>30</td>
<td>2125640 (12137.50)</td>
<td>1505170 (8594.61)</td>
</tr>
</tbody>
</table>
5. Conclusion

In summary, a simple criterion has been derived that will allow a designer to choose the optimal locations for placing vibration supports. This will narrow the design problem to that of determining the required stiffness to achieve a desired fundamental natural frequency.

Acknowledgment

This work was supported by the Office of Naval Research. Arlington, Virginia.

References
