AD-A119 177 NATIONAL AERONAUTICS AND SPACE ADMINISTRATION CLEVEL--ETC F/G 13/9
THE OPTIMAL DESIGN OF INVOLUTE GEAR TEETH WITH UNEQUAL ADDENDA.(U)
1982 M SAVAGE, J J COY, D P TOWNSEND
UNCLASSIFIED NASA-E-1235 NASA-TM-82866 NL

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THE OPTIMAL DESIGN OF INVOLUTE GEAR TEETH WITH UNEQUAL ADDENDA

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ABSTRACT

The design of a gear mesh is treated with the objective of minimizing the gear size for a given gear ratio, pinion torque, pressure angle, and allowable tooth strengths. The gear tooth strengths considered are scoring, pitting fatigue, and bending fatigue. Kinematic involute interference is also avoided. The design variation on standard spur gear teeth called the long and short addendum system is considered. In this system, the mesh center distance and pressure angle are maintained as is the ability to manufacture the teeth with standard tooling. However, the pinion and gear tooth proportions are altered in order to obtain fewer teeth numbers for the same ratio as standard gears without kinematic involute interference. The effect of this nonstandard gearing geometry on tooth strengths and gear mesh size are studied. For a 2:1 gearing ratio, the optimal nonstandard gear design is compared with the optimal standard gear design.

BACKGROUND

The use of long and short addendum teeth for an involute spur gear mesh has long been considered an improvement in design over standard involute spur gear geometry (1). This belief comes from the fact that larger teeth are stronger in bending fatigue than smaller teeth (2). The first problem encountered in the minimum tooth design is kinematic involute interference on the pinion, where the tip of the gear tooth contacts the pinion tooth below the base circle on the pinion. To take advantage of the increased bending strength of the larger teeth and avoid the kinematic interference problem without resorting to special tooling, the long and short addendum system has been adopted (1). Design procedures for gear meshes utilizing unequal addenda have been recently presented by Tucker (3) and Estrin (4). Both procedures are excellent procedures which consider many factors - kinematic, structural, and manufacturing. However, they do not consider the Hertzian contact pressure at the tip of the gear tooth which is a prime contributor to gear scoring. The authors (5) have more recently presented an optimization method

for standard spur gears which includes consideration of this effect. It is the purpose of the work presented herein to extend the results of (5) to unequal addenda gears.

THE DESIGN PROBLEM

A gear set is normally used to transmit an input torque from a shaft turning at one speed to an output torque on a shaft turning at a lower speed. The basic parameters describing this situation are the gear ratio, m_g, the input pinion torque, T_p, and the pinion speed, Ω_p.

The size of the gear is defined by the gear mesh center distance, C. A standard gear mesh with 22 teeth on the pinion and 44 teeth on the gear is shown in figure 1. For a 2:1 gear ratio, these numbers of pinion and gear teeth result in an optimal design configuration as shown in reference (5). By tying the gear face width, f, to the pinion pitch diameter by the ratio, λ,:

$$\lambda = \frac{fP_d}{N_p} \tag{1}$$

where P_d is the diametral pitch of the gear set and N_p is the number of teeth on the pinion, the center distance C is made a measure of the gear blank's volume as well as the gear area. This length to diameter ratio, λ, insures that a uniform load is present across the tooth face. Many gear meshes have λ values in the neighborhood of 0.25, although meshes with extremely stiff shafts can have values as high as 0.6. In addition to these parameters, the gear and pinion teeth are defined by the pitch line pressure angle, ϕ, and the addenda, a_p and a_g, and the dedenda, d_p and d_g, of the teeth. These teeth are shown in figure 2 for the AGMA standard values of

$$a_p = a_g = 1/P_d \tag{2}$$

and

$$d_p = d_g = 1.25/P_d \tag{3}$$

In the long and short addendum system, the addendum of the pinion and the dedendum of the gear are increased by the tool shift, e, while the addendum of the gear and the dedendum of the pinion are decreased by the tool shift, e.

The parameters to be determined in a design are the number of pinion teeth, N_p, the diametral pitch, P_d, and the tool shift, e, for a given gear ratio, m_g, pinion torque, T_p, pressure angle, ϕ, pinion width to diameter ratio, λ, and the material strengths.

The material properties of importance for both the pinion and the gear are their elastic modulii, E_p and E_g, their Poisson's ratios, ν_p and ν_g, and their bending fatigue strengths and their surface endurance strengths.

The optimal solution to the long and short addendum design problem is a design with an adequate contact ratio (usually greater than 1.4), no kinematic involute interference, balanced bending and surface endurance strengths, standard diametral pitch and minimum center distance.

LONG AND SHORT ADDENDUM TEETH

Long and short addendum teeth are shown in figure 3 for a 2:1 gear ratio with 14 teeth on the pinion and 28 teeth on the gear, a pressure angle of 20 degrees and a tool shift of $0.4/P_d$. The addenda changes in this tooth system cause the tooth thickness of the pinion tooth to increase and the tooth thickness of the gear to decrease. This increases the bending strength of the pinion tooth but decreases the bending strength of the gear tooth.

In addition, the top land of the pinion tooth is reduced. This reduction in pinion top land thickness places an upper bound on the amount of tool shift possible, at which point the pinion tooth becomes pointed. Since kinematic involute interference produces a lower bound on the tool shift, these two factors of minimum top land and involute interference place practical limits on the amount of tool shift possible in a given design.

DESIGN COMPARISON

One can study the effect of various tool shifts on the strength characteristics of an unequal addendum gear mesh by computer simulation. The maximum pinion bending stress and the maximum gear bending stress are plotted versus tool shift in figure 4. For this figure and the next, a face width of 0.5 inches (12.7 mm), a diametral pitch of 10, a pressure angle of 20 degrees, a gear ratio of 2 and 14 pinion teeth were selected. The gear ratio of 2 was chosen as a representative reduction. The fewer the teeth number, the larger the teeth for a given center distance. The number of pinion teeth was selected as 14 to come down to a number of teeth low enough to produce kinematic interference with standard teeth and to display a design surfficiently different from the standard optimum design to illustrate the strengths and weaknesses of long and short addendum gearing.

The pinion torque for this figure was set at 1,000 lb-in (113 N-m). The material chosen was steel with a modulus of 30×10^6 psi (200 GP_a) and a Poisson's ratio of 0.25. In this figure, one can see that the gear stress increases while the pinion stress decreases as the tool shift increases.

Figure 5 is a plot of the Hertzian contact pressure between the teeth at the tip of the gear tooth and at the highest point of single tooth contact on the gear. Both pressures are plotted versus tool shift for the same data as used in figure 4. The intersection of the two curves indicates the tool shift required to produce equal contact pressure at the gear tip and at the highest point of single tooth contact on the gear tooth. These are the two locations of highest contact pressure between the teeth. Since the contact pressures are much higher than the bending stresses, this amount of tool shift produces the optimum long and short addendum design for 14 pinion teeth and a 2:1 gear ratio. Dividing this tool shift by the diametral pitch produces a dimensionless tool shift which produces this balanced design state for all gear sizes.

The Hertzian contact pressure at the tip of the gear tooth is a measure of how close to the pinion base circle the contact with the gear tooth is initiated. It is the relative imbalance of this stress (5) which forces the optimal standard tooth design to have a large number of pinion teeth. This same factor requires the tool shift to be relatively high for the unequal addenda design.

It is interesting to note that for equal pinion length to diameter ratios and equal Hertzian contact pressures, the optimal design indicated by figure 5 with a tool shift of $0.4/P_d$ has a center distance similar in size to that of the optimal standard tooth design with 22 pinion teeth shown in figures 1 and 2. Figure 3 shows the tooth proporitions of the unequal addendum optimal design.

If one sets 200 ksi (1.34 MP_a) as an acceptable contact pressure, then the 14 tooth pinion design has a diametral pitch of 6, a face width of 0.5 inches (12.7 mm), a center distance of 3.5 inches (88.9 mm), and a maximum bending stress of 24 ksi (161 kP_a). Figure 6 is a plot of the Hertzian contact pressure versus the involute length on the pinion tooth from the base circle for this design. In comparison, the standard tooth design from reference (5) has a diametral pitch of 10, a face width of 0.55 inches (14 mm), a center distance of 3.3 inches (83.8 mm), and a maximum bending stress of 7.9 ksi (53 kP_a) for a maximum contact pressure of 200 ksi (1.34 MP_a). This is due to the fact that the surface curvature is a function of the base circle size for efficient designs. These two designs have nearly the same base circle sizes. If the diametral pitches were exactly proportional to the tooth numbers, the face widths and maximum contact pressures would be equal for the two designs.

SUMMARY AND CONCLUSIONS

The optimum design for a minimum center distance gear mesh with long and short addendum teeth was presented in this paper. In considering this design, the following results were obtained:

(1) by going to the long and short addendum tooth system, no size improvement can be obtained over an optimally designed standard spur gear mesh,

(2) the optimal design of a long and short addendum gear mesh is determined by the balance of Hertzian contact stress between the teeth at the gear tip and at the highest point of single tooth contact on the gear tooth, and

(3) the optimal design of a long and short addendum gear mesh is defined by the gear ratio, pressure angle, number of pinion teeth and tool shift ratio and is valid for any physical size of gear set.

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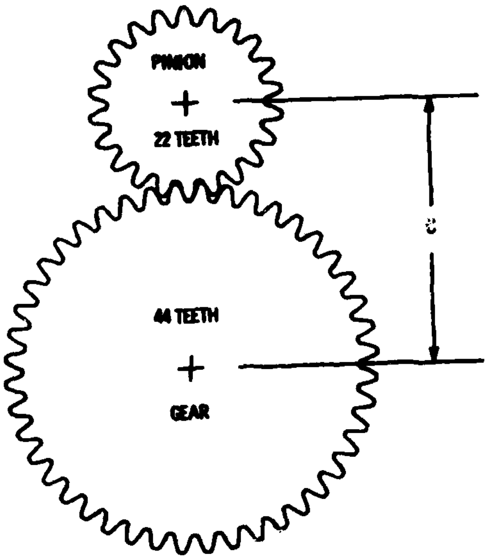

Figure 1. - Standard gear mesh.

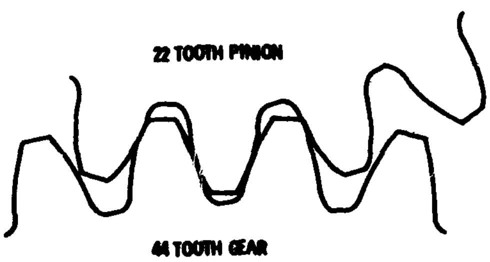

Figure 2. - Standard involute mesh geometry.

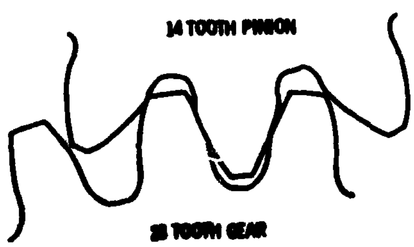

Figure 3. - Unequal addenda mesh geometry.

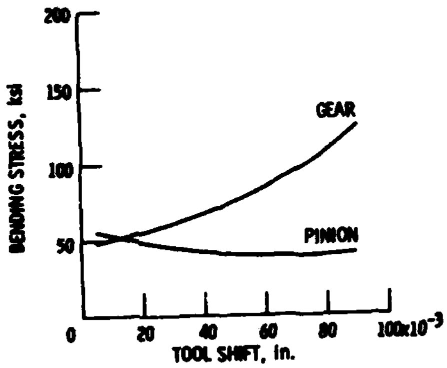

Figure 4. - Tooth bending stress comparison.

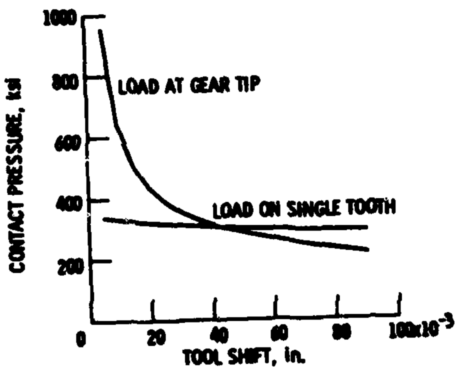

Figure 5. - Hertzian contact pressure comparison.

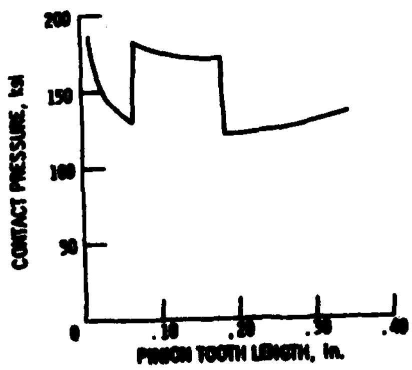

Figure 6. - Optimal design Hertzian contact pressure.

1. Report No. NASA TM-82866 AVRADCOM TR-82-C-7	2. Government Accession No. AD-A119177	3. Recipient's Catalog No.
4. Title and Subtitle THE OPTIMAL DESIGN OF INVOLUTE GEAR TEETH WITH UNEQUAL ADDENDA		5. Report Date
		6. Performing Organization Code 505-32-42
7. Author(s) Michael Savage, John J. Coy, and Dennis P. Townsend		8. Performing Organization Report No. E-1235
		10. Work Unit No.
9. Performing Organization Name and Address NASA Lewis Research Center and AVRADCOM Research and Technology Laboratories Cleveland, Ohio 44135		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20545 and U.S. Army Aviation Research and Development Command, St. Louis, MO 63166		14. Sponsoring Agency Code

15. Supplementary Notes

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17. Key Words (Suggested by Author(s)) Spur gears; Machine design; Optimal design; Optimization		18. Distribution Statement Unclassified - unlimited; STAR Category 37	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages	22. Price*

* For sale by the National Technical Information Service, Springfield, Virginia 22161