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TEST PLANNING, COLLECTION AND ANALYSIS
OF PRESSURE DATA RESULTING FROM ARMY
WEAPON SYSTEMS. VOL III - A CORRELATION
WINDOW FOR THE M198 HOWITZER

Final Report

Steve Slinker
Henry C. Evans

May 1980

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Del Mar, CA 92014

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JAYCOR

16 August 1982

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Contract DAMD 17-78-C-8087 as per letter instruction from USA MRDC dated 28
September 1981.*

Sincerely,
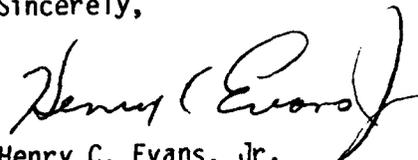
Henry C. Evans, Jr.
Program Manager
Fluid Dynamics Division

Enclosures

HCE/cvl

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Skewness and Kurtosis
Shock Tube versus Gun Overpressures
Front to Back Correlation

Back to Front Correlation
Side to Side Correlation
Correlation Coefficient

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
JAYCOR examined various pressure-time histories of the M198 Howitzer and the M203 Charge and Shock Tube waveforms. The examination consisted of Correlations of individual shots ensembles, various positions and methods of making a comparison, such as, Front-to-Back Correlation, Back-to-Front Correlation and Side-to-Side Correlation. Also addressed in the appendix of the report is a statistical derivation of correlation coefficients, Skewness and Kurtosis.

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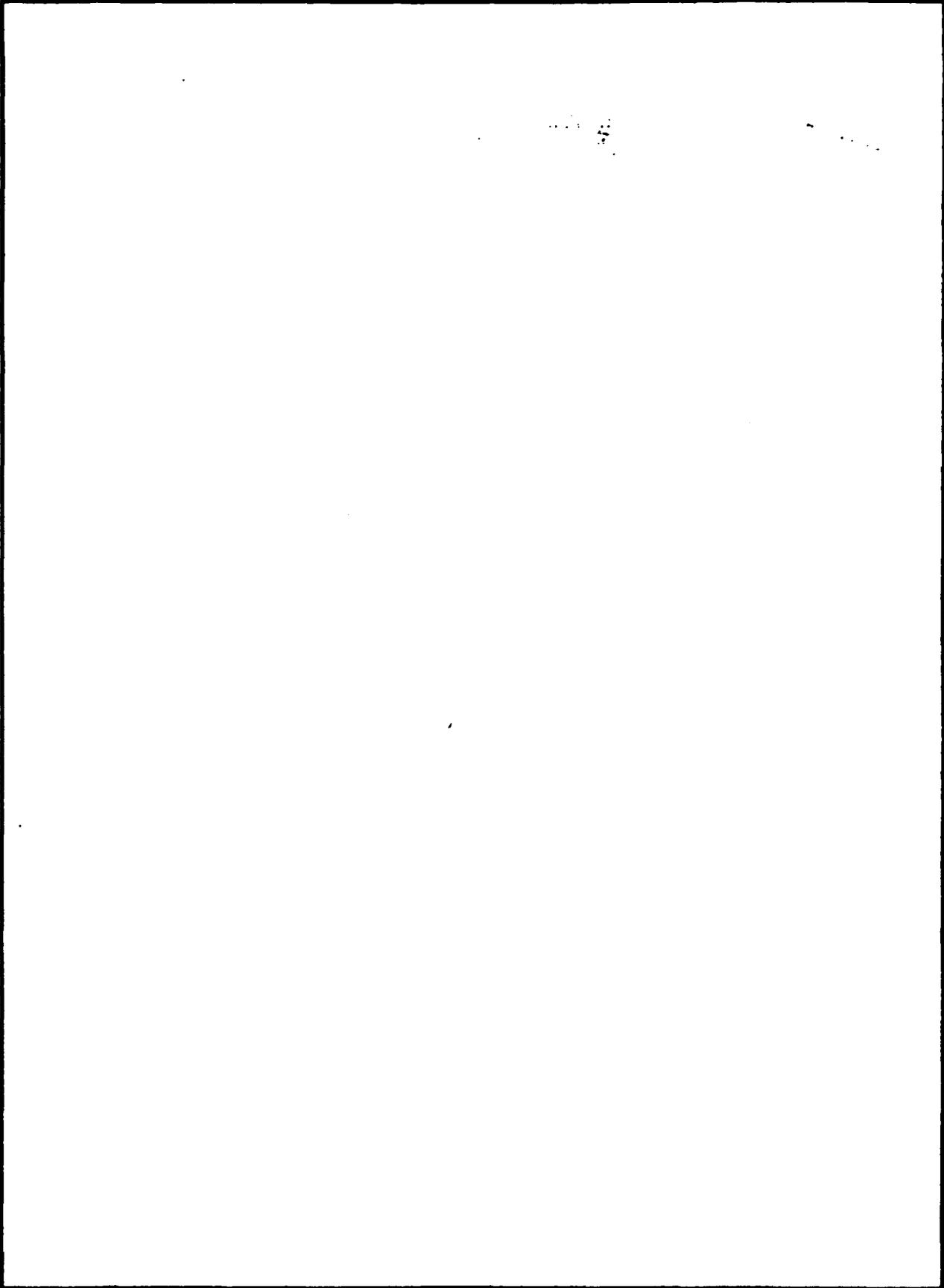
SUMMARY

This report examines various pressure-time histories in the far field produced by the M198 155mm howitzer using the M203 charge. These readings are then examined and compared to shock tube waveforms produced by the Lovelace Medical Research Inhalation and Toxicology Foundation, Kirtland Air Force Base, New Mexico. The examination consists of correlations from individual shot ensembles and various aspects of these ensembles. The methods of correlation were; front-to-back correlation back-to-front correlation and side-to-side correlation. The statistical derivation of correlation coefficients, skewness and kurtosis are explained to support the examination.

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SECTION 1
EXECUTIVE SUMMARY

1-1   INTRODUCTION AND BACKGROUND.

Development of the M198 155mm towed howitzer followed an evolutionary process dating from early efforts to field a lightweight unarmored, self-propelled howitzer capable of achieving increased ranges to counter enemy capabilities. During initial concept definition, the requirements were changed from a self-propelled to a towed howitzer. This towed howitzer (XM-198) was approved as an advanced development project in September 1968.

Results of the US Army Operational Test and Evaluation Agency (OTEA) evaluation of the M198 howitzer conducted at Ft. Sill, July through December 1975, indicated that at least two persons, the gunner and assistant gunner, had headaches from the howitzer firings. Members of OTEA's Test Directorate also made comments indicating that they were distressed to be in the immediate area of the howitzer when firing the M203 propelling charge for periods of one to four hours. The overpressures causing the problems to the crew were promulgated by the propellant and the muzzle brake for the howitzer. The muzzle brake was designed with acceptable recoil attenuation and having crew station blast overpressure levels of 3 psi or less. However, the overpressure durations from the howitzer determined by testing were in excess of MIL STD 1474 (MI).

In order to fully understand the physics of blast overpressures and the phemonema involved, Walter Reed Army Institute for Research (WRAIR) contracted JAYCOR to assist in the characterization of blast physics, data collection, reduction and analysis.

Besides providing descriptions of the blast overpressure fields around the guns and the shock tube, a major purpose of the data analysis is to answer three questions:

1. What parameters of a pressure wave are most responsible for causing respiratory damage in animals and men?

2. How well does the shock tube simulate the gun?

3. How reproducible are the pressure waves from the gun and the shock tube?

At present, there is no satisfactory data available to answer the first question. Yet its solution is intimately related to the answers of questions 2 and 3. If the relative importance of various parameters of a pressure wave are not known, a judgment cannot be made as to the quality of the shock tube simulation of the gun or the amount of shot-to-shot or day-to-day variance allowable in basing any conclusions on the validity of a particular experiment. For example, if it is found that lung damage is most sensitive to the amount of energy at 100 hertz and the peak pressure encountered, but relatively insensitive to the A-duration and impulse, then more emphasis is placed on the first two parameters when judging the quality of a shock tube simulation.

The thrust of JAYCOR's data analysis program has been to develop tools and techniques for evaluating reproducibility and simulation of pressure waves. The first step is the determination of major parameters associated with a given pressure curve, such as the maximum and minimum overpressures, A- and B-Durations, impulses, location and strength of the reflected pulses, frequency content, etc. Next, standard statistical measures of the pressure curves are taken with major emphasis placed on the correlation coefficient. The correlation coefficient is a simple and commonly used statistical device employed in the comparison of different sets of data. Other measures under examination are the standard deviation, skewness and kurtosis of a pressure time history. The definitions of these quantities are given in Appendix A.

1-2   DISCUSSION AND RESULTS.

The results of the first application of these techniques are contained in the JAYCOR Final Report for Contract DAMD 17-78-C-8062. It was found during the effort that the shot-to-shot correlations at both the gun and the shock tube were generally around 0.9 for record lengths of 150 milliseconds (ms). In correlating different locations at the gun and in correlating the shock tube with the various locations around the gun (see Figure 1-1), coefficients near 0.8 were obtained.
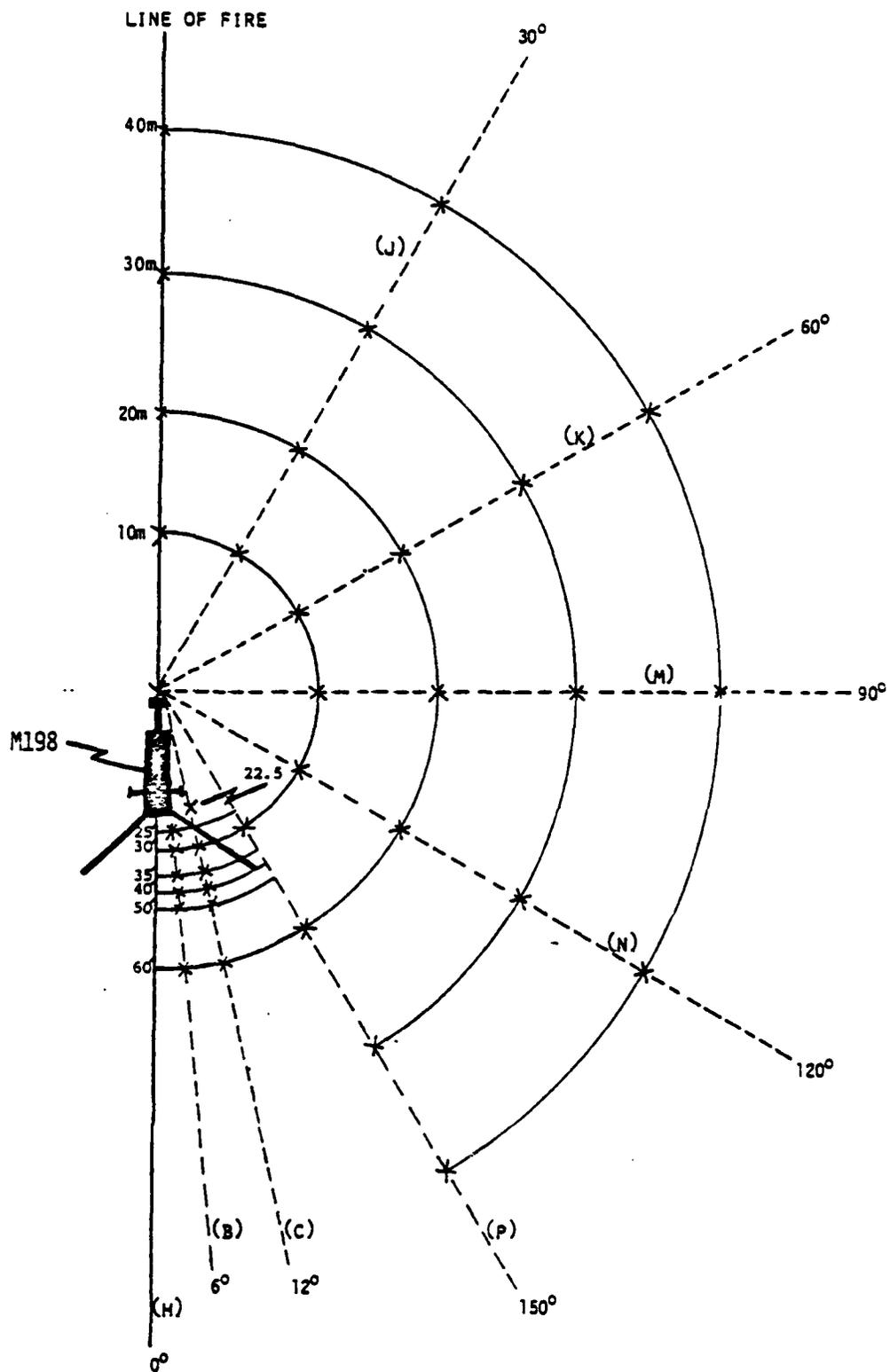
Figure 1-1. Transducer Locations for the M198 Firings

Though this appears satisfactory it was noted that the correlation coefficient was rather insensitive to differentiating between various locations (at least at a record length of 150 ms) and it was suggested the the variation of correlation coefficient with record length be studied. The details of this study are contained in Section 2.

The results of this analysis indicate that the correlation coefficient is most sensitive to the difference between typical pressure curves if a record length of 5-15 ms from pulse onset is used. Consequently, in future use of the correlation coefficient, this region should be particularly emphasized.

The reasons for the sensitivity of the correlation coefficient to the 5-15 ms record length can easily be seen by examining typical pressure time histories (See Figures 1-2 and 1-3). The major features of a typical blast overpressure wave around the M-198 are a large sharp positive spike then succeeded by a reflected spike followed by a region of negative overpressure. The total time duration of the two positive spikes is 5-10 ms while the much gentler region of negative overpressure lasts 20-40 ms. The reason the correlation coefficient between two curves at different gun sites drops at around 5-15 ms is due to the presence of the reflected pulse. At different locations, the distance between the path of the reflected pulse and the path of the direct pulse varies. Consequently, their relative arrival times are different. The drop in correlation reflects this change.

In comparisons between shots at the same location the correlation coefficient tends to drop somewhat in this region, but not nearly as much as in the case for two different locations.

After some 20 ms, the correlation rises to almost the same level for all locations and then slowly decays with increasing record length.

1-3   CONCLUSIONS.

In conclusion, the correlation coefficient is most useful as a means of distinguishing two pressure time histories if a record length of 5-15 ms from pulse onset is used because of structure relating to the reflected pulse. It is

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B-25 ENSEMBLE

4' 8,000

Figure 1-2

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C-30 ENSEMBLE

4' 0.000

PRESSURE (PSI)

Initial
Peak

Reflected
Pulse

Region of Negative Overpressure

TIME (MS)

Figure 1-3

6
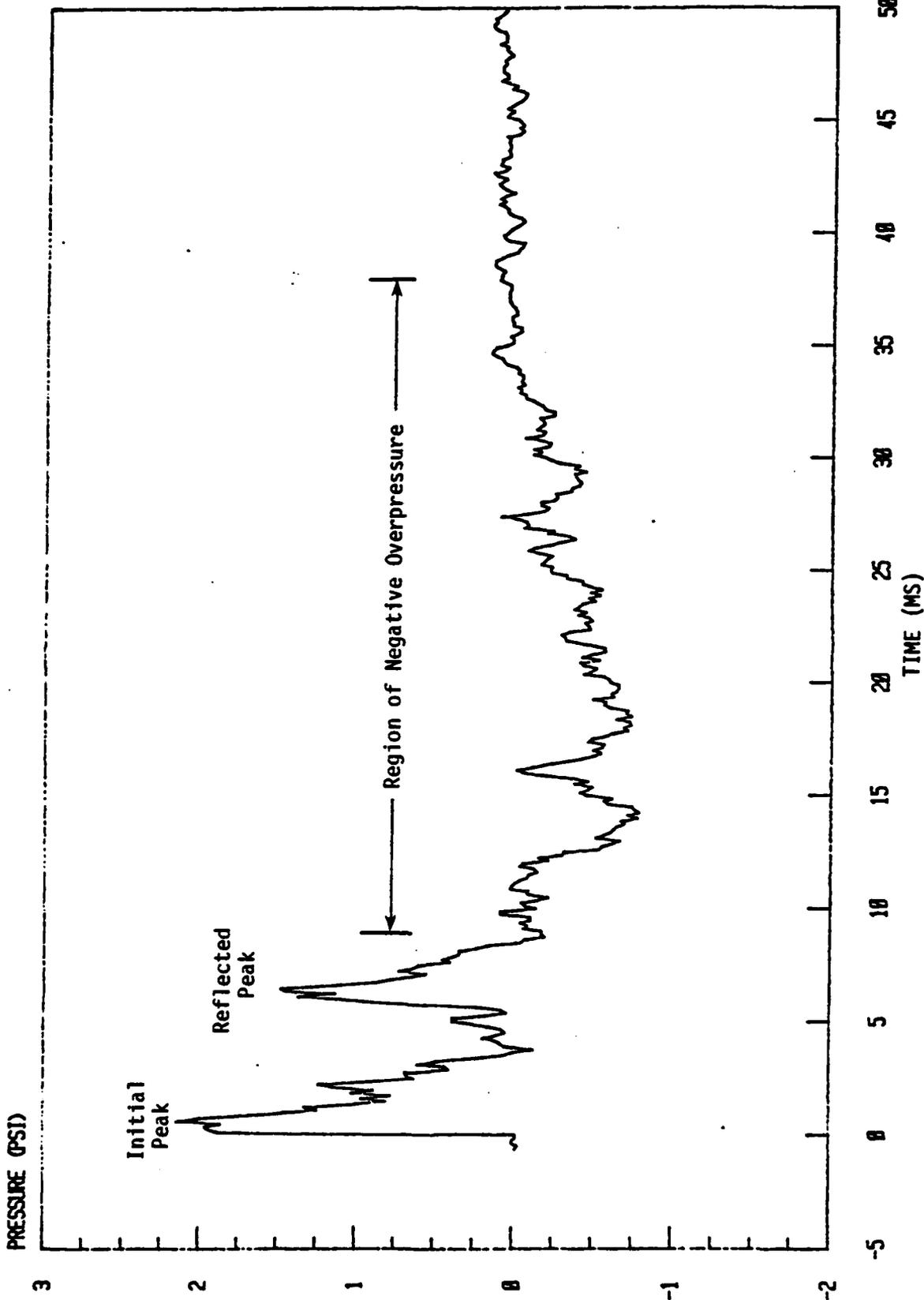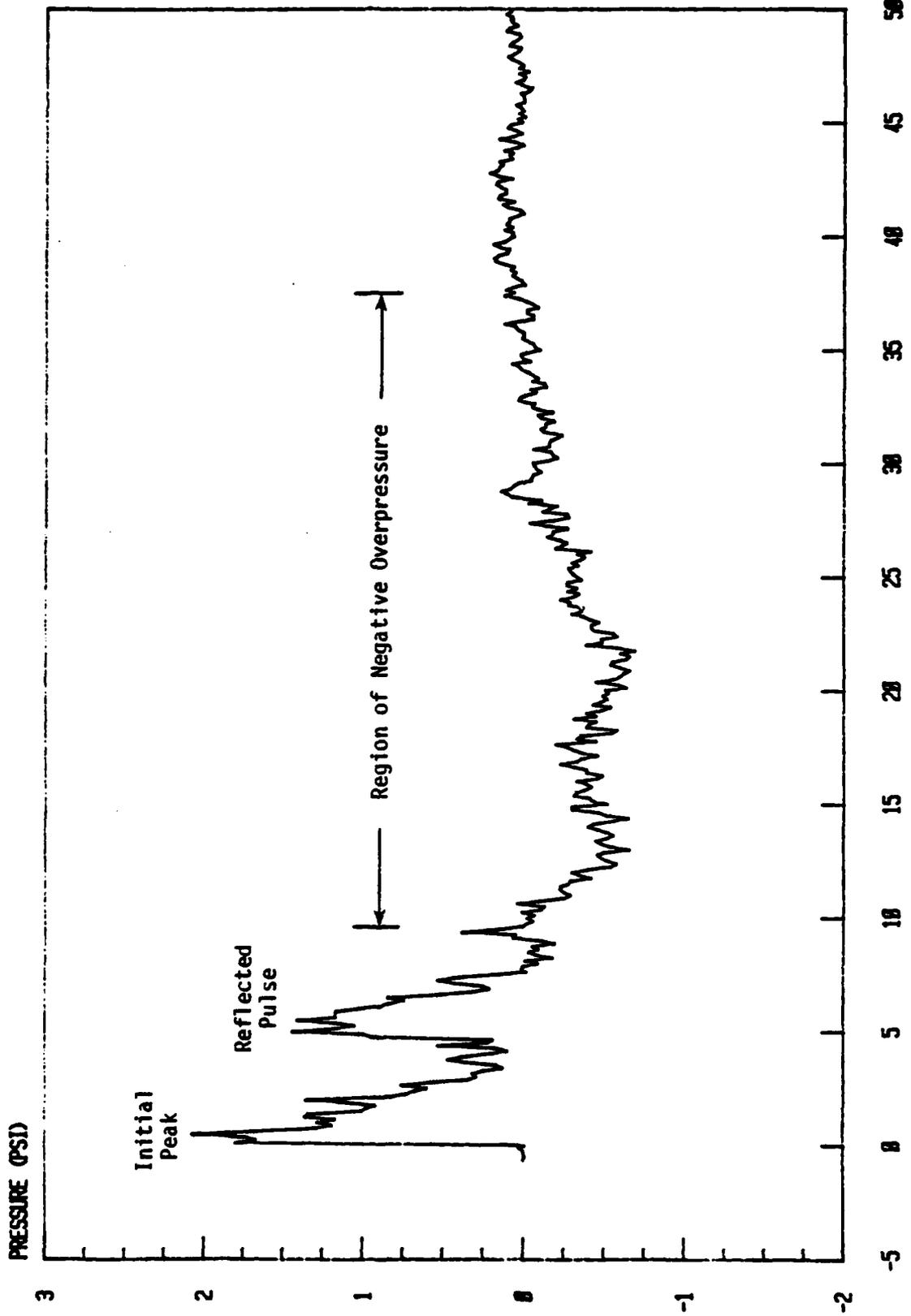
not known what affect the location and strength of the reflected pulse has on biological systems.

The standard deviation, skewness and kurtosis (see Appendix A) of a pressure time history were also investigated. Though these measures differ significantly between various locations at the gun and between the gun and shock tube, acceptable criteria as to the ranges they may acquire cannot be stated until human dose response data is available.

SECTION 2
DETAILS OF DATA ANALYSIS

2-1   INTRODUCTION.

The primary goal of this study is to determine the usefulness of the cor-
relation coefficient as a measure for comparing the overpressure waves from
blasts. Two pressure waves with a "high" correlation would be expected to produce
nearly the same effect on a physiological system. Though this hypothesis can only
be verified by experiment, it is of some importance to understand how wave shape
and record length affect the correlation. For example, it would be useful to know
at what point in time (if any) two typical pressure waves tend to have the lowest
correlation. This point, if it exists, would be the best point at which to apply
the correlation coefficient, i.e., the correlation is the most sensitive here, and
consequently, it is best able to distinguish between the curves.

The results of the first application of correlation techniques to the
study of blast overpressures are contained in JAYCOR's Final Report DAMD 17-78-C-
8062, dated 13 August 1979. In this report, individual shots at the same trans-
ducer location, at both the 30 November - 1 December 1978 firing of the M-198
howitzer and the March 1979 firings of the Lovelace shock tube, were correlated.
Comparisons were also made between the ensemble averages of shots with a given gun
or tube configuration. Record lengths of 75 ms and 150 ms were used. Some of the
results are contained in Table 2-1.

The relatively high correlations between different locations and between
the gun and the shock tube were surprising considering the variety of the pressure
wave forms and the variation in other parameters between shots. For example, the
shock tube was designed to simulate the pressure field at location C22 of the M-
198 with the howitzer at 0 mil azimuth and 267 mils quadrant elevation. Yet the
correlations between the shock tube and other howitzer locations were nearly the
same.

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Table 2-1. Correlation

CORRELATION OF INDIVIDUAL SHOTS (150 ms RECORDS)

M-198 Location C22 4' 0,267

	Correlation
SHOTS 14,15	.93
SHOTS 14,17	.93
SHOTS 15,17	.94

Shock Tube. Gauge 2 Day 4

SHOTS 3,15	.93
SHOTS 3,22	.92
SHOTS 15,22	.93

CORRELATION OF SHOCK TUBE ENSEMBLE OF DAY 1
GAUGE 2 WITH VARIOUS M-198 LOCATIONS

	Correlation
WITH C22 3' 0,267	.78
C22 4' 0,267	.80
C22 4' 0,800	.79
B30 4' 0,800	.81
C30 4' 0,267	.82

2-2   TYPES OF CORRELATION EXAMINED.

Thus, a study of the variation of correlation coefficient with record
length was suggested. This was accomplished in three ways:

Front-to-Back correlation which fixes the beginning point of correlation
at pulse onset and then increases the length of the record compared, e.g., the
Front-to-Back 10 ms correlation coefficient is the correlation between the first
10 ms (from pulse onset) of the two curves.

Back-to-Front correlation fixes the end point of correlation relative at
pulse onset and correlates backward in time from there, e.g., Back-to-Front 30 ms
(on a 50 ms record) is the correlation between the last 20 ms of the records.

Side-to-Side which compares corresponding parts of the curves as the cor-
relating window slides forward in time, e..g., the Side-to-Side 17.5 ms correla-
tion coefficient (with a 2.5 ms window) is the correlation between 15 to 17.5 ms
portions of the curves.

2-3   CORRELATIONS OF INDIVIDUAL SHOTS.

Results from applying the three types of correlation to three shots at
location C22 of the cannon and to three shots at the shock tube are given in Table
2-2. Figures 2-1 through 2-8 are plots of this data.

The Front-to-Back coefficient reaches a minimum around the time of
arrival of the reflected pulse, rises to a maximum around 15-20ms after pulse
arrival and then slowly decays as the correlation length increases. The Back-to-
Front and Side-to-Side coefficients both show that, after the main positive por-
tions of the pressure time history, the correlation between waves is generally
low.

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Table 2-2. Correlation Windows.

M-198 Time	SHOTS 14,15			SHOTS 14,17			SHOTS 15,17		
	Front-to-Back	Back-to-Front	Side-to-Side	Front-to-Back	Back-to-Front	Side-to-Side	Front-to-Back	Back-to-Front	Side-to-Side
2.5	.91	.93	.91	.90	.93	.90	.96	.94	.96
5	.92	.91	.93	.82	.91	.80	.90	.91	.87
7.5	.93	.85	.77	.92	.86	.87	.92	.87	.77
10	.95	.85	.48	.94	.86	.42	.95	.87	.73
12.5	.95	.85	-.10	.94	.87	.17	.95	.87	.11
15	.96	.86	.64	.95	.88	.29	.95	.88	.48
25	.95	.50	.43	.95	.54	.57	.95	.56	.68
50	.94	.42	.08	.94	.30	-.09	.94	.33	.04
75	.93	.50	.21	.93	.40	.06	.94	.42	.10
100	.93	.29	.10	.93	.25	.39	.94	.18	-.03
150	.93	-.14	-.14	.93	.25	.25	.94	.36	.36
Max F-B	.958 17.5ms			.954 20ms			.958 17.5ms		
Min F-B	.912 2.5ms			.823 5ms			.901 5 ms		

SHOCK TUBE Gauge 2 Day 4

Time	SHOTS 3,15			SHOTS 3,22			SHOTS 15,22		
	Front-to-Back	Back-to-Front	Side-to-Side	Front-to-Back	Back-to-Front	Side-to-Side	Front-to-Back	Back-to-Front	Side-to-Side
2.5	.97	.94	.97	.98	.92	.98	.97	.93	.97
5	.91	.86	.44	.86	.84	.09	.92	.85	.46
7.5	.94	.81	.68	.91	.80	.49	.94	.78	.45
10	.95	.79	.59	.93	.78	.16	.95	.76	.24
12.5	.96	.79	.36	.94	.78	.46	.96	.77	.70
15	.97	.79	.38	.95	.79	.39	.97	.77	.24
25	.96	.74	-.15	.96	.72	.20	.97	.70	.28
50	.97	.67	.04	.95	.62	.08	.96	.58	.34
75	.95	.65	.74	.94	.60	.58	.95	.52	.72
100	.94	.69	.42	.94	.62	.25	.94	.55	.08
150	.94	.05	.05	.93	.01	.01	.93	.04	.04
Max F-B	.974 25ms			.982 25ms			.971 1.5ms		
Min F-B	.911 5ms			.865 5ms			.916 5ms		

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FRONT-TO-BACK CORRELATION

C22 4' 8.267
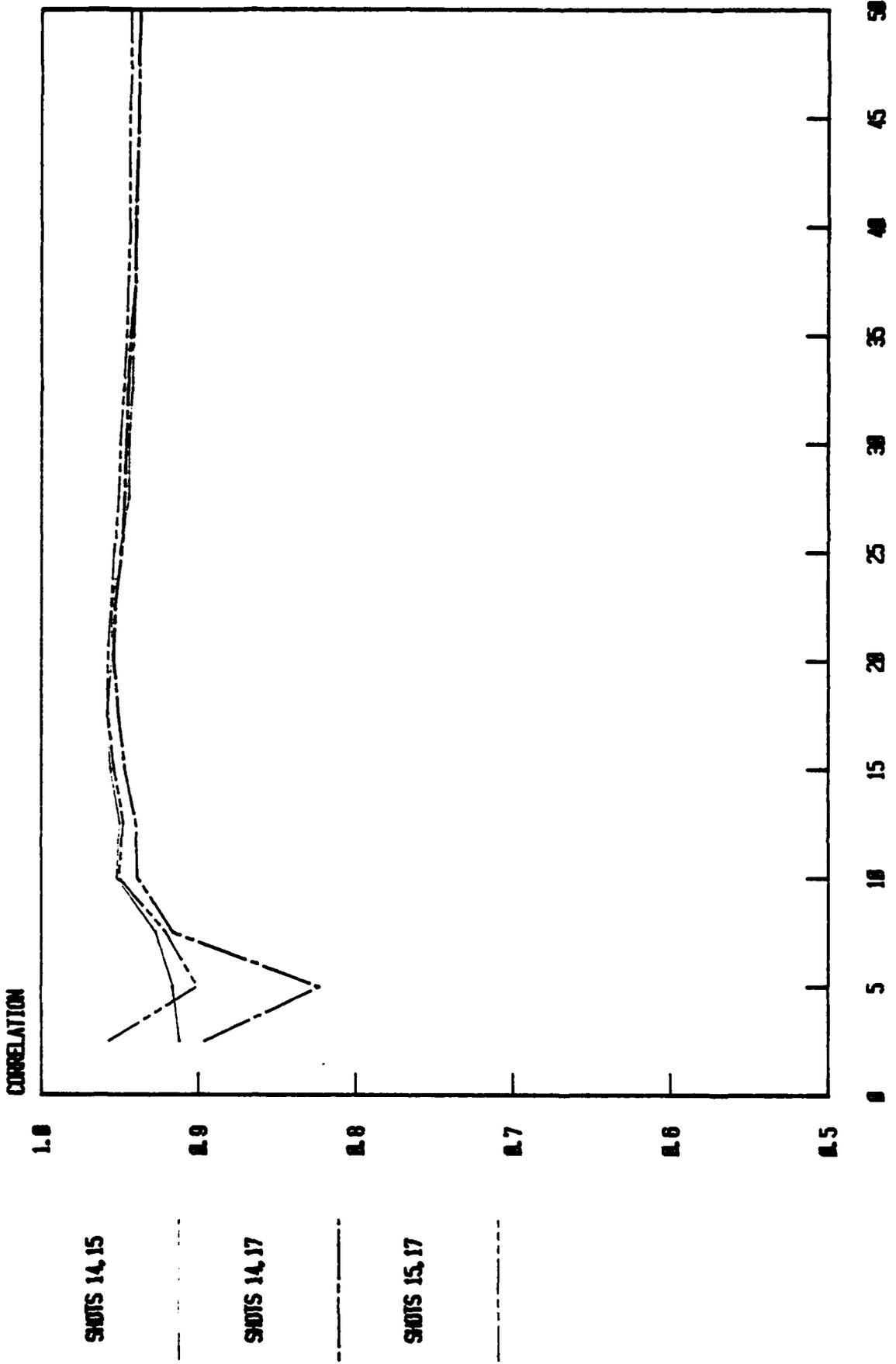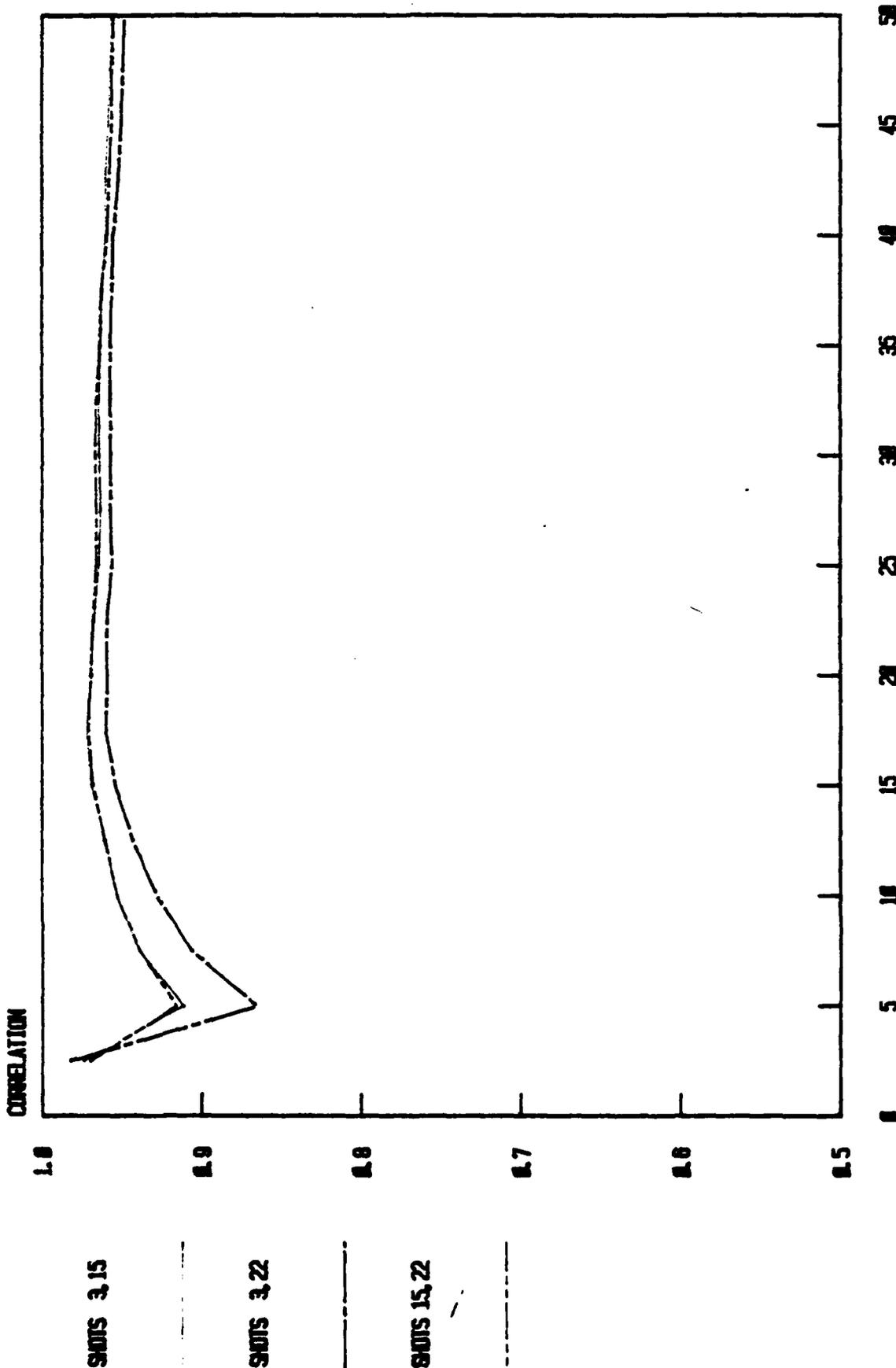
CORRELATION

TIME (MS)

Figure 2-1

SHOTS 14,15

SHOTS 14,17

SHOTS 15,17

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FRONT-TO-BACK CORRELATION
SHOCK TUBE DAY 4 GAUGE 2

Figure 2-2

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BACK-TO-FRONT CORRELATIONS

C2 4' 8,287

Figure 2-3

TIME (MS)

CORRELATION

SHOTS 14,15

SHOTS 14,17

SHOTS 15,17

14
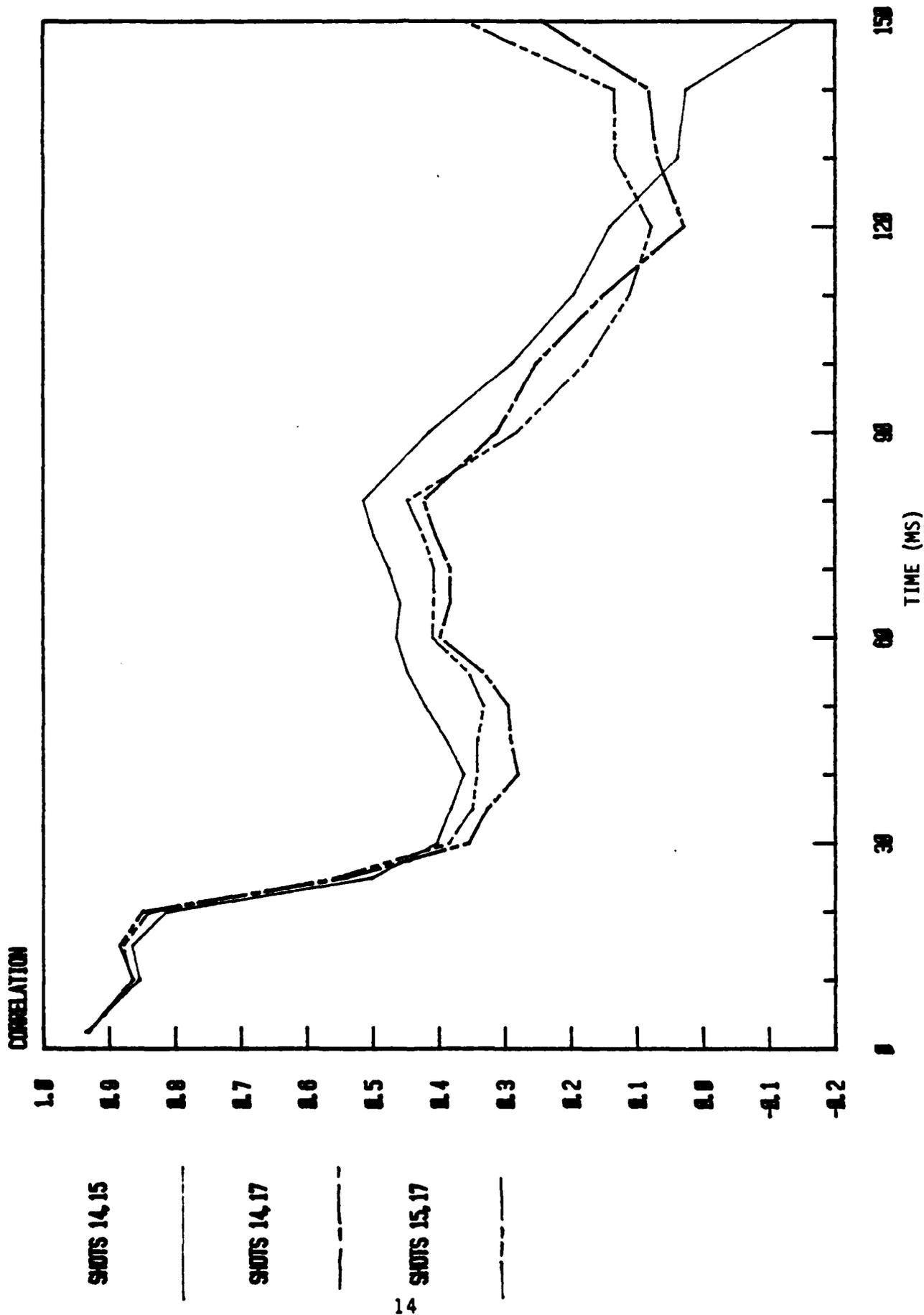
BACK-TO-FRONT CORRELATIONS

SHOCK TUBE DAY 4 GAUGE 2

Figure 2-4

15
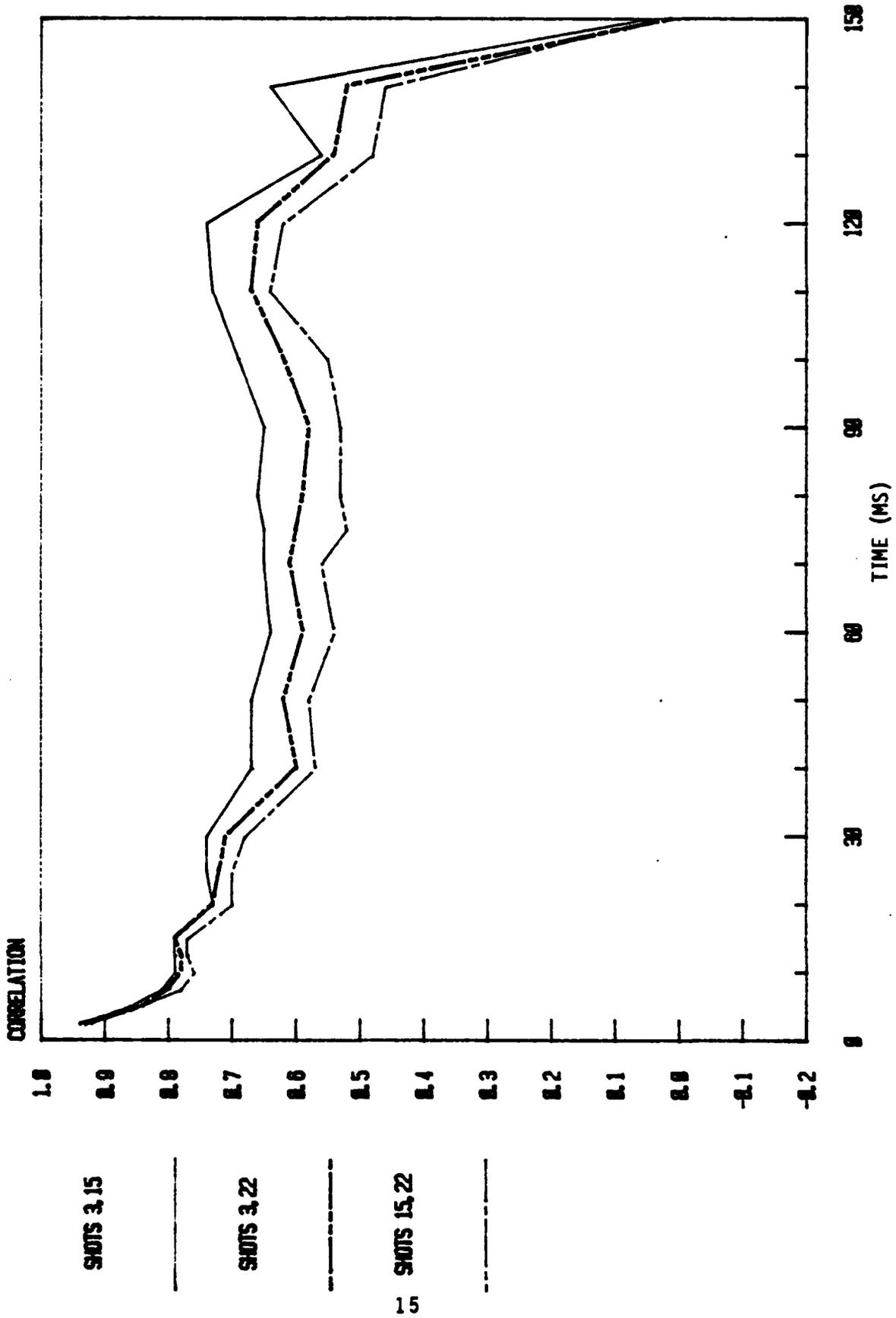
SIDE-TO-SIDE CORRELATION

C22 4' 0.267

CORRELATION

TIME (MS)

Figure 2-5

SHOTS 14,15

SHOTS 14,17

SHOTS 15,17

16

SIDE-TO-SIDE CORRELATION

SHOCK TUBE DAY 4 GAUGE 2


SHOTS 3,15

SHOTS 3,22

SHOTS 15,22

Figure 2-6

17
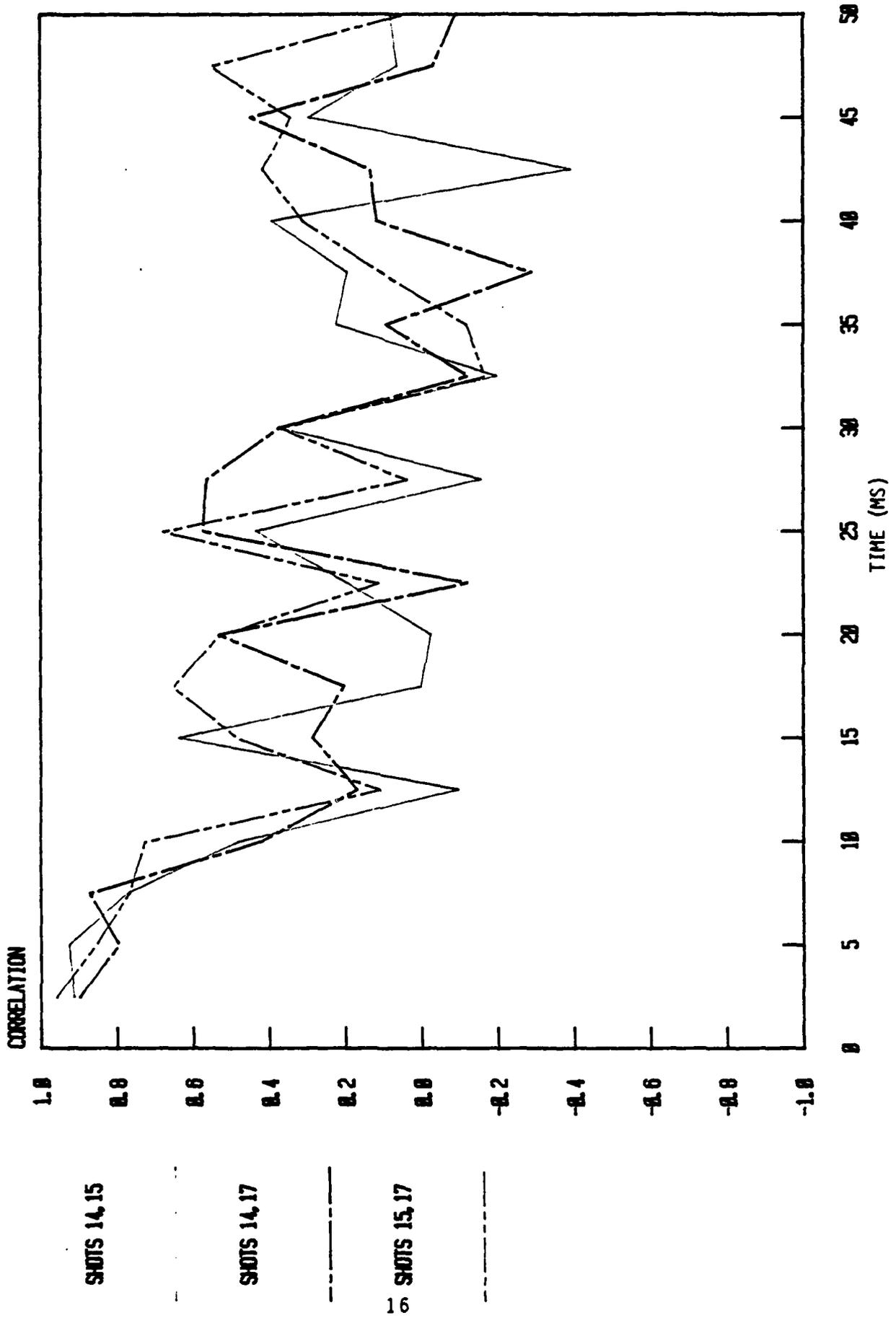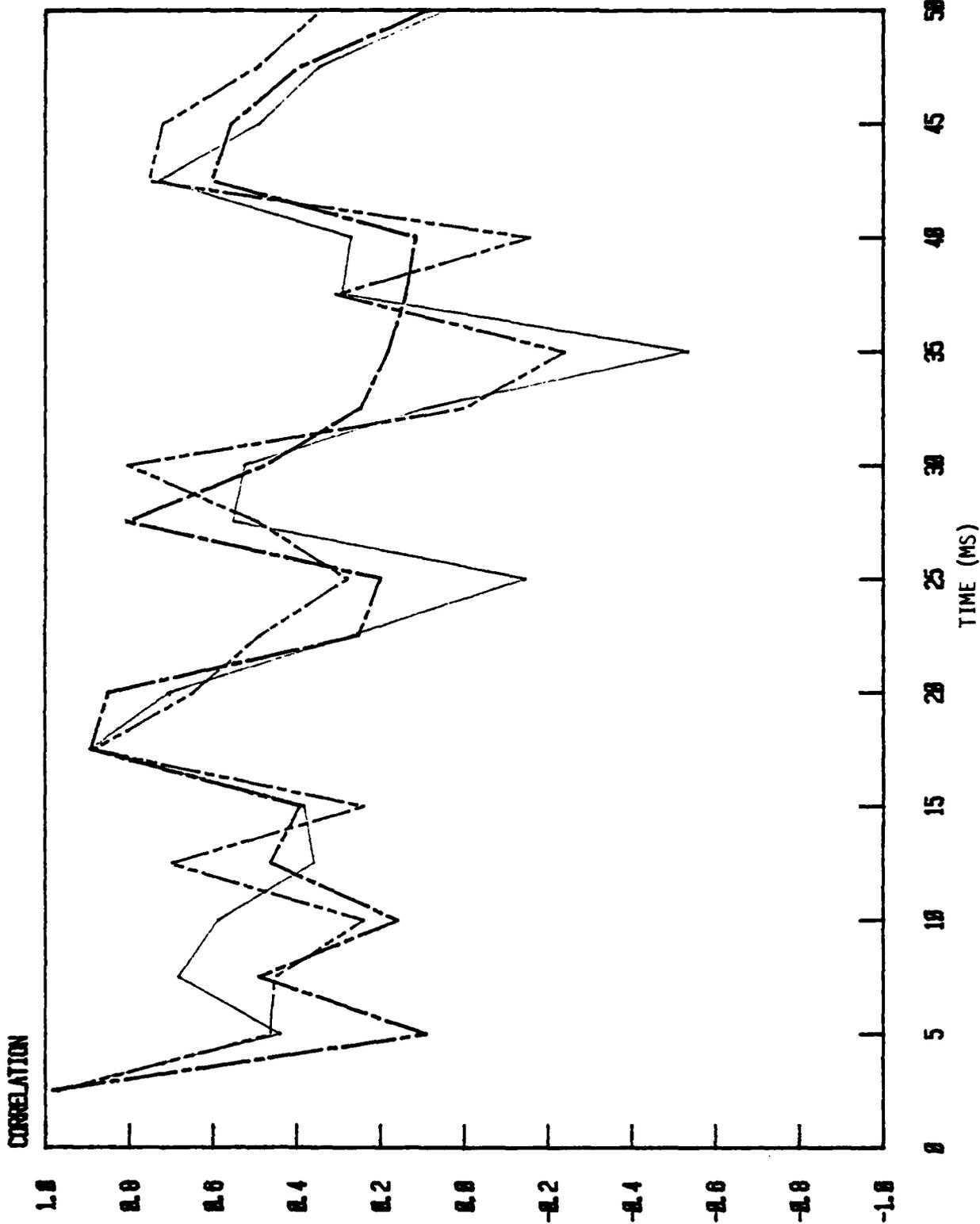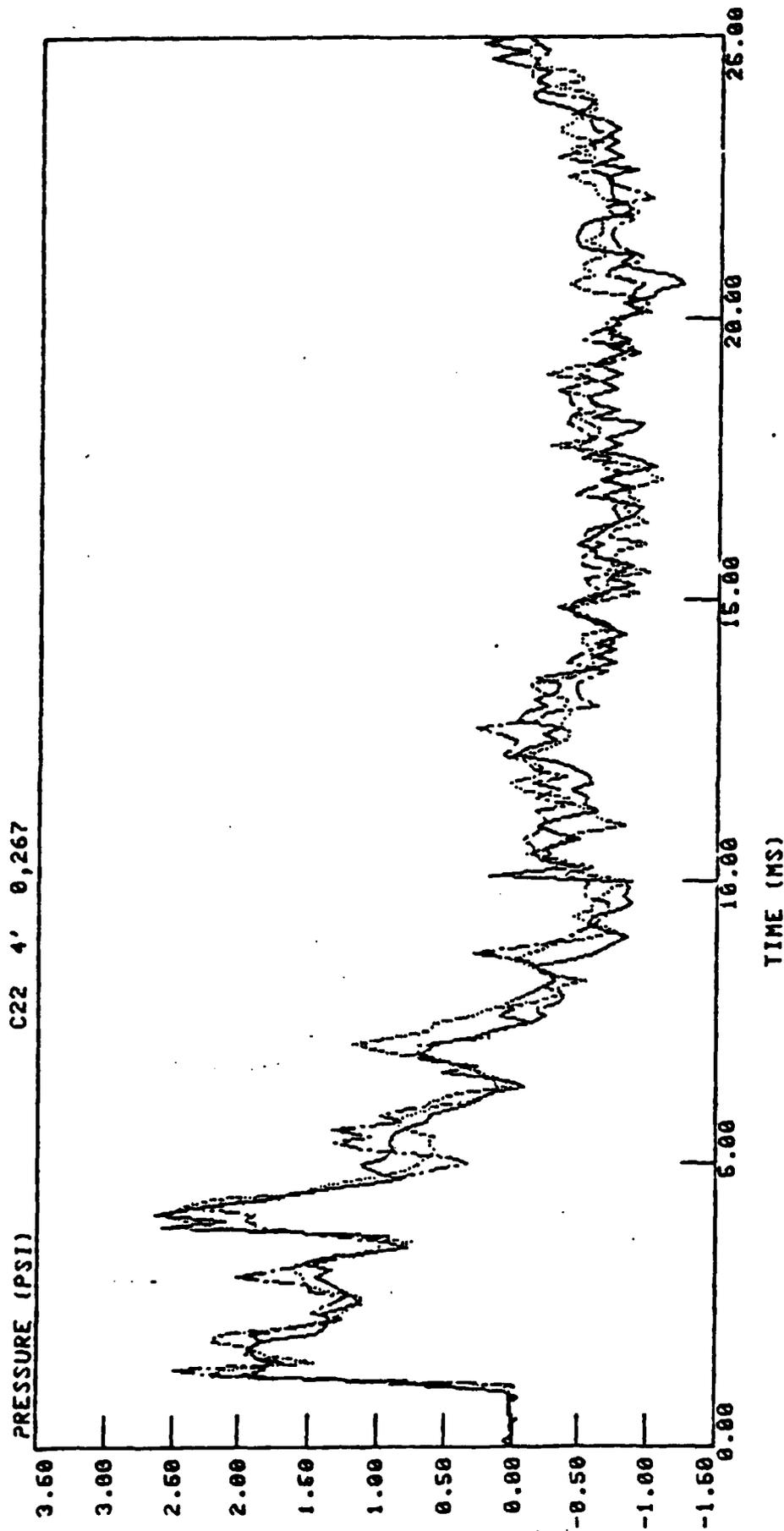
Figure 2-7. Overlay of Shots 14, 15, 17 M198
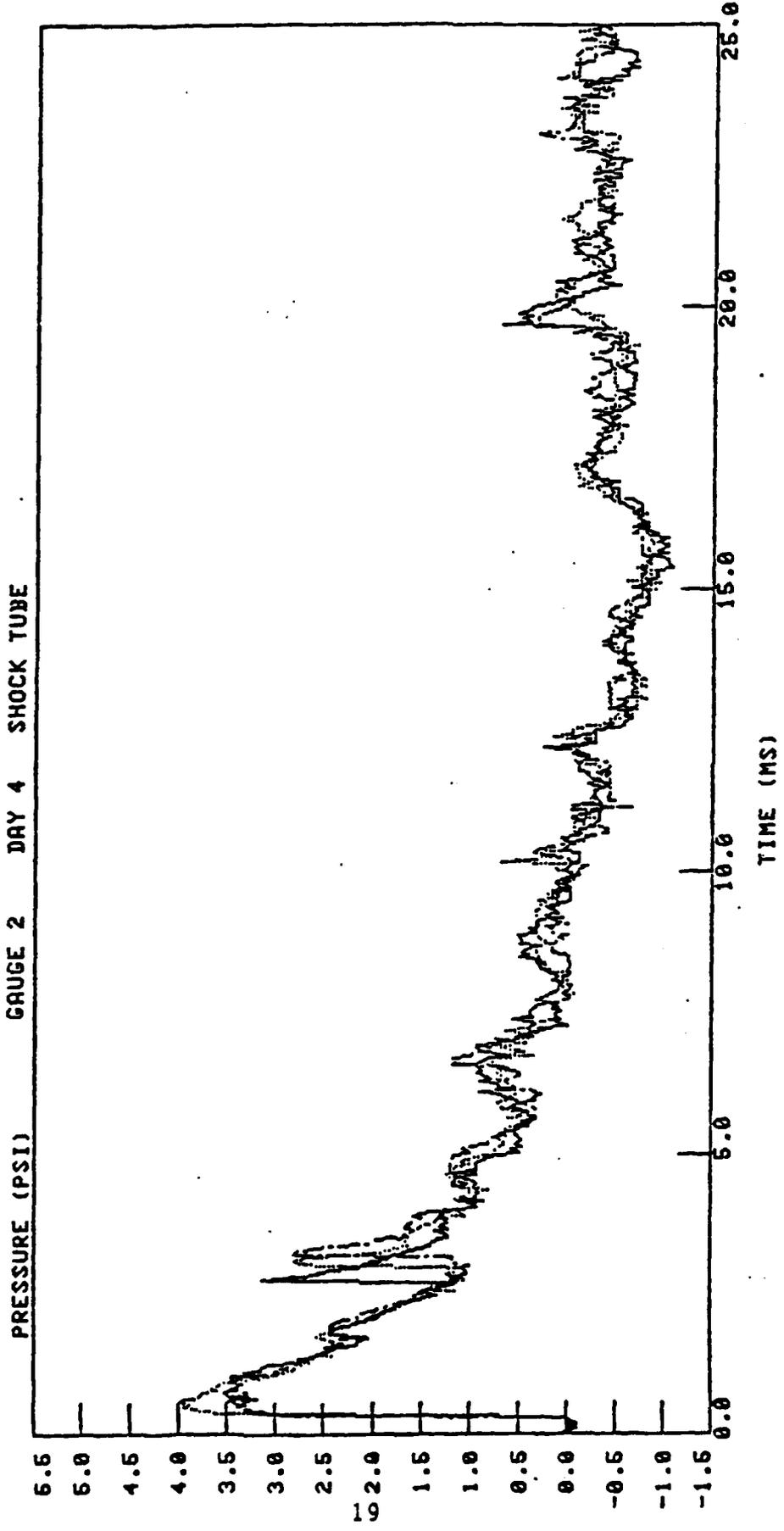
Figure 2-8. Overlay of Shots 3, 15, 22 Shock Tube

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2-4   CORRELATION OF ENSEMBLES.

Next, three shot ensemble averages were made at various locations, and these were correlated in the same manner. Some of these results are provided in Table 2-3 and are plotted in Figures 2-9 through 2-13.

As these figures demonstrate, many of the conclusions drawn from the shot-to-shot correlation window data are also available here. As expected, the minimum correlation which occurs from 5-15ms from pulse onset is lower than in the shot-to-shot case, reflecting the different wave form structure due to the ground reflected pulse. In particular, the comparison with C22 and the shock tube pressures, as contrasted to the other howitzer measuring locations, is noteworthy. At 50ms all the correlation coefficients were between .81 and .86. However, if the minimum correlation coefficient is used, then the C22 locations correlate much better than the other locations. At 800 mils the minimum correlation coefficient was .803 for the C22 location while the largest minimum for the other locations was .696 for B25. The results were similar for the 267 mil data. (See Table 2-3.)

2-5   EFFECTS OF TIME SHIFTS.

In computing the correlation windows, the curves were considered lined up in time when they showed the largest correlation for a long record length. In other words, the curves were visually aligned and shifted in time relative to one another until the highest correlation was obtained. In all cases, this corresponded to closely matching the first pulses of the waves. To see what effect time shifting had on the latter portions of a pressure time history, correlation windows were computed for time shifted B30, C30 howitzer location ensembles. The results are given in Figures 2-14 through 2-17. Shift 1 corresponds to a time shift of .125ms, Shift 2 to .25ms and Shift 3 to .375ms.

Time shifting principally affects the correlation of the first pulse in the windowed correlation. After 7.5ms the Back-to-Front and Side-to-Side-shifted curves are indistinguishable from the unshifted ones. On the Front-to-Back plot the shifted curves are slightly lower in correlation after 15ms, but they all have

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Table 2-3. Correlations of Ensemble of Day 4 Transducer 2
At Lovelace With Various Aberdeen Locations

TIME (ms)	4' 0,800					4' 0,267			
	C22	C30	B25	B30	N45	C22	C30	B25	B30
2.5	.92	.93	.93	.97	.98	.90	.89	.96	.95
5	.85	.86	.90	.91	.81	.74	.70	.71	.67
7.5	.82	.57	.70	.61	.65	.83	.80	.78	.80
10	.80	.70	.75	.73	.77	.87	.86	.84	.86
12.5	.84	.78	.80	.80	.83	.88	.89	.88	.89
15	.88	.83	.85	.85	.87	.91	.91	.91	.92
17.5	.89	.85	.86	.87	.88	.92	.92	.92	.92
20	.89	.86	.86	.87	.89	.92	.92	.91	.92
30	.88	.84	.86	.86	.89	.87	.88	.88	.88
40	.86	.83	.84	.85	.88	.85	.86	.86	.87
50	.84	.81	.82	.83	.86	.84	.85	.85	.85
Max	.921	.927	.926	.969	.977	.918	.924	.962	.952
Time	2.5	2.5	2.5	2.5	2.5	17.5	17.5	2.5	2.5
Min	.803	.566	.696	.613	.651	.740	.700	.712	.670
Time	10.0	7.5	7.5	7.5	7.5	5.0	5.0	5.0	5.0

TUBE-GUN FRONT-TO-BACK CORRELATION
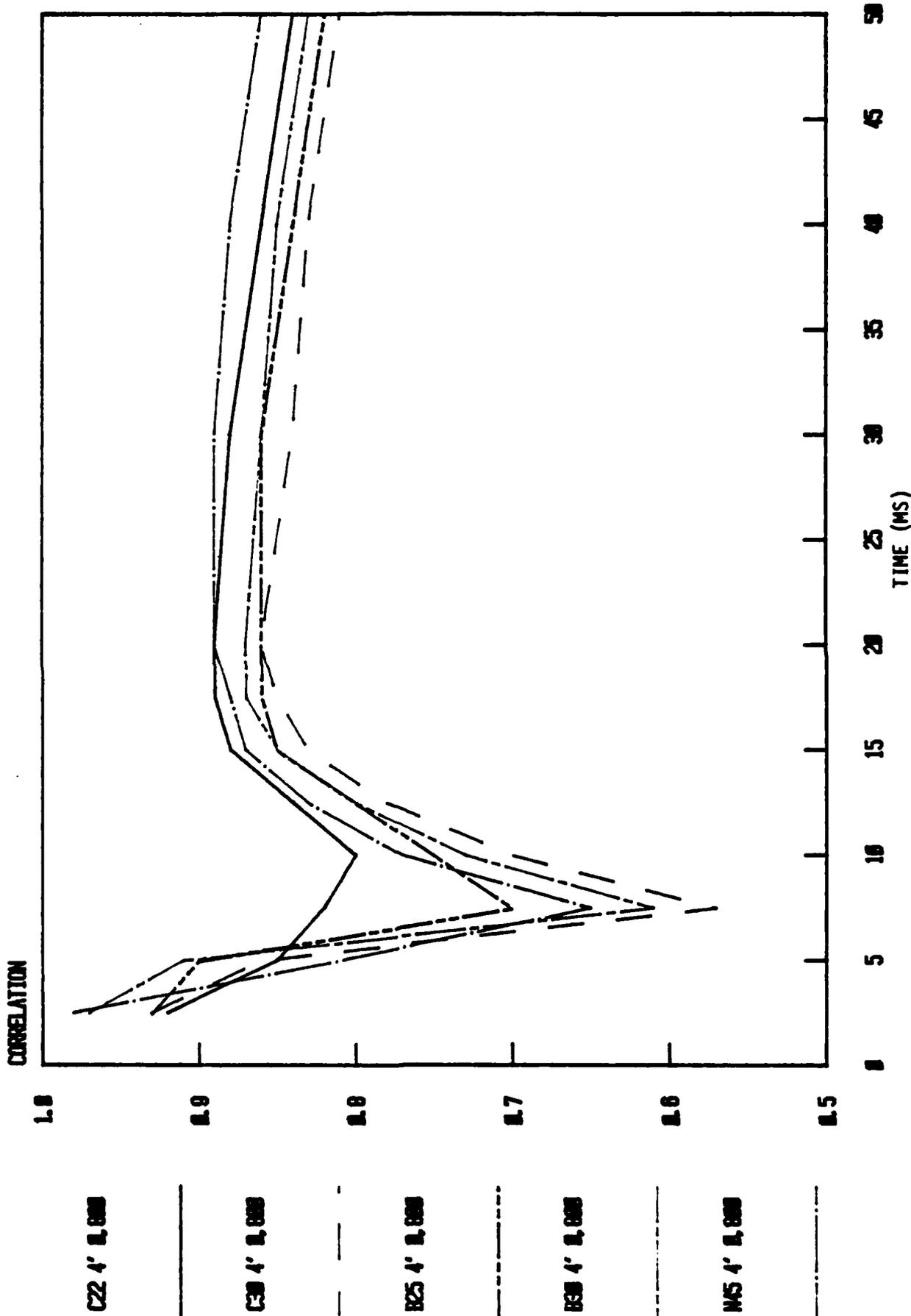
TUBE ENS. DAY 4 GAUGE 2 SHOTS 3,15,22

Figure 2-9
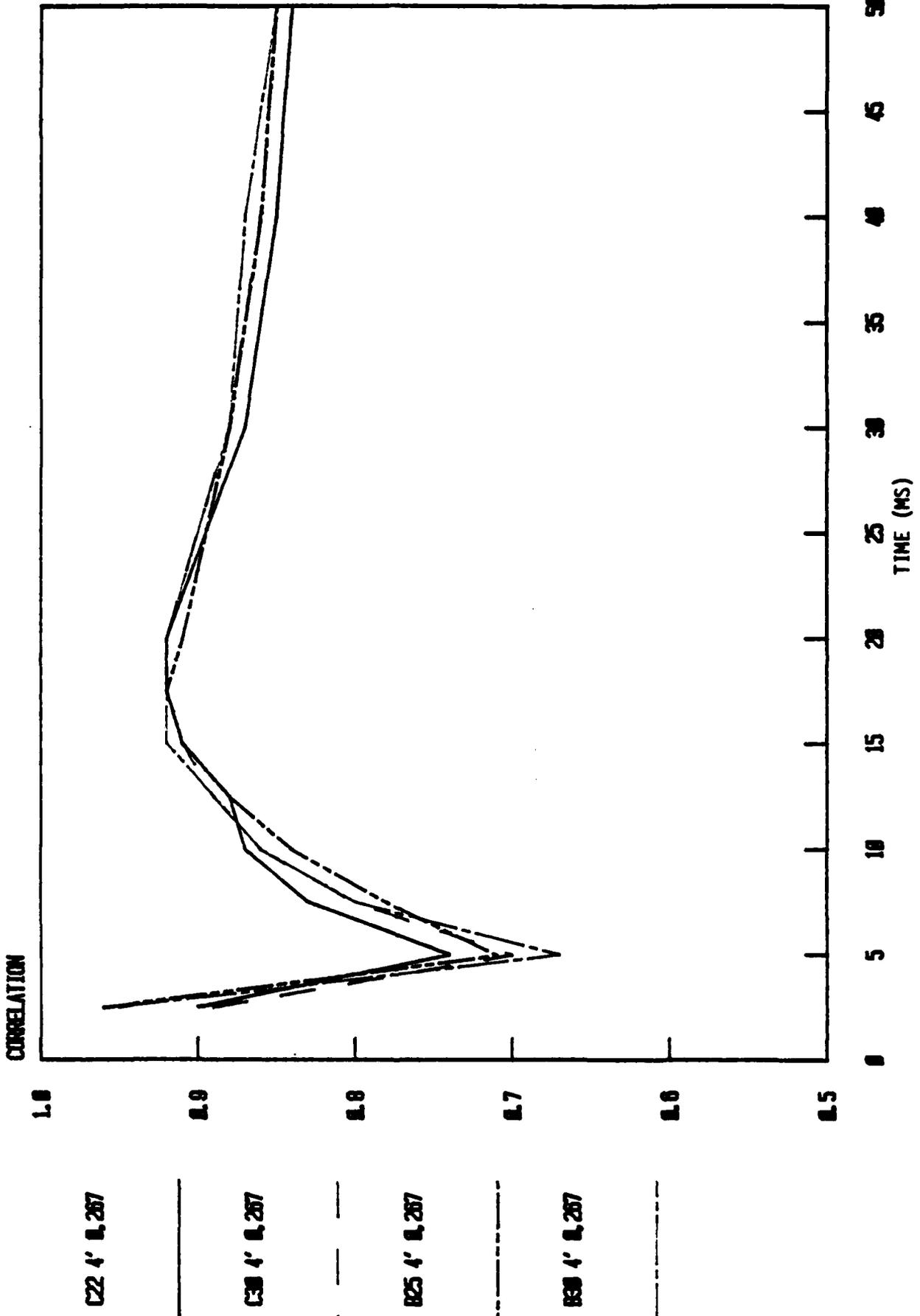
22

TUBE-GUN FRONT-TO-BACK CORRELATION

TUBE ENS. DAY 4 GAUGE 2 SHOTS 3,15,22

Figure 2-10

23

COMPARISON OF SHOCK TUBE WITH POSITION C22 4' 2670E

PRESSURE (PSI)

Tube Ensemble Day 4 Gauge 2

C22
Ensemble

TIME (MS)

Figure 2-11

24

COMPARISON OF SHOCK TUBE WITH POSITION C22 4' 8000E

PRESSURE (PSI)

Tube Ensemble   Day 4   Gauge 2

C22
Ensemble

4

3

2

1

0

-1

25

-5   0   5   10   15
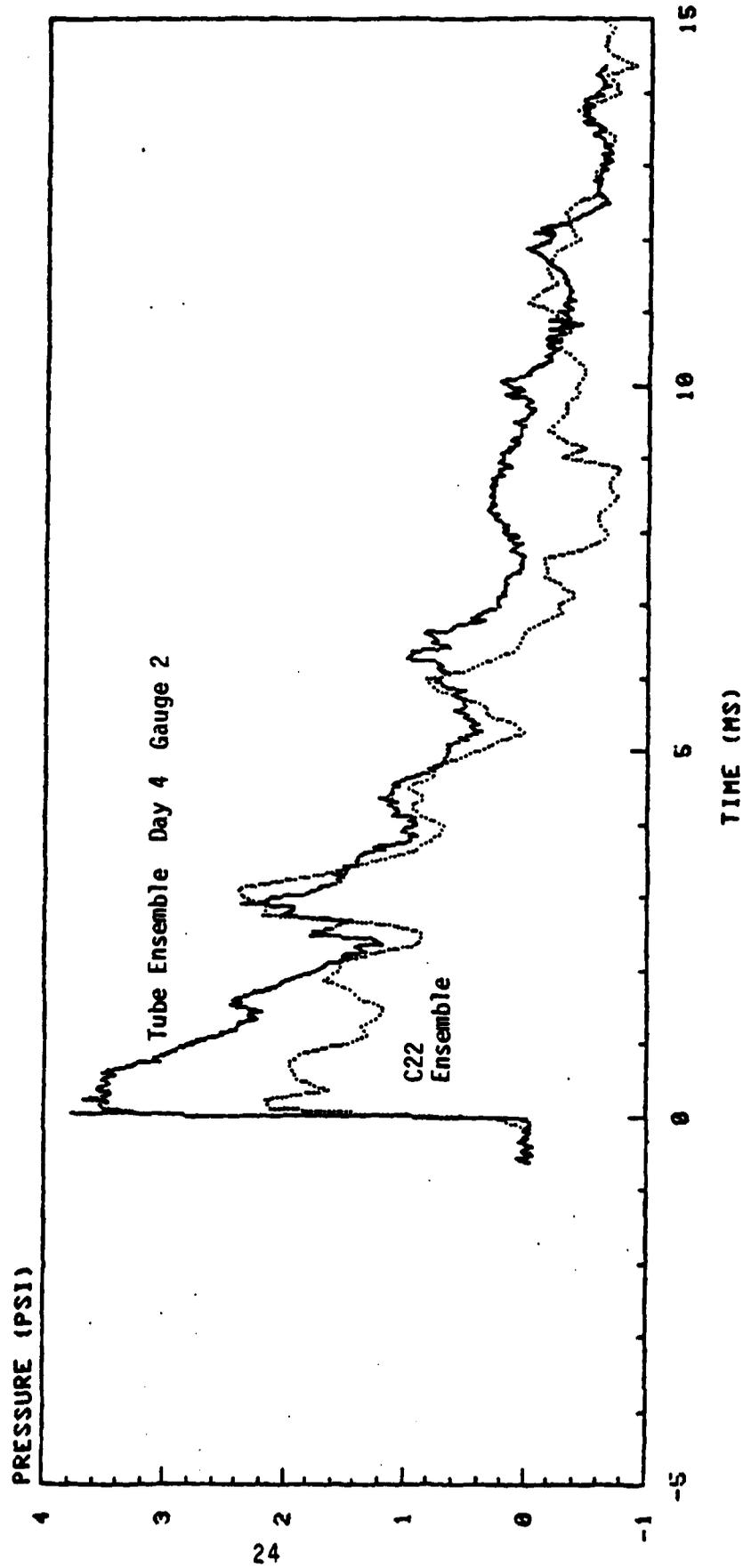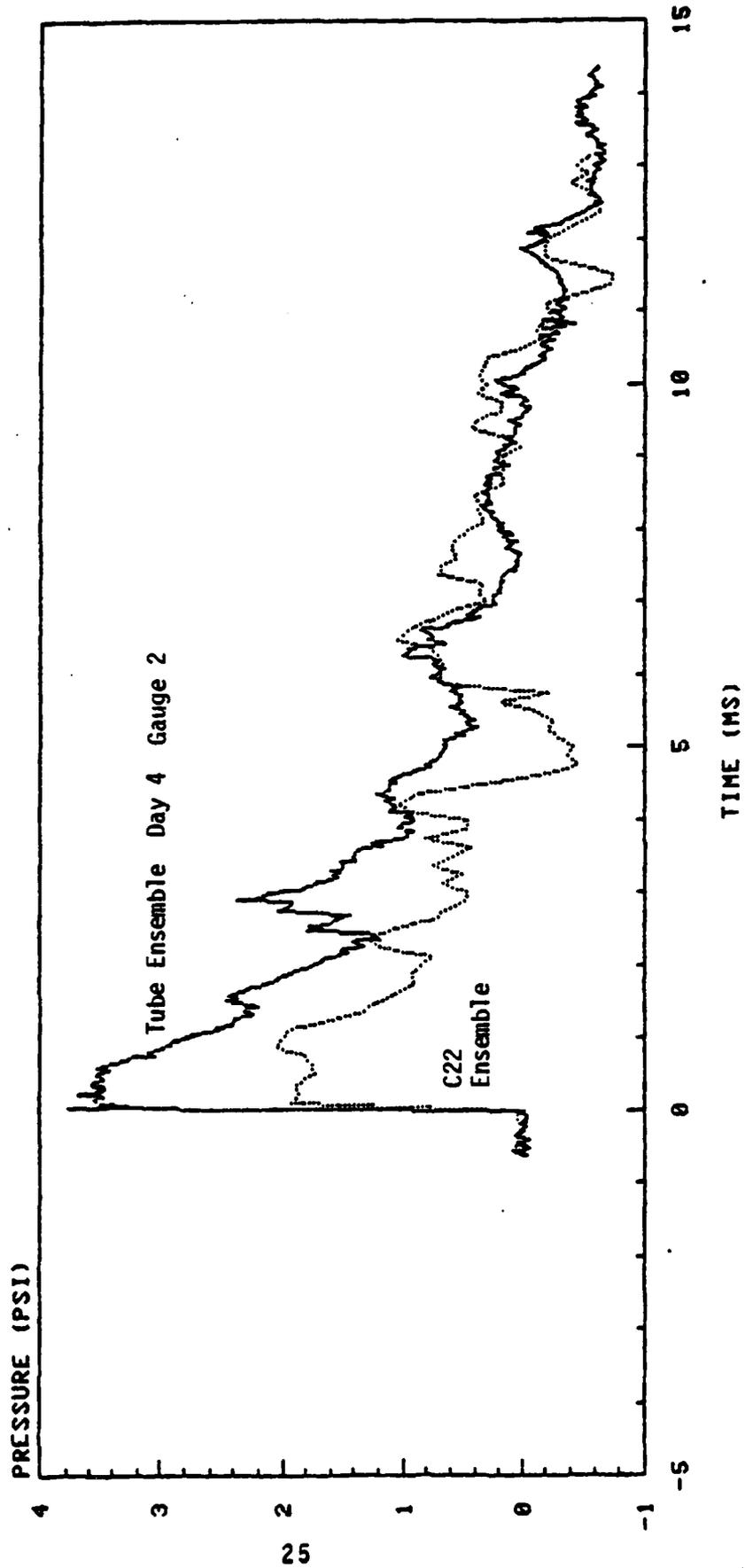
TIME (MS)

Figure 2-12

COMPARISON OF SHOCK TUBE WITH POSITION B30 4' 267QE

PRESSURE (PSI)

Tube Ensemble   Day 4   Gauge 2

B30
Ensemble

TIME (MS)

Figure 2-13

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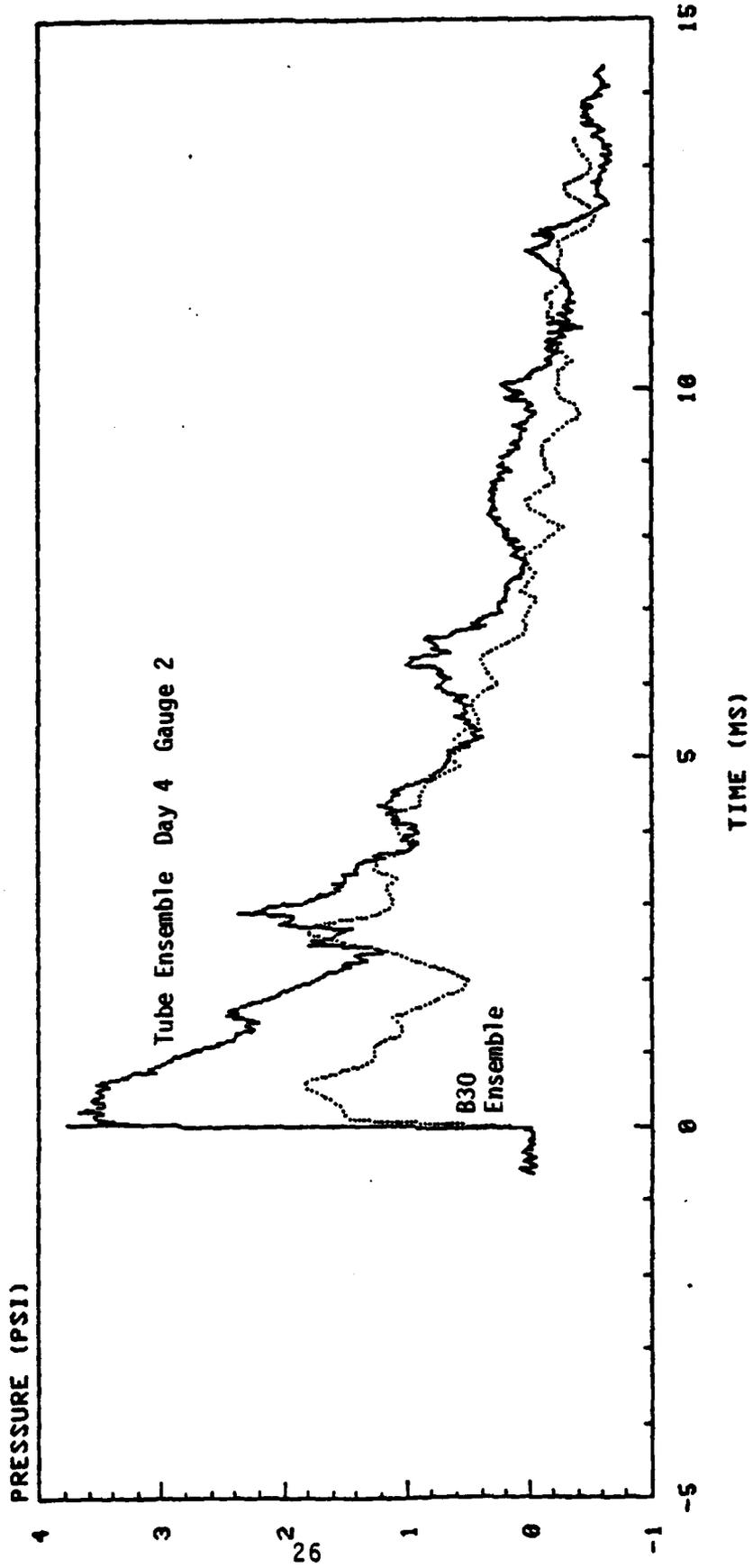
B30 C30 ENSEMBLES

BACK-TO-FRONT 2.5 MS WINDOWS

CORRELATION

TIME (MS)

Figure 2-15

SHIFT 0

SHIFT 1

SHIFT 2

SHIFT 3

27
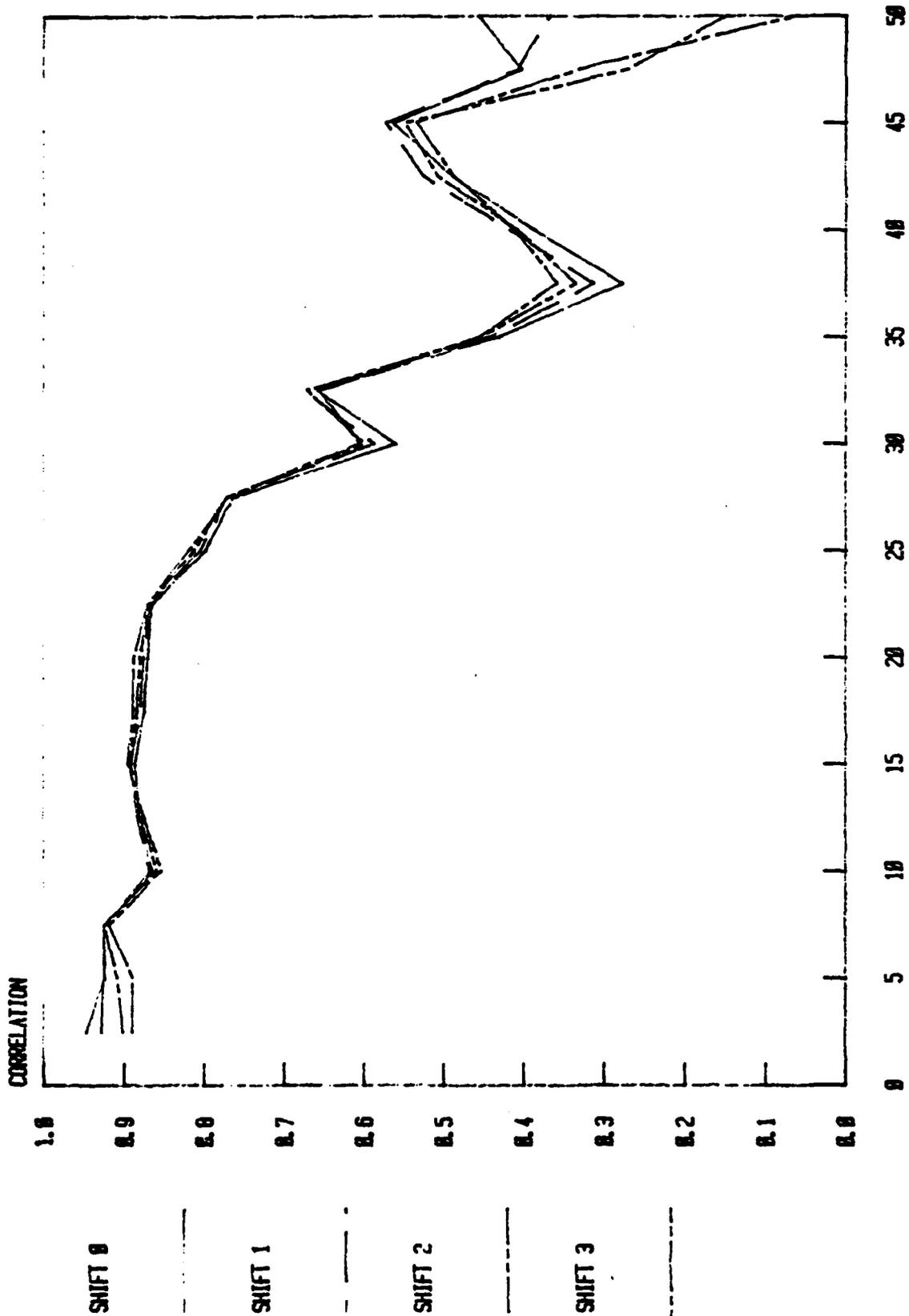
B30 C30 ENSEMBLES

FRONT-TO-BACK 2.5 MS WINDOWS

CORRELATION

SHIFT 0

SHIFT 1

SHIFT 2

SHIFT 3

TIME (MS)

Figure 2-14

28
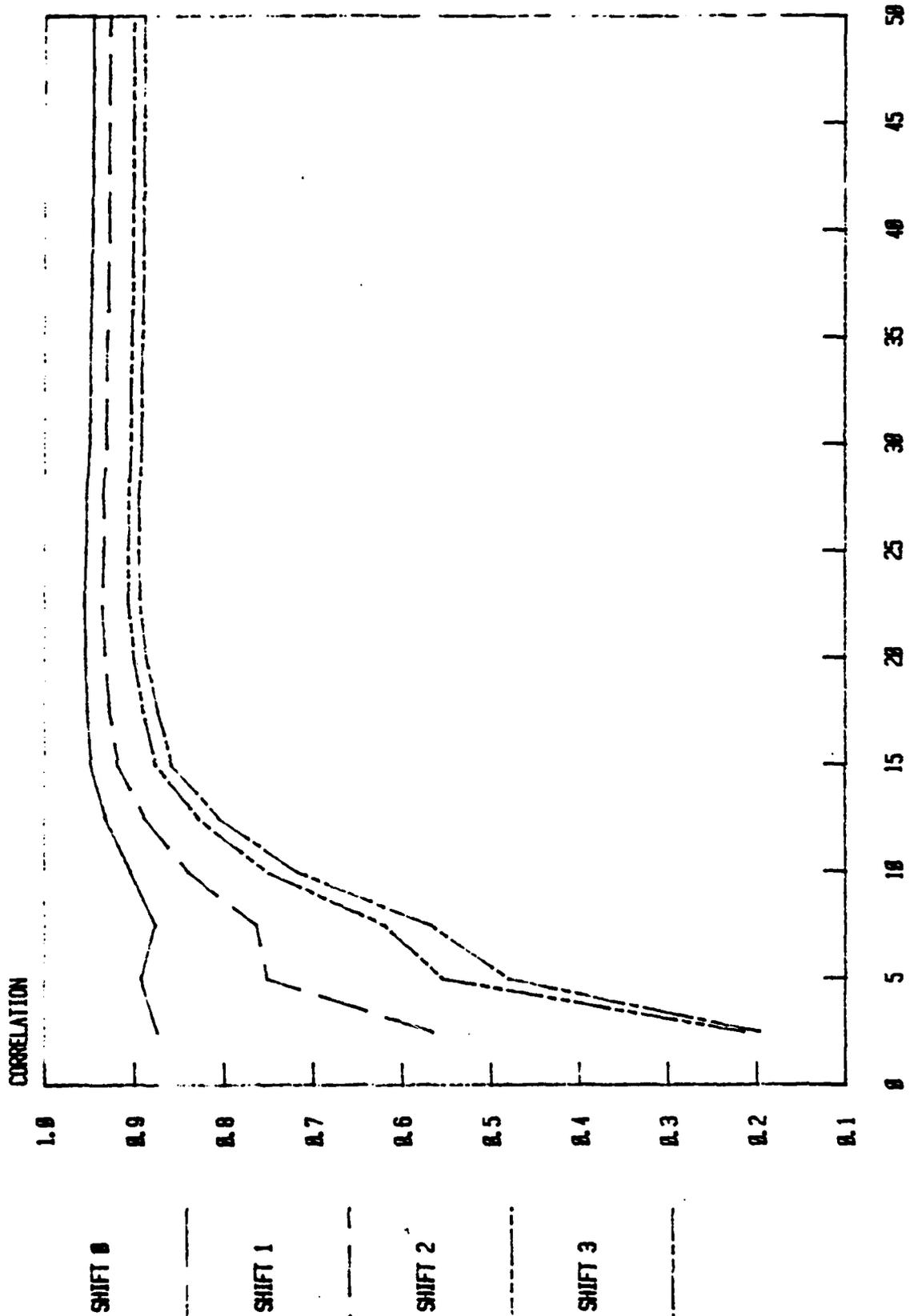
B30 C30 ENSEMBLES
SIDE-TO-SIDE 2.5 MS WINDOWS

CORRELATION

Figure 2-16

SHIFT 0

SHIFT 1

SHIFT 2

SHIFT 3

29

TIME (MS)
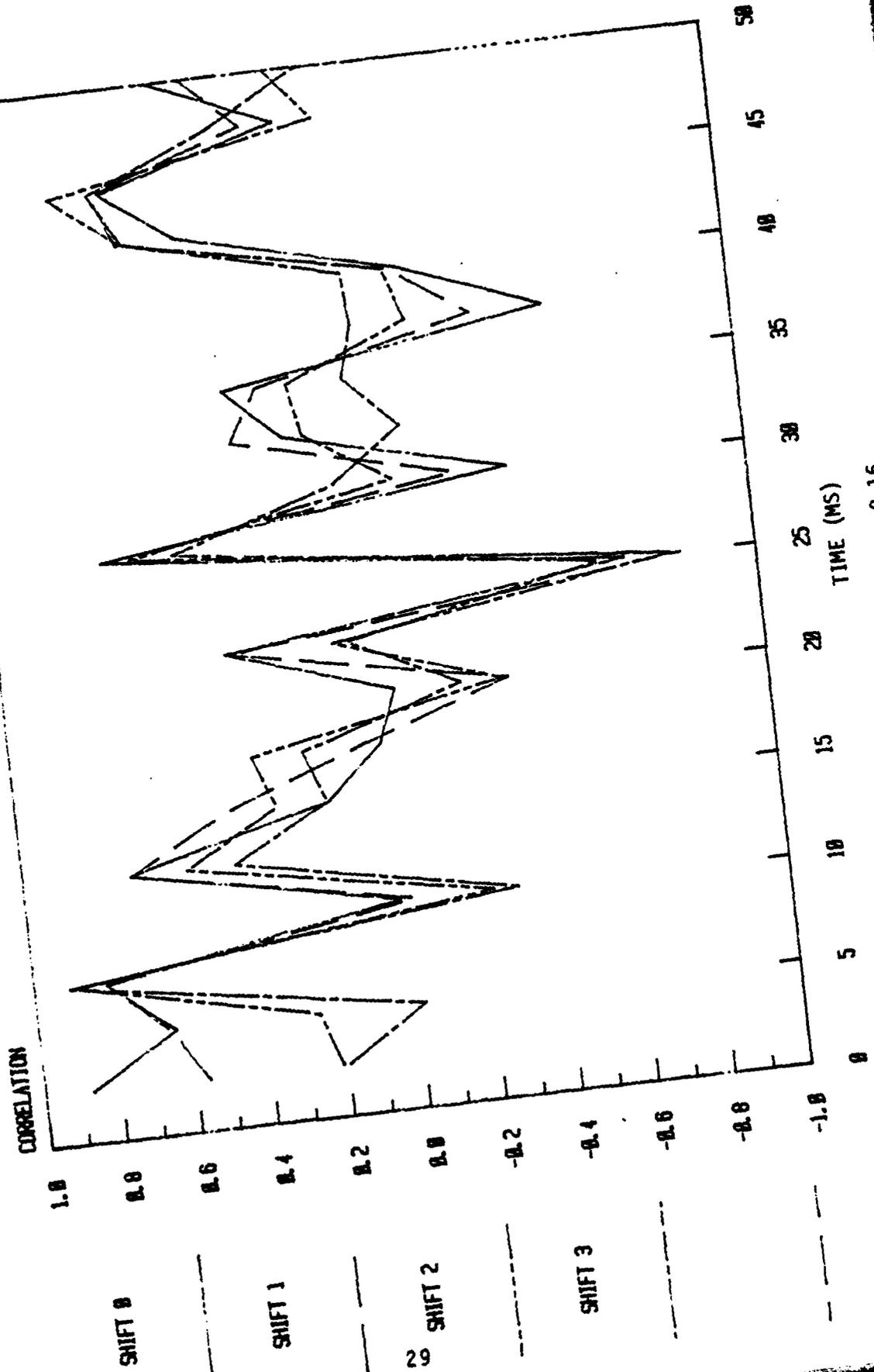
B-30 C-30 ENSEMBLES

4' 0.800

PRESSURE (PSI)

TIME (MS)

Figure 2-17

30
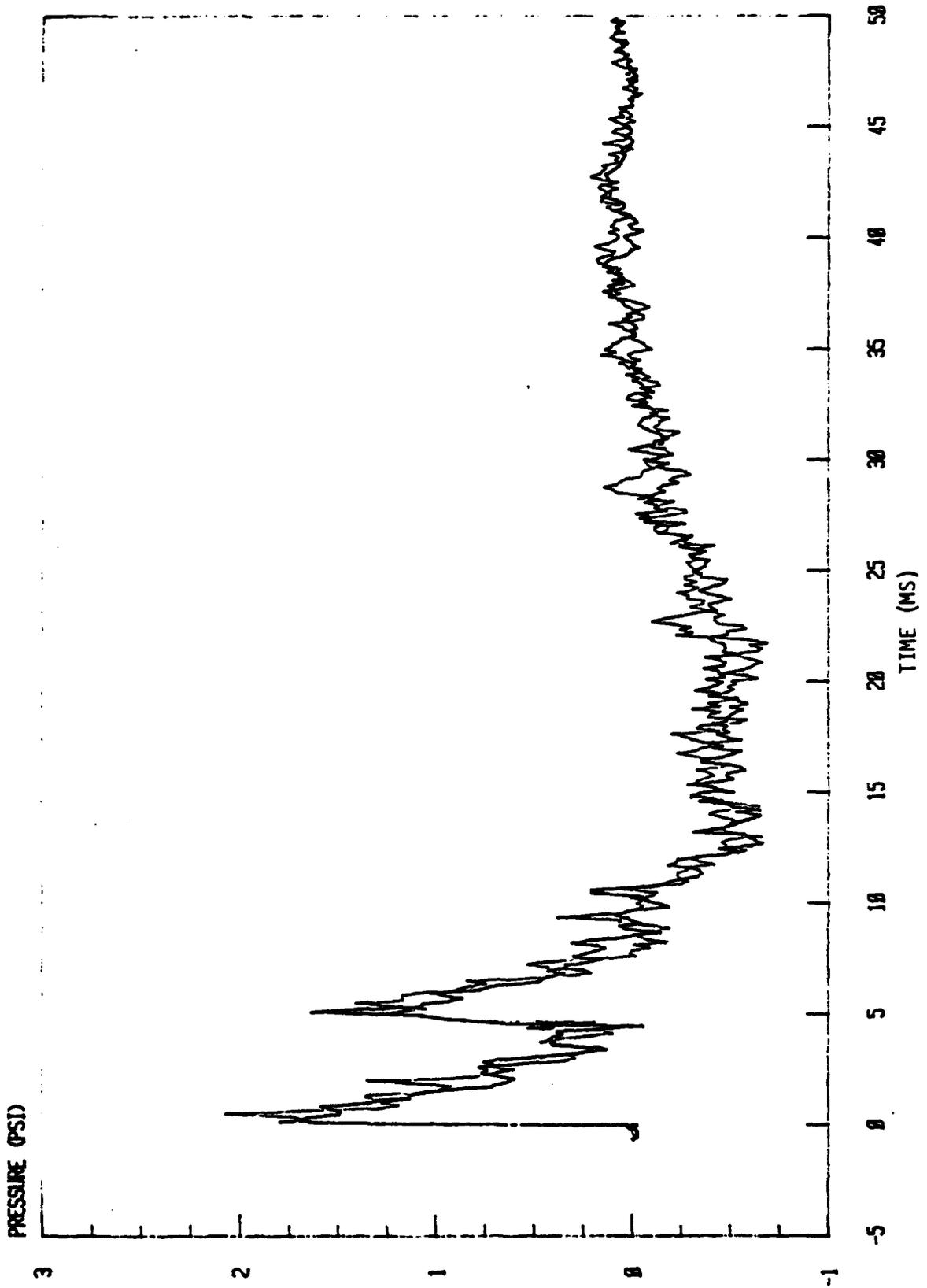
the same structure. These results confirm the hypothesis that the first 5-15ms of the wave form are the most critical in determining the correlation coefficient.

2-6   OTHER STATISTICAL PARAMETERS.

In conjunction with the correlation window study, the time-average pressure, the standard deviation, the skewness, and the kurtosis were computed as functions of time.   (For definitions of these terms, see Appendix A.)   These statistical parameters are a measure of the distribution of pressure values occurring in a given blast wave and, as such, are harder to interpret than the correlation coefficient and other more familiar parameters.   It should be noted that, even at 150ms-record-length (where the correlation between gun sites and shock tube were all in the .8 range), the skewness and kurtosis varied considerably between them.   For the various howitzer locations listed the skewness ranged from 2.2 to 2.6 and the kurtosis from 16 to 19.   The shock tube ensemble had a skewness of 4.0 and a kurtosis of 28.   (See Table 2-4.)   Though this may prove to be significant, further investigation of these parameters is probably not warranted until there is evidence of correlations between them and induced physiological damage.

Table 2-4. Skewness and Kurtosis

INDIVIDUAL SHOTS

M-198 C22 4' 0,267 (150 ms)

	SKEWNESS	KURTOSIS
SHOT 14	2.2	18.2
SHOT 15	2.6	20.0
SHOT 17	2.6	18.6

Shock Tube Day 4 Gauge 2

SHOT 3	4.0	26.6
SHOT 15	4.3	30.5
SHOT 22	4.0	26.5

ENSEMBLE COMPARISON OF DAY 1 GAUGE 2 WITH VARIOUS
CANNON ENSEMBLES

			SKEWNESS	KURTOSIS
SHOCK	TUBE	ENSEMBLE	4.0	28
C22	3'	0,267	2.2	16
C22	4'	0,267	2.6	19
C22	4'	0,800	2.4	19
B30	4'	0.267	2.3	16
B30	4'	0,800	2.4	18

APPENDIX   A

DEFINITIONS OF THE STATISTICAL

PARAMETERS USED IN THE STUDY

APPENDIX A
DEFINITIONS OF THE STATISTICAL PARAMETERS
USED IN THE STUDY

Correlation Coefficient

A standard statistical measure of the relationship between two sets of data is the (linear) correlation coefficient. The correlation coefficient is a number between -1 and 1 which indicates how well one can predict a second set of data knowing only the first. A correlation coefficient close to 1 in magnitude suggests that the corresponding data points in two sets of data are closely related; for example, if the correlation is very near 1 and the measurement of one of the quantities increases with time, then the other quantity will also increase. On the other hand, if the correlation between two quantities in close to 0 in magnitude, knowing one of them does not help predict the other. In this case, the quantities are said to be uncorrelated.

Let x_1, x_2, ..., x_N and y_1, y_2, ..., y_N be the two quantities to be correlated when N is the number of data samples taken. It is assumed that the two quantities are sampled using the same time interval.

The average or means of the data samples are defined by:

$$\bar{x} = \frac{1}{N} (x_1 + x_2 + \ldots + x_N) = \frac{1}{N} \sum_{i=1}^{N} x_i$$

and

$$\bar{y} = \frac{1}{N} \sum_{i=1}^{N} y_i.$$

The standard deviations are defined by:

$$\sigma_x = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2} \qquad \sigma_y = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \bar{y})^2}$$

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The standard deviation is a measure of how much the data deviates from its average value. This is seen by noting that the square of the differences between the respective data points, and their average is summed in the definition of the standard deviation.

The dimensional units of the average and the standard deviation of a set of data samples are the same as that of the data samples.

The correlation coefficient, r_{XY}, can now be defined:

$$r_{xy} = \frac{\frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{x})(y_i - \overline{y})}{\sigma_x \sigma_y}$$

It can be shown that $-1 \leq r_{xy} \leq 1$. The quantity r_{XY} is dimensionless.

If $y_i = x_i$ for every value of i, then $y = x$ and $\sigma_y = \sigma_x$. The correlation coefficient becomes:

$$r_{xy} = r_{xx} = \frac{\frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{x})(x_i - \overline{x})}{\sigma_x \sigma_x} = \frac{\frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{x})^2}{\sigma_x^2} = \frac{\sigma_x^2}{\sigma_x^2} = 1.$$

The correlation coefficient is defined so that it does not depend on a constant additive shift or multiplicative scaling of the data. To see this, consider a third set of data points z which are related to the x quantities by:

$$z_i = ax_i + b$$

where a and b are constant numbers.

Then the average value of z is:

$$z = \frac{1}{N} \sum_{i=1}^{N} (ax_i + b) = a \frac{1}{N} \sum_{i=1}^{N} x_i + b \frac{1}{N} \sum_{i=1}^{N} 1$$

$$= a\overline{x} + b.$$

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The standard deviation of z is:

$$\sigma_z^2 = \frac{1}{N} \sum_{i=1}^{N} (z_i - \bar{z})^2 = \frac{1}{N} \sum_{i=1}^{N} (ax_i - b - (a\bar{x} + b))^2$$

$$= \frac{1}{N} \sum_{i=1}^{N} (ax_i - a\bar{x})^2 = \frac{1}{N} \sum_{i=1}^{N} a^2(x_i - \bar{x})^2$$

$$= a^2 \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2 = a^2\sigma_x^2 \qquad \text{or } \sigma_z = a\sigma_x$$

The correlation coefficient between z and y can now be computed:

$$r_{zy} = \frac{\frac{1}{N} \sum_{i=1}^{N} (z_i - \bar{z})(y_i - \bar{y})}{\sigma_z \sigma_y}$$

$$= \frac{\frac{1}{N} \sum_{i=1}^{N} (ax_i + b - (a\bar{x} + b))(y_i - \bar{y})}{a\sigma_x \sigma_y} = \frac{\frac{1}{N} \sum_{i=1}^{N} a(x_i - \bar{x})(y_i - y)}{a\sigma_x \sigma_y} = r_{xy}.$$

An interesting way of illustrating the meaning of the correlation coefficient is through the use of a scatter plot. A scatter plot is a point plot in the plane which uses the data points (x_i, y_i) i = 1, ..., N as coordinates of the plotted points. Suppose a least squares it to a straight line is made to these points. The slope of this line gives the correlation coefficient of the data points.

Figure A-1 is a scatter plot of two gun shot ensembles (the points (x_i, y_i) are plotted rather than $(x_i$, y_i).

Normalized Skewness and Kurtosis:

The skewness and kurtosis of a set of data points is most easily interpreted if a frequency distribution plot of the data is made. A frequency distribution plot is made by finding the fractional number of times a given data value occurs. For example, if the set of data values were 2113121313, the data value 1 occurs 5 times in 10 samples, or $\frac{1}{2}$ of the time. The frequency of occurrence of each data value is obtained, and then these values are plotted against the corresponding data values. The frequency distribution plot of an overpressure wave is given in Figure A-2. Fifty milliseconds of the record were used in making this plot.

N45 C30 SCATTER PLOT
4' 0.000

N45

C30

Figure A-1

37
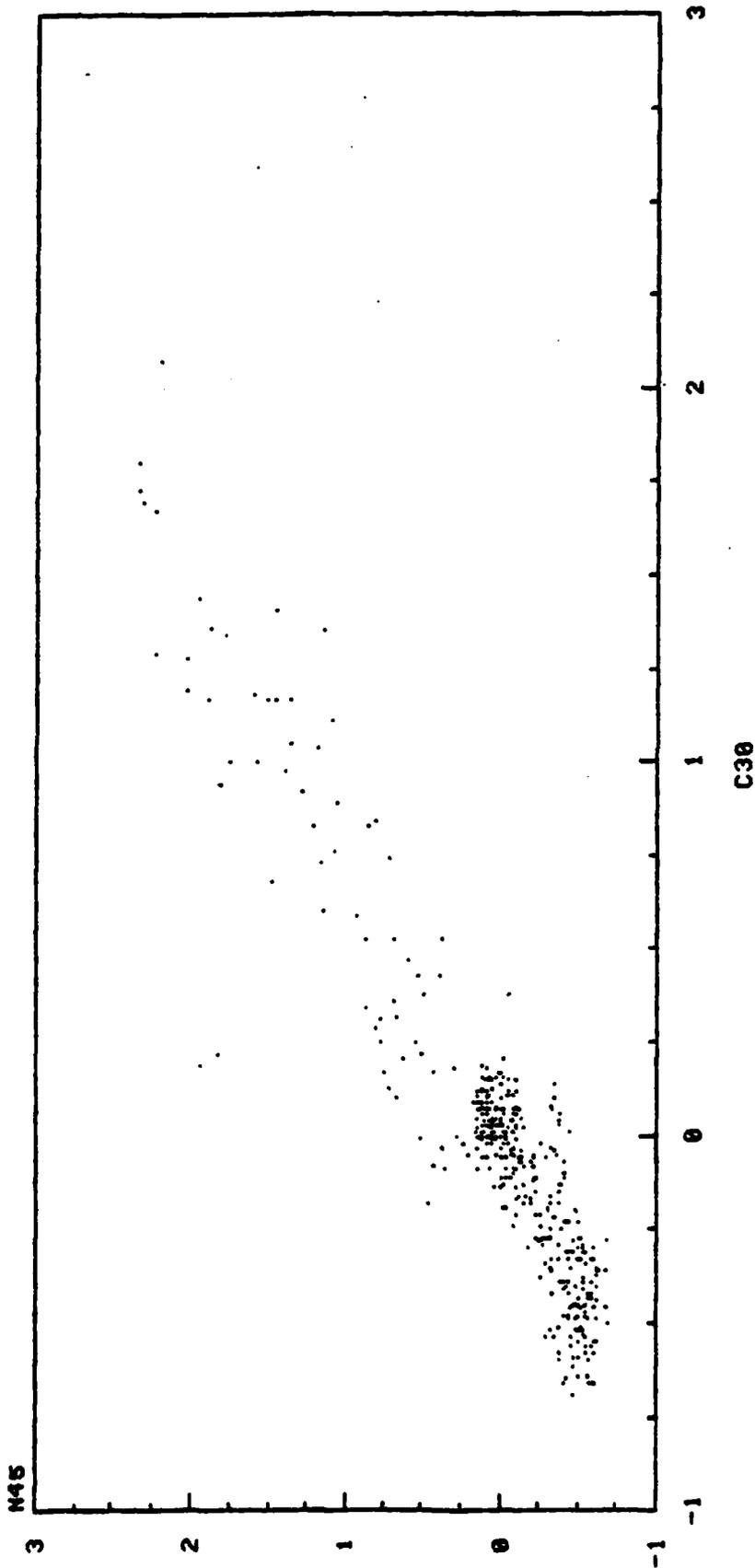
C-22 ENSEMBLE DISTRIBUTION

4' 0,000 SIMS
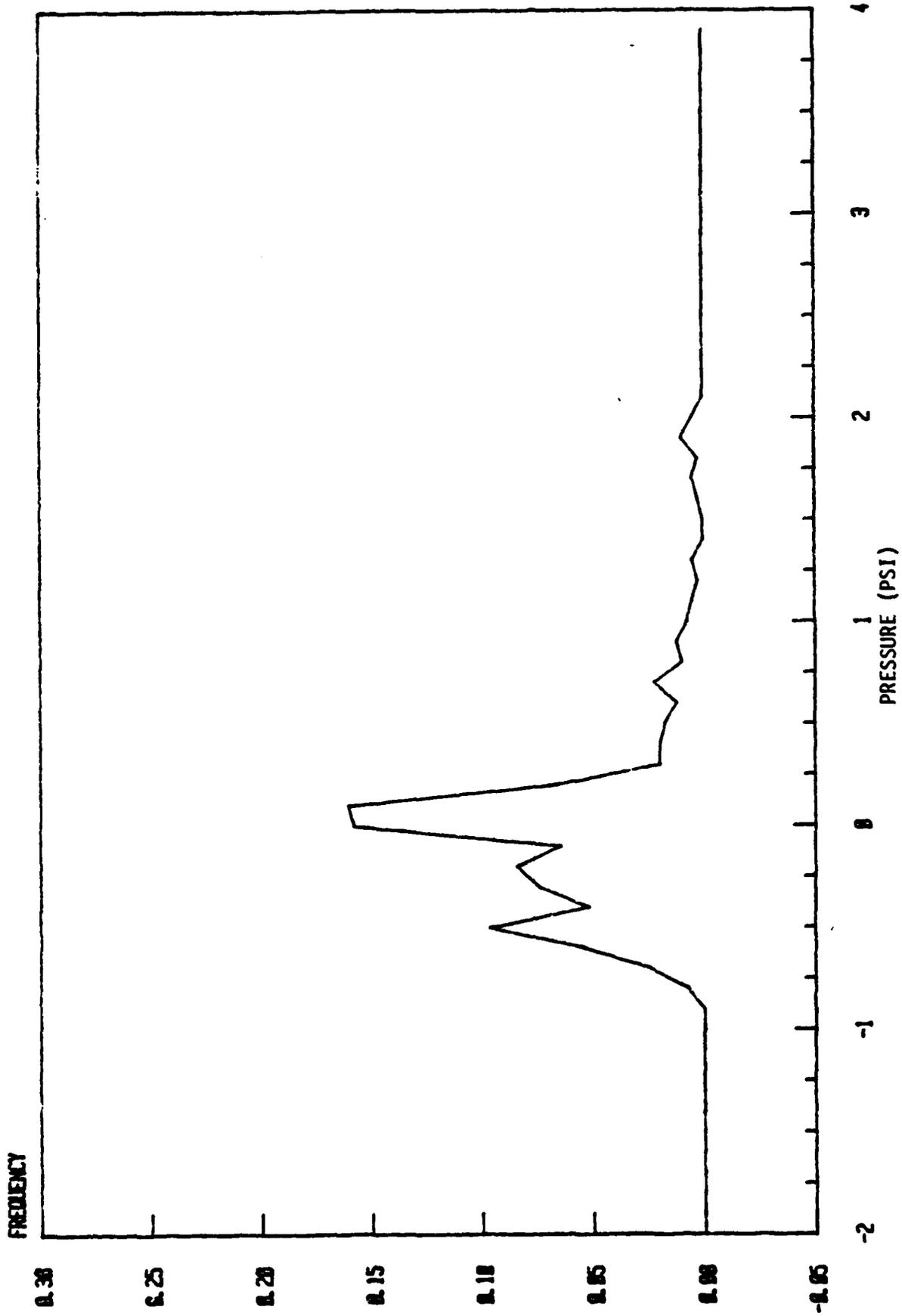
Figure A-2

In terms of the frequency distribution plot, the sign of the skewness tells whether the "tail" of the curve is to the right (positive skewness) or the left (negative skewness) of the most probable value.

The kurtosis is a measure of the degree of flattening, or peakness, of the frequency distribution function near the most probable value. The normalized kurtosis of a normally distributed curve is 3. If K_X is less than 3, the distribution curve of X is "flatter" than the normal distribution. If it is greater than 3, its curve is more peaked.

The normalized skewness and kurtosis are two statistical measures of a set of data. They are related to the standard deviation in that they describe the distribution of data values around the average.

The definitions of the normalized skewness S_X and kurtosis K_X of a set of data samples $x_i, \ldots, x_N$ are as follows:

$$S_x = \frac{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^3}{\sigma_x^3} \qquad\qquad K_x = \frac{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^4}{\sigma_x^4}$$

Since they are normalized, both the skewness and kurtosis are unitless.

CONTRACT PUBLICATION AND PERSONNEL

Publications and personnel supported by this contract. Test Planning collection, and Analysis of Pressure Data Resulting from Army Weapon Systems - are listed in chronological order by volume, subject matter and personnel contributing to the effort.

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                                     Mr. Henry C. Evans, Jr.

April 1980 - Volume II -   Modeling of Far Field Data
                                     Dr. J. Stuhmiller
                                     Dr. F. Chan
                                     Dr. P Masiello
                                     M(s) K. Tani

May 1980 - Volume III -   A Correlation Window for the M198 Howitzer
                                     Dr. Steve Slinker
                                     Mr. Henry C. Evans, Jr.

May 1980 - Volume IV -   Data Analysis of the M198 and M109 May 1979 Firings
                                     Dr. Steve Slinker
                                     Dr. Henry C. Evans, Jr.

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