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MODE ANALYSIS IN A
MISALIGNED UNSTABLE RESONATOR

THESIS

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MODE ANALYSIS IN A
MISALIGNED UNSTABLE RESONATOR

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

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PREFACE

The purpose of this study was to analyze the effects of mirror misalignment on the transverse modes and beam steering of an unstable laser resonator. The analysis was developed for any general unstable resonator design with rectangular apertures and did not allow for inclusion of a gain medium. The final result, a computer program, can be used to calculate mode eigenvalues and subsequently the intensity and phase in the plane of the feedback mirror for a desired mode. The slope of the phase, from which a beam steering angle can be determined, is also calculated for the lowest loss mode. The resulting tilt in the phase front is due to diffraction, and is consequently a beam steering angle additional to the geometric misalignment. The code is basically an extension, or modification, to a previous computer model developed by J.E. Rowley, and follows similar work done by P. Horwitz.

Although a major portion of this work may be found elsewhere, specific details and applications throughout the text are generally not available. They are provided here in order to present a complete and clear progression of the analysis. A large number of equations and derivations are required since the topic is analytical in nature rather than experimental. This sometimes leads to trivial substitutions and algebraic steps being included for the sake of continuity, but it is hoped that these are minimal. Physical interpretations and definitions are included where possible for better understanding.

I would like to express my gratitude to my advisor, Lt. Col. John Erkkila, for his time, patience, guidance, and especially his enthusiasm which inspired me continually throughout.
Table of Contents

Preface .......................................................... ii
List of Figures .................................................. v
Abstract .......................................................... vii

I. Introduction ..................................................... 1
   Background ..................................................... 1
   Objectives .................................................... 6
   Assumptions .................................................. 7
   Procedure and Organization .............................. 8

II. Development of the Integral Equation from the Fresnel -
    Kirchhoff Diffraction Formula .......................... 9
   1-D Analysis .................................................. 9
   Introduction of Phase Lag ................................. 12

III. The Integral Equation Appropriate to the Misaligned
    Resonator ................................................... 19
    Effects of Mirror Misalignment ........................ 19

IV. Solution of the Generalized Integral Equation ........ 24
    Stationary Phase Approximation ........................ 24
    The Polynomial Equation ................................. 25
    Second Order Approximation ............................. 30

V. Beam Steering ................................................ 33
    Geometrical Beam Steering ............................... 33
    Beam Steering Angles Due to Diffraction ............ 35

VI. Results, Conclusions, and Recommendations .......... 38
    Results ...................................................... 38
    Conclusions ................................................ 39
    Recommendations .......................................... 40

Bibliography .................................................... 46
Appendix A (Solution of definite integral in equation (2.23)) 48
Appendix B (Final form of the integral equation) .......... 51
Appendix C (The polynomial equation) ..................... 53
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix D (The plots of higher order modes)</td>
<td>55</td>
</tr>
<tr>
<td>Appendix E (Program listing)</td>
<td>61</td>
</tr>
<tr>
<td>Vita</td>
<td>75</td>
</tr>
</tbody>
</table>
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>Stable and unstable resonator geometries</td>
<td>2</td>
</tr>
<tr>
<td>1-2</td>
<td>Equivalent lens system of an unstable resonator</td>
<td>3</td>
</tr>
<tr>
<td>2-1</td>
<td>Geometry of rectangular plane mirrors</td>
<td>10</td>
</tr>
<tr>
<td>2-2</td>
<td>Typical unstable resonator geometry (for phase lag)</td>
<td>11</td>
</tr>
<tr>
<td>3-1</td>
<td>Geometry of an unstable laser resonator (misaligned)</td>
<td>20</td>
</tr>
<tr>
<td>5-1</td>
<td>Equivalent asymmetric cavity of the misaligned resonator</td>
<td>35</td>
</tr>
<tr>
<td>6-1</td>
<td>Intensity plot for the lowest loss mode with $\delta = 0.0$</td>
<td>41</td>
</tr>
<tr>
<td>6-2</td>
<td>Phase plot for the lowest loss mode with $\delta = 0.0$</td>
<td>41</td>
</tr>
<tr>
<td>6-3</td>
<td>Intensity plot for the lowest loss mode with $\delta = 0.2$</td>
<td>42</td>
</tr>
<tr>
<td>6-4</td>
<td>Phase plot for the lowest loss mode with $\delta = 0.2$</td>
<td>42</td>
</tr>
<tr>
<td>6-5</td>
<td>Intensity plot for the lowest loss mode with $\delta = 0.5$</td>
<td>43</td>
</tr>
<tr>
<td>6-6</td>
<td>Phase plot for the lowest loss mode with $\delta = 0.5$</td>
<td>43</td>
</tr>
<tr>
<td>6-7</td>
<td>Phase slope vs. mirror misalignment (magnification = 2.0 and Equivalent Fresnel number = 9.6)</td>
<td>44</td>
</tr>
<tr>
<td>6-8</td>
<td>Phase slope vs. mirror misalignment (magnification = 2.9 and Equivalent Fresnel number = 16.4)</td>
<td>45</td>
</tr>
<tr>
<td>D-1</td>
<td>Intensity plot for mode #2 and $\delta = 0.0$</td>
<td>56</td>
</tr>
<tr>
<td>D-2</td>
<td>Phase plot for mode #2 and $\delta = 0.0$</td>
<td>56</td>
</tr>
<tr>
<td>D-3</td>
<td>Intensity plot for mode #2 and $\delta = 0.5$</td>
<td>57</td>
</tr>
<tr>
<td>D-4</td>
<td>Phase plot for mode #2 and $\delta = 0.5$</td>
<td>57</td>
</tr>
<tr>
<td>D-5</td>
<td>Intensity plot for mode #3 and $\delta = 0.0$</td>
<td>58</td>
</tr>
<tr>
<td>D-6</td>
<td>Phase plot for mode #3 and $\delta = 0.0$</td>
<td>58</td>
</tr>
<tr>
<td>D-7</td>
<td>Intensity plot for mode #3 and $\delta = 0.5$</td>
<td>59</td>
</tr>
<tr>
<td>D-8</td>
<td>Phase plot for mode #3 and $\delta = 0.5$</td>
<td>59</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>D-9</td>
<td>Intensity plot across the feedback mirror only using the first order approximation</td>
<td>60</td>
</tr>
<tr>
<td>D-10</td>
<td>Phase plot across the feedback mirror only using the first order approximation</td>
<td>60</td>
</tr>
</tbody>
</table>
The integral equation that describes mode structure of an unstable resonator with rectangular apertures is developed from scalar diffraction theory. This equation, modified to account for misalignments, is solved by applying the asymptotic methods developed by Horwitz. A second order approximation of the method of stationary phase is then employed to calculate phase and intensity values for all points in the output plane. The phase front is also curve fitted to a straight line over the geometrical region for the lowest loss mode. From the slope of the straight line, a direction of propagation can be attributed to the wave. This is a diffracted beam steering angle and is additional to the geometric steering angle (i.e., the beam steering angle due to the geometric misalignment of either or both mirrors).

Plots of intensity and phase for various degrees of misalignments are presented as results of a computer program that utilizes the derived expressions. Also included are graphs of the phase slope versus mirror misalignment.
I. Introduction

Background

In any laser cavity there are two modes of oscillation: longitudinal (or axial) and transverse (or radial). The longitudinal modes coupled with a specific transverse mode determine the frequency of radiation emitted, while the transverse modes alone determine the intensity distribution across the laser beam. In either case, the existing modes can be found by applying the appropriate boundary conditions to the wave equation. In a stable resonator with spherical mirrors and edge effects (diffraction effects due to the finite size of the mirrors) neglected, the transverse solutions to the wave equation become Hermite-Gaussian or Laguerre-Gaussian functions for mirrors with either rectangular or circular symmetry respectively (Ref. 1:1324).

For the unstable resonator, edge effects must be included since the output is a diffraction-coupled beam passing around, rather than through the output mirror. An iterative approach to finding the resonant modes of the cavity accounts for these edge effects and was introduced by Fox and Li (Ref. 2). This method is to first assume an initial field distribution in the resonator and then apply the Fresnel-Kirchhoff diffraction formula to the problem of infinite strip mirrors.

Figure 1-1 shows differences between a stable resonator and an unstable resonator. Any ray originating in the stable resonator and striking one of the mirrors will remain inside the cavity, even for an infi-
finite number of passes. In the unstable resonator however, the ray will eventually pass out of the cavity. For the case under consideration, \( a_1 \) assumed large in comparison with beam spot size on mirror 1, the ray will pass around the much smaller feedback mirror. It can therefore be seen that only edge effects from the smaller mirror need to be considered.

The more familiar criterion for stable and unstable resonators is given by the product of \( g_1 \) and \( g_2 \), where

\[
g_i = 1 - \frac{L}{R_i} \quad i = 1, 2
\]  

(1.1)

Here \( L \) = cavity length and \( R_i \) = radius of curvature of the \( i \)th mirror. Also, \( R_i \) is defined as positive for concave mirrors and negative for convex. For stable resonators the product of the \( g \) parameters lies inside the range of
Unstable resonators however, are characterized by either $g_1 g_2 > 1$ or $g_1 g_2 < 0$.

The equivalent lens train of a typical unstable resonator is shown in Figure 1-2. As stated earlier, an initial field distribution is assumed at $F_2$ and propagated through one round trip to $F'_2$ using the Fresnel-Kirchhoff diffraction formula. With the restriction that the field reproduce itself after the round trip and introducing phase lag due to mirror curvature, the resulting integral equation is

$$v g(x) = \sqrt{\frac{i \pi}{\tau}} \int_{-1}^{1} g(y) e^{-i \tau (y - x/M)^2} dy$$

(1.3)

Here $x$ is the spatial coordinate, $y$ is a dummy variable of integra-
tion, \( i = \sqrt{-1} \), \( M \) = cavity magnification, \( g(x) \) is the field distribution on the output mirror, and \( t \) is defined in equation (B.10). The transverse field distribution of the resonator modes are then given by the eigenfunctions of the integral equation. The multiplicative constant, \( \nu \), is the eigenvalue associated with the eigenfunctions and gives the diffraction loss and phase shift of the mode (Ref. 1:1325).

Rowley (Ref. 3), following the analysis of Horwitz (Ref. 4), developed a computer code to numerically solve the integral equation. He assumed that the field on the mirror before the round trip, \( g(y) \), consisted of a unit amplitude cylindrical wave plus a series of edge diffracted waves, and is given by

\[
g(y) = 1 + \sum_{n=1}^{N} c_n H_n(y) \tag{1.4}
\]

(Ref. 3:17). Therefore, the field on the mirror after the round trip, \( g(x) \), would differ only by the multiplicative constant \( \nu \).

Letting

\[
c_n H_n(y) = a_n F_n(y) + b_n G_n(y) \tag{1.5}
\]

the integral equation to be solved is

\[
\nu \left[ 1 + \sum_{n=1}^{N} \left[ a_n F_n(x) + b_n G_n(x) \right] \right] = \sqrt{\frac{it}{\pi}} \int_{-1}^{1} e^{-it(y - x/M)^2} \left[ 1 + \sum_{n=1}^{N} \left[ a_n F_n(y) + b_n G_n(y) \right] \right] dy \tag{1.6}
\]

The method of stationary phase is used to approximate this integral.
It states that an integral of the form

\[ I = \int_{a}^{b} e^{-itp(y)} q(y) \, dy \]  

(1.7)

can be expressed as a series when \( t \) is large and \( q(y) \) is a slowly varying function. The first two terms of the series are given by

\[ I \approx e^{-it/4} q(y_0) e^{-itp(y_0)} \sqrt{\frac{2\pi}{tp''(y_0)}} + \]

\[ \frac{1}{t} \left[ \frac{q(b)}{p'(b)} e^{-itp(b)} - \frac{q(a)}{p'(a)} e^{-itp(a)} \right] \]  

(1.8)

(Ref. 3:19; 14:1073). This is a first order approximation to the integral equation and is used in Chapter 4 for determination of eigenvalues. A second order approximation is also given in Chapter 4 and is required for intensity and phase calculations outside the mirror edges. In equation (1.8) the prime indicates the derivative of the function with respect to \( y \), and \( y_0 \) is the stationary phase point, i.e.,

\[ p'(y_0) = 0 \]  

(1.9)

Before equation (1.6) can be solved, explicit forms for \( F_n(y) \) and \( G_n(y) \) are needed. They are

\[ F_n(y) = -\sqrt{\frac{M_{n-1}}{4\pi t}} \exp\left[ -it(1 - y/n_n) p_{M_{n-1}} \right] \frac{1 - y/n_n}{1 - y/n_n} \]  

(1.10)

and
\[ g_n(y) = -\sqrt{\frac{M_{n-1}}{4\pi t}} \cdot \text{exp}(-it(1 + y/M_n^2)/M_{n-1}) - \frac{1}{1 + y/M_n}. \]  

(1.11)

where the variable \( M_n \) is defined in equation (3.12). Horwitz determined the functional form of the \( F_n \)'s and \( G_n \)'s by an asymptotic expansion of the integral equation (Ref. 4). The specific set of functions were then chosen for their reproducibility, i.e., after making the stationary phase approximation to equation (1.6) the same set of functions are arrived at. Equating coefficients then leads to a \( 2N + 1 \) degree polynomial in \( \nu \), which is solved by a generalized root finding routine. Once the eigenvalues have been determined, the field is calculated by first specifying a particular eigenvalue or mode and then calculating the constants \( a_n \) and \( b_n \) (a detailed solution is left to Chapter 4).

A limitation to this analysis is the fact that it is only valid for cavities with perfectly aligned mirrors. Since mirror alignment is critical to the effectiveness of any laser system, it is desirable to be able to determine the sensitivity of the system for a specified misalignment. This sensitivity is measured in terms of the power out, beam steering, and mode distortion (i.e., the beam quality). Several investigations into mode distortion due to mirror misalignment have been reported (Ref. 5, 6, 7, 8), however, only a few have dealt with the problem of beam steering (Ref. 8).

Objectives

The purpose of this study is to analyze the effects of mirror misalignment on the transverse modes and beam steering of an unstable laser resonator. The analysis is developed for a bare strip resonator with
rectangular apertures following similar work done by Horwitz (Ref. 5).

A computer program was written (a modified version of a code developed by Rowley) to facilitate the study. The computer model calculates mode eigenvalues and subsequently evaluates the intensity and phase in the plane of the feedback mirror for a desired mode. The slope of the phase, from which the direction of propagation can be determined, is also calculated for the lowest loss mode. The resulting tilt in the phase front is due to diffraction, and is a beam steering angle additional to the geometric misalignment.

Assumptions

The basic assumptions of Reference 3 are still invoked since the analysis is an extension of this previous work. In addition to these is the added restriction of small misalignments (Assumption 5). They are (Ref. 3:2):

1. That scalar diffraction theory can be used if a) the diffracting aperture is large compared to wavelength and b) the diffracted fields are not observed too close to the aperture. The first constraint is easily attained at optical wavelengths and the second is achieved at moderate cavity lengths.

2. That the diffraction integrals and mode eigenfunctions are separable allowing a 1-D strip resonator to be used in the foregoing analysis. When the mirror separation is very much larger than the mirror dimensions, the problem of the rectangular mirrors reduces to a one-dimensional problem of infinite strip mirrors (Ref. 2:457).

3. Diffraction effects from only the feedback mirror are considered if the larger mirror is considered infinite in comparison with the beam
spot size on that mirror.

4. That the modes in the strip resonator consist of a fundamental cylindrical wave modified by a finite number of edge diffraction effects. This assumption is supported by early analysis of unstable resonators (Ref. 9:279).

5. That the optical axis (the line joining the centers of curvature of the two mirrors) does not pass too close (within a Fresnel zone) to the edge of the small mirror (Ref. 5:167).

Procedure and Organization

In Chapter 2 the Fresnel-Kirchhoff diffraction formula is modified to include phase lag due to mirror curvature for a strip resonator; and in Chapter 3 the resulting integral equation is generalized to include effects of small misalignments. Chapter 4 then deals with its solution using the stationary phase approximation. In Chapter 5 the diffracted beam steering angle is presented and the magnitude compared to the geometric beam steering angle, while the results and conclusions are left to Chapter 6.
II. Development of the Integral Equation from the Fresnel-Kirchhoff Diffraction Formula

Equation (1.3) is derived by first reducing the problem of rectangular mirrors to a one-dimensional problem of infinite strip mirrors. This analysis follows the work of Fox and Li (Ref. 2). The next step is to account for phase lag due to mirror curvature, which can be found in References 3 and 4. Although this derivation can be found in the references cited, it is included here for easy access and completeness of the study.

1-D Analysis (Ref. 2:456-457, 484-486)

In a Cartesian coordinate system, the field $E_p$ due to an illuminated aperture $A$ is given by the surface integral

$$E_p(x,y) = \frac{ik}{4\pi} \int_A E_a(x,y) e^{-ikr} (1 + \cos \theta) dS$$  \hspace{1cm} (2.1)

where $E_a$ is the aperture field, $k$ is the propagation constant, $r$ is the distance from a point on the aperture to the point of observation, and $\theta$ is the angle which $r$ makes with the unit normal to the aperture. From Figure 2-1, where propagation is from right to left, this can be written as

$$E(x_1,y_1) = \frac{i}{2\lambda} \int_{-c}^{c} \int_{-a}^{a} E(x_2,y_2) e^{-ikr} (1 + \frac{l}{r}) dx_2 dy_2$$  \hspace{1cm} (2.2)
Also $\lambda$ is the wavelength of the radiation and $r$ is the distance defined by

$$r = \sqrt{L^2 + (x_2 - x_1)^2 + (y_2 - y_3)^2}$$  \hspace{1cm} (2.3)$$

If $L$ is large compared to the dimensions of the diffracting aperture, mirror 2, then the binomial expansion of the above square root can be approximated by retaining only the first two terms of the expansion, i.e.

$$r \approx L \left[1 + \frac{1}{2} \left( \frac{x_2 - x_1}{L} \right)^2 + \frac{1}{2} \left( \frac{y_2 - y_3}{L} \right)^2 \right]$$  \hspace{1cm} (2.4)$$

where $L^2$ was factored out of the square root before the expansion. If again $L$ is considered large compared to dimensions $a$ and $c$, then $\cos \theta \approx \frac{x}{L} \approx 1$. It is seen here that even though $r \approx L$, the exponential
will be more sensitive to this round-off, therefore equation (2.4) is substituted into equation (2.2) to become

\[ B(x_1, y_a) = \frac{\gamma L}{e^{\frac{-ikL}{2}}zL} \int \int E(x_2, y_2) e^{-\frac{ikL}{2}[(x_2 - x_1)^2 + (y_2 - y_1)^2]} dx_2 dy_2 \]

(2.5)

The resonant modes of any laser cavity are characterized by reproducible fields after only one round trip through the resonator. From Figure 2-2 this is given by

\[ \gamma E(x'_2, y'_2) = E(x_2, y_2) \]

(2.6)

Here \( E(x'_2, y'_2) \) is the field after the round trip at \( P'_2 \), \( E(x_2, y_2) \) is the original field at \( P_2 \), and \( \gamma \) is a complex constant. When the mirrors have rectangular geometry the fields are best represented in a Cartesian.
coordinate system where the two orthogonal components become separable.

\[ E(x,y) = U(x) U(y) \]  

(2.7)

Substitution of equation (2.7) into equation (2.5) yields

\[ U(x_1)U(y_1) = \frac{ie^{-ikL}}{\lambda L} \int_{a}^{b} U(x_2) e^{-\frac{ik}{2L}(x_2 - x_1)^2} dx_2 \int_{c}^{d} U(y_2) e^{-\frac{ik}{2L}(y_2 - y_1)^2} dy_2 \]

(2.8)

Restricting the analysis to a one-dimensional strip resonator with the integral defined over the region of the diffracting aperture, mirror 2, the equation for the one-way diffraction of a wave from \( P_2 \) to \( P_1 \) becomes

\[ U(x_1) = \sqrt{\frac{i}{\lambda L}} e^{\frac{-ikL}{2}} \int_{a_2}^{b_2} U(x_2) e^{-\frac{ik}{2L}(x_2 - x_1)^2} dx_2 \]

(2.9)

Similarly, the one-way diffraction formula for propagation from \( P_1' \) to \( P_2' \) is

\[ U(x_2') = \sqrt{\frac{i}{\lambda L}} e^{\frac{-ikL}{2}} \int_{a_1}^{b_1} U(x_1') e^{-\frac{ik}{2L}(x_1' - x_2')^2} dx_1' \]

(2.10)

Introduction of Phase Lag

For equations (2.9) and (2.10) to adequately represent propagation in the resonator, the phase lag due to mirror curvature must be accounted for. Considering equation (2.9), or propagation from mirror 2 to mirror 1 in Figure 2-2, the total phase lag at some position \( x \) is
\[ \phi(x) = k\left[\Delta_1 - \Delta(x_1) + \Delta_2 - \Delta(x_2)\right] \quad (2.11) \]

or

\[ \phi(x) = k\left[R_1 - \sqrt{R_1^2 - x_1^2} + R_2 - \sqrt{R_2^2 - x_2^2}\right] \quad (2.12) \]

The maximum free space region between plane \( P_i \) and the \( i \)th mirror is \( \Delta_i \), while \( \Delta(x_i) \) is the free space region between plane \( P_i \) and the \( i \)th mirror at some distance \( x_i \). It should be emphasized that this is the phase lag for propagation in one direction only.

We can assume the paraxial approximation to be valid over the entire mirror if the radius of curvature is large and the physical dimensions are small compared to the resonator length. Therefore, if the above square roots are expanded in a binomial series as before, only the first two terms are retained.

\[ \sqrt{1 - x_1^2/R_1^2} \approx 1 - x_1^2/2R_1 \quad (2.13) \]

Equation (2.12) is now written

\[ \phi(x) = k(x_1^2/R_1 + x_2^2/R_2) \quad (2.14) \]

Writing \( 1/R_1 \) in terms of the \( g \) parameter from equation (1.1), the phase lag takes the form

\[ \phi(x) = k\left[x_1^2(1 - g_1) + x_2^2(1 - g_2)\right] \quad (2.15) \]

This phase term, seen to be positive, is actually a phase advance-
merit: but when added to the negative phase of equation (2.9) it gains the awkward notation of phase lag. The total phase for the one-way diffraction of a wave as it applies to a resonator with spherical mirrors is then

\[ \phi_{\text{total}} = -\frac{kL}{2L} x_1^2 + x_2^2 - 2x_1x_2 - x_1^2(1 - \varepsilon_1) - x_2^2(1 - \varepsilon_2) \]

(2.16)

After combining some terms, the diffraction formula for propagation from \( P_2 \) to \( P_1 \) is given by

\[ U(x_1) = \sqrt{\frac{1}{2L}} e^{-\frac{ikL}{2} \int_{-a_2}^{a_2} U(x_2) e^{-\frac{ik}{2L}(x_1^2 \varepsilon_1 + x_2^2 \varepsilon_2 - 2x_1x_2)} \, dx_2} \]

(2.17)

Similarly equation (2.10) becomes

\[ U(x_2') = \sqrt{\frac{1}{2L}} e^{-\frac{ikL}{2} \int_{-a_1}^{a_1} U(x_1') e^{-\frac{ik}{2L}(x_1'^2 \varepsilon_1 + x_2'^2 \varepsilon_2 - 2x_1'x_2')} \, dx_1'} \]

(2.18)

If the reproducibility argument of equation (2.6) is invoked, i.e.,

\[ \gamma'U(x_1') = U(x_1) \]

(2.19)

then equations (2.17 - 2.19) can be combined to yield the round trip diffraction formula as it applies to a laser resonator with spherical mirrors and rectangular apertures. In the equation below, the constant phase term has been absorbed into the complex constant \( \gamma' \) to become \( \gamma \).

The integral equation is

\[ \int \]
This equation, although derived from the geometry of Figure 2-2, is
equally valid for the resonator of Figure 1-2 and, in general, any laser
resonator. This can be seen from the sign convention chosen for the ra-
dius of curvature, \( R_1 \). In the above example \( R_1 \) and \( R_2 \) are defined
as positive (refer to page 2) leading to a direct substitution, from
equation (2.14) to equation (2.15), of \( 1/R_1 \) in terms of \( \varepsilon_1 \). From
Figure 1-2 however, equation (2.14) takes the form

\[
\phi(x) = k\left(\frac{x_1^2}{2R_1} - \frac{x_2^2}{2R_2}\right)
\]  

(2.21)

But with \( 1/R_2 = (\varepsilon_2 - 1)/L \), the phase again takes the form of equation
(2.15). Thus equation (2.20) is a general result where the sign of \( R_1 \)
has been absorbed into the \( \varepsilon \) parameter.

The next step is to consider diffraction effects from the feedback
mirror only. This permits the limits of integration over \( P_1 \) to go to
infinity since the beam spot size on mirror 1 is assumed small compared
to mirror dimensions. If in addition to setting the limits of integra-
tion over mirror 1 to \( \pm \infty \), we let

\[
\begin{align*}
x_1' &= x_1 \\
x_2' &= x \\
x_2 &= y
\end{align*}
\]  

(2.22)
the integral equation becomes

\[ yU(x) = \frac{i}{\lambda L} \int_{-\infty}^{a_2} \int_{a_2}^{\infty} U(y) e^{-\frac{ik}{2L}(x^2g_1 + x^2g_2 - 2x_1y)} \, dx \, dy \quad (2.23) \]

The justification for equations (2.22) is seen by following a single ray from mirror 2 to mirror 1 and then back again. As the ray strikes mirror 1 the point of incidence must be the same as the point of reflection, i.e., \( x_1 = x'_1 \). However, the ray leaving mirror 2 will in general be displaced from the original position giving \( x_2 \neq x'_2 \). Therefore, if \( x'_2 \) is the \( x \) position at which the field is to be calculated after the round trip and \( x_2 = y \) (where \( y \) is a dummy variable of integration over the diffracting aperture), equation (2.20) becomes equation (2.23) above.

The interior integral is extracted with its evaluation relegated to Appendix A; the result being the complete kernel of the integral equation.

\[ K = \frac{i}{2\lambda Lg_1} \sqrt{\frac{2\pi}{2\lambda Lg_1}} e^{-\frac{in}{2\lambda Lg_1} [(2g_1g_2 - 1)(x^2 + y^2) - 2xy]} \quad (2.24) \]

Now if

\[ \epsilon = 2g_1g_2 - 1 \quad (2.25a) \]

and

\[ F = \frac{F_2}{2g_1} = \frac{a_2^2}{2\lambda Lg_1} \quad (2.25b) \]

are introduced, the round trip diffraction formula becomes
\[ fU(x) = \frac{1}{\pi F_0} \int_{-a_2}^{a_2} U(y) e^{-\frac{i\pi F_0}{a_2^2}(x^2 + y^2)} dy \quad (2.26) \]

All quantities are the same as defined previously with the addition of \( F_0 \). Here \( F_0 \) is the ordinary Fresnel number of the smaller mirror.

The ordinary Fresnel number is physically interpreted as the additional path length per pass in half wavelengths for a ray traveling from one mirror's center to the other mirror's edge, compared to one traveling from mirror center to mirror center (Ref. 11:159-161).

Normalizing the coordinate system such that \( a_2 = 1 \), the resulting integral equation is

\[ fU(x) = \frac{1}{\sqrt{iF}} \int_{-1}^{1} U(y) e^{-i\pi F\left[x^2 + y^2\right] - 2xy} dy \quad (2.27) \]

Further simplification is obtained by defining

\[ N_{eq} = \frac{F}{2}(M - 1/M) \quad (2.28) \]

and

\[ U(x) = e^{-i\pi N_{eq}x^2} g(x) \quad (2.29) \]

Several different interpretations for the equivalent Fresnel number, \( N_{eq} \), are available (Ref. 11:159-161; 12:360). Here the equivalent Fresnel number is defined as the distance, in half wavelengths, between the outer edge of the output mirror and the nearest point on the outgoing geometrical wave when that wave just touches the mirror center (Ref. 12:360). That geometrical wave is assumed to be a cylindrical wave of the
\[ e^{-i\pi N_{eq}x^2} \]  

Substitution of equations (2.28) and (2.29) into equation (2.27) yields

\[ \psi_0(x) e^{-i\pi F(M - \frac{1}{M})x^2} = \sqrt{4F} \int_{-1}^{1} g(y) e^{-i\pi F(M - \frac{1}{M})y^2} dy - \pi F[g(x^2 + y^2) - 2xy] e^{-i\pi F(g(x^2 + y^2) - 2xy)} dy \]

After some manipulation, detailed in Appendix 5, this equation simplifies to the final form

\[ \psi_0(x) = \sqrt{\frac{4}{\pi}} \int_{-1}^{1} g(y) e^{-it(y-x/M)^2} dy \]

where from Appendix B, \( t = \pi MF \) and \( \psi = \gamma \sqrt{M} \).
III. The Integral Equation Appropriate to the Misaligned Resonator

In Chapter 2 the integral equation was developed for the case of a perfectly aligned resonator. It has been shown (Ref. 5:167-168; 10:2241-2242) that the equation appropriate to the general misaligned resonator differs from the usual one only in the limits of integration. This is true regardless of which mirror is tilted.

The limits of integration are determined by the angle through which the mirror is tilted, which mirror is misaligned, and the cavity geometry. The results of this section (i.e., the limits of integration) pertain only to the geometry of Figure 3-1 and cannot be generalized to include other configurations.

Effects of Mirror Misalignment

This analysis is basically a geometry problem with the development restricted to misalignment of the feedback mirror.

In Figure 3-1 the optic axis of the perfectly aligned resonator is the line $bcde$. When mirror 2 is tilted around point c by an angle $\theta$, the optic axis of the resonator becomes $hgef$. The center of curvature of mirror 1 is at $e$, while the centers of curvature of mirror 2 before and after the tilt are $d$ and $f$, respectively. The angle between the new optic axis and the old is $\alpha$. For small angles $\theta$, $\alpha$ is given by

$$\alpha = \frac{\theta P}{R_1 - R_2 - L}$$

(3.1)
This, however, neglects the fact that $R_2$ by convention is negative. Therefore, substitution of a negative $R_2$ and rewriting in terms of the $g$ parameter, equation (3.1) becomes

$$\omega = \frac{g_1 - 1}{1 - g_1 g_2}$$  \hspace{1cm} (3.2)

The degree of tilt, as expressed in distance across the feedback mirror, is given by the line $\overline{ce}$. This distance is

$$\overline{ce} = \omega(R_1 - L') = \omega(R_1 - L)$$  \hspace{1cm} (3.3)

If we again write $R_1$ in terms of $g_1$ and use the results of equation (3.2), it becomes
\[
\bar{c}_g = \frac{\theta Lg_1}{g_1g_2 - 1} 
\]  

(3.4)

The problem now is to express the distance \( \bar{c}_g \) in terms of the mirror dimension \( a_2 \).

\[
\frac{\bar{c}_g}{a_2} = \frac{Lg_1}{a_2} \frac{g_1}{g_1g_2 - 1} 
\]  

(3.5)

From equation (2.25) this is written

\[
\frac{\bar{c}_g}{a_2} = \frac{\theta}{2F} \frac{a_2/\lambda}{g_1g_2 - 1} 
\]  

(3.6)

With the following definitions

\[
N_{eq} = \frac{1}{2} F(M - 1/N) 
\]  

(3.7a)

and

\[
M = \frac{(g_1g_2)^{1/2} + (g_1g_2 - 1)^{1/2}}{(g_1g_2)^{1/2} - (g_1g_2 - 1)^{1/2}} 
\]  

(3.7b)

equation (3.6) takes the form

\[
\delta = \frac{\bar{c}_g}{a_2} = \frac{a_2 \delta}{\lambda} \frac{M + 1}{N_{eq} M - 1} 
\]  

(3.8)

When the angle \( \theta \) is small, the tilted resonator is equivalent to an aligned resonator with an asymmetric mirror relative to the resonator axis (Ref. 10:2242), refer to Figure 5-1. Since the origin of the x-axis is defined as the point of intersection of the optic axis and mir-
ror 2, the extent of the asymmetric mirror is from \(-1 + \delta\) to \(1 + \delta\).

The limits of integration are now

\[ \alpha = -1 + \delta \quad (3.9a) \]

and

\[ \beta = 1 + \delta \quad (3.9b) \]

Although \(\delta\) is in general a linear function of \(\theta\) for any cavity configuration, its exact form must be determined from the specific geometry.

The generalized integral equation is now written as

\[ \psi_l(x) = \frac{\sqrt{\it{n}}}{\pi} \int_{\alpha}^{\beta} g(y) e^{-\it{t}(y - x/M)^2} dy \quad (3.10) \]

The series functions \(F_n\) and \(G_n\) also have minor changes due to the misalignment, i.e.,

\[ F_n(x) = -\sqrt{\frac{M_{n-1}}{4\pi}} \frac{\exp[-\it{n}(\beta - x/M)^2/M_{n-1}]}{\beta - x/M} \quad (3.11a) \]

and

\[ G_n(x) = \sqrt{\frac{M_{n-1}}{4\pi}} \frac{\exp[-\it{n}(\alpha - x/M)^2/M_{n-1}]}{\alpha - x/M} \quad (3.11b) \]

(Ref. 5:168-169), where again \(M\) is the cavity magnification and

\[ M_n = \sum_{k=0}^{n} M^{-2k} \quad (3.12) \]

As previously stated, \(\delta\) is a function of which mirror is tilted.
If mirror 1 had been misaligned by the same angle \( \theta \), then it would have been found (Ref. 8:579) that

\[
\frac{\theta R_1 R_2}{L - R_1 - R_2} = \frac{\theta L}{\varepsilon_2 - 1}
\]  

(3.13)

It is evident from equations (3.4) and (3.13), that the geometry of Figure 3-1 is less sensitive to misalignment (by a factor of \( \varepsilon_1 \)) for a tilt of mirror 2 vs. the same tilt of mirror 1.
IV. Solution of the Generalized Integral Equation

In Chapter 3 the integral equation was modified to include effects of small misalignments. This chapter is concerned with its solution by applying the method of stationary phase.

Since the problem is basically algebraic and requires numerous manipulations, the scope of this section is limited to a detailed outline of the solution.

Stationary Phase Approximation

The integral equation to be solved is

\[ \nu \, g(x) = \sqrt{\frac{\pi}{\nu}} \int_{a}^{b} g(y) \, e^{-ix(y - x/M)^2} \, dy \]  

(4.1)

where \( g(x) \) is the field distribution on the output mirror after the round trip, \( g(y) \) is the original field distribution, and \( \nu \) is the eigenvalue associated with the eigenfunctions.

Following Chapter 1,

\[ g(y) = 1 + \sum_{n=1}^{N} c_n H_n(y) \]  

(4.2)

where it was assumed that \( g(y) \) consisted of a unit amplitude cylindrical wave plus a series of edge diffracted waves. The basis for this assumption is that the original field is composed of the primary cylindrical wave plus diffraction effects from the previous \( N \) reflections. The series terminates with \( H_n(y) \), the last function to have an effect on the
field. The addition of one more function (or reflection) to the series would add only to the amplitude and not the spatial distribution or shape of the field, i.e., $H_{N+1}(y)$ is a constant. A good approximation is to let

$$N = \frac{\ln(250 N_{eq})}{\ln M} \quad (4.3)$$

(Ref. 4:1533).

Combining equations (1.5), (4.1), and (4.2) yields

$$v \left(1 + \sum_{n=1}^{N} [a_nF_n(x) + b_nG_n(x)]\right) =$$

$$\sqrt{\frac{i\pi}{n}} \int_{\alpha}^{\beta} e^{-it(y - x/M)^2} \left(1 + \sum_{n=1}^{N} [a_nF_n(y) + b_nG_n(y)]\right) dy \quad (4.4)$$

This is equivalent to equation (1.6) with the limits of integration changed to account for the misalignment. Substitution of the $F_n$'s and $G_n$'s from equation (3.11) allows the stationary phase approximation to be applied, refer to equation (1.8). Defining the quantities

$$I_0 = \sqrt{\frac{i\pi}{n}} \int_{\alpha}^{\beta} e^{-it(y - x/M)^2} dy \quad (4.5a)$$

$$I_n = \sqrt{\frac{i\pi}{n}} \int_{\alpha}^{\beta} [a_nF_n(y) + b_nG_n(y)] e^{-it(y - x/M)^2} dy \quad (4.5b)$$

equation (4.4) becomes

$$v \left(1 + \sum_{n=1}^{N} [a_nF_n(x) + b_nG_n(x)]\right) = \sum_{n=0}^{N} I_n \quad (4.6)$$
The problem is to now apply the stationary phase approximation to the individual \( I_n \)'s and then sum the results. Starting with \( I_0 \) and comparing with equation (1.7), it is seen that

\[
q(y) = \sqrt{\frac{it}{\pi}} \quad (4.7a)
\]

and

\[
p(y) = (y - x/M)^2 \quad (4.7b)
\]

Also, from the stationary phase point definition, equation (1.9),

\[
y_o = x/M \quad (4.8)
\]

Employing equation (1.6) and with some manipulation, it can be shown that

\[
I_0 = 1 - \sqrt{\frac{1}{4\pi nt}} \left[ e^{-it(\beta - x/M)^2} - e^{-it(\alpha - x/M)^2} \right] \quad (4.9)
\]

When written in terms of \( F_n \) and \( G_n \) from equation (3.11), this becomes

\[
I_0 = 1 + F_1(x) + G_1(x) \quad (4.10)
\]

Applying the same technique to \( I_1 \), where

\[
I_1 = \sqrt{\frac{it}{\pi}} \int_{\alpha}^{\beta} \left[ a_1 F_1(y) + b_1 G_1(y) \right] e^{-it(y - x/M)^2} dy \quad (4.11)
\]

the approximation yields
One more iteration is necessary in order to show a general trend. Again, using the stationary phase approximation to evaluate $I_2$, the result is

$$I_2 = a_2 F_2(x) + b_2 G_2(x) + F_1(x)[a_2 F_1(\xi) + b_2 G_1(\xi)] + G_1(x)[a_2 F_1(\alpha) + b_2 G_1(\alpha)]$$

(4.12)

From equations (4.10), (4.12), and (4.13) the resulting summation can be generalized to include all $I_n$'s, i.e.,

$$\sum_{n=0}^{N} I_n = 1 + F_1(x) + G_1(x) + \sum_{n=1}^{N} [a_n F_{n+1}(x) + b_n G_{n+1}(x)]$$

$$+ F_1(x) \sum_{n=1}^{N} [a_n F_n(\xi) + b_n G_n(\xi)]$$

$$+ G_1(x) \sum_{n=1}^{N} [a_n F_n(\alpha) + b_n G_n(\alpha)]$$

(4.14)

The equivalence of equations (4.6) and (4.14) gives rise to

$$v \left[ 1 + \sum_{n=1}^{N} [a_n F_n(x) + b_n G_n(x)] \right] = 1 + F_1(x) + G_1(x)$$

$$+ \sum_{n=1}^{N} [a_n F_{n+1}(x) + b_n G_{n+1}(x)]$$

$$+ F_1(x) \sum_{n=1}^{N} [a_n F_n(\xi) + b_n G_n(\xi)] + G_1(x) \sum_{n=1}^{N} [a_n F_n(\alpha) + b_n G_n(\alpha)]$$

(4.15)
The Polynomial Equation

From equation (4.15) a polynomial equation with determinable coefficients is developed. This eigenvalue polynomial (where \( \nu \) is the eigenvalue) is of order \( 2N + 1 \) and is solved numerically. Each root is then associated with a different resonant mode of the cavity.

Referring to equation (4.15), the first step is to equate coefficients. For \( n \neq 1 \) we have

\[
v_{a_{n+1}} = a_n = a_N \nu^{N-n}
\]

and

\[
v_{b_{n+1}} = b_n = b_N \nu^{N-n}
\]

Here the right equality is obtained from

\[
a_{n+1} = \frac{a_n}{\nu} = \frac{a_1}{\nu^n}
\]

and letting \( n = N - 1 \), i.e.,

\[
a_N = \frac{a_1}{\nu^{N-1}} = \frac{a_n \nu^{n-1}}{\nu^{n-1}} = a_n \nu^{n-N}
\]

The next step is to equate constant terms, which gives

\[
\nu = 1 + a_N F_{N+1} + b_N G_{N+1}
\]

Here the \( x \) dependence has been dropped since, by definition, the last functions to contribute to the spatial distribution of the field are \( F_N(x) \) and \( G_N(x) \), or that \( F_{N+1}(x) \) and \( G_{N+1}(x) \) are constant for all \( x \).
Equating coefficients of $F_n(x)$ and $G_n(x)$ allows us to write

$$v_{a_1} = 1 + \sum_{n=1}^{N} [a_n F_n(\beta) + b_n G_n(\beta)]$$  \hspace{1cm} (4.20a)

and

$$v_{b_1} = 1 + \sum_{n=1}^{N} [a_n F_n(\alpha) + b_n G_n(\alpha)]$$  \hspace{1cm} (4.20b)

Substitution for $a_n$, $b_n$, $a_1$, and $b_1$ from equation (4.16) and rewriting equation (4.19) we obtain

$$a_n v^n = 1 + \sum_{n=1}^{N} [a_n F_n(\beta) + b_n G_n(\beta)]$$  \hspace{1cm} (4.21a)

$$b_n v^n = 1 + \sum_{n=1}^{N} [a_n F_n(\alpha) + b_n G_n(\alpha)]$$  \hspace{1cm} (4.21b)

$$v = 1 + a_n F_{N+1} + b_n G_{N+1}$$  \hspace{1cm} (4.21c)

In the above equations the summations are understood to be from $n = 1$ to $n = N$. Although not readily apparent, equations (4.21) can be combined to yield a polynomial equation in $v$ of order $2N + 1$. The expression is left to Appendix C, where it is seen that the coefficients are calculated from a knowledge of $F_n(\alpha)$, $F_n(\beta)$, $G_n(\alpha)$, $G_n(\beta)$, $F_{N+1}$, and $G_{N+1}$. Therefore, by specifying the quantities $M$, $\delta$, and $N_{eq}$ (refer to equations (3.9), (3.11), (3.12), and (4.3)), the roots of the polynomial are eventually determined.

Once the eigenvalues have been calculated, the constants $a_n$ and $b_n$ are determined for a particular mode (again refer to Appendix C). The resulting field is then evaluated at incremental positions across the feedback mirror, i.e.,
\begin{equation}
    g(x) = 1 + \sum_{n=1}^{N} \left[ a_n f_n(x) + b_n g_n(x) \right]
\end{equation}

Given the field, \( g(x) \), the phase is easily calculated from
\begin{equation}
    \varphi(x) = \arctan \left( \frac{\ddot{x}}{\dddot{x}} \right)
\end{equation}

where \( g(x) \) is the complex number \( x + iy \).

\textbf{Second Order Approximation}

A higher order approximation to the method of stationary phase is required, due to the singularities involved, whenever \( y_0 \) approaches the endpoints of the integral. For \( I_0 \) (refer to equation (4.8)) and the case of a perfectly aligned resonator, this occurs when \( x \) approaches the shadow boundaries.

Since the \( \nu \)'s were determined from a valid first order approximation over the region \( \alpha < x < \beta \), then the eigenvalues are acceptable for all \( x \). This is justified by noting that \( \nu \) is a constant. The series constants \( a_n \) and \( b_n \) are also valid from the first approximation, refer to Appendix C, where it is seen that they are a function of the constants \( \nu^{N-n}, F_n(\alpha), G_n(\alpha), F_{n+1}, \) and \( G_{n+1} \). This implies that in order to calculate the phase and intensity for all \( x \) values, a new approximation to the integral
\begin{equation}
    I = \int_{\alpha}^{\beta} q(y) e^{-itp(y)} \, dy
\end{equation}

is all that is required.
The higher order approximation to equation (4.24) can be simplified by letting

\[ u(x) = e^{-itp(x)} q(x) \sqrt{\frac{n}{tp''(x)}} \exp \left[ \frac{it \left[ p'(x) \right]^2}{2p''(y)} \right] \]  

(4.25a)

and

\[ v(x) = \sqrt{\frac{t}{p''(x)}} p'(x) \]  

(4.25b)

Depending on the location of the stationary phase point, \( y_o \), the approximation takes one of three forms. They are: 1) for \( y_o \leq \alpha \)

\[ I = u(\xi) \left[ E^*[v(\xi)] - \frac{1-i}{2} \right] - u(\alpha) \left[ E^*[v(\alpha)] - \frac{1-i}{2} \right] \]  

(4.26)

2) for \( \alpha \leq y_o \leq \beta \)

\[ I = e^{-i\pi/4} q(y_o) e^{-itp(y_o)} \sqrt{\frac{2\pi}{tp''(y_o)}} + u(\xi) \left[ E^*[v(\xi)] - \frac{1-i}{2} \right] \\
- u(\alpha) \left[ E^*[v(\alpha)] + \frac{1-i}{2} \right] \]  

(4.27)

and 3) for \( y_o \geq \beta \)

\[ I = u(\xi) \left[ E^*[v(\xi)] + \frac{1-i}{2} \right] - u(\alpha) \left[ E^*[v(\alpha)] + \frac{1-i}{2} \right] \]  

(4.28)

(Ref. 3:29-30). In the above equations, \( E^* \) is the complex conjugate of the Fresnel integral.

Since the mathematics are again quite involved and tedious, the application of equations (4.26-4.28) will not be given. It is, however,
sufficient to say that the method of solution is similar to that of the first order approximation.
V. Beam Steering

The major effects of mirror misalignment in unstable resonators are beam steering and mode distortion. Mode distortion is easily verified by observation of the results presented in the next chapter. The geometrical beam steering angle, \( \gamma_g \), is however not determined due to the mathematical construct of the misaligned resonator, i.e., the misaligned resonator is modeled as an aligned asymmetric resonator (see Figure 5-1). Additional information is required in order to calculate \( \gamma_g \) (refer to Chapter 3) since this analysis depends only on a knowledge of \( \delta \), \( M \), and \( N_{eq} \).

A diffracted beam steering angle, \( \gamma_d \), is observed by comparison of Figures 6-2, 6-4, and 6-6. The phase fronts are seen to shift slightly with variations in the parameter \( \delta \). Since the normal to the phase front determines the direction of propagation, a straight line curve fit of the phase is desired.

This chapter calculates order of magnitude quantities for \( \gamma_g \) and \( \theta \) (the mirror tilt angle). Also, a least squares curve fit is discussed for determination of \( \gamma_d \).

Geometrical Beam Steering

The beam steering angle due to the geometric misalignment is the angle which the new optic axis makes with the old. From Figure 3-1

\[
\gamma_g = \omega = \theta \frac{\xi_1 - 1}{1 - \xi_1 \xi_2}
\]  

(5.1)

Combining equations (3.6), (3.7), and (2.25) the tilt angle of the mirror
can be written as
\[ \theta = \frac{a_1}{4L_0^2} \left[ \frac{(M - 1)^2}{M} \right] b \quad (5.2) \]

Substitution of equation (5.2) into (5.1) gives \( \gamma_g \) in terms of \( \delta \).

\[ \gamma_g = \frac{b_1}{1 - b_1 b_2} \left[ \frac{a_1}{4L_0^2} \right] \left[ \frac{(M - 1)^2}{M} \right] b \quad (5.3) \]

In order to make a comparative analysis later, we specify \( M = 2.0, \)
\( N_{eq} = 9.6, \) and choose \( L \) to be 2.0 meters. From equations (B.6) and (2.25)

\[ \beta_1 \beta_2 = \frac{1}{4} (M + 1/M + 2) = 1.125 \quad (5.4) \]

Calculating \( R_1 \) in terms of \( R_2 \) is possible by substitution of equation (1.1) in the above expression. The cavity configuration of Figure 5-1 is also desired (i.e., \( R_1 \) is positive and \( R_2 \) is negative), which gives

\[ R_1 = \frac{L + R_2}{1 - (0.125)b_1 b_2/L} \quad (5.5) \]

Having already specified \( R_1 \) as positive, requires

\[ \frac{(0.125)b_1 b_2}{L} < 1.0 \quad (5.6) \]

or \( R_2 < 16 \) meters. Therefore, letting \( R_2 = 10 \) meters the value of \( R_1 \)
is set at 32 meters. The maximum geometrical beam steering angle, \( \gamma_{e_{\text{max}}} \)
occurs for \( \delta = 1.0 \). For \( \delta > 1 \) there is no longer an axis within the
resonator with the symmetry of the original resonator, and there is no possibility of exciting modes characteristic of the aligned unstable resonator (Ref. 8:579-580). With $a_2 = 0.015$ meters, $y_{\text{gmax}}$ is equal to 0.5 mrad. This is equivalent to a mirror tilt of approximately 1 mrad as calculated from equation (5.2).

**Beam Steering Angles Due to Diffraction**

The diffracted beam steering angle is first determined by finding the slope of the phase over the geometrical region. Refering to Figure 5-1, the region is seen to be from $-M+\delta M$ to $M+\delta M$. This is easily obtained by noting that the cavity magnification is a constant depending only on $R_1$, $R_2$, and $L$. The transverse magnification of the perfectly aligned resonator is defined as
\[ M = \frac{x_1}{a_2} = x_1 \]  \hspace{1cm} (5.7)

where the dimensions are normalized such that \( a_2 = 1.0 \). When mirror 2 is misaligned the equivalent resonator of Figure 5-1 has the same cavity magnification, and is given by

\[ M = \frac{x_2}{a_2 + o} = \frac{x_2}{1 + o} \]  \hspace{1cm} (5.8)

or \( x_2 = M + \delta M \). The extent of the geometrical region in the negative direction can similarly be shown to be \( -M + \delta M \).

A straight line curve fit of the phase is achieved by the method of least squares. Given \( n \) sets of points \((x,y)\), where \( y \) is the phase in radians and \( x \) is the normalized distance, the best straight line \( y = mx + b \) is determined by solving the two normal equations

\[ mn + m \sum x_j = \sum y_j \]  \hspace{1cm} (5.9)

\[ b \sum x_j + m \sum x_j^2 = \sum x_j y_j \]

(Ref. 15:683), where the summation is from \( j = 1 \) to \( j = n \). The diffracted beam steering angle is then

\[ \tan \gamma_d \approx \gamma_d = \frac{d}{r} \]  \hspace{1cm} (5.10)

The quantities \( d \) and \( r \) are the change in \( y \) and \( x \) respectively in MKS units, i.e., \( d \) is obtained from \( e^{ikd} = e^{iy} \) and \( r = a_2 x \). Therefore, the diffracted beam steering angle is
\[
\gamma_d = \frac{\lambda y}{2n_a x}
\]  
(5.11)

From the results of the next chapter, the slope \((y/x)\) is determined from the phase calculations for various values of \(\delta\) and plotted in Figures 6-7 and 6-8. Analyzing the case of \(M = 2\) and \(N_{eq} = 9.6\) (Figure 6-7), \((y/x)_{max}\) is seen to be approximately 0.14. Also, from equations (2.28) and (2.25)

\[
\frac{\lambda}{a_2} = \frac{a_2 (M - 1/M)}{4\epsilon_1 N_{eq} L}
\]  
(5.12)

which gives

\[
\gamma_d = \frac{y}{x} \left[ \frac{a_2 (M - 1/M)}{4\epsilon_1 N_{eq} L} \right]
\]  
(5.13)

Using the values from the previous cavity, \(a_2 = 0.015\ m\) and \(L = 2.0\ m\), \(\gamma_d_{max} \approx 7\ \mu\text{rad}\). The maximum value of 0.14 is seen to occur at \(\delta = 0.225\), therefore; \(\gamma_d\) is equal to 0.225 \(\gamma_{\epsilon_{max}}\) or \(\approx 0.1\ \text{mrad}\). It is clearly seen that the diffracted beam steering angle is approximately 7\% of the geometrical beam steering angle, and is a significant contribution when considering propagation over long distances.
VI. Results, Conclusions and Recommendations

Results

The main result of this study is the development of the computer program, BARC 2, as listed in Appendix E along with a description of input variables. This code calculates intensity and phase in the plane of the feedback mirror by specifying $\beta$, $M$, and $N_{eq}$. Also, the slope ($y/x$) is determined for the lowest loss mode only, where the diffracted beam steering angle is related to the slope by equation (5.13).

Plots of intensity and phase for the first three modes of a cavity with $M = 2.0$ and $N_{eq} = 9.6$ are at the end of this section and in Appendix D. For various $\beta$, the intensity and phase distortion is clearly evident. The phase of the lowest loss mode (Figures 6-2, 6-4, and 6-6) is seen to remain relatively uniform, but has a definite slope or directionality associated with it as $\beta$ changes. This slope is plotted in Figure 6-7 for a range of $\beta$ between 0.0 and 0.25.

For the cavity configuration chosen in Chapter 5, the diffracted beam steering angle is a significant portion of the total beam steering angle for specific values of $\beta$. Again referring to Figure 6-7, for $\beta = 0.01125$, the slope is $0.1304$. This gives $\gamma_d \approx 6.5 \mu\text{rad}$ and $\gamma_g = 0.01125 \gamma_{\text{Cmax}} \approx 5.6 \mu\text{rad}$, which shows that $\gamma_d$ is of the order of $\gamma_g$ when $\beta = 0.01$. This value is, however, totally dependent on cavity geometry and must be evaluated for each resonator as discussed in Chapters 3 and 5. More importantly, $\gamma_d$ is seen to vary with no apparent regularity as $\beta$ varies.
Conclusions

By comparison of the phase and intensity plots with those in Reference 5, the basic conclusion reached is that program BARC2 produces valid results. Also, for the case of a perfectly aligned resonator, i.e. \( \delta = 0 \), the results are consistent with those of the previous study (Ref. 3).

As shown in Chapter 3, the geometrical beam steering angle is a linear function of the mirror tilt angle for small misalignments. This has been confirmed from a previous analysis by Krupke and Sooy (Ref. 8). Also, the diffracted beam steering angle is of the order of the geometrical beam steering angle for \( \delta \ll 1.0 \); however the exact limit depends on which mirror is misaligned and the cavity geometry.

The irregularities associated with the diffracted beam steering angle versus mirror misalignment (Figures 6-7 and 6-8) is a result of the structure in the phase. Had the feedback mirror been illuminated by a plane wave of infinite extent, the diffracted beam steering angle would have been zero for all \( \delta \) since the mathematical model is simply a translation of the diffracting obstacle (see Figure 5-1). Referring to Figure 6-2, when the mirror is translated to the right (the dashed lines indicate the position of the mirror) it intercepts a further advanced wave on the right than on the left, and a definite slope is observed in the phase front. Specifically, at \( \delta \approx 0.2 \) (again refer to Figure 6-2) the extent of the mirror becomes -0.8 to 1.2 where a peak in the phase is intercepted on the right and a minimum on the left. This produces the maximum beam steering angle of Figure 6-7.

Further inspection of Figure 6-2 would require \( \gamma_{d,\text{max}} \) to occur at \( \delta \approx 0.9 \). This is exactly the case and the slope at this point is 0.519. The reason for terminating the plots in Figures 6-7 and 6-8 at \( \delta = 0.25 \)
is due to the excessive amount of computer time required to generate them. Therefore, relative values for the diffraction beam steering angle can be estimated from the phase of the perfectly aligned resonator.

**Recommendations**

The computer model, developed from the previous analysis, calculates intensity and phase in the plane of the feedback mirror for the case of a bare strip resonator. A beam steering angle due to diffraction is also determined for the lowest loss mode.

An obvious extension would be to determine beam steering angles for all possible modes of the cavity. In addition, the model might be modified to account for the presence of a saturated or non-uniform gain medium. Since the analysis was restricted to mirrors with rectangular apertures, the case of circular mirrors would be another topic for investigation.

Still another area of consideration would be the analysis of mirror tilt and its effect on higher order aberrations; or the change in the phase slope vs. mirror misalignment curves for higher Fresnel numbers.
Fig. 6-1. Intensity plot for the lowest loss mode with $\delta = 0.0$.

Fig. 6-2. Phase plot for the lowest loss mode with $\delta = 0.0$. 
Fig. 6-3. Intensity plot for the lowest loss mode with $\delta = 0.2$.

Fig. 6-4. Phase plot for the lowest loss mode with $\delta = 0.2$. 
Fig. 6-5. Intensity plot for the lowest loss mode with $\delta = 0.5$.

Fig. 6-6. Phase plot for the lowest loss mode with $\delta = 0.5$. 
Fig. 6-7. Phase slope vs. mirror misalignment, $\theta$. INCDEL is the increment value of $\theta$ for which the slope was calculated.
Fig. 6-8. Phase slope vs. mirror misalignment. INCDEL is the increment value of \( \delta \) for which the slope was calculated.


Appendix A is devoted to the evaluation of the definite integral in equation (2.23).

\[
\frac{i}{\lambda L} \int_{-\infty}^{\infty} e^{-\frac{i k}{2L} \left( x_1^2 \varepsilon_1 + x_1^2 \varepsilon_2 - 2x_1 x \right) - \frac{i k}{2L} \left( x_1^2 \varepsilon_2 + y_1^2 \varepsilon_2 - 2x_1 y \right)} \, dx_1 \quad (A.1)
\]

The exponentials can be combined to yield

\[
\frac{i}{\lambda L} e^{-\frac{i k}{2L} \varepsilon_2 (x^2 + y^2)} \int_{-\infty}^{\infty} e^{-\frac{i k}{2L} 2x_1^2 \varepsilon_1 - 2x_1 (x + y)} \, dx_1 \quad (A.2)
\]

Completing the square in the exponent, the integral becomes

\[
\frac{i}{\lambda L} e^{-\frac{i k}{2L} \varepsilon_2 (x^2 + y^2)} e^{-\frac{ik}{4L \varepsilon_1} (x + y)^2} \int_{-\infty}^{\infty} e^{-\frac{i k}{L} x_1 \sqrt{\varepsilon_1} - x_1 (x + y) + \frac{(x + y)^2}{4 \varepsilon_1}} \, dx_1 \quad (A.3)
\]

Instead of (A.3), we write the integral as

\[
\frac{i}{\lambda L} \exp\left[ -\frac{i k}{2L \varepsilon_2} (x^2 + y^2) - \frac{(x + y)^2}{2 \varepsilon_1} \right] \int_{-\infty}^{\infty} \exp\left[ -\frac{i k}{L} x_1 \sqrt{\varepsilon_1} - x_1 \frac{x + y}{2 \sqrt{\varepsilon_1}} \right] \, dx_1 \quad (A.4)
\]

Now, if we let

\[\beta = \frac{k}{L} = \frac{2\pi}{\lambda L}\]

and

\[V = (x_1 \sqrt{\varepsilon_1} - \frac{x_1 + y}{2 \sqrt{\varepsilon_1}})\]

\[\text{48}\]
then the resulting integral is

\[ \int_{-\infty}^{\infty} \frac{e^{-i\beta V^2}}{\sqrt{\beta_1}} dV \]  \hspace{1cm} (A.6)

where the constant term in front of \((A.4)\) has been dropped. With the additional definitions

\[ \omega = \sqrt{\beta} V \]
\[ d\omega = \sqrt{\beta} dV \]  \hspace{1cm} (A.7)

the definite integral of \((A.6)\) is written

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i\omega^2} d\omega = \frac{2}{\sqrt{\beta_1}} \int_{0}^{\infty} e^{-i\omega^2} d\omega = \frac{n}{\sqrt{\beta_1}} \]  \hspace{1cm} (A.8)

(Ref. 13:380); evaluation of the integral can be found in any handbook of mathematics. Substituting the value of \(\beta\), from \((A.5)\), into \((A.8)\) yields the constant

\[ \sqrt{\frac{L\lambda}{2\beta_1}} \]  \hspace{1cm} (A.9)

With the integral evaluated, \((A.4)\) becomes

\[ \sqrt{\frac{1}{2\lambda \beta_1}} \exp\left[-\frac{iK_0}{2\beta_1} \left(x^2 + y^2\right) - \frac{(x + y)^2}{2\beta_1}\right] \]  \hspace{1cm} (A.10)

If the exponential is expanded with a common denominator of \(4\beta_1\), then
(A.10) takes the final form of the kernel when the substitution of $2\pi/\lambda$ is made for $k$, i.e.,

$$
\sqrt{\frac{i}{2\Lambda \zeta_1}} \exp\left[-\frac{ix}{2\Lambda \zeta_1} \left(2\zeta_2 \zeta_2 - 1\right)(x^2 + y^2) - 2xy\right]
$$

(A.11)
Before equation (2.31) takes the form of equation (1.3), substitution of some variables must be accomplished.

\[

g(x) = \frac{\text{in} F(M - \frac{1}{M}) x^2}{2} = \sqrt{\text{in} F} \int_{-1}^{1} g(y) e^{-\frac{\text{in} F}{2}(M - \frac{1}{M}) y^2 - \text{in} F[g(x^2 + y^2) - 2xy]} \, dy
\]

Given

\[
M = \frac{\sqrt{g + 1} + \sqrt{g - 1}}{\sqrt{g + 1} - \sqrt{g - 1}}
\]

(Ref. 11:157) and solving for \( g \), we get

\[
M = g + \sqrt{g^2 - 1}
\]

\[
(M - g)^2 = g^2 - 1
\]

\[
g = \frac{M^2 + 1}{2M}
\]

Substitution of this in equation (B.1), the integral becomes

\[
g(x) = \sqrt{\text{in} F} \int_{-1}^{1} g(y) e^{-\frac{\text{in} F}{2}(M - \frac{1}{M})(y^2 - x^2) + (x^2 + y^2)(M + \frac{1}{M}) - 4xy} \, dy
\]
\[ Yg(x) = \sqrt{1F} \int_{-1}^{1} g(y) e^{-i\pi F \left( \frac{x^2}{M} + My^2 - 2xy \right)} \, dy \]  \hspace{1cm} (B.8)

\[ Yg(x) = \sqrt{1F} \int_{-1}^{1} g(y) e^{-i\pi MF(y - x/M)^2} \, dy \]  \hspace{1cm} (B.9)

If we define

\[ t = \pi MF \]  \hspace{1cm} (B.10)

then equation (B.9) can be written

\[ Yg(x) = \sqrt{\frac{i\pi}{M}} \int_{-1}^{1} g(y) e^{-it(y - x/M)^2} \, dy \]  \hspace{1cm} (B.11)

Letting

\[ v = \gamma \sqrt{M} \]  \hspace{1cm} (B.12)

equation (1.3) is arrived at, i.e.,

\[ v g(x) = \sqrt{\frac{i\pi}{\pi}} \int_{-1}^{1} g(y) e^{-it(y - x/M)^2} \, dy \]  \hspace{1cm} (B.13)
APPENDIX C

The coefficients of the polynomial are not readily determined from the form listed below; however, with the following definitions it is the most concise form available.

\[
\begin{align*}
F_\alpha &= \Sigma v^{N-n} F_n(\alpha) \\
F_\beta &= \Sigma v^{N-n} F_n(\beta) \\
G_\alpha &= \Sigma v^{N-n} G_n(\alpha) \\
G_\beta &= \Sigma v^{N-n} G_n(\beta)
\end{align*}
\]

(C.1)

Here it is understood that the summation is from \( n = 1 \) to \( n = N \).

The polynomial is now written as

\[
\begin{align*}
&v^{2N+1} - v^{2N} - v^{N+1}(F_\beta + G_\alpha) + v^N(F_\beta - F_{N+1} + G_\alpha - G_{N+1}) \\
+ &v(F_\beta G_\alpha - F_\alpha G_\beta) + (F_\alpha G_\beta - F_\beta G_\alpha) \\
+ &F_{N+1}(G_\alpha - G_\beta) - G_{N+1}(F_\alpha - F_\beta) = 0 
\end{align*}
\]

(C.2)

Also, the \( F_{N+1} \) and \( G_{N+1} \) are the series functions with the \( x \) dependence neglected.

The constants \( a_n \) and \( b_n \) are then calculated, once the eigenvalues have been determined, by specifying a particular mode. The first two equations of (4.21) are combined to yield
\[ b_n = \frac{1 + \frac{v-1}{F_{N+1}} \sum v^{N-n} F_n(\alpha)}{v^N - \sum v^{N-n} G_n(\alpha) + \frac{G_{N+1}}{F_{N+1}} \sum v^{N-n} F_n(\alpha)} \]  

(C.3)

Here again the summation is from $n = 1$ to $n = N$.

The $a_n$'s and $b_n$'s are coupled through the third equation of (4.21), and from equations (4.16) the resulting series constants are determined, i.e.,

\[ b_n = b_n v^{N-n} \]  

(C.4)

\[ a_n = \frac{v^{N-n}(v-1) - b_n G_{N+1}}{F_{N+1}} \]
APPENDIX D

The plots presented here are a few of the higher loss modes, and are included for additional verification of the computer program by comparison with the graphs found in Reference 5. All but the last two plots are a result of the second order approximation. Figures D-9 and D-10 are seen to be the intensity and phase across the feedback mirror only, and are identical in structure to Figures 6-1 and 6-2.
Fig. D-1. Intensity plot for mode #2.

Fig. D-2. Phase plot for mode #2.
Fig. D-3. Intensity plot for mode #2 and $\delta = 0.5$.

Fig. D-4. Phase plot for mode #2 and $\delta = 0.5$. 
Fig. D-5. Intensity plot for mode #3 and δ = 0.0.

Fig. D-6. Phase plot for mode #3 and δ = 0.0.
Fig. D-7. Intensity plot for mode #3 and $\delta = 0.5$.

Fig. D-5. Phase plot for mode #3 and $\delta = 0.5$. 

59
Fig. D-9. Intensity plot across the feedback mirror only using the first order approximation.

Fig. D-10. Phase plot across the feedback mirror only using the first order approximation.
APPENDIX E

A listing of program BARC2 employing the analysis and derivations of the preceding chapters is given here. Also included is a list of input variables required for program operation. Unless otherwise stated, the input variables follow normal Fortran convention for being either real or integer values.

**BARC2 Inputs** (variables are listed in order required) -

- **MAG**: Cavity magnification (real)
- **NEQ**: Equivalent Fresnel number (real)
- **DELTA**: Mirror misalignment (offset in fraction of mirror radius)
- **NBIG**: Desired number of terms in field series
- **MTEST1**: Input 0 to list eigenvalues.
- **MTEST2**: Input 0 to continue with other options, 1 to do new cavity (MAG, NEQ, DELTA), or 2 to exit.
- **MODE**: Desired mode number for phase and intensity calculations (1 to 2*NBIG + 1). The eigenvalues, λ, are listed according to loss, i.e., mode #1 is the lowest loss mode.
- **MTEST3**: Input 0 to calculate intensity and phase, 1 to continue with other options, or 2 to exit.
- **MTEST4**: Input 0 to calculate intensity over expanded range. If MTEST4 equals 0, control is to subroutine ALLINT with variables INCX and MTEST5 skipped.
- **INCX**: Increment value of x for phase and intensity calculation
- **MTEST5**: Input 0 to list field, phase, and intensity across the feedback mirror.
- **MTEST6**: Input 0 to plot constants a_n and b_n versus NBIG, 1 to return, or 2 to exit (return is to MTEST2). If MTEST6 equals 0, the plots are generated and control is automatically returned to MTEST2.

*Note - INCX is the integer number of points between consecutive whole x values. For example, INCX = 100 specifies 100 points between x = 0 and x = 1 for the phase and intensity calculations.*
Subroutine ALLINT Inputs -

XMIN: Minimum x value over which intensity and phase are calculated.

XMAX: Maximum x value over which intensity and phase are calculated.

INCX: Increment value of x

NTEST1: Input 0 to list field, phase, and intensity.

NTEST2: Input 0 to plot intensity and phase.

Program operation is returned to MTEST6 in BARC2.

Note - For the best results let:  

\[
\begin{align*}
XMIN &= -MAG + DELTA*MAG - 0.5 \\
\text{and} & \\
XMAX &= +MAG + DELTA*MAG + 0.5
\end{align*}
\]

- Since ZCPOLY (the root finding routine) limits the degree of the polynomial to 49, the maximum value for NBIG is 24.
PROGRAM HARC2 (DATA, INI III »UUTPUT TAPE8-0UTPUT.

COMMON STOREX<700)»PHASE(700)»FILLN(700)

REAL NEQ, MAG, MSUN(51), MSUPN(51)

COMPLEX EYE, CI, CDUM, AN1, AN2, X1, X2, Y1, Y2, Z1, Z2

COMPLEX ROOT, CONSTH(51), PCONA(51), PCONB(51), FIX, GXY

COMPLEX FBETA, GBA(51), GBA2, X1, X2, Y1, Y2, Z1, Z2

DIMENSION RINDEX(51), LABEL(25)

DATA LABEL/25/10H

C*************************************************************
THIS PROGRAM COMPUTES RESONATOR MODE EIGENVALUES AND
SUBSEQUENTLY EVALUATES INTENSITY VALUES FOR POINTS
ACROSS THE OUTPUT PLANE OF A STRIP LASER RESONATOR.
THE PROGRAM DEALS ONLY WITH A BARE CAVITY.
OUTPUT CONSISTS OF AN EIGENVALUE LIST, WITH PHASE
AND MAGNITUDE, FIELD VALUES FOR A SELECTED MODE
FUNCTIONS OF WEIGHTING CONSTANTS, AND PLOTS OF INTENSITY
ACROSS THE OUTPUT PLANE WITH EITHER LIMITED OR EXTENDED
RANGE.

INPUT QUANTITIES ARE AS FOLLOWS:

MAG = MIRROR MISALIGNMENT (OFFSET IN FRACTION OF MIRROR RADIUS)
MAC = MAGNIFICATION
MSUPN = EQUIVALENT FRESNEL NUMBER
NOTE: FVMAG DENOTES EIGENVALUE MAGNITUDE, AND FVPH DENOTES
EIGENVALUE PHASE.
OUT THIS PROGRAM ALSO REQUIRES IMSL ROUTINE ZCPLOT562.

C*******************************************************************************

C**********************************************************************************

PI=2.*AS IN(1.0)
EYE=CMPLX(0.,1.)

994 FORMAI (F6.2,4X)
990 FORMAT(1X.F5.4.4X)

777 WRITE(P.997'
999 FORMAT(1H1**, INPUT MAG, NEQ, AND OFFSET:**/)

540 READ **MAG,NEQ,DELTA
550 WRITE(0,900)MAG,NEQ,DELTA
560 WRITE(4)MAG,NEQ,DELTA
570 WRITE(5)NEQ, MODE, EIG
580 WRITE(6)=10HENVALUE;

600 C*******************************************************************************

610 C MSUN(I)=MAG**2(I-1)
620 C MSUN(I)=1+1/MAG**2 + ... +1/MAG**(2*I-2)
630 C

640 C*******************************************************************************

650 C
660 C
670 C
680 C
690 C
700 = CONTINUE
710 = T=2*NEQ*PE*MAG**2/(MAG**2-1.)
720 = RNRIG=ALOG(T)/ALOG(MAG)
730 = IF(RNRIG<.61,10) GO TO 11
**Program Description:**

The program is designed to compute the coefficients of a polynomial. It involves initializing variables, reading input, and performing calculations based on given formulas. The output includes the coefficients of the polynomial.

**Algorithm Summary:**

1. Initialize variables and read input values.
2. Compute the polynomial coefficients using given formulas.
3. Output the calculated coefficients.

**Key Steps:**

- **Line 740:** Write initial values to a file.
- **Line 750:** Go to line 777.
- **Line 770:** Format statement to output calculated NBIG value.
- **Line 870:** Compute coefficients of the polynomial using given formulas.
- **Line 900:** Continue the computation process as needed.
1300 = NA=NBIG-1
1390 = NB=NBIG-1
1400 = DO 20 I=1,NC0EF
1410 = X2=CMPLX(0.,0.)
1420 = Y2=X2
1430 = IF (1.EQ.NC0EF) GO TO 23
1440 = DO 22 J=1,NA
1450 = KA=NBIG+JB-NH
1460 = Y1=ALPHA(M-JA)*GBETA(KK)-FBETA(M-JB)*GALPHA(KK)
1470 = Y2=Y1+Y2
1480=22 CONTINUE
1490 = IF (1.EQ.NC0EF) GO TO 25
1500=23 DO 24 JJ=1,NH
1510 = XK=NBIG+JB-NH
1520 = Y1=ALPHA(M-JH)*GBETA(KK)-FBETA(M-JB)*GALPHA(KK)
1530 = Y2=Y1+Y2
1540=24 CONTINUE
1550 = NR=NH-1
1560 = GO TO 27
1570=25 DO 26 JB=1,NHIG
1580 = NK=NBIG-JB+1
1590 = Y1=ALPHA(JB)*GBETA(KK)-FBETA(JB)*GALPHA(KK)
1600 = Y2=Y1+Y2
1610=26 CONTINUE
1620=27 Z1=FBETA(M)*(GALPHA(I-M-1)-GALPHA(I-M-1))
1630 = Z2=GALPHA(H)*(FBETA(I-M-1)-FBETA(I-M-1))
1640 = COEF(I)=X2*Y2+Z1-Z2
1650 = NA=NA-1
1660=29 CONTINUE
1470=C******************************************************************************
1680 = C
1690 = C COMPUTE ROOTS OF POLYNOMIAL WITH IMSL ROUTINE ZCPOLY. TO
1700 = C OBTAIN THE EIGENVALUES. AND THEN ORDER EIGENVALUES BY
1710 = C SIZE.
1720 = C
1730 = C******************************************************************************
1740 = CALL ZCPOLY(COEF,ND0G,LAMBDA,IER)
1750 = IF (TEST1.LE.0) WRITE(8,89)
1760 = I=1
1770 = DO 70 I=2,ND0G
1780 = SIZE= REAL(LAMBDA(i))**2+AIMAG(LAMBDA(i))**2
1790 = K=I
1800 = DO 75 J=I+1,ND0G
1810 = SIZE1=REAL(LAMBDA(J))**2+AIMAG(LAMBDA(J))**2
1820 = IF (SIZE1.LT.SIZE) GO TO 75
1830 = K=J
1840 = SIZE=SIZE1
1850=75 CONTINUE
1860 = CDUM=LAMBDA(I)
1870 = LAMBDA(I)=LAMBDA(K)
1880 = LAMBDA(K)=CDUM
1890 = CL(I)=LAMBDA(I)
1900 = EUPH=ATAN2(AIMAG(CL(I)),REAL(CL(I)))*180./PI
1910 = SMA=REAL(CL(I))**2+AIMAG(CL(I))**2
1920 = SMAG=SQRT(SMA)
1930 = IF (TEST1.LE.0) WRITE(8,333)I,LAMBDA(I),SMAG,EUph
1940=333 FORMAT(1X,19,1X,4(14.7,1X),/)
1950 = J=I
1960=70 CONTINUE
1970 = EUPH=ATAN2(AIMAG(LAMBDA(NDEG)),REAL(LAMBDA(NDEG)))*180./PI
1980 = SMA=REAL(LAMBDA(NDEG))**2+AIMAG(LAMBDA(NDEG))**2
1990 = SMAG=SQRT(SMA)
2000 = IF (TEST1.LE.0) WRITE(8,333)NDEG,LAMBDA(NDEG),SMAG,EUph
NOW CALCULATE THE CONSTANTS FOR THE FUNCTION SUM FOR A PARTICULAR MODE. THEN LOOP TO CALCULATE THE FIELD AT A SELECTED NUMBER OF POINTS.

NOW CALCULATE THE CONSTANTS FOR THE FUNCTION SUM FOR A PARTICULAR MODE. THEN LOOP TO CALCULATE THE FIELD AT A SELECTED NUMBER OF POINTS.
CALL ALLINT(MAG, MSHBN, MSUPN, CONSTA, CON81, T, N1G, N01)
1 ALPHA, BETA, LABEL, DELTA, MODE
GO TO 102
WRITE(8, 992)
FORMAT(1X, *INPUT INCREMENT OF X FOR PLOT: */)
READ *, INCX
WRITE(8, 979) INCX
WRITE(8, 950)
FORMAT(1X, *INPUT 0 TO LIST FIELD, PHASE, AND INTENSITY: */)
READ *, INCX
WRITE(8, 975) IF(MTEST5.NE.0) GO TO 556
WRITE(8, 954)
NDATA = 0
X = ALPHA
BRIGHT = 0.
N1 = N1 + 1
STOREX = X
X1 = CMPLX(0., 0.)
X2 = X1
WRITE(8, 970) IF(X1.EQ.0) GO TO 31
N1 = N1 + 1
N2 = 0
PHASE(N1) = 0.
THE = ABS(PHASE(I1) - PHASE(I))
IF(THETA.GT.300.) N2 = N2 + 1
IF(N2.GT.1) GO TO 38
CALL HGRAFI(STOREX, XININT, NDATA, LABEL, 3, 0, 0)
GO TO 102
CALL HGRAFI(STOREX, PHASE, NDATA, LABEL, 3, 0, 0)
WRITE(8, 991)
FORMAT(1X, *TYPE 0 TO PLOT CONSIS VS N, 1 TO RETURN, OR 2 TO EXIT: */)
READ *, MTEST6
WRITE(8, 976) IF(MTEST6 .EQ. 1) GO TO 103
WRITE(8, 970) IF(LABEL(9) .EQ. 1) GO TO 101
IF(LABEL(10) .EQ. 10) HCONSISTANT
THIS SUBROUTINE FOLLOWS PROGRAM BARCAN AND COMPUTES BEAM INTENSITIES IN THE OUTPUT PLANE. INTERMEDIATE POINTS FOR EVALUATION ARE INPUT WHILE ALL OTHER REQUIRED QUANTITIES ARE CARRIED THROUGH IN THE ARGUMENT LIST AS FOLLOWS:

MAG = CAVITY MAGNIFICATION
MSUBN = ARRAY FOR PARTIAL SUMS OF INVERSE POWERS OF MAG
MSUPN = ARRAY FOR MAG TO SOME POWER
CONST = ARRAY OF CONSTANTS IN THE ASYMPTOTIC SERIES
T = QUANTITY DEFINED IN BARCAN PER HORMITZ
NBIG = # TERMS IN THE SERIES
ROOT = MODE EIGENVALUE
LABEL = PLOT LABELING ARRAY

CALL EXIT
END
3920=901 FORMAT(1X,*INPUT MIN AND MAX X VALUES AND 1 POINTS BETWEEN: */)
3930= READ *,XMIN,XMAX,INCX
3940= WRITE(8,902)XMIN,XMAX,INCX
3950=902 FORMAT(1X,*INPUT VALUES ARE: *F5.2*F5.2*F5.2*F5.2*/)
3960=903 FORMAT(1X,*INPUT VALUE IS: */)
3970= WRITE(8,904)
3980=904 FORMAT(1X,*INPUT 0 TO LIST FIELD, PHASE, AND INTENSITY: */)
3990= READ *,NTEST1
4000= WRITE(8,905)NTEST1
4010= IF(NTEST1.NE.0) GO TO 20
4020= WRITE(8,906)
4030=905 FORMAT(17X,*FIELD*120X,*INTENSITY*3IX,***.1IX,*PHASE (DEG)*)
4040= X-XMIN
4050= ALLFUN*(0..0.)
4060= DO 310 I=1,NBIG
4070= M1NV=1./MSUPN(I)
4080= VAR1=2.*(1.+1./MSUFN(2*I)/MSUBN(I))
4090= VAR2=5*TPT(4.*T/PI/VAR1)
4100= VAP =MINV/MSUBN(I)
4110= STAPHA=X/MA8+BETA*VAR3)/(.5*VAR1)
4120= STAFHB=(X/MAli + ALPHA*VAR3)/(.5*VAR1)
4130= INAPHL=(BETA-X/MAG)**2+< BETA-BE FA*M1NV)\-2/MSUBN(I)
4140= INARG2=(BETA-X/MAG-ALPHA*<1,-MINV)*VAR3)**2/.5*VAR1)
4150= APART1=CEXP(EYE*T)*AARG1)*(-CONSTA(I))/(BETA-ALPHA*MINV)
4160= INARG7=(ALPHA-X/MAG)**2+(ALPHA-ALPHA*MINV)**2/.5*VAR1)
4170= INARG8=(ALPHA-X/MAG-ALPHA*MINV)**2/.5*VAR1)
4180= DARP2=INARG8-TNARG7
4190= BPART1=CEXP(BARG1*EYE*T)*(CONSTB(I))/(ALPHA-BETA*MINV)
4200= INARG0= (BETA-L'r CON* (APART1*BPART1)
4210= APART2=CEXP(EYE*T**VAR3)**2/MSUBN(I)
4220= INARG6= (BETA-X/MAG-ALPHA*MINV)**2/.5*VAR1)
4230= INARG9= (ALPHA-X/MAG)**2+(ALPHA-ALPHA*MINV)**2/.5*VAR1)
4240= INARG11= (ALPHA-X/MAG-ALPHA*MINV)**2/.5*VAR1)
4250= INARG3= (ALPHA-X/MAG-ALPHA*MINV)**2/.5*VAR1)
4260= INARG5= (ALPHA-X/MAG-ALPHA*MINV)**2/.5*VAR1)
4270= INARG9= (ALPHA-X/MAG-ALPHA*MINV)**2/.5*VAR1)
4280= INARG1= (ALPHA-X/MAG-ALPHA*MINV)**2/.5*VAR1)
4290= INARG2= (ALPHA-X/MAG-ALPHA*MINV)**2/.5*VAR1)
4300= OUTCON=SORT(MSUBN(I)/PI/T/VAR1)/2./SQR1
4310= FRSA1=VAR2*(BETA-X/MAG-ALPHA*VAR3)/MSUBN(I)
4320= FRSA2=VAR2*(ALPHA-X/MAG-BETA*VAR3)
4330= FRSB1=VAR2*(BETA-X/MAG-ALPHA*VAR3)
4340= FRSB2=VAR2*(BETA-X/MAG-ALPHA*VAR3)
4350= EYEF1=EYEFAC
4360= EYEF2=EYEFAC
4370= IF(STAPHA.GT.BETA) EYEF2=-EYEFAC
4380= IF(STAPHA.GT.BETA) EYEF1=EYEFAC
4390= IF(STAPHA.GE.BETA) AND STAPHA.LE.BETA) EYEF2=EYEFAC
4400= IF(IN=APART1*(FRESL(FRS1)-EYEF1)-APART2*(FRESL(FRS2)-EYEF2)
4410= IF(FRES1=CEXP(-EYE*(PI/4.+T*(STAPHA-X/MAG)**2*(BETA-
4420= STAPHA)**2/MSUBN(I)))/(BETA-STAPHA*MINV)*(-CONSTA(I))
4430= IF(STAPHA.GE.BETA) AND STAPHA.LE.BETA) AFUN=AFUN*SPNT1*SORT(2.)
4440= IF(1 = EYEF1=EYEFAC
4450= EYEF2=EYEFAC
4460= IF(STAPHA.GT.BETA) EYEF1=EYEFAC
4470= IF(STAPHA.GT.BETA) EYEF2=EYEFAC
4480= IF(STAPHA.GE.BETA) AND STAPHA.LE.BETA) EYEF2=EYEFAC
4490= IF(STAPHA.GE.BETA) AND STAPHA.LE.BETA) EYEF2=EYEFAC
4500= SPNT12=CEXP(-EYE*(PI/4.+T*(STAPHA-X/MAG)**2*(ALPHA-BETA)*
4510= SPNT2=CEXP(-EYE*(PI/4.+T*(STAPHA-X/MAG)**2*(ALPHA-BETA)*
4520= IF(STAPHA.GE.BETA) AND STAPHA.LE.BETA) HEUN=HEUN+SPNT12*SORT(2.)
4530= ALLFUN=OUTCON*AFUN*AFUN+ALLFUN
4540= CONTINUE
4550= EYEF1=EYEFAC
4560 = EYEF2=EYEFAC.
4570 = IF (X/MAG.GT.BETA) EYEF1=EYEFAC.
4580 = IF (X/MAG.GT.BETA) EYEF2=EYEFAC.
4590 = IF (X/MAG.GT.BETA) EYEF1=EYEFAC.
4600 = FRSU01=SQRT(2.*T/PI)*(BETA-X/MAG).
4610 = FRSU02=SQRT(2.*T/PI)*(ALPHA-X/MAG).
4620 = VAR4=LSQRT(EYEF2/EYEFAC).
4630 = UOX=VAR4*((FRESL(FRSU01)-EYEF1)-(FRESL(FRSU02)-EYEF2)).
4640 = SHN=UOE*EYEF1/4.*VAR4*SQRT(2.).
4650 = IF (X/MAG.GT.BETA) AND .X/MAG.LE.X/MAG.LE.BETA) UOX=UOX+SHN=UOG.
4660 = ALLFUN=ALLFUN+UOX.
4670 = FIELD(NDATA)=ALLFUN.
4680 = STOREX(NDATA)=X.
4690 = XINTEN(NDATA)=AIMAG(ALLFUN)**2+REAL(ALLFUN)**2.
4700 = PHASE(NDATA)=1/2*(AIMAG(ALLFUN)+REAL(ALLFUN)).
4710 = PHASE(NDATA)=PHASE(NDATA)*180./PI.
4720 = PHASE2(NDATA)=ATAN(-AIMAG(ALLFUN)-REAL(ALLFUN)).
4730 = IF (XINTEN(NDATA).GT.BRIGHT) BRIGHT=XINTEN(NDATA).
4740 = IF (NTEST1.NE.0) GO TO 400.
4750 = WRITE(8.906) ALLFUN,XINTEN(NDATA),STOREX(NDATA),PHASE(NDATA).
4760 = WRITE(8.906) ALLFUN,XINTEN(NDATA),STOREX(NDATA),PHASE(NDATA).
4770 = 1 PHASE2(NDATA).
4790 = X=X+1./INCX.
4800 = IF (X>MAG.XMAX) GO TO 500.
4810 = NDATA=NDATA+1.
4820 = GO TO 50.
4830 = N1=NDATA+1.
4840 = PHASE(N1)=0.
4850 = DO 510 I=1,NDATA.
4880 = XINTEN(I)=XINTEN(I)/BRIGHT.
4890 = K=O.
4900 = J1=J(24).
4910 = J2=J(25).
4920 = DO 520 I=1,J1.
4930 = THETA2=ABS(PHASE2(I+1)-PHASE2(I)).
4940 = THETA=ABS(PHASE(I+1)-PHASE(I)).
4950 = IF (THETA2.GT.300.) NCOUNT(2)=NCOUNT(2)+1.
4960 = IF (THETA.LE.300.) GO TO 520.
4970 = K=K+1.
4980 = J(K)=1.
4990 = NCOUNT(1)=NCOUNT(1)+1.
5000 = CONTINUE.
5010 = DO 530 I=1,51.
5020 = MSUPN(I)=MSUPN(I)/MAG.
5030 = WRITE(8,907).
5040 = READ **NTES2.
5050 = WRITE(8,907) **NTES2.
5060 = IF (NTEST2.NE.0) GO TO 550.
5070 = CALL HGRAPH(STOREX,XINTEN,NDATA,LABEL,1,0,0).
5080 = CALL HGRAPH(STOREX,XPHASE,NDATA,LABEL,3,0,0).
5090 = GO TO 550.
5100 = CONTINUE.
5110 = DO 535 I=1,535.
5120 = IF (NCOUNT(2).GT.1) GO TO 537.
5130 = CALL HGRAPH(STOREX,XPHASE,NDATA,LABEL,3,0,0).
5140 = CALL HGRAPH(STOREX,XPHASE,NDATA,LABEL,3,0,0).
5150 = GO TO 550.
5160 = NCOUNT(1)=NCOUNT(1)+1.
5170 = J1=J(K).
5180 = J2=J(K+1).
5190 = IF (K.EQ.NCOUNT(1)) J2=J(25).
5200 DO 545 J=1,J2
5210 IF (PHASE(J1).GT.0.0) PHASE(J1)=PHASE(J1)+360.
5220 IF (PHASE(J1).LT.0.0) PHASE(J1)=PHASE(J1)-360.
5230 CONTINUE
5240 K=J2
5250 IF (K.LE.NCOUNT(1)) GO TO 540
5260 CALL HGRAFI(STOREX*PHASE+NDATA+LABEI*J+1,0,0)
5270 J1=J(J2)
5280 J2=J(J2)
5290 IF (PHASE(J1).GT.0.0) CONTINUE
5300 K=M2
5310 IF (K.LT.1.NC0UNI(1)) 60 TO 540
5320 CALL HGFAPH(STOREX,PHASE+NDATA+LABEI*J+3,0,0)
5330 J1=J(24)
5340 SYI=SYI/PHAZE
5350 = SYX*SYX*STOX*PHASE+PHAZE
5360 NCOUNT(3)=NCOUNT(3)+1
5370 SLOPE=ABS((SY1*SY1-SXY**2-SX2*NCOUNT(3))/SX1**2-SX2*NCOUNT(3))
5380 WRITE(*,910)SLOPE
5390 910 FORMAT('X* SLOPE = **G14.7/
5400 WRITE(4),NDATA+1,INCX
5410 DO 580 I=1,NDATA
5420 WRITE(4),FIELD(I),STOREX(I+INCX)
5430 WRITE(4),FORMAT (1X.*SLOPE
5440 920 FORMAT('X*COMPLETED CALCULATION AND PLOT* EXTENDED,*/
5450 RETURN
5460 END
5470 C**************************************************************************************************************
5480 SUBROUTINE HGRAFI(X,Y,N,1D,N0,NP,NS)
5490 DIMENSION X(1),Y(1),ID(25) * IF (NO.EQ.0) CALL PLOT(-1.05,-1.5,-3)
5500 IF (NO.EQ.0) GO TO 30 * IF (NO.LT.0) GO TO 10
5510 CALL SCALE(X,1,N1) * CALL SCALE(Y,5,N2)
5520 10 CALL PLOT(0,-1.2) * CALL PLOT(8.5,1.2)
5530 CALL PLOT(1.35,1.35,-3) * CALL PLOT(0,0.30,-2)
5540 IF (ID.EQ.0) GO TO 25 * IF (NO.LT.0) GO TO 11
5550 CALL PLOT(-1.1,-1.3) * CALL PLOT(0,-2.-2)
5560 CALL SYMBOL(25,3,07,ID(1),90,20) * CALL SYMBOL(45,3,07,ID(3),90,20)
5570 CALL SYMBOL(65,3,07,ID(13),90,20) * CALL SYMBOL(85,3,07,ID(15),90,20)
5580 CALL SYMBOL(105,3,07,ID(5),90,20) * CALL SYMBOL(115,3,07,ID(7),90,20)
5590 CALL SYMBOL(135,3,07,ID(9),90,20) * CALL SYMBOL(135,3,07,ID(11),90,20)
5600 CALL SYMBOL(145,3,07,ID(13),90,20) * CALL SYMBOL(155,3,07,ID(15),90,20)
5610 CALL PLOT(0.9,0,3) * CALL PLOT(1.25,0.2)
5620 CALL PLOT(1.25,2.2) * CALL PLOT(0,0.30,-3)
5630 CALL PLOT(-1.1,-1.3) * CALL PLOT(0,-2,-2)
5640 CALL PLOT(5.8,0,0,2) * CALL PLOT(-5.8,0,0,2)
5650 CALL PLOT(0.8,0,0,2) * CALL PLOT(-0.8,0,0,2)
5660 CALL PLOT(0.8,0.2) * CALL PLOT(-0.8,0.2)
5670 CALL PLOT(0.8,0.2) * CALL PLOT(-0.8,0.2)
5680 CALL PLOT(5.8,0.2) * CALL PLOT(-5.8,0.2)
5690 CALL PLOT(5.5,0.2) * CALL PLOT(-5.5,0.2)
5700 CALL PLOT(0,0.2) * CALL PLOT(0.9,0.2)
5710 CALL AXIS(0,0.,10,1,20,5.,180.,Y(N1)+Y(N2)) * CALL AXIS(0,0.,10,1,20,5.,180.,Y(N1)+Y(N2))
5720 CALL AXIS(0,0.,10,1,20,5.,180.,Y(N1)+Y(N2)) * CALL AXIS(0,0.,10,1,20,5.,180.,Y(N1)+Y(N2))
5730 CALL AXIS(0,0.,10,1,20,5.,180.,Y(N1)+Y(N2)) * CALL AXIS(0,0.,10,1,20,5.,180.,Y(N1)+Y(N2))
5740 Y(N1)-Y(N2) * CALL LINE(Y(N1)+Y(N2),0,NF,NS)
5750 Y(N1)=Y(N2) * CALL PLOT(1.05,2,10,-3)
5760 RETURN * ENU
5770 SUBROUTINE AXIS(X0,Y0,0,0,1,N0,0,0,1,ANG,RMIN,DF)
5780 DIMENSION L(1) * ANG=3.14159/100. * DF=1.*COS(A) * I=1*SIN(A)
5790 IC=ISIGN(1,NL) * NNC=IABS(NC) * R=1. * N=1 * X=0 * Y=0
5800 CALL PLOT(0.0,2) * CALL PLOT(X+1,0,2)
5810 CALL PLOT(0.1475,0,2) * CALL PLOT(Y+1,0,2)
5820 IF (N.EQ.5) CALL PLOT(X-5,0,2) * CALL PLOT(Y+1,0,2)
5830 IF (N.EQ.10) CALL PLOT(X+5,0,2) * CALL PLOT(Y+1,0,2)
5840 71
5840 N=MOD(N+10) + K=K+1 IF(N.LT.KL) GO TO 10
5850 A=ANG-(IC+1)*45.1 IX=10.100 IF=N.N11
5860 C=1.25+1.2511 $ I=I*19.511
5870 X=X0+IX*DX1 Y=Y0+IC*DY1
5880 R=MAX(ABS(R0MIN),ABS(RMIN+DR*KL))1 K=ALOG10(R)
5890 K=INT(ABS(K))1 IX=(IX+1)1 I=IX=MOD(IX,3)
5900 ENCOD(7+105)1 M0E (XY+0.475,7)1 R1=K4HR1
5910 X=X+HDX $ Y=Y+HY1 R=R11 IF(N.LT.KL) GO TO 10 20
5930 K=KL-1X NNC)/2. I C=1.14.511
5940 X=X0+IX*DX1 Y=Y0+IC*DY1
5950 CALL SYMBOL(X,Y,1X,ANG,NNC)1 IF(I.R.EQ.0) RETURN
5960 ENCOD(5+102)5 CALL SYMBOL(999,999,10,5,ANG,5)1
5970 CALL WHERE(X,Y,A)
5980 ENCOD(3+103)3 IR CALL SYMBOL(X,Y,07,3,ANG,3)
5990 ENCOD(101)1 M0E (7.2)
6000 ENCOD(5H*10)
6010 ENCOD(13)
6020 RETURN END
6030 C****************************************************************
6040 C SUBROUTINE SCALE (DATA, LENGTH, N, K)
6050 C REAL DATA(N), LENGTH, SF(5)
6060 C DATA(N) = DATA(N) + 1
6070 C INTEGER N = NUMBER OF DATA POINTS
6080 C INTEGER LENGTH = LENGTH OF THE PLOT AXIS (E.G. IN INCHES)
6090 C INTEGER K = UNUSED PARAMETER INCLUDED FOR COMPATIBILITY
6100 C WITH THE EQUIVALENT CALCOMP SUBROUTINE
6110 C THE FOLLOWING VALUES ARE RETURNED:
6120 C DATA(N+1) = ADJUSTED DATA MINIMUM
6130 C DATA(N+2) = "NICE" SCALE FACTOR IN DATA UNITS PER LENGTH UNIT (E.G. VOLTS/INCH)
6140 C SUBROUTINE SCALE (DATA, LENGTH, N, K)
6150 C REAL DATA(N), LENGTH, SF(5)
6160 C DATA(N) = DATA(N) + 1
6170 C INTEGER N = NUMBER OF DATA POINTS
6180 C INTEGER LENGTH = LENGTH OF THE PLOT AXIS (E.G. IN INCHES)
6190 C INTEGER K = UNUSED PARAMETER INCLUDED FOR COMPATIBILITY
6200 C WITH THE EQUIVALENT CALCOMP SUBROUTINE
6210 C THE FOLLOWING VALUES ARE RETURNED:
6220 C DATA(N+1) = ADJUSTED DATA MINIMUM
6230 C DATA(N+2) = "NICE" SCALE FACTOR IN DATA UNITS PER LENGTH UNIT (E.G. VOLTS/INCH)
6240 C C COMPUTE THE RAW SCALE FACTOR
6250 C CMIN=IMAX=DATA(1)
6260 DO 10 I=1,N
6270 IF(DA(I).LT. CMIN) CMIN = DATA(I)
6280 IF(DA(I).GT. CMAX) CMAX = DATA(I)
6290 CONTINUE
6300 C EXCLUDE TRIVIAL ERROR CASES
6310 C DATA(N+1) = CMIN
6320 C DATA(N+2) = (CMAX - CMIN)CMAX = DATA(I)
6330 C CMAX = DATA(N+2)
6340 C CMAX = DATA(N+2) IF (LENGTH .LE. 0.0, OK. CMAX .EQ. CMAX) RETURN
6350 C RAWSF = (CMAX - CMIN) / LENGTH
6360 C RAWSF = SFMAN1 * 10. ** SFEXP, WHERE 1 .LE. SFMAN1 .LT. 10
6370 C SFEXP = AMIN(ALOG10(RAWSF))
6380 C IF (RAWSF .LT. 1.0) SFEXP = SFEXP - 1.0
6390 C SFMAN1 = RAWSF * 10.0 ** (-SFEXP)
6400 C LOCATE NEXT LARGER "NICE" SCALE FACTOR
6410 C DO 20 I=1.5
**Source Code**

6480=20 IF ( SF(I) .GT. SFMANT ) GO TO 30
6490= PRINT*,' SCALE: SCALE FACTOR ERROR ... ' RETURN
6500=30 SFNICE = SF(I) * 10.0 ** SFEXP
6510=
6520=C COMPUTE ADJUSTED DATA MINIMUM
6530=
6540= A => M = AINT ( XMIN / SFNICE ) * SFNICE
6550= IF ( A => M .GT. RMIN ) A => M = A => M - SFNICE
6560= IF ( (OMAX - A => M) / SFNICE .LT. LENGTH ) GO TO 40
6570=
6580=C NEED TO USE THE NEXT LARGER SCALE FACTOR
6590=
6600= IF ( I .LT. 5 ) SFNICE = SF(I+1) * 10.0 ** SFEXP
6610= IF ( I .GT. 5 ) SFNICE = 20.0 * 10.0 ** SFEXP
6620= A => M = AINT ( XMIN / SFNICE ) * SFNICE
6630= IF ( A => M .GT. RMIN ) A => M = A => M - SFNICE
6640=40 CONTINUE
6650= DATA(N+1) = A => M
6660= DATA(N+2) = SFNICE
6670= RETURN
6680= END
6690=*******************************************************************************
6700= COMPLEX FUNCTION CERF(ZZ)
7120= IF (REAL(Z).LT.0.) CERF = -CERF
7130= 70 RETURN
7140= END
7150=******************************************************************************
7160= COMPLEX FUNCTION FRESL(X)
7170= COMPLEX EYE, Z, CERF
7180= EYE = (0., 1.) * F1 = 2.*ASIN(1.)
7190= Z = SQRT(F1) * X * (1. - EYE) / 2.
7200= FRESL = (1. + EYE) / 2. * CERF(Z)
7210= FRESL = CONJG(FRESL)
7220= RETURN $ END

74
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**Abstract:**

The integral equation that describes mode structure of an unstable resonator with rectangular apertures is developed from scalar diffraction theory. This equation, modified to account for misalignments, is solved by applying the asymptotic methods developed by Horwitz. A second order approximation of the method of stationary phase is employed to calculate phase and intensity values for all points in the output plane. The phase front is also curve fitted to a straight line over the geomet-
rical region for the lowest loss mode. From the slope of the straight line, a direction of propagation can be attributed to the wave. This is a diffracted beam steering angle and is additional to the geometric steering angle (i.e., the beam steering angle due to the geometric misalignment of either or both mirrors).

Plots of intensity and phase for various degrees of misalignments are presented as results of a computer program that utilizes the derived expressions. Also included are graphs of the phase slope versus mirror misalignment.