AN EXPERIMENTAL TECHNIQUE FOR THE STUDY OF VELOCITY PROFILES IN--ETC(U)

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THESIS

AN EXPERIMENTAL TECHNIQUE FOR THE STUDY OF VELOCITY PROFILES IN A GROWING DROPLET USING A PULSED NITROGEN LASER

by

August Frederick Pellin III

March 1982

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An Experimental Technique for the Study of Velocity Profiles in a Growing Droplet Using a Pulsed Nitrogen Laser

by

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ABSTRACT

An ultraviolet beam (337.1 nm) from a pulsed nitrogen gas laser fired through a fluid containing a photochromic dye produces opaque traces in the fluid along the path of the laser beam. Subsequent movement of the fluid deforms the opaque traces and produces a record of the fluid flow. A high powered laser has been designed, built, and tested. Velocity distributions have been obtained in slowly forming droplets of solvent in an immiscible ambient fluid.
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I. INTRODUCTION

Valuable insight into the hydrodynamics of fluid flow can be obtained by "seeding" the flow with some tracer material and by analyzing the subsequent movement of the tracer. Aluminum flakes have been used to demonstrate surface flows in liquids [1]. Food dyes are often used in water and smoke streaks can provide histories of air flow. These flow visualization techniques provide excellent quantitative measurements of fluid flow phenomenon, and, through the use of photography, qualitative measurements of flow field velocity distribution can be obtained. In cases where solid particles are used as tracer materials, errors can result when tracer particles do not accurately follow the flow streamlines.

In the present work, the use of a relatively new flow visualization and measurement technique is described. The tracer chosen was a photochromic pyrospiran dye that, while dissolved in a hydrocarbon solvent, changed in color from transparent to deep violet when exposed to ultraviolet light. A pulsed nitrogen gas laser directed at a fluid containing the dye was repeatedly fired into the fluid producing violet traces along the path of the laser beam. Subsequent motion of the fluid deformed the traces and produced a time history of the hydrodynamics.
Analysis of the data included photographing the flow, digitizing the position of the traces, correcting for optical refraction, and computing the local flow velocities.

A high powered laser was constructed based on a Blumenthal circuit [2] producing ultraviolet light at a wavelength of 3371 Angstroms. The internal flow of dispersed droplets of a hydrocarbon solvent growing in an immiscible ambient fluid was analyzed to verify the use of the technique.
II. PREVIOUS WORK

Flow visualization through the use of photochromic dye was first introduced by Popovich and Hummel in 1966 [3]. They found that by dissolving 0.1% by weight of 2(2,4-dinitro benzyl)-pyridine (DNBP) in 95% ethanol, blue tracer lines would result in the initially clear fluid when exposed to the focused beam from a high intensity xenon flash tube. Quartz plates were used in the test section to minimize the absorption of the ultraviolet light from the flash tube. The pyridine dye shifted in color from transparent to violet within 3 microseconds and, if not reexposed, reverted back to its transparent state within several milliseconds. The technique was used to look at the viscous sublayer in turbulent pipe flow. The non-invasiveness of the technique was noted.

The production of DNBP and its optical characteristics are explained in references [4] and [5]. It changes color when exposed to light in the relatively wide range of ultraviolet to 4000 Angstroms.

Popovich and Hummel [6] employed their "flash photolysis" technique using pyridine dye in ethanol flowing through a smooth-walled rectangular aluminum duct. Photographs were taken of 1 mm long traces produced in the ethanol by a xenon flash tube 1.4 milliseconds after the
creation of the trace. The deformation of the traces were
analyzed to yield the velocities in the laminar sublayer
and in the buffer layer. It was noted that measurements
beyond \( Y^+ = 17 \), where

\[
Y^+ = Y \sqrt{T/\rho/v}
\]  

(1)

were not always possible. Results were compared with
Reichardt's and Diesler's velocity curves and showed good
agreement.

Due to the toxicity of DNBP, the use of 1,3,3-tri-
methylindoline-6'-nitro-benzospiropyran (TNBSP) was prefer-
able. This dye also dissolved easily in organic solvents
and responded to ultraviolet light in approximately the
same manner as DNBP after ultraviolet irradiation. TNBSP
transformed from a transparent state to a violet color
(5250 Angstroms) and reverted back again in a matter of
seconds. Production and characteristics of TNBSP are ex-
plained by Berman, et al [7].

Humphrey, et al [8] documented the fact that high
fluxes of energy from an ultraviolet light source can
dramatically alter a fluid flow. In their study, a giant
pulse ruby laser beam fired into a pendant droplet of
chlorobenzene containing TNBSP produced violent disturbances
in the droplet. Disturbances were attributed to tiny gas
bubbles created along the path of the laser beam in the
fluid.
Humphrey, Hummel, and Smith [11] adapted a commercial multipulse nitrogen laser to replace the xenon flashtube used to excite the photochromic dye. The flow visualization technique was used to obtain flow patterns inside of growing droplets of solvents in an immiscible ambient fluid. Equations were given to facilitate the computation of velocity profiles in droplets from dye traces. Correcting errors in these trace positions induced by optical refraction were discussed in reference [9]. It was shown that errors in trace position could be very large if the effects of refraction were ignored. Reference [11] contained computed velocity vector distributions along the central core of a slowly growing droplet of an organic liquid in water. The sparse distribution of points sampled within the droplet allowed approximate pressure profiles to be computed.

In Culbreth, Johnson, and Marschall [12], a custom built pulsed nitrogen laser and photochromic dye were used to study droplets of mineral spirits rising in water as part of a study of direct-contact heat transfer. High speed movies at 400 frames per second were made of developing droplets as the pulsed laser produced violet traces in the droplet interiors. The laser design was used in the present work and consisted of a high-voltage power supply, a source of pressurizing nitrogen gas, a vacuum pump, the laser channel made up of plexiglas box with copper electrodes and associated electronics.
A brief history of the pulsed nitrogen laser used for this study begins with the development of an inexpensive, powerful, portable device by James Small reported in reference [13]. His simple laser design involved discharging two copper plates across a 1 cm wide gap in an enclosure containing nitrogen gas at 100 torr of pressure. The two plates were separated by several sheets of mylar plastic dielectric above a ground plate. The upper plates were charged initially to approximately 20000 volts and one plate was periodically grounded producing rapid discharges of electrons through the enclosure containing the nitrogen gas. Collisions between electrons excited the nitrogen gas into emitting photons with a wavelength of 3371 Angstroms. Photon emission occurred in pulses 9 nanoseconds in duration during which 50 to 100 kilowatts of ultraviolet radiation were released. The design described in reference [13] relied on a spark gap to periodically ground out one of the upper plates and, subsequently, fire the laser. Commercial lasers capable of producing pulses of light at these intensities and wavelength cost typically on the order of $10000. In reference [2], the construction of a small pulsed nitrogen gas laser is described costing about $30.

Details of the laser constructed for the present study are discussed later. The spark gap used to fire the laser described above was replaced by a thyratron tube and its
associated electronics as described by Devlin [10]. Devlin developed a liquid cooled pulsed nitrogen laser for non-linear spectroscopic measurements.

The history of the pulsed nitrogen-laser/photochromic dye flow visualization technique has been described. The method appeared to be of value in studying a wide range of fluid dynamics problems where non-intrusive velocity measurements are desired. A facility has been developed here at NPS as a part of this work using this measurement technique and the strengths and weaknesses of the technique as a measurement tool have been assessed. Velocity distributions have been obtained in growing droplets and the necessary computer codes for digitizing and reducing data are presented. The equations necessary for correcting optical distortion and for computing the velocity vectors from dye traces are given. Construction, operation, and safety considerations are explained for the pulsed nitrogen laser built for this project.
III. EXPERIMENTAL PROCEDURE

The apparatus shown in Figure 1 consisted principally of a vertical 3"x3"x10" flat wall, plexiglas, rectangular parallelepiped containing the continuous phase (water). The whole of the experimental equipment was constructed of plexiglas and PVC tubing. Contamination of the system by dust and surface active agents such as chemicals and grease was carefully avoided. Droplets of mineral spirits were formed from a single polished plexiglas nozzle immersed in the continuous phase. The nozzle was pointed so that the dispersed phase did not wet the nozzle tip; therefore, the nozzle diameter could be considered constant. A variable speed positive displacement pump was used to circulate the dispersed phase (mineral spirits) through the system. The flow rate through the nozzle was regulated by a combination of pump speed, recirculation valve position, and cutout valve position. A fluidic capacitor was placed in line downstream of the pump and upstream of the flowmeter as shown in Figure 2 to dampen the pulses that were generated by the pump and to provide a storage area for the mineral spirits. The flow rate was measured using a previously calibrated flowmeter. The mineral spirits was recirculated through the system until equilibrium temperature was achieved and the flow rate steadied out at a previously
selected rate. Upon formation, the droplets were exposed to the ultraviolet radiation produced by the nitrogen laser. The ultraviolet light from the laser was focused through a 12 cm focal length lens until only a very thin laser line was focused in the droplet just above the nozzle exit. Focusing of the laser beam was easily accomplished by putting a few drops of florishene in the water and running the laser at a very high frequency. This allowed a visual check on both the focus of the beam and the exact position that the beam hit the droplet. Upon irradiation of the droplet, the trace was present throughout the entire history in a plane containing the axis of symmetry of the drop. A high resolution television camera focused through a 10 power microscope was used to see the droplet and subsequent laser traces as they were formed. A polaroid picture was taken of the television monitor to record the laser traces. Flow rates were varied and frequency of the laser was varied to obtain the best overall "picture" of the forming drop.
IV. NITROGEN LASER

A. DESIGN CONSIDERATIONS

This thesis required the use of a high powered ultraviolet nitrogen pulsed laser to cause the photochromic dye to change color. The wavelength of the laser output was 337.1 nm with peak power of 200-300 megawatts within a pulse width of approximately 9 nanoseconds. The pulsing frequency was variable so that at low flow rates, the droplets would not saturate causing the laser traces in the droplet to merge. A choice of a high voltage thyratron to discharge a capacitor was therefore made.

B. LASER PRINCIPLES

The basic design criteria for a nitrogen laser is to excite as many molecules as possible and to do it in a short period of time (10 nanosec). The closely spaced group of laser transitions near 337.1 nanometers in the nitrogen molecule may be thought of as arising from a simple four level electronic group. See Figure 3. An electron in the nitrogen molecule is promoted from the ground state to the upper laser level by a collision with a free electron in the high voltage discharge. The molecule then relaxes to the lower laser level emitting a quantum of light at the 337.1 nanometer wavelength. A very high voltage is used to pump the nitrogen molecule up to the upper laser level. The
laser chosen consisted of two 25 cm long parallel copper plates separated by a 1 cm gap. A rectangular plexiglas enclosure was placed around the plates near the gap. The enclosure was filled with nitrogen gas at low pressures to provide the lasing medium. Glass microscope slides with a thickness of 1.33 mm were attached to both ends of the enclosure to allow the laser beam to pass out of the enclosure. The laser plates were separated by a thin mylar sheet with a high dielectric constant from a ground plate. The plates were configured as capacitors by an electronic circuit first described by Blumenthal [2]. By pulsing very high voltages across the gap between the two copper plates surrounded by nitrogen gas, the gas was bombarded by electrons producing the plasma which emitted a quantum of light in the ultraviolet range. The surface area of the copper plates determined the magnitude of the output signal. The larger the surface area, the larger the capacitance resulting in a larger output signal. However, increasing the area caused the time to charge the plates to increase.

C. LASER CONSTRUCTION

1. Laser Channel

The copper electrode plates of the capacitor are an integral part of the laser channel. Figure 4 shows the dimensions of the electrode plates. Plates of 6.35 millimeter thick plexiglas were used to construct all the sides
of the laser channel. This reduced the amount of ultraviolet light emitted to the surrounding area since the plexiglas absorbed all of the ultraviolet light produced by the laser that did not go through the output windows. The dimensions of the plexiglas sides are shown in Figure 5. Figure 6 shows how the plexiglas pieces and the electrode plates go together to form the laser channel. A front surface mirror was placed at the back of the laser channel to double the intensity of the laser output by reflecting the ultraviolet light back through the laser channel and out the front window. Silicon sealer was used to seal all joints and make the laser channel vacuum tight.

2. **Capacitor**

The capacitor was made from thin copper plates and layers of 0.0075 inch thick mylar plastic sheets. Figure 6 shows how the capacitor was assembled. The thyratron lead was soldered to the top capacitor plate and a ground strap was soldered to the bottom left capacitor plate. The assembled capacitor was then bolted together using phenolic particle board to hold the copper plates and mylar sheets tightly together. The measured capacitance was found to be 0.0045 microfarads. Details of the final assembly are shown in Figure 7.
3. **Electronics**

This laser design utilized an EG&G HY1102 grounded grid thyratron. Its placement in the laser circuit is shown in Figure 8. All electronic components were installed in a cart underneath the laser channel to limit the lengths of the high voltage leads to the laser channel and to make the laser semi-portable. Figures 9 and 10 show the details of the triggering circuits for the thyratron.

4. **Laser Container**

The laser components were installed in the cart as shown in Figure 11. This cart was grounded to provide the operator protection from high voltages and any static electricity buildup. The cart also attenuates any Radio Frequency energy generated by the electronic components.

D. **AUXILIARY SYSTEMS**

1. **Vacuum System**

A Welch model 1402 DUO-SEAL vacuum pump was used to maintain the laser channel between the ideal 26-28 inches of vacuum. The suction line was installed at the rear of the laser channel and a 0-30 inch Hg vacuum gauge was placed between the laser channel and the vacuum pump close to the laser channel.

2. **Nitrogen Gas Supply**

Nitrogen gas was supplied to the front of the laser channel via a reducing valve and cutout valve. Nitrogen
pressure to the cutout valve was set for 10 psi and the cutout valve was used to regulate the laser channel pressure.

3. Power Supplies

The thyratron reservoir filament and cathode filament heaters were powered by a 115 v AC voltage source through a transformer which reduced the voltage to 6.2 volts. The SCR anode voltage was supplied by an HP 711A 0-300 VDC power supply. A WAVETEC pulse generator provided the +/-10 V square wave signal at various frequencies to the cathode decoupler network. High voltage was supplied by a HIPPOTRONICS 0-30 kV power supply. Power supply connections are shown in Figure 8.

E. LASER OPERATION

1. Operational Settings

The SCR anode voltage was set to 250 volts. The pulse generator was at the desired frequency between 1 and 30 Hertz. Channel pressure varied between 2-4 inches absolute of mercury. Alignment of the output mirror was done by removing the mirror and sighting down the laser channel to the target. The mirror was installed and sighting down the laser channel in the opposite direction allowed for its alignment.

2. Firing Sequence

An understanding of the firing sequency is essential to the safe operation of the electronic circuitry.
Initially the top capacitor plates are charged to a voltage of about 18,000 V and the bottom plate is grounded. A large negative pulse is applied to the thyatron cathode. This large negative pulse causes a discharge between the thyatron grid and anode. After discharge, the top left capacitor plate becomes negatively charged with respect to the top right capacitor plate. A high potential between the two electrode plates will result in breakdown and thus excitation of the nitrogen molecule by electron collisions. An inductor is placed in line between the left and right capacitor plates to allow the charge buildup to occur on the right top plate and to allow the right top plate to ground completely before the next pulse is applied to the left plate.

3. **Turn-On Procedure**

   a) Plug in the power cord for the thyatron heater elements. A 10 minute warmup time is required to prevent damage to the thyatron when operating.

   b) Evacuate the laser channel.

   c) Connect the 67.5 VDC battery to the terminals.

   d) Adjust the nitrogen gas supply to maintain 26-28 inches Hg vacuum in the laser channel.

   e) Set the SCR anode voltage for 250 Volts.

   f) Turn on the pulse generator and set to the desired frequency.
g) Energize the high voltage power supply. Slowly raise the voltage until lasing occurs at approximately 15 kV. Do not exceed 20 kV. The thyatron can be damaged if the voltage exceeds 20 kV. NOTE: The high voltage will increase as pulsing frequency decreases.

4. Turn-Off Procedure

To turn off the laser follow the turn-on procedure in reverse order. It is important that the high voltage be lowered before lowering the frequency because you can exceed 20 kV. Turn off the nitrogen supply and let the pressure bleed down before turning off the vacuum pump to eliminate the possibility of creating a pressure inside the laser channel greater than atmospheric. This situation can lead to pinhole leaks forming in the laser channel.

F. LASER SAFETY

Ultraviolet radiation can cause serious damage to the operator's eyes. To reduce the hazard, the laser was set up in a controlled access area. Only those people wearing plastic safety glasses were allowed in the room while the laser was in operation.

Since very high voltages were used, the laser container was grounded to prevent a shock hazard. It also prevented a static electricity buildup on any components.
V. DATA ANALYSIS

The photograph showing the clearest laser traces was digitized using an HP-9845 computer system. Figure 12 shows the data points used in the analysis.

The first step was to correct all the data points for the difference in refraction between the mineral spirits and water. The correction is necessary because a curved surface separating two medias of different indices of refraction will cause the apparent displacement of any point within one or the other mediums when viewed through the interface.

A. ASSUMPTIONS

The following assumptions are made:

1) The drop is symmetrical about its vertical axis and its profile can be accurately described by the empirical equation given by Poutanen and Johnson [2] for growing bubbles.

\[ R^n \theta = 1 \]  

(2)

where:  
- \( R, \theta \) = polar coordinates of the profile
- \( n \) = arbitrary parameter chosen to fit any particular droplet

Figure 13 shows the drop geometry for Equation (2).
2) The point object is located in a plane containing the drop axis; the said plane being perpendicular to the normal of the camera lens.

3) All rays of light proceeding from point objects within the drop, upon crossing the interface can be assumed to follow paths parallel to the normal of the camera lens.

4) The center of gravity of the growing drop and the camera lens central axis remain at the same height during drop growth.

B. CALCULATION OF OPTICAL DISPLACEMENTS

The direct application of Equation (2) in polar form is very inconvenient for the present analysis. A transformation into the Z-Y coordinate system was made taking the apex of the drop as the origin. The result of the transformation is:

\[(Y^2 + Z_1^2) \arctan \left( \frac{Y}{Z_1} \right) = l^n \] (3)

where:
\[Z_1 = Z - R_0\] (4)
\[l = \frac{D_m}{d'}\] (5)
\[R_0 = \text{distance from the polar coordinate origin to the new origin}\]
\[n, d' = \text{empirical constants determined as in [9]}\]
It can be shown that the optical corrections can be expressed as:

\[ \Delta Z = \tan(\hat{r}-\hat{i}) \sqrt{C^2-R_2^2}/\tan(\hat{r}) \]

\[ \Delta Y = (\tan(\hat{r}-\hat{i})/\tan(\hat{r})) \times R_2 \]

where:

\[ \hat{r} = \arccos\left(\frac{C}{(R_1 \sqrt{1+(1/m_1)})}\right) \]

\[ \hat{i} = \arcsin\left(\frac{n_r}{n_i} \sin(\hat{r})\right) \]

\[ C = \sqrt{R_1^2 - R_2^2} \]

\[ n_r = \text{Index of refraction of water} \]

\[ n_i = \text{Index of refraction of mineral spirits} \]

\[ m_1 = \frac{N+2}{R_1(R_1^2+(Z_f-R_0)^2)^{N/2}} \cos^2\left(\frac{N(R_1^2+(Z_f-R_0)^2)^{N/2}+n_i n_R_1(Z_f-R_0)^2}{N+2}\right) \]

\[ Z_f = \text{Vertical coordinate of point F} \]

The optically corrected data points are then determined by:

\[ X = X + \Delta Y \]

\[ Z = Z + \Delta Z \]

Figure 14 shows the position of the corrected data points. Figure 15 shows the position of the corrected and uncorrected
data points. Figure 18 is the HP-9845 computer program that was used to solve the above equations for the optical corrections.

C. VELOCITY VECTOR DETERMINATIONS

Due to symmetry, only the right half of a droplet need be considered as shown in Figure 16. The Y axis lies along the center line of the nozzle and is given the value 0 at the nozzle exit, Y is positive in the direction of flow. The radial coordinate is labeled X, and is positive away from the centerline.

Quantitative measurements of fluid flow inside the droplet were limited to a core along the Y axis with the same diameter as the inside diameter of the nozzle. In this well defined region, the shape of any induced trace i, after correction for optical distortion, was found to be excellently represented by a polynomial of degree 3 i.e.,

\[ F(X) = a(I)X^3 + b(I)X^2 + c(I)X + d(I) \quad (14) \]

where \( a(I), b(I), c(I), \) and \( d(I) \) are time dependent and are calculated by a least squares fit of Equation (14) to the experimental values.

The fact that flow within the growing droplet is not undirectional except at very high flow rates complicates the calculation of fluid element velocities from the induced traces. Two traces within the drop at time \( t \) are shown as
dotted lines in Figure 16. At a later time $t+\Delta t$, the same traces are shown as continuous lines. It is possible to follow the change of position of an element of fluid from point A to point B using a mass balance technique as follows. Consider the displacement of an element of fluid ($P_1, P_2, P_3, P_4$) contained between traces 1 and 2 during an interval of time $\Delta t$ as shown in Figure 16. As the element moves forward to its new position it is distorted, some of the fluid from the moving element being squeezed out in the radial direction. Provided the fluid element is small, an approximation to its vector velocity is:

$$V = \frac{\Delta \mathbf{AB}}{\Delta t}$$

(15)

and the direction of its movement can be characterized by the angle $\alpha$.

Given trace 1 and 2,

Let $A_x = X$ coordinate of point A
Let $A_y = Y$ coordinate of point A

Then

$$A_y = \frac{1}{2}[(a(2)-a(1))A_x^3 + (b(2)-b(1))A_x^2 + (c(2)-c(1))A_x + d(2)-d(1)] + F(A_x)$$

(16)

Though distorted due to a combined radial and axial motion of the fluid, the volume of the fluid element remains the same during an interval of time $\Delta t$. Since the trace containing the distorted element can also be fitted to
equations of the type given by Equation (14), where the
constants are now found for that element of volume, the
volumes are set equal to each other and the polynomial is
solved for the X coordinate of point B. Therefore from the
theorem of PAPPUS:

Let \( B_X \) = X coordinate of point B
Let \( B_Y \) = Y coordinate of point B

\[
V(1,2) = (a(2)-a(1))A_X^5 + (b(2)-b(1))A_X^4 + (c(2)-c(1))A_X^3 + (d(2)-d(1))A_X^2 \quad (17)
\]

\[
V(3,4) = (a(4)-a(3))B_X^5 + (b(4)-b(3))B_X^4 + (c(4)-c(3))B_X^3 + (d(4)-d(3))B_X^2 \quad (18)
\]

Now, since \( V(1,2) = V(3,4) \) Equation (18) can be solved for
\( B_X \). Once this value is found, substitution into the follow-
ing equation gives \( B_Y \).

\[
B_Y = 1/2[(a(4)-a(3))B_X^3 + (b(4)-b(3))B_X^2 + (c(4)-c(3))B_X + (d(4)-d(3))] + F(B_X) \quad (19)
\]

The velocity can now be expressed as:

\[
V = \sqrt{(B_X-A_X)^2 + (B_Y-A_Y)^2} \quad (20)
\]

and the angle \( \alpha \) can be found by:

\[
\alpha = \arctan((B_Y-A_Y)/(B_X-A_X)) \quad (21)
\]

30
Figure 19 is the HP-9845 computer program that was used to solve the above equations for the velocity vectors and angles.
VI. RESULTS

Figure 17 shows the velocity profiles for the 4 traces that were obtained during this procedure. As can be seen, the velocity decreased as the fluid moved away from the nozzle in the vertical direction; and the velocity along a given trace also decreases as you move away from the centerline. The magnitude of the peak velocity coming out the nozzle was computed to be 97.6 micrometers per second. This corresponds to a Reynolds number of approximately 0.05. The velocity profile looked very much like the laminar velocity profile at the nozzle exit which is parabolic in shape. Table I shows the coordinates and magnitude and direction of the velocity vectors using laser traces 1-2, 2-3. The origin of the coordinate system is the center of the nozzle at the nozzle exit. The magnitude of the velocity is unitless and is equal to the computed velocity divided by the peak velocity in the nozzle. Likewise, Table II shows the results using laser traces 2-3, 3-4 and Table III shows the results using laser traces 1-3, 2-4.
A. ERROR ANALYSIS

All positional measurements in the droplets were based on the observed nozzle bore diameter. The accuracy of the bore diameter was approximately 2.7%. The droplet profiles and position of points on traces were digitized from a polaroid picture of droplets obtained through a videoscreen. The effects of parallax were minimized by placing the camera as far from the inlet nozzle as possible. Humphrey and Hummel [9] reported that parallax error is negligible if the droplet traces are observed from more than 25 cm and readings are confined to within 3 mm of the center of gravity of the droplet. In this study, the camera was placed approximately 15 cm away from the droplet and traces were viewed within 1 mm of the droplet center. The error caused by the nozzle bore diameter far exceeded the digitizing error of 0.008 mm.

The error induced in determining each point on a trace in (X,Y) coordinates was influenced by errors in maximum droplet diameter, point coordinates \( R_z \) and \( Z_p \) described earlier, and the indices of refraction of mineral spirits and water.

The errors in indices of refraction were assumed to be small and in the correction for optical refraction, positional errors predominated. An analysis was done to compute the error in position for points on the traces in the
droplets after correcting for refraction. The uncertainty analysis technique detailed by Holman [14] was used. The points obtained from photochromic dye traces that were used to compute velocities were all confined within an area no wider than the inlet nozzle diameter and far from the axis of the droplet. Within this area, point positions were known within ± 0.005 mm after correcting for refraction.

The corrected traces were curve fitted using a third order polynomial. The fit was very good with a standard deviation of under 0.001 mm. The time interval between firings was accurately controlled by a crystal based function generator with an accuracy of about ± 0.0001 seconds. All computations on the HP-9845 were done in double precision arithmetic to minimize errors.

The estimated accuracy of the fluid velocities was, on the average, 0.07 mm/s or 6.3% of the average fluid velocity at the nozzle tip.

It should be noted that errors will rise very rapidly if data is taken near the periphery of the droplet due to the traces appearing to be very close to one another.
VII. CONCLUSIONS

Velocities were computed using a nitrogen laser/flow visualization technique. The technique was used to investigate the velocity profile in the core of a slowly forming droplet ($Re = 0.05$). Velocities along the centerline of the droplet were seen to decrease from a peak velocity in the nozzle of 2.22 mm/s to 95.6%, 87.8%, and 68.1% of the peak velocity as one moves toward the droplet apex. Peak velocity in the nozzle equaled twice the average velocity as determined from parabolic laminar profile in a circular pipe. Computed velocities agreed well with expected velocities and with the restrictions of laminar flow and small flow geometry, this measurement technique appears to be accurate and useful.
VIII. **RECOMMENDATIONS**

Since a "home-made" laser was used in this experiment, only simple hydrodynamic problems can be solved. The absorption of the laser light by the mineral spirits necessitated the use of a very small diameter nozzle. Only with high powered commercial grade lasers can this technique be expanded to more challenging hydrodynamic problems. Since only polaroid pictures were taken the rate of droplet growth was severely limited. This, in turn, limited the laser frequency so that saturation did not occur.
Figure 1

Experimental Test Cell
Figure 2
Line Diagram - Test Assembly
Figure 3

Nitrogen Electronic Excitation Levels
Figure 4
Copper Electrode Plates
Figure 5
Laser Channel Plexiglas Pieces
Figure 6

Laser Channel
Figure 7
Final Assembly
Figure 8

Thyratron Placement in Laser Circuit
Figure 10
Triggering Circuit
Figure 11

Electronic Component Configuration
Figure 12
Photochromic Dye Traces in Growing Droplet
Figure 13
Geometry for Equation (2)

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Figure 15
Optically Corrected and Uncorrected Data Points
Figure 16

Velocity Vector Computations
Figure 17
Velocity Profile inside the Droplet
10 OPTION BASE 1
11 DEG
20 DIM R2(100),R1(100),Zf(100),X(100),Y(100)
30 ASSIGN #1 TO "BUBL1"
40 READ #1,1
50 FOR I=1 TO 24
60 READ #1;X(I),Y(I)
70 R2(I)=X(I)
80 Zf(I)=4039-Y(I)
90 NEXT I
91 ASSIGN #1 TO "R11:R8",0
92 READ #1,1
100 FOR I=1 TO 24
110 READ #1;R1(I)
120 NEXT I
130 L=2010
140 N=50
150 R0=1608.9
160 Etau=1.48
170 Etai=1.41
180 Delta=L/N
190 CREATE "BUBL1:F9",1,16*50
200 ASSIGN #1 TO "BUBL1"
210 READ #1,1
220 FOR I=1 TO 24
230 C=SQR(R1(I)^2-R2(I)^2)
240 Alpha=Zf(I)-R0
250 Beta=R1(I)^2+Alpha^2
251 Q=Alpha*Beta^3.5
260 M0=Q*COS(Delta*Beta*(-N/2))^2
270 M0=M0+N*Delta*R1(I)+Alpha^2
271 Q1=R1(I)+Beta^3.5
280 M2=Q1*COS(Delta*Beta*(-N/2))^2
290 M2=M2-N*Delta*Alpha^3
300 M1=M0/M2
310 Rhat=ACSC(C/(R1(I)+SQR(1+M1*(-2))))
320 Ihat=ASNC(Etau/Etai*SIN(Rhat))
330 Deltax=TAN(Rhat-Ihat)/TAN(Rhat)*R2(I)
340 Deltax=TAN(Rhat-Ihat)*SQR(C^2-R2(I)^2/TAN(Rhat)^2)
350 X(I)=X(I)-Deltax
360 Y(I)=Y(I)-Deltaz
370 PRINT #1;X(I),Y(I)
380 PRINT X(I),Deltax,Y(I),Deltaz
390 NEXT I
400 DISP "COMPLETE"
410 STOP
420 END

Figure 18
Program to Determine Optical Corrections
```plaintext
10 A1=-9.0056E-7
20 A2=1.08851E-6
30 A3=-5.5430478E-7
40 A4=9.657368E-8
50 B1=.00005203
60 B2=.0003121675
70 B3=-.00027479
80 B4=-.0011586063
90 C1=-.17388898
100 C2=-.2513398
110 C3=-.8571831
120 C4=.23956036
130 D1=.1971.8256
140 D2=2425.3585
150 D3=.2716.75829
160 D4=2493.295854
170 Z1=A2-A1
180 Z2=B1-B1
190 Z3=C2-C1
200 Z4=02-D1
210 Z5=R3-R2
220 Z6=R3-B2
230 Z7=C3-C2
240 Z8=D3-D2
250 CREATE "BVEL1",1,2898
260 ASSIGN #1 TO "BVEL1"
270 READ *1,1
280 L2=0
290 L1=0
291 DEG
292 BUFFER #1
300 FOR I=1 TO 89
320 Coef=Z1*L1-5*Z2*L1+4*Z3*L1-3*Z4*L1-2
330 FOR J=1 TO 2000
350 IF Coef THEN GOTO 390
360 L2=L2+1
370 NEXT J
390 Ab=.25*SQRT(L2-L1-2*(By-Ay)^2)
400 IF L2-L1=0 THEN GOTO 430
410 Theta=ATH((By-Ay)/(L2-L1))
420 GOTO 440
430 Theta=90
440 PRINT L1,Ay,L2,By,Ab,Theta
450 PRINT #1;L1,Ay,L2,By
451 IF L2<200 THEN L2=L2+10
452 IF L2>200 THEN L2=L1-200
460 L1=L1+10
470 NEXT I
480 DISP "COMPLETE"
490 ASSIGN #1 TO *
500 STOP
510 END
```

Figure 19
Program to Determine Velocity Vectors
Table I. Numerical results of velocity vector computations for laser traces 1-2, 2-3.

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Table II. Numerical results of velocity vector computations for laser traces 2-3, 3-4.
### Table III. Numerical results of velocity vector computations for laser traces 1-3, 2-4.

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