DEPTH CONSTANCY IN STEREOSCOPIC AFTERIMAGES: EFFECTS OF VIEWING—ETC(U)

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UNCLASSIFIED N14-0001-82C-0003
DEPTH CONSTANCY IN STEREOSCOPIC AFTERIMAGES: EFFECTS OF VIEWING DISTANCE AND MEASUREMENT METHOD

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August 1982
Technical Report

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Stereopsis
Depth
Depth constancy
Afterimages

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Stereoscopic depth constancy refers to the proposition that perceived depth signaled by retinal disparity remains constant despite changes in viewing distance. There has been some controversy as to whether stereoscopic depth constancy can operate at distances greater than a few meters. A previous study used a stereoscopic afterimage technique to determine whether depth constancy holds as fixation distance varies. After obtaining an afterimage containing depth information in the form of retinal disparity,
observers set a depth probe equal in apparent distance to the disparate afterimage. The results showed that depth constancy persisted up to 27 meters.

The present study was conducted to address two issues raised by the previous work. First, it is important to establish that stereoscopic depth constancy can be confirmed by measures which do not themselves depend on retinal disparity as the depth probe method does. Second, it is only beyond 27 meters of viewing distance that there are large departures in the predictions made by different models of depth constancy.

Observers judged the apparent depth in a stereoscopic afterimage in three ways. The depth probe was employed for one series of judgments. For another, the observers reported how far from themselves the afterimage appeared in feet. Finally, the observers estimated the proportion of the total distance to the fixation point between themselves and the afterimage. Observations were made with fixation distances ranging from 5 meters to essentially infinity. For distances greater than 20 meters, observations were made outdoors with familiar objects as fixation points.

The results show that stereopsis can provide veridical depth information at large fixation distances. They further demonstrate that depth constancy can be measured by methods which do not depend upon disparity matching. These findings suggest that any theory of stereoscopic depth constancy, or for that matter any theory of stereopsis, must include a mechanism for rescaling retinal disparity in accord with viewing distance.
Depth Constancy in Stereoscopic Afterimages: Effects of Viewing Distance and Measurement Method

Stereopsis refers to that particularly compelling perception of depth resulting from the fact that our two eyes view the world from different locations. As a consequence, the eyes receive slightly different patterns of stimulation. The images of objects at or near the fixation distance fall on corresponding locations on the two retinas. Objects nearer or farther than fixation cast images on non-corresponding points and these differences in locations are referred to as retinal disparity. Retinal disparity is typically expressed in degrees of visual angle. The images of objects nearer than fixation are said to be in negative or crossed disparity while those of objects farther than fixation are in positive or uncrossed disparity. Since they vary with the depth interval the retinal disparities of the images of objects in the visual field carry depth information. Indeed retinal disparity can serve as a cue to depth, a fact known since the time of Wheatstone (1838).

Just as the size of the image of an object is not sufficient by itself to determine the size of the object, so retinal disparity alone is an ambiguous cue to depth. The image size of an object varies with its distance from the observer, and must be recalibrated as that distance changes. This rescaling of size as a function of apparent distance is known as size constancy and results in stable size perception in the face of constantly varying image sizes.

Depth perception too is stable in spite of the fact that the retinal disparity produced by a given depth interval varies with the distance from
which the depth interval is viewed. By analogy with size constancy, this stability of depth perception is termed depth constancy. When depth intervals are signaled by retinal disparity it is called stereoscopic depth constancy, and requires that retinal disparity be recalibrated according to distance. However, while image size varies inversely with observation distance, retinal disparity varies in a more complex way. This is described in more detail below.

The nature of the recalibration of retinal disparity and the range over which stereoscopic depth constancy operates is of considerable interest and controversy. A recent review (Ono and Comerford, 1977) concluded that depth constancy holds only within a few meters of viewing distance.

In an earlier report (Cormack, 1982) data were presented that suggested depth constancy holds at distances up to 27 meters. These data were obtained through the use of a novel technique employing stereoscopic afterimages. Such afterimages, containing depth information in the form of retinal disparity, make it possible to vary fixation distance while holding retinal disparity absolutely constant. In the previous study perceived distance was measured as a function of viewing distance for two different retinal disparities. Observers set a depth probe to the same apparent distance as the afterimage. The results showed that the amount of depth seen in a stereoscopic afterimage could be predicted by the geometry of retinal disparity. These results are reproduced from the earlier report in Figure 1.

Two questions arise immediately with regard to these data. The first concerns the method used to measure perceived depth in the afterimage. After obtaining an afterimage, the observers fixated a target at some specified distance. A movable rod was then adjusted until it appeared to be in the plane of the afterimage. This experimental approach is illustrated in
Figure 1. Apparent depth in stereoscopic afterimages as a function of fixation distance for two retinal disparities. Solid curves represent predicted values. Dashed lines have a slope of 2, indicating a squared relation. Adapted from Cormack (1982).
Figure 2. The task required of the observer was a simple one, and yielded reliable results. The question is whether the observers were setting the depth probe to the apparent distance of the afterimage, or were instead simply making a disparity match between the probe and the afterimage. If the latter were the case, it could be argued that the technique employed did not provide information about the depth seen by the observer, but only about how well the observers could set the disparity in the probe equal to that in the afterimage. The skeptic could even argue that little or no depth was seen in the afterimage, or that the perceived depth was unrelated to viewing distance. The observers reported that the apparent depth in the afterimage was clear and compelling. It is also the case that they were asked to set the probe so that it looked at the same distance as the afterimage. Nevertheless, it is important to establish that the relationship observed between fixation distance and apparent depth can be measured with methods that are not dependent on disparity matching.

With regard to depth constancy the second question is a more interesting, and certainly more important one theoretically. It concerns the range over which depth constancy occurs, the cues that make it possible, and the degree to which it leads to veridical depth perception. Those who have championed the operation of depth constancy have generally held that the scaling of depth signaled by retinal disparity occurs primarily at near distances, i.e. distances smaller than 5 meters (Ono and Comerford, 1977). At these distances, given a constant disparity as viewing distance increases, observed depth increases as its square. Thus if the viewing distance is doubled, the observed depth should be quadrupled. If fixation distance is tripled, the depth should increase by a factor of nine. This notion dates back to von Kries (1924), and has been advanced by others more recently (eg., Wallach...
Figure 2. Depiction of probe technique. Observer fixates on tip of distant rod. Other rod serves as a depth probe and is moved back and forth until it appears in the plane of the circles in depth.
and Zuckerman, 1963). The values predicted by the square law for perceived depth in the afterimages are indicated in Figure 1 by the straight dashed lines with a slope of 2.

Von Kries, and Wallach et. al. as well as others have pointed out that the square law can at best be a rough approximation to what can occur in depth constancy. Note in Figure 1 that for each disparity value, there is a point (indicated by an arrow) beyond which the depth exceeds the fixation distance. This would mean that for crossed disparities, the image would appear behind the head. Clearly this cannot occur, and the fact that it is predicted by a simple square law merely demonstrates that the square law could only operate over a limited range.

Actually, the geometry of retinal disparity is described by a slightly more complex trigonometric function than the square law. With the eyes pointed straight ahead and the stimuli within a few degrees of fixation, the depth indicated by a given crossed disparity is determined by the following equation:

\[
\text{Depth} = D - (.5P \cdot (\text{TAN}(\text{ATN}(D/.5P) - .5R)))
\]

where \( P \) = interpupillary distance, \( D \) = fixation distance, and \( R \) = retinal disparity. See appendix A for a derivation of the equation. This yields the solid curved lines in Figure 1 for the crossed disparities indicated.

It is clear from Figure 1 that the results parallel the predicted values shown by the solid lines. It is also evident that the obtained values end just where the prediction begins to deviate from a straight line with a slope of two. In order to show that the perceived depth in a stereoscopic after-image is veridical and, in fact, follows the geometry of retinal disparity,
rather than some other function, it is necessary to make observations at distances far greater than those reported in the earlier study.

A finding of veridical depth constancy at large fixation distances could be important for several reasons. First it would serve as a demonstration of the range of useful depth perception. Second, it would specify the rule or rules by which retinal disparity is recalibrated. Third, it would require that distance cues that can operate beyond a few meters be involved in the recalibration of disparity.

In this report data are presented relevant to both questions raised above. To address the question of disparity matching, measures of apparent depth in stereoscopic afterimages were made using three different techniques. The probe technique used previously and described above was repeated for control purposes. A second technique required the observer to estimate in feet, how far away the disparate targets appeared to be. The third approach asked the observer to consider the distance to the fixation point to equal 100 and to report at what percent of that distance the disparate target appeared to reside.

With regard to the second question, in order to make observations at extreme viewing distances, it was necessary to move out of the laboratory. The largest viewing distance possible in the laboratory was 27 meters. This is not sufficient to discriminate between the square law and the trigonometric function describing retinal disparity. Therefore, a set of observations were made outside in the open where very large fixation distances could be employed.

METHOD

Subjects

Four observers were employed. All had 20/20 vision or wore glasses which corrected to 20/20. All scored within normal limits on stereo-acuity as
measured by the orthorater and the Julesz random element stereogram test (Julesz, 1971). Two observers, P. H. and R. R. were naive as to the purpose of the experiment and were unaware of the geometry of retinal disparity. The others are both experienced psychophysical observers. Observers requiring glasses wore them during trials.

Apparatus

The apparatus for generating afterimages is shown in Figure 3. It was a hand held portable device suitable for outdoor use. A forehead rest maintained a constant fixation distance. The eye ports provided a clear view of the stimulus in depth. The stimulus slides held transparancies. The target contained in the stimulus slides is shown in Figure 4. The fixation bars were visible in the horizontal aperture bisecting the target. The circles were in front of the fixation bars by either .93 cm or .40 cm. The depth was controlled by spacers and produced approximately 16.3' or 4.5' of retinal disparity. The exact disparity depended on the interpupillary distance of the observer. In any case it was that disparity produced by .93 cm (or .40 cm) of depth at a viewing distance of 35 cm (or 45 cm).

Behind the stimulus slides was a diffusing screen and behind that, at a distance of 60 cm from the observers' eyes, was a photographic flash gun. This flash gun could be triggered by depressing, with the thumb, a switch located just below the handle.

For trials in which the test viewing distance was 20 meters or less, measures were made indoors in a long hallway. At one end a white screen was illuminated by a strobe light flickering at approximately 4 hz. This flickering light served to maintain a clear view of the afterimage. The fixation target was 85 cm in front of the screen. It consisted of a clearly
Figure 3. Device used to produce stereoscopic afterimages. a-flashgun, b-stimulus in depth, c-scale, d-handle, e-pushbutton, f-viewing ports.
Figure 4. Inspection stimulus. Circles and horizontal dashed bar are translucent. Circles stand out in depth from the horizontal bar. Background is opaque.
visible rod 30 cm high by 4 cm wide mounted at eye level on a tripod. The hallway was dimly illuminated, but various cues to distance (lights, doorways, tiles, cement blocks, etc.) were visible.

For the outdoor trials no flickering light could be provided to enhance the visibility of the afterimage. Instead, observers blinked their eyes rapidly or flickered their view by moving their hand up and down in front of their eyes. The fixation targets consisted of a triangular red emergency traffic reflector (16 inches on a side) located at one end of an open field 250 meters from the observer, a red light on a radio tower viewed from 6 km, a similar light on a radio tower viewed from 7.8 km, and a star (at least $10^{13}$ km distance). All outdoor observations were made at night. Distance cues such as trees, shrubs, grass, buildings in the distance, and the like were dimly visible.

Procedure

For all trials, the induction of the afterimage was identical. After dark adapting for a couple of minutes, the observer held the afterimage generator to the eyes. The observer was instructed to fixate carefully on the fixation point (the vertical line in the center of the fixation bar). When good fixation was achieved, the observer pressed the button which triggered the flashgun. Immediately the observer lowered the afterimage generator and fixated on the test fixation point. Observers were instructed to abort the trial if a clear afterimage was not obtained, or if the fixation bar did not appear in the plane of the test fixation point.

The three methods for measuring the apparent depth in the afterimage will be called the "probe" technique, the "proportion" or "percent" technique, and the "foot estimate" technique. The details of the probe technique have been described elsewhere (Cormack, 1982). Briefly, while the observer
fixated the test fixation point, the experimenter held a six foot vertical rod in the field of view of the observer immediately adjacent to the afterimage. The observer instructed the experimenter to move the vertical rod nearer or farther away, until it appeared in the plane of the circles in depth. Then the distance from the rod to the observer was measured with a tape measure.

For the percent technique, the observer was told to consider the distance to the test fixation point to be 100. The observer was asked to state what percent of that distance represented the distance to the circles in depth. Thus if the circle appeared to be one fourth of the way from the observer to the fixation point, then the observer would say 25 percent. If the circles appeared to be more than half way to the fixation circles, say 2/3 of the way, then the observer would say 67 percent. If there were no depth in the display, the observer would say 100 percent.

The final measurement method was the foot estimate technique. In this approach, the observer was simply to report the number of feet from himself to the apparent position of the circles in depth.

Probe and foot estimate measurements were made at all seven fixation distances: 5; 10; 20; 250; 6000; 7800 and $10^{16}$ meters. The proportion technique was used only at the three smallest fixation distances. These were done indoors and the three distances were randomized across trials within any experimental session. Except for the two radio towers which could be seen from the same vantage point, each of the longer distances required a different location and so they were run in blocks of trials. In every session the measurement techniques used were randomized across trials. Only one disparity value was employed in any one session.

Intertrial intervals were at least 2 minutes and in every case were sufficient to allow the afterimage to fade to invisibility. Frequent breaks
were taken to rest the eyes. Observers made judgements individually; in some cases one observer would make a judgement while another readapted. Four observers made 8 judgements for each combination of fixation distance and measurement technique employed.

RESULTS

The results are expressed in terms of the depth interval between the fixation point and the apparent location of the afterimage circles. For data from the probe technique this distance could be calculated by subtracting the distance of the probe to the observer from the total fixation distance. In the foot estimate technique the observer gave an estimate of the distance from himself to the circles in depth, so by subtracting this estimate from the total fixation distance, the depth could be determined. In the case of the proportion technique, the observer reported the percent of the fixation distance occupied by the apparent interval between himself and the circles in depth. Therefore the depth was determined by multiplying the fixation distance by the reported percent and subtracting this value from the fixation distance. In all cases, the depth has been converted to centimeter values.

Table 1 provides a comparison of the three methods for measuring the apparent depth in stereoscopic afterimages at three fixation distances for -16.3' and 4.5' of retinal disparity. Two points can be appreciated immediately. First, in general, the apparent depths from the three methods agree. Second, the probe technique shows the smallest variability. The standard errors given are those between subjects, and the same trend is found in the within subjects variability.

Figures 5 and 6 provide a further comparison between the probe technique and the foot estimate technique. Two things are illustrated clearly
TABLE 1

Means and Standard Errors for the Apparent Depth in Stereoscopic Afterimages as Measured by Three Techniques: Probe, Foot Estimate, and Percent
All Measures in Centimeters

<table>
<thead>
<tr>
<th>Fixation Distance</th>
<th>Measure</th>
<th>4.5 Min. Disparity</th>
<th>16.3 Min. Disparity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Method</td>
<td>Method</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Probe</td>
<td>Foot Estimate</td>
</tr>
<tr>
<td>500</td>
<td>Mean</td>
<td>148</td>
<td>128</td>
</tr>
<tr>
<td></td>
<td>S.E.</td>
<td>7.7</td>
<td>24.9</td>
</tr>
<tr>
<td>1000</td>
<td>Mean</td>
<td>417</td>
<td>374</td>
</tr>
<tr>
<td></td>
<td>S.E.</td>
<td>2.3</td>
<td>15.3</td>
</tr>
<tr>
<td>2000</td>
<td>Mean</td>
<td>1098</td>
<td>915</td>
</tr>
<tr>
<td></td>
<td>S.E.</td>
<td>5.5</td>
<td>12.1</td>
</tr>
</tbody>
</table>
Figure 5. Comparison of probe technique (circles) and foot estimate technique (squares) for measuring the apparent depth in a stereoscopic afterimage with 16.3' crossed disparity. Solid line shows predicted values based on geometry of retinal disparity. Dashed line has slope of 2 showing squared relation.
Figure 6. Comparison of probe technique (circles) and foot estimate technique (squares) for measuring the apparent depth in a stereoscopic afterimage with 4.5' crossed disparity. Solid line shows predicted values based on geometry of retinal disparity. Dashed line has a slope of 2 showing squared relation.
here. First, in general, the two approaches provide the same results. Second, at small fixation distances, the differences in the results from the two methods are more apparent. For the most part, this is an artifact of the log-log coordinates. Although errors increased with fixation distance, they did not increase by orders of magnitude as they would have to do if they were to show up on log coordinates.

Figure 7, together with Figures 5 and 6, presents the essential findings of this study. Here we see the growth of depth signalled by retinal disparity as a function of fixation distance. The dashed straight lines with a slope of 2 indicate the prediction based on a square law. That is, they show what would be predicted by a law which says that depth signalled by a given disparity grows as the square of the fixation distance. The solid curved lines indicate the depth predicted by the geometry of retinal disparity. The data points clearly follow the predicted values. There is a clear tendency to overestimate the depth. This is most evident at smaller fixation distances in the graph. The tendency is equally large at greater fixation distances but does not show up because of the log-log coordinates.

It will be noted that the data collected while using a star as the fixation point is not included in the graph. This is because the actual distance to the star is not known, so the depth interval cannot be calculated. However, if the distance from the observer to the apparent location of the afterimage is subtracted from any stellar distance, say 15 light years, and the result is plotted on the graph, it falls on the prediction line. Thus however far the star was, the computed depth is in line with the predictions based on the geometry of retinal disparity.
Figure 7. Apparent depth measured by the probe technique in a stereoscopic afterimage as a function of fixation distance for 16.3' (circles) and 4.5' (squares) crossed disparity. Solid curves show depth expected from the geometry of retinal disparity. Dashed lines have a slope of 2, showing a square relation.
DISCUSSION

One very important issue in psychophysics concerns the effects of different measurement techniques on psychophysical judgements. This issue is important because there is no way for an experimenter to observe directly the perceptions of the subject and so check on the accuracy of phenomenal reports. Indeed, for many sensory and perceptual continua, the various psychophysical techniques yield significantly different results (Jones, 1974; Stevens, 1974). This places on the experimenter the responsibility of calibrating the measurement techniques used in any perceptual study. This calibration may involve comparing different measurement methods, to discover whether they give the same results. If they do, this constitutes evidence that they are both reasonable measures. Even if they do not, assuming each gives consistent results, it may be possible to equate the measures by performing an appropriate transformation on one of them.

The close agreement between the results for probe technique, the foot estimate technique and the percent distance technique indicates clearly that the amount of depth reported in stereoscopic afterimages is not greatly affected by the measurement methods employed in this study. Observers reported about the same amount of depth no matter how they were asked to report it. This suggests that the probe technique used in previous work provided data that accurately reflected the depth perceived by the observers. It also suggests that any of the methods explored in this study may be used where the conditions of an experiment allow it. This is important, for there are experimental situations where one or another of the techniques is inappropriate or impossible to implement. For instance, as noted earlier, when the fixation distance was very great, observers found it very difficult to make per-
cent distance judgements. In many situations it might be impossible to arrange a depth probe between the observer and the fixation target. In such a case, the foot estimate or the percent distance technique might be used.

It is clear that as fixation distance grows, the depth as measured in this study grows essentially without limit. However, before concluding that a rescaling of the depth signalled by retinal disparity continues without limit, it would be well to consider an alternative interpretation.

Recall first that what was measured in the present study was egocentric distance. In every case the observer was asked in one way or another to indicate how far from himself the circles in depth appeared to be. Now the predictions from the geometry of retinal disparity regarding egocentric distance have been detailed in a previous report (Cormack, 1982). They show that as fixation distance increases, at first egocentric distance to the afterimage increases rapidly, and then at about 10 cm it levels off and assumes a virtually constant value. The point at which egocentric distance levels off, by the way, is the same point at which the growth of depth declines to a linear function.

What this means is that once the depth interval has grown as a function of fixation distance to the point where the egocentric distance of the afterimage no longer increases, no further rescaling is required. For any given disparity, there is a maximum egocentric distance that an afterimage, or any target, carrying that disparity can assume. Once a disparate target or afterimage has reached that egocentric distance, no further recalibration of retinal disparity is required.

A further complication is introduced by the findings regarding apparent distance. It has long been recognized that apparent distance is not infinite. Very early it was noted that the stars do not appear to be as far away as they
might. In fact, a moment's reflection will confirm that although some stars are orders of magnitude farther away than others, they do not appear to be so. In fact, much more important than their actual distance is their elevation in the heavens, in determining their apparent distance. Ptolemy noted that the vault of the heavens appears to be, not a hemisphere, but a flattened arch, nearer at the zenith than at the horizon. Thus while ever more distant fixation distances may be selected (such as stars), it may be that it is their apparent distance, not their objective distance, that rescales disparity. This view has been defended and elaborated by Foley (1980).

Gilinsky (1951) has studied the question of large apparent distances experimentally. She finds that there is a maximum apparent distance of about 100 feet. In her study, she used a variety of approaches including fractionation, apparent size estimation, and distance production. In each case, the results agreed with the general equation:

\[ d' = \frac{A(D)}{A+D} \]

where \( d' \) = apparent distance, \( D \) = physical distance, and \( A \) = maximum apparent distance. The implication of this equation is that when physical distance is small relative to the maximum apparent distance, then apparent distance will be very close to physical distance. As physical distance grows apparent distance approaches the maximum apparent distance asymptotically.

Gilinsky's results suggest that no matter how far away a fixated object may be, the rescaling of retinal disparity will not exceed that required for roughly 100 feet. This makes sense when one considers the requirements of the primate visual system. For the most part, it is only at relatively close distances that relative depth intervals are of any consequence to the organism.

This notion also makes sense when the cues to distance are examined.
closely. For virtually all the depth cues their potency or ability to provide a clear metric decreases with fixation distance. Convergence and retinal disparity become a matter of seconds of arc. Accommodation becomes a matter of small fractions of a diopter. The perspective cues and texture gradients become vanishingly small. Shadows are no longer discernable and lighting becomes irrelevant. Interposition and height in the visual field never did give more than order information. The only cue left is movement parallax, and that follows the same rules as retinal disparity, i.e., it becomes negligible unless one is moving at extremely high speeds.

What are the consequences of such a view for the interpretation of the results of the present study? Such a view suggests that as fixation distance grows, there will come a point (presumably at about 100 feet) where further increases will make no difference in the perceived egocentric distance of the afterimage. This is because, no matter what the physical distance to fixation, the apparent depth cannot exceed the apparent fixation distance. As fixation distance grows, apparent fixation distance saturates yielding a constant apparent depth and consequently a constant apparent egocentric distance to the afterimage.

This effect of maximum apparent fixation distance together with the geometry of retinal disparity, leads to a possible explanation of the overestimation of depth found in this study and in the previous study of depth in afterimages (Cormack, 1982). It would occur in this way. Let us take the 2000 cm fixation distance used in the present study. According to the geometry of retinal disparity, an afterimage containing \( -16.34' \) of retinal disparity should show a depth of 1188 cm or an egocentric distance of 812 cm. But according to the results of Gilinsky, a fixation distance of 2000 cm should appear to be about 1207 cm. The geometry of retinal disparity predicts
that at 1207 cm an afterimage with -16.34' of retinal disparity should appear at an egocentric distance of 641 cm. If this egocentric distance of 641 cm is subtracted from 2000 cm (the physical fixation distance) the result is a depth of 1359 cm. In the present study, at a physical fixation distance of 2000 cm, a measured apparent depth of 1395 cm was obtained. This agrees more closely with a prediction corrected according to Gilinsky's findings than with an uncorrected prediction based on the physical fixation distance.

All the depth values computed from egocentric measures in the present study conform to expectations based on the assumption of a maximum apparent distance better than they do to expectations omitting such an assumption.

It appears then, that veridical depth perception based on stereopsis and occurring at large fixation distances may be made possible by the fact that at these fixation distances a target with a given crossed disparity takes up a constant egocentric distance. This obviates the need for continued rescaling of retinal disparity as fixation distance grows without limit. The apparent overestimation of depth may be due to the existence of a maximum apparent fixation distance. This causes apparent depth values computed from egocentric measures to be inflated.

It is concluded that depth constancy holds for fixation distances as large as it is possible to obtain. This depth constancy leads to veridical depth judgements across the full range of fixation distances and thus is in accord with the geometry of retinal disparity. This implies that depth cues such as accommodation or convergence whose saliency is limited to near distances, cannot be the sole sources of information for the rescaling of retinal disparity.
APPENDIX

The calculations of retinal disparity and predictions of apparent depth in this paper assume that convergence is symmetrical and that targets in depth are within a few degrees of fixation. Both these conditions were met by the stimuli employed. Given these conditions, the geometry may be understood by reference to Figure A.

The right and left eyes are shown as "R.E." and "L.E." respectively. Interpupillary distance is represented by "P". The letter "D" refers to the fixation distance. Lower case "d" stands for the depth interval. Angle "b" is the convergence angle of the right eye fixated on the triangle. Since convergence is symmetrical, total convergence will be twice angle "b". Similarly, angle "c" equals the disparity of the square in the right eye and thus half the total retinal disparity. It is clear that angle "c" is also the difference between angles "a" and "b".

The ratio of "D" to one-half of "P" is the tangent of angle "b", and so angle "b" is given by ATN(D/.5P). By the same logic, angle "a" is given by ATN((D-d)/.5P). Since one-half the retinal disparity is equal to "c", and since "c" is equal to angle "b" minus angle "a", we arrive at the following formula for disparity (R):

\[ R = 2 \left( \text{ATN}(D/.5P) + \text{ATN}((D-d)/.5P) \right) \]

Note that in this case, since disparity is crossed, R typically will be expressed as a negative value.

It is possible to rearrange the terms of this formula to solve for d (the depth interval) when R (retinal disparity) is given. Conceptually, it may be clearer to derive such a formula separately. First, note that angle "a"
Figure A. The geometry of retinal disparity. Labels are identified in the text.
is equal to angle "b" minus angle "c", or to put it another way, angle "a" is equal to angle "b" minus half the retinal disparity. Since the tangent of angle "b" is equal to fixation distance (D) divided by one-half the interpupillary distance (P), angle "a" is given by ATN(D/.5P)-.5R. The tangent of angle "a" gives the ratio of (D-d) to .5P, so multiplying that tangent by P and subtracting the result from D gives us the depth interval. This is expressed as the following formula:

\[ d = D - (0.5P \times \text{TAN(ATN(D/0.5P)-0.5R)}) \]

As with disparity, by convention, d will be expressed as a negative value since disparity is crossed.
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28
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