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12

Research Report CCS 429

A SEA-SHORE ROTATION GOAL PROGRAMMING MODEL FOR NAVY USE

by

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1. Introduction

This is a report on the current status and projected work on Sea/Shore Rotation Models for the U.S. Navy. For perspective, we begin with a discussion of the nature of the problem and some preceding efforts as follows.

The problem of sea/shore rotation arises on a large scale because the Navy has some 250 "distributable communities." Each such community represents a specific skill area that the Navy has to manage. For example, "gunners' mates" represents one such skill area. Within the area of gunners' mates there are various sub-specialties which must also be separately identified and managed. The class of ASROC (Anti-Submarine Rockets) gunners' mates represents one such sub-speciality.[1] Within each such sub-speciality there are also 7 pay grades that also need to be separately identified and managed.

There are fleet units and shore units which must be balanced relative to one another. In a personnel shortage situation, such as at present, the priorities must lie in planning for the fleet units. Some of the specialty areas have a preponderance of billets at sea and very few at shore. Other specialty areas, such as "data processing technicians," have most of their billets on shore.

Navy policy requires that no person should spend more than five years at sea and less than two years on shore. This policy is to be implemented no matter what sea/shore billet ratio obtains in any area.

[1]Every ship which has an ASROC has to have two ASROC gunners' mates at a minimum.

There are further complications in that other Navy manning policies must also be considered. For example, three year tours of duty apply to recruiters and recruit company commanders. Recruiters and recruit company commanders, and other similar groups, are handled as sub-specialties while they are ashore and treated as a different specialty when they return to sea duty. Thus, for instance, a gunner's mate may spend five years at sea followed by three years of shore duty as a recruiter. If, however, the same person were to come ashore in a gunner's mate billet he might only spend two years ashore before being returned to sea.

The preceding example represents only one of numerous variations that may occur. These all need to be explicitly considered in any detailed model that is to be useful in guiding actual Navy planning of sea/shore rotation schedules.

Other factors also need to be considered. These include attrition, accession, retention, promotion, and demotion. Additionally, planners must be able to balance all of these movements with the changing technologies (and their related requirements) which are constantly occurring in the fleet (and elsewhere). For example, in a brief ten year span the Navy had to go from having no need for gas turbine technicians to the present situation in which thousands of them are needed. This occasions a need for training and retraining of personnel with consequent requirements that impinge on sea/shore rotation schedules.

2. Background

Sea/shore rotation has traditionally been accomplished with very extensive and laborious uses of hand calculations within either one specialist area, or one pay grade within one such area. The calculations are done on a static basis without consideration of further repercussions.

There have been three attempts to model and computerize the process. The first such effort, called a Dynamic Flow Model, was done by NPRDC (Navy Personnel Research and Development Center) in the early 1970's. This model was not up to the job of handling all of the variables. The second effort was therefore undertaken by the Center for Naval Analysis, circa 1978-79, which turned out to be too cumbersome and expensive for practical use. Finally, a third effort, pointed primarily at "economic policy analysis" of sea/shore rotation, could not supply the detail needed for actual scheduling.[1]

3. Present Model

The present "system model" was designed and developed to provide direct access to the highly developed technologies that are now available for models of "network type." As in the case of other types of manpower planning models,[2] some of the relations and constraints are not of pure network types. As was also the case in these other models, however, devices were developed in the course of our research to make the resulting models amenable to methods of pure network calculation and solution. For such networks one has advantages of two orders of magnitude (and more) in terms of the size of the models and speed of computation. Access to advantages like these is evidently essential for an ability to handle the Navy's sea/shore rotation system in adequate detail.

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[1] For purposes of abbreviation, we may refer to these as the NPRDC, CNA and DCF (Discounted Cash Flow) models, respectively.

[2] See [4] - [6].

To conform with Navy requirements and practices, the models are developed to apply to each specialty area separately.[1] Starting with a given distribution of personnel by pay-grade and LOS (length of service) and by status in a rotation sequence, the effects of attrition, promotion, accession are handled while working through the flows over a pre-set time horizon toward goals of desired levels of manning, both at the horizon and at various points en route which are deemed to be important. This has yielded a constrained network model of goal programming variety.

Because the model is of a goal programming variety and treatable in an equivalent linear programming form,[2] the results of linear programming theory are available through highly efficient computer software. This provides ready access to tradeoff information that would not be available from other routes. This information is applicable for use in policy evaluation and guidance as well as for use in the management of the sea/shore rotations under given policies. This makes it possible to study the potential value of altering present policies relative to future requirements without losing contact with the present needs for scheduling.

An example of the tradeoff evaluations and related "what-if" possibilities that are desired from such a modeling effort may be made by reference to issues of fleet readiness. A quantitative increase in present fleet readiness capabilities may have repercussions for the manning of shore facilities that will adversely affect future capabilities of the fleet. These are the kinds of trade-offs that may need to be considered

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[1] The combinations previously alluded to of different sea and shore specialties are, however, treatable in this same model format.

[2] See [1].

(in varying detail) between the present and the future. Others take the form of trade-offs by reference to repercussions for performance between different Navy functions (and installations) in a single time dimension. These trade-offs and "what if" possibilities are not merely averages or expected values from a series of computer runs on, say, a simulation model employing Monte Carlo routines. In the models developed here, they are effected by reference to optimum levels of achievement on each of the goal possibilities that might be considered.

A numerical example will shortly be provided and full mathematical details (with the related theory and methodology) are supplied in the Appendix that accompanies this report. Although the numerical example is kept very small and simple, this is only to facilitate exposition and understanding. The point to bear in mind is that the full-scale problem is to be accommodated with accompanying routines of computation that will be feasible and economical to use. In addition the kinds of tradeoffs that are needed for policy and evaluation guides that have just been indicated will also be available for use without extra cost or effort.

4. Numerical Example

As an illustration of how the "sea/shore rotation systems model" works, a small numerical example is provided with data representative of that of an actual detailing community. This example considers only paygrade 3 and LOS levels 1 and 2 over a two-period horizon. The rotation policy under consideration is five periods at sea/two periods on shore (5/2).

Additionally, at each rotation point, a percentage of personnel will move to tours such as recruiting and instructing. Personnel that are on such tours will be designated as being in the category "other."

At the beginning of the first period, the manning levels in paygrade 3/LOS 1 are given in Table 1.

TABLE 1

INITIAL MANNING LEVELS

PG	LOS	ACTIVITY	NBR. OF PERSONNEL
3	1	Shore	54
		Sea	843
		Other	53

Figure 1 gives a network flow diagram of how personnel move through the network in the two period example. The designations of nodes and arcs are that of our newly developed BIGNET code and are defined in section 3 of the Appendix.

At twelve of the nodes represented in Figure 1, the number of personnel can be decreased by attrition or promotion and increased by accession or demotion from a higher paygrade. These are the nodes labeled with the suffixes 1S, 1P, 2N, and 2P. No demotions are possible from paygrade 3 since it is the lowest paygrade level. From the Navy data supplied to us, we derived the percentages for losses, accessions, promotions and demotions in paygrade 3/LOS 1 that are given in Table 2.

TABLE 2

PG	LOS	ACTIVITY	% LOSSES-ACCESSIONS	% PROMOTED	% DEMOTED
3	1	Shore	0	51	0
		Sea	0	48	0
		Other	0	24	0

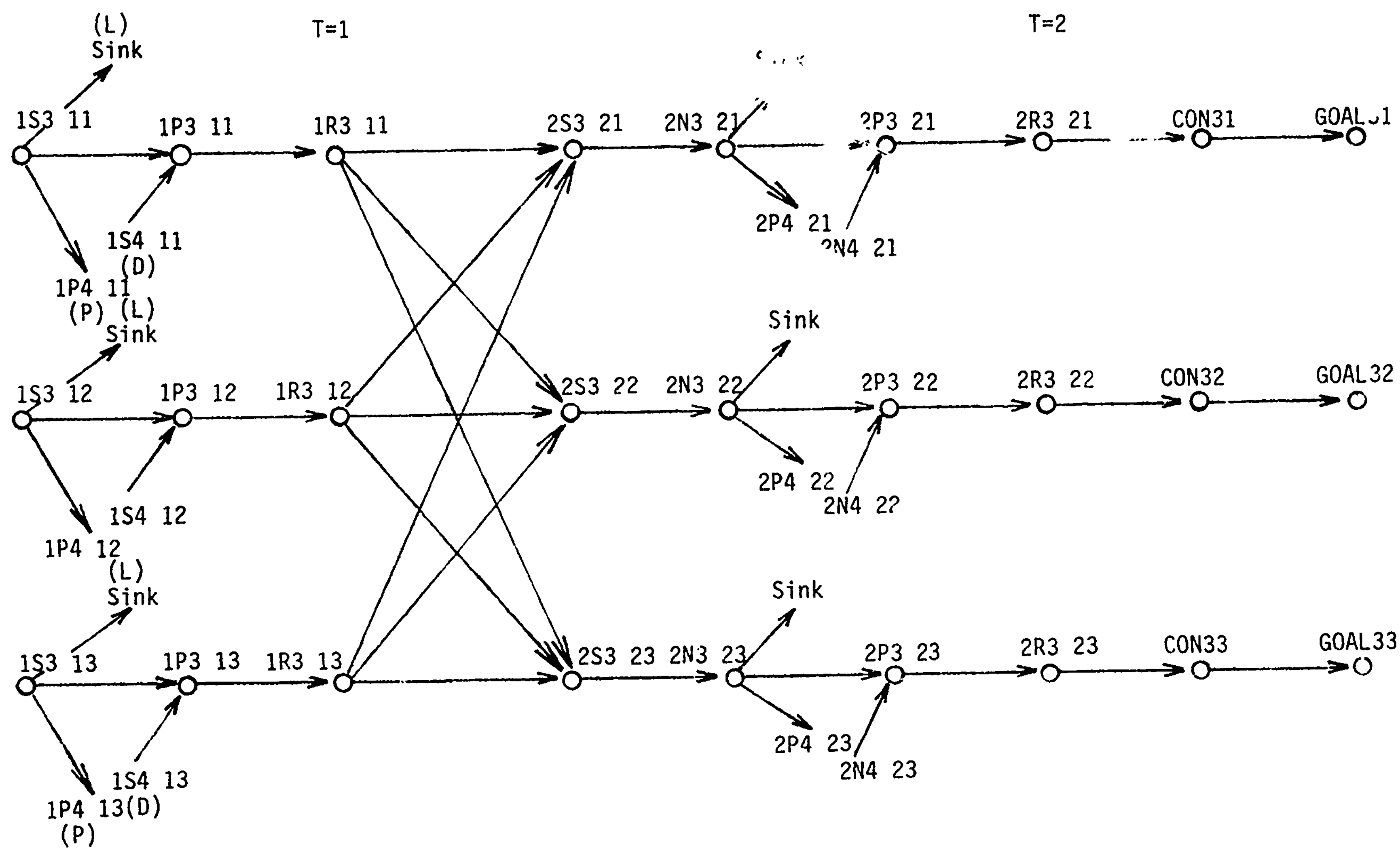
T=1
T=2
(L) Sink
1S3 11
1P3 11
1R3 11
2S3 21
2N3 21
2P3 21
2R3 21
CON31
1S4 11 (D)
1P4 11 (P)
2P4 21
(L) Sink
1S3 12
1P3 12
1R3 12
2S3 22
2N3 22
2P3 22
2R3 22
CON32
GOAL32
Sink
1S4 12
1P4 12
2P4 22
(L) Sink
1S3 13
1P3 13
1R3 13
2S3 23
2N3 23
2P3 23
2R3 23
CON33
GOAL33
Sink
1S4 13
1P4 13(D) (P)
2P4 23
2N4 23

FIGURE 1

We relate these data to Figure 1 as follows. At node 1S3Ø11, 51% of the personnel will be promoted to paygrade 4 and move to node 1P4Ø11. None of the personnel are lost to attrition, so 49% continue to node 1P3Ø11. If any personnel were demoted to paygrade 3 from paygrade 4, they would move, for example, from 1S4Ø11 to 1P3Ø11. The actual flows in the numerical example are provided in Table 3.

TABLE 3

			NUMBER OF PERSONNEL				
PG	LOS	ACTIVITY	AT START	PROMOTED OUT	LOST	DEMOTED IN	BEFORE ROTATION
3	1	Shore	54	28	0	0	26
		Sea	843	411	0	0	432
		Other	53	13	0	0	40

At the end of the first period, some of the personnel are rotated. This example uses a 5 periods at sea/2 periods on shore rotation policy. In this example, at a rotation node, the assumption has been made, for purposes of estimating rotation proportions, that the personnel in this arc were distributed in equal proportions over tour length. In this example, it was assumed that 20% of the total were in their first year at sea, 20% are in their second year, and so forth, so that equal proportions applied for each of this 5 period tour length. Similarly, for a two year shore tour, it was assumed that 50% of the personnel were in their first year and 50% were in their last year of shore duty. The "other" category also had a two year average tour length in this example. The rotation percentages given in Table 4 represent the 5 years at sea/2 years on shore rotation policy under consideration.

TABLE 4

ROTATION PERCENTAGES

FROM/TO	SHORE	SEA	OTHER
Shore	49	50	1
Sea	19	80	1
Other	14	36	50

Referring again to Figure 1, we proceed further as follows. At the end of the period 1, personnel in paygrade 3/LOS 1 will increase their LOS by 1 and move to paygrade 3/LOS 2. Some of the personnel will be rotated as well. For example, personnel at node 1R3Ø11 can continue on shore and move to 2S3Ø21, or they can be rotated to sea and move to 2S3Ø22 or they can be rotated to some "other" duty and move to 2S3Ø23. (See the 3 arrows indicating movement out of node 1R3Ø11.) Thus, utilizing the model in this manner we would obtain the personnel balance (after rotation) shown in Table 5.

TABLE 5

PERSONNEL MOVEMENTS AND BALANCE

PAYGRADE 3/LOS 2

	FROM/TO	SHORE	SEA	OTHER
PAYGRADE 3/LOS 1	Shore	13	113	0
	Sea	87	342	4
	Other	6	14	20
	Total	106	369	24

The personnel have now moved into the second period of this two period example. During the second period, some of the personnel in paygrade 3/LOS 2 will be promoted or lost and new personnel will be taken in by either accession or demotion. This is similar to what occurred in period one. However, the percentages for these various events will differ in these two periods since the personnel are now in their third year of service.

Taking account of the differences in the two periods and proceeding toward numerically specified goals such as are symbolized on the right of Figure 1, the model produces the results that are displayed in Table 6.

TABLE 6

SECOND PERIOD SUMMARY FOR PAYGRADE 3/LOS 2

ACTIVITY	AFTER ROTATION	PROMOTED	LOST	DEMOTED IN	BEFORE ROTATION
Shore	106	78	11	0	17
Sea	369	243	30	7	104
Other	24	6	4	0	14

The results of model computation, as applied to this particular example, show that at the end of two periods (before rotation) the strength in paygrade 3, LOS 2 would be seventeen persons on shore, 104 at sea, and fourteen in the category "other." Referring back to Table 1, these results may be compared to the initial numbers in paygrade 3/LOS 1 of 54 on shore, 843 at sea, and 53 in the other category. The decline is due mainly to promotions to the next paygrade. For paygrade 3/LOS 2 the promotion rate is above 48%, while that for paygrade 3/LOS 2 is in excess of 60%.

5. Algorithm and Computation

The preceding results are based upon an example whose network configuration contains 140 nodes and 280 arcs. Figure 1 shows only a small section of this network. The size of the example model is only a fraction of that of the complete GPSSR model, which includes data for all paygrades and all LOS possibilities. Because our priority was to develop as fast as possible the computer capability for the much larger GPSSR model, the example was run on our initial software for a full-scale GPSSR model.

Due to the large scale nature of the GPSSR model, it was necessary to maintain some of the data on external storage devices as well as within central memory. Hence, at each iteration, a considerable amount of time was spent on retrieving/storing this data. It can be observed in the statistics below that the I/O time is a significant contributor to total computer time.

The computer time required to run the example model is broken down into 2 task categories--1) network algorithm execution and data modifications between iterations, 2) input/output of problem data. The following table shows the actual breakdown in terms of total time (in TM seconds) and percent of total time.

TABLE 7

Task Category	TM Seconds	% of Total
Algorithms	1.08	19
I/O	4.67	81
Total	5.75	100

Though the ratio of I/O time to total time will decrease as the problem size increases, it is nevertheless worthwhile to reduce the I/O time as much as possible.

Currently, research is underway to eliminate much of the I/O now existing. Adoption of a more efficient data storage scheme can lead to the removal of intermediate I/O tasks. Only the initial data input and final data output will be necessary. Modifications such as this can substantially reduce the I/O time and hence the total computer time.

6. Conclusions and Recommendations

Larger and more complex examples are now being used to test the algorithm and the GPSSR model. These examples are also being used as prototypes for testing some of the varieties of problems that can be comprehended within our GPSSR formulation. It is expected that these prototype tests will be sufficiently advanced by the end of the present summer so that the tasks and organization arrangements for ensuring full and effective implementation should now begin to be considered.

Present plans call for implementation on the NIH IBM machines. Project staff who have had prior experience in this area will therefore now begin to look into what is likely to be needed to effect the transfer from the present CDC software and facilities. This should be accompanied by administrative arrangements on the part of the Navy to ensure that the implementation is facilitated in a way that ties directly into potential Navy uses. Planning for this next phase of the operation should commence immediately.

From the beginning, the work on sea/shore rotation modeling and implementation was conceived of as the area of immediate highest priority for the Navy. The methodology and modeling efforts were also directed, however, to possible further extensions to other areas. This included officer and enlisted personnel slating systems, optimal force structure and distribution projections. Navy arrangements for continuation of this work should take this into account. In addition, the possible uses of the GPSSR model for policy evaluations as well as for rotation scheduling also need to be provided for if the capabilities that are presently being developed are to be fully exploited by the Navy.

APPENDIX

A1. Introduction

The Navy's Sea-Shore rotation is formulated as a new type of constrained network goal programming (CNGP) model. These CNGP models are here specialized to a form that we shall refer to as GPSSR (=Goal Programming Sea-Shore Rotation) models which are specifically designed to accomodate problems involved in the navy's Sea-Shore rotation scheduling not only for implementation but also for policy (and goal setting) evaluations.

In this GPSSR model we start with known numbers of personnel in various PG-LOS (Paygrade-Length of Service) categories at either sea or shore location. To each of these PG-LOS combinations is assigned a corresponding node of the network. From these nodes there are arcs (carrying personnel flows) leading to other nodes which correspond to the various PG-LOS sea or shore locations at the different time periods in the planning horizon.

Between certain nodes we may have multiple arcs called "goal arcs" which serve to represent goals for personnel flows between nodes designating various types of PG-LOS-SS (paygrade-length of service-sea shore) status.[1] These goal arcs serve to represent the (nonlinear) goal attainment objectives in network format.[2]

We also introduce a collection of what we shall refer to as "nozzle nodes" which serve to distribute personnel to attrition, continuing status, promotion or rotation. For convenience of analysis and computation, we collect the flows actually leading to a nozzle node into a preceding

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[1]The numbers and types of goal arcs used in the model will depend on the policy and operational scheduling questions to be examined.

[2]See [4] - [6] for further discussion and examples.

"assembly node" and thus have only a single arc--a "hose arc"--leading to a nozzle node. We designate the arcs leading out of a nozzle node as "spray arcs." See Figure 2 below.

The requirement that the flows on spray arcs be a specified proportion of the hose arc flow (e.g., attrition proportion and continuation, promotion or rotation proportions) constitute additional constraints which take the GPSSR model out of the category of "pure network type" models. The historical or posited proportionalities, unmodified, also give rise to an additional difficult problem when integer numbers of personnel are required to be manifest in the flows. Thus, we need also to provide for a flexibility in these transfers of status which will render, as does the real process, integer numbers of transfers while corresponding closely to the proportionalities desired.

We are able to meet both the integrality requirements and the reduction of solution to that of pure network problems by introduction of a special representation of additional "hose-spray" constraints together with a new iterative procedure that we have developed which is also applicable to a large class of constrained network problems. In this procedure, the model is solved iteratively by means of a succession of pure network models. Our algorithm develops a series of pure network models to solve the GPSSR model even though the latter is not a model of pure network type.[1] This use of pure networks as the "workhorse" provides substantial advantages not only in computational efficiency but also substantial improvement in our ability to deal with the large-scale modeling requirements that are involved in dealing with sea-shore rotation scheduling in adequate detail.

[1] See the discussion of the model approximation and solution procedures as discussed in [2] and [3].

A2. New Model Structure

We now turn to an explicit analytic formulation of the new structure involved in our aforementioned integer relaxation of the proportionality constraints imposed on the hose and spray arc flows in our basic goal programming network model.

Starting with the goal programming network model in the form

$$\min \sum_{j=1}^{n} c_j y_j$$

s.t.

$$(2.0) \qquad \sum_{j=1}^{n} \varepsilon_{ij} y_j = E_i , \quad i = 1, \ \ldots, m$$

$$L_j \leq y_j \leq U_j$$

where y_j is the flow on arc j, ε_{ij} is the incidence number[1] of arc j on node i, E_i is the efflux (or influx) at node i, L_j and U_j are the lower and upper bounds on arc flow y_j, we develop additional constraints for "nozzle" nodes supplied by "hose" arcs from "assembly" nodes and leading into "spray" arcs as in the diagram.

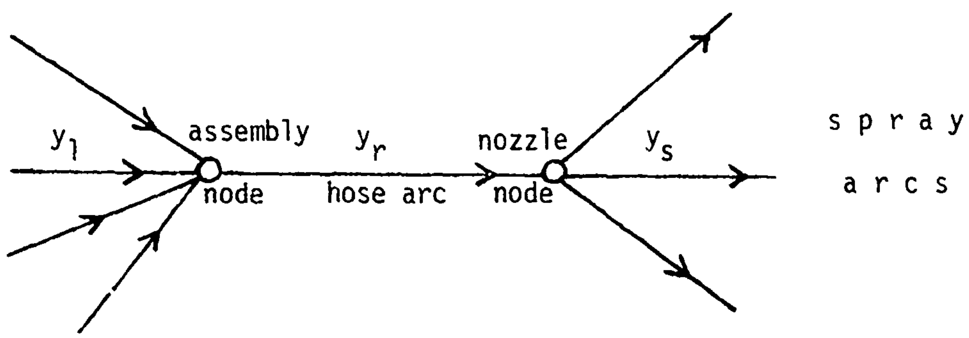

Figure 2

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[1]See Chapter XVII in [3].

The pure network constraints on hose arc flow at an assembly node and its nozzle node are

$$(2.1) \qquad -\sum_{\ell \in \mathcal{A}(r)} y_\ell + y_r = 0$$

$$(2.2) \qquad -y_r + \sum_{s \in \mathcal{S}(r)} y_s = 0$$

where

$\mathcal{A}(r)$ = { set of antecedent arcs to hose arc r }

$\mathcal{S}(r)$ = { set of spray (or successor) arcs to hose arc r }

We represent attrition, promotion, rotation, etc. by spray arc flows which are required to be historical or posited proportions of the hose arc flow, e.g.

$$(2.3) \qquad -p_s y_r + y_s = 0 \ , \ s \in \mathcal{S}(r)$$

where $p_s \geq 0$ and $\sum_{s \in \mathcal{S}(r)} p_s = 1$

Because of (2.3), $c_s y_s = c_s p_s y_r$ so that we can replace c_s and c_r by $c_s' = 0$ and $c_r' = c_r + \sum_{s \in \mathcal{S}(r)} p_s c_s$

Next (2.3) itself can be replaced by

$$(2.4) \qquad -p_s y_r + y_s \geq 0 \ , \ s \in \mathcal{S}(r)$$

for if we had some $y_{s'} > p_{s'} y_r$, we would have

$$\sum_{s \in \mathcal{S}(r)} y_s > \sum_{s \in \mathcal{S}(r)} p_s y_r = y_r$$

contradicting (2.2).

We do want integer flows y_s when y_r is integer, i.e. we want rotations, promotions, etc. to be in terms of whole numbers of personnel. This cannot be assured if we require (2.3) to hold since there is no reason why $p_s y_r$ is always an integer when $0 < p_s < 1$. Thus, we make the (nonlinear) integer relaxation of (2.4),

(2.5) $\quad y_s \geq \lfloor p_s y_r \rfloor \;, \; s \in \mathcal{S}(r)$

where $\lfloor a \rfloor$ = the largest integer less than or equal to a.

Our algorithm replaces (2.5) by the sequence

(2.6) $\quad y_s \geq \lfloor \rho^{(k)} p_s \bar{y}_r^{(k-1)} \rfloor$

of lower bounds on iterations k = 1, ..., K, where $0 = \rho^{(1)} < \cdots \rho^{(k)} < \cdots \rho^{(K)} \leq 1$, and $\bar{y}_r^{(k-1)}$ is the solution value of y_r at the (k-)st iteration.

Our problem at the kth iteration then becomes the pure network problem

(2.7)
$$\begin{aligned} \min \quad & \sum_j c_j y_j \\ \text{s.t.} \quad & \sum_j \varepsilon_{ij} y_j = E_i \;, \; i = 1, \ldots, n \\ & L_j \leq y_j \leq U_j \;, \; j = 1, \ldots, n \\ & y_s \geq \lfloor \rho^{(k)} p_s \bar{y}_r^{(k-1)} \rfloor, \; s \in \mathcal{S}(r), \text{ for all hose arcs } r. \end{aligned}$$

A3. Network Model Notation of the "BIGNET" Code

As mentioned earlier, both a new model type and a new algorithmic method for solving this new model type have been developed for solving the GPSSR model. In order to perform the data manipulations and mathematical programming calculations required for solving the GPSSR models, new software has been developed in the form of what we shall refer to as the "BIGNET code." The large scale nature of the data involved in tracking PG-LOS-type duty transitions requires the development of an efficient computerized data management capability. Thus, we develop the data representation in terms of the following network model notation.

For a specified detailing community, our network flow model characterizes the flow or movement of personnel over time. We are interested in observing this movement relative to various status categories which are defined in terms of 1) paygrade, 2) LOS, 3) type duty.

The primary elements of any directed network model are the nodes and the arcs. In our model, each node represents a particular status, which may be defined in terms of external characteristics, e.g. "paygrade 6," or in terms of internal model classification, e.g. "source" or "goal."

The flow in an arc connecting a pair of nodes (a pair of states) represents the number of personnel who were in the first category, but moved to the second category without intervening states. This flow is determined by the model under conditions which seek to provide final status totals which are "as close as possible"[1] to pre-specified category goals, but do not violate other structural constraints, i.e. historical (or posited) rates of promotion.

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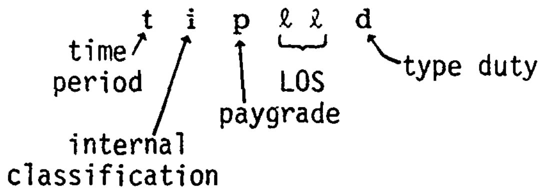
[1] This term is to be interpreted in the goal programming context discussed in Section 4 of this report.

The three external characteristics are incorporated into a 6-character <u>node name</u> as follows:



$t \in \{1,2,\ldots,T\}$

$i \in \{S,N,P,R\} \equiv$ {Start,Net,[1] Promotion,Rotation}

$p \in \{3,\ldots,9\}$

$\ell\ell \in \{00,01,\ldots,31\}$

$d \in \{1,2,3\} \equiv$ {shore,sea,other}

Other node names not following this convention exactly are:

CONV<u>pd</u>	Note: Specific to paygrade, type duty only
GOAL<u>pd</u>	
SOURCE	
SUSINK	

"CONVpd" nodes serve to automatically sum the flows into certain other nodes. The "GOALpd" nodes are used in conjunction with the "CONVpd" nodes to achieve the model's goal structure. Two "goal arcs" join the CONVpd/GOALpd pair. "SOURCE" and "SUSINK" are used for internal (computer code) reasons only. Refer to the example below for further elaboration and to Figure 2 for the relative positioning of these nodes within the network. Note that each pair comprises an arc within the network.

For illustrative purposes, we continue with the example used in the text of this report. Consider the following situation and the corresponding model output:

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[1] This refers to "net" number of personnel after accessions, losses, promotions, and demotions (used for internal purposes only).

At the beginning of period 1, there were 843 personnel classified as paygrade 3, LOS 1, sea duty -

SOURCE 1S3ø12 flow = 843

During the period, 411 were promoted -

1S3ø12 1P4ø12 flow = 411

Also, 13 persons from paygrade 4, LOS 1, sea duty are not promoted -

1S4ø12 1P4ø12 flow = 13

Of those promoted from paygrade 3 and not promoted from paygrade 4, 86 go to shore duty, but at the end of the period, the LOS has been incremented by 1 -

(1P4ø12 2S4ø21 flow = 86)

There actually is no occurrence of the arc 1P4ø12 2S4ø21. We have instead introduced the nozzle node, 1R4ø14, which does not alter the network solution, but is necessary for the creation of the hose arc which we use in our iterative approach. We thus have the set of arcs:

		with flow
SOURCE 1S3ø12		843
1S3ø12 1P4ø12		411
1S4ø12 1P4ø12		13
1P4ø12 1R4ø12	} hose arc	424
(assembly node →) 1R4ø12 2S4ø21	← nozzle node	86

Because the goals are specific to paygrade and type duty only, we sum the "2R" nodes over LOS.

For example:

2R4ø01 CONV41 flow = 0

2R4ø11 CONV41 flow = 31

2R4ø21 CONV41 flow = 104

Here, the flow into CONV41 is the sum of the flows into 2R4ø11, 2R4ø21, and 2R4ø31.

See Figure 3 for a small sub-network diagram incorporating the above mentioned nodes and arcs.

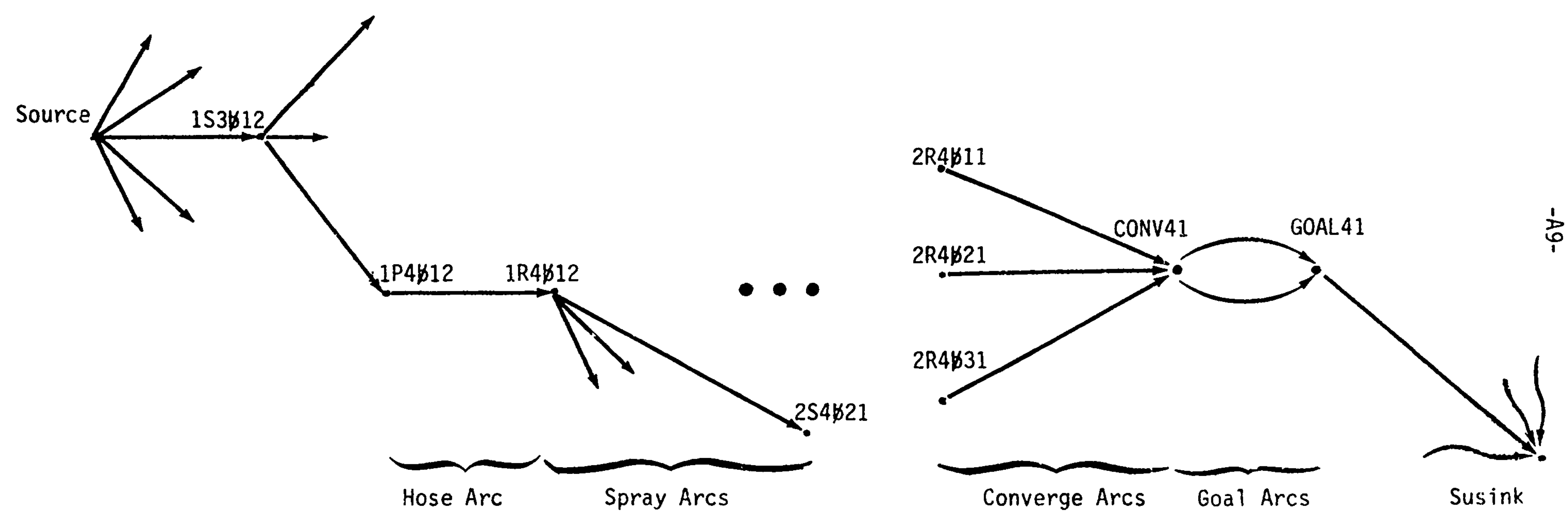
Source
1S3012
1P4012
1R4012
2S4021
2R4011
2R4021
2R4031
CONV41
GOAL41
Hose Arc
Spray Arcs
Converge Arcs
Goal Arcs
Susink

FIGURE 3

We conclude by briefly discussing the model notation relative to the model structure described in the previous section.

As noted before, each node as input data is designated by a 6-character name. Correspondingly, each arc is designated by a pair of nodes and therefore by a 12-character name. The computer program, however, converts these unique character names into unique node and arc numbers as in the mathematical description of the model.

To illustrate this correspondence, we provide an example (Figure 4) which is extracted from materials from Figure 3. Node 1P4b̸12, a nozzle node, is the 33rd node in the network. Arc numbers were assigned to arcs "incident" upon node 1P4b̸12 as follows:

arc	arc no.
1S3b̸12 1P4b̸12	36
1S4b̸12 1P4b̸12	62
1P4b̸12 1R4b̸12	89

Using this conversion of node/arc names to node/arc numbers, the 33rd constraint equation of (2.0) is the following:

(3.0) $$- y_{36} - y_{62} + y_{89} = 0$$

Hence, $\epsilon_{33} = 0$

and $\epsilon_{33j} = \begin{cases} -1 & \text{for } j = 36, 62 \\ 1 & \text{for } j = 89 \\ 0 & \text{otherwise} \end{cases}$

Referring to the more specific equation from (2.1), we have

$$- \sum_{\ell \epsilon \mathcal{A}(89)} y_\ell + y_{89} = 0$$, which is equivalent to the previous equation.

Here, $\mathcal{A}(89)$, the set of indices of antecedent arcs to arc 89 is defined to be {36,62}

The equations from (2.3) relative to "hose" arc 89 are as follows:

$$\text{(3.1)}\qquad \begin{aligned} -\ 19\ y_{89} + y_{127} &= 0 \\ -\ .80\ y_{89} + y_{128} &= 0 \\ -\ .01\ y_{89} + y_{129} &= 0 \end{aligned}$$

Note that $\mathcal{S}(89)$, the set of indices of spray arcs corresponding to "hose" arc 89 is defined as {127,128,129}. (These arc numbers can be obtained from the model output and correspond to spray arcs 1R4b̸12 2S3b̸21, 1R4b̸12 2S3b̸22, and 1R4b̸12 2S3b̸23, respectively.)

Also, observe that $p_{127} = .19$, $p_{128} = .80$, $p_{129} = .01$,

$$\sum_{s\,\varepsilon\,\mathcal{S}(89)} p_s = 1 \quad \text{and} \quad p_s \geq 0,\ s\ \varepsilon\,\mathcal{S}(89)$$

The following diagram, which was used in the above discussion, may prove useful in other ways as well.

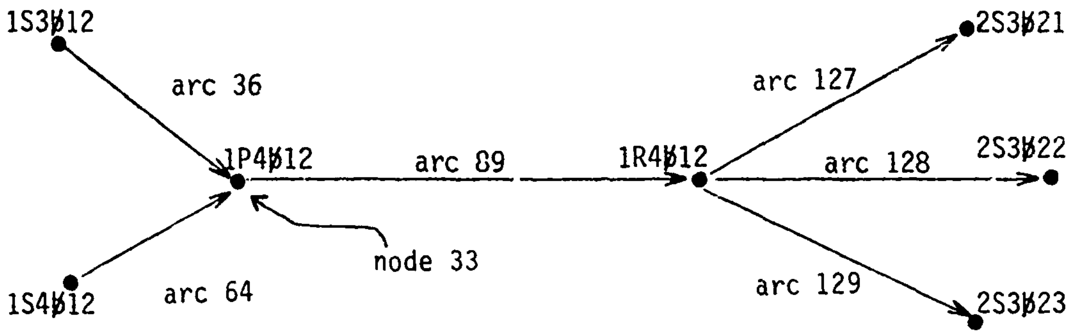

FIGURE 4

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Relations between sea duty and shore duty in the U.S. Navy involves large numbers of variables and constraints, along with projected rates of attrition, promotion, demotion, and transfer, which need to be simultaneously considered over a series of periods. Mechanisms for evaluating possible changes in policies and other requirements are also required. This paper reports on a constrained network goal programming model with an accompanying algorithm and a computer code (called BIGNET) which makes it possible to deal with all of

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20. ABSTRACT (continued)

these aspects of the Navy's sea-shore rotation problems in the ultra high speed processing times that are characteristic of network codes. An accompanying appendix contains a numerical example with accompanying details of the way the model can be implemented.

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