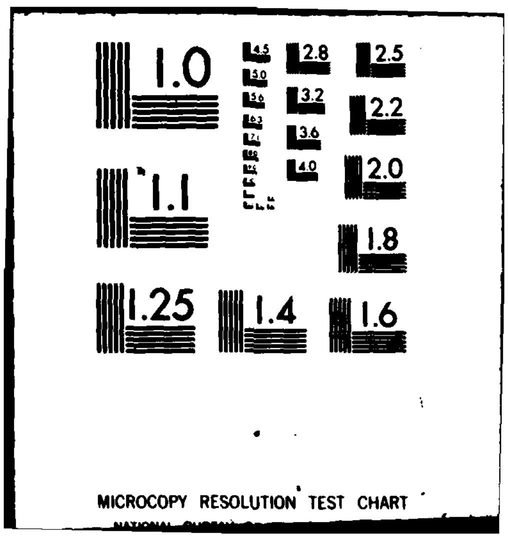
AD-A117 314 ABERDEEN PROVING GROUND MD F/G 19/6
METHODS FOR EVALUATING GUN-POINTING ANGLE ERRORS AND MISS DISTA--ETC(U)
JUN 82 P M SCHWARTZ
UNCLASSIFIED NL

END
DATE
FILMED
8-82
DTIC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6
MICROCOPY RESOLUTION TEST CHART

SCHWARTZ

(1)

18 JUN 1982

AD A117314

METHODS FOR EVALUATING GUN-POINTING ANGLE ERRORS AND MISS DISTANCE PARAMETERS FOR AN AIR DEFENSE GUN SYSTEM (U)

Contents:

*PAUL M. SCHWARTZ, PhD
US ARMY ABERDEEN PROVING GROUND
ABERDEEN PROVING GROUND, MD. 21005

PART I. Coordinate frames, system and instrumentation data; gun angles. p 4

COORDINATIZATION. The basic reference frame used to coordinatize target position and resolve gun-pointing direction into azimuth and elevation components every .1 second is an (east, north, gravity vertical up) system centered in the vehicle at the point C on the turret axis at the ground height of the gun trunnion, $F(C/\mathcal{X},\mathcal{Y},\mathcal{Z})$. Government trackers at known range coordinates acquire target position in their own frames. For a stationary pass, each vehicle sits on a prescribed pad, its turret axis approximately over a point on the pad with known range coordinates. These sets of coordinates, together with the trunnion height, permit the conversion from tracker frame coordinates to reference coordinates, which are further subject to a low-pass digital filter [1, Sec 5.1]. For a moving vehicle pass, in addition vehicle position is provided by a government tracker (also filtered), and $F(C/\mathcal{X},\mathcal{Y},\mathcal{Z})$ becomes a moving frame.

All velocity-related parameters are computed solely from target coordinates available every .1 second in the reference frame. Target velocity components $(\dot{x},\dot{y},\dot{z})$ are associated to each target position vector (x,y,z) by differentiation of a moving fitted polynomial arc. The so-called level plane is spanned by $C,\mathcal{X},\mathcal{Y}$; target azimuth α_T in the level plane has vertex C, initial ray direction $\mathcal{Y}$, with a positive clockwise (viewed from above) sense of rotation to its terminal side; target elevation ε_T has vertex C, and initial ray in the level plane. So $\tan\alpha_T = x/y$, $\tan\varepsilon_T = z/\sqrt{x^2+y^2}$, and time differentiation yields the angular rates $\dot{\alpha}_T$, $\dot{\varepsilon}_T$ in terms of x,y,z.

DTIC FILE COPY

DTIC ELECTE JUL 22 1982 S D B

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

82 07 19 221

The computation of the components of the range rate and range-angular velocity resolution of target velocity runs as follows. Set $\varrho=(x,y,z)$ and $\vartheta=(\dot{x},\dot{y},\dot{z})$; for a non-zero vector α let $\|\alpha\|$ denote its length and α^0 the unit vector in the direction of α . Resolving ϑ into two components, one along the range vector ϱ and the other perpendicular to ϱ in the plane spanned by ϱ & ϑ, get $\vartheta=\frac{1}{\|\varrho\|^2}\left[(\varrho\cdot\vartheta)\varrho+\varrho\times(\vartheta\times\varrho)\right]$ (*) . On the other hand, from a standard elementary mechanics set-up, get $\vartheta=\dot{r}\varrho^0+r\dot{\theta}\boldsymbol{\theta}$, where $r=\|\varrho\|$, $\boldsymbol{\theta}$ is that vector in the plane spanned by ϑ, ϱ leading ϱ by 90°, and $\dot{\theta}$ is the angular velocity of the range vector. Indeed, (*) shows that the coefficient of $\varrho^0=\varrho\cdot\vartheta/\|\varrho\|$, precisely the time derivative $\dot{r}$ of the range. From the definition of $\boldsymbol{\theta}$, it follows that $\boldsymbol{\theta}=\mathrm{sgm}(\dot{x}y-x\dot{y})\left[\varrho\times(\vartheta\times\varrho)\right]^0$; as $\|\varrho\times(\vartheta\times\varrho)\|=\|\varrho\|\cdot\|\vartheta\times\varrho\|$, $\dot{\theta}=\mathrm{sgm}(\dot{x}y-x\dot{y})\frac{\|\vartheta\times\varrho\|}{r^2}$.

GUN AZIMUTH & ELEVATION ANGLES. To obtain gun-pointing direction errors and miss distance results, it is necessary to resolve real-time gun-pointing direction into azimuth and elevation angles wrt the reference frame. Usually this is just a matter of out-of-level compensation. A representative situation for an air defense tank is discussed here.

The turret frame $F(C/\mathfrak{X}_T,\mathfrak{Y}_T,\mathfrak{Z}_T)$ is defined as follows: $\mathfrak{Y}_T$ is the turret axis direction given by the gun vector direction at 0 turret elevation, $\mathfrak{X}_T$ points in the direction of the trunnion to the right of $\mathfrak{Y}_T$ (looking down), $\mathfrak{Z}_T=\mathfrak{X}_T\times\mathfrak{Y}_T$ (turret vertical up). The attitude of the turret frame wrt the reference frame is determined only to the extent of specifying turret pitch and roll relative to the gravity vertical axis $\mathfrak{Z}$.

Accession For
NTIS GRA&I ☑
DTIC TAB ☐
Unannounced ☐
Justification
By
Distribution/
Availability Codes
Dist | Avail and/or Special
A

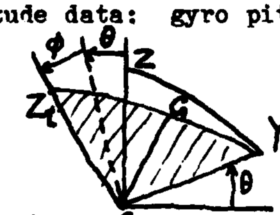
DTIC COPY INSPECTED 2

Gun azimuth and elevation in the reference frame is computed using the following data:

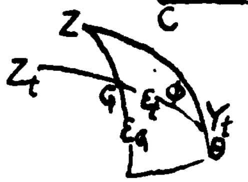
Pitch angle θ , with typical sign conventions + or - according as the turret front moves up or down;
Roll angle ϕ , with typical sign conventions + or - according as the right side moves up or down;
Gun resolver elevation ε_t off the turret plane;
Target angles wrt turret frame---
α_t = target azimuth wrt turret frame (lead traverse), with typical sign conventions + or - according as the target lies to the right or left of the gun elevation plane;
e_T = target elevation off turret plane;
Target position in the reference frame.

The desired azimuth α_G and elevation ε_G are computed in the following 3 steps.

Step 1. Quadrant elevation ε_G (signed) from $\varepsilon_t, \theta, \phi$
Attitude data: gyro pitch θ ,roll ϕ



$\overrightarrow{CZ_t}$ represents turret vertical
$\overrightarrow{CZ}$ represents gravity vertical (on reference unit sphere)
$\overrightarrow{CY_t}$ represents nominal turret centerline (gun pointing direction at 0 turret el)

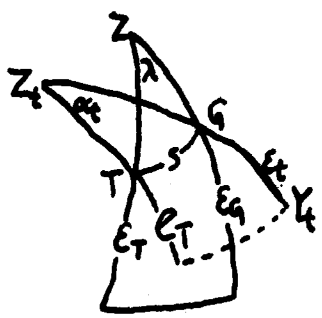


Gun elevates in plane CZ_tY_t

ϕ = dihedral angle between planes Z_tCY_t & ZCY_t

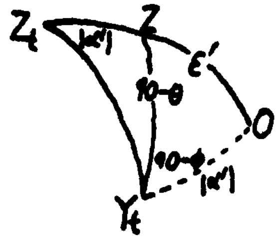
$$\therefore \sin \varepsilon_G = \cos \varepsilon_t \sin\theta + \sin\varepsilon_t \cos\theta \cos\phi$$

Step 2. Lead azimuth magnitude λ from $\varepsilon_G, \varepsilon_t, e_T, \varepsilon_T, \alpha_t$



$\overrightarrow{CT}$ represents target direction
S = target-gun angle in slant plane
From $\triangle TZ_tG$
$$\cos S = \sin\varepsilon_t \sin e_T + \cos\varepsilon_t \cos e_T \cos\alpha_t$$
From $\triangle TZG$
$$\cos S = \sin\varepsilon_T \sin\varepsilon_G + \cos\varepsilon_T \cos\varepsilon_G \cos\lambda$$
$\therefore \lambda$ (unsigned) is determined

Step 3. α_G. It remains only to determine on which side of the vertical plane ZCG containing the gun direction the target lies. Let the true vertical point Z have turret angles α', ϵ' . Note that α' and ϕ have opposite signs.



From $\Delta Y_t Z O$

$$\sin\epsilon' = \cos\theta\cos\phi,$$
$$\cos\epsilon'\cos\alpha' = \sin\theta;$$
$$\sin|\alpha'| = \tan\epsilon'\tan\phi, \text{ so}$$
$$\cos\epsilon'\sin\alpha' = -\cos\theta\sin\phi.$$
$$\therefore \overrightarrow{CZ} = -\cos\phi\sin\theta\,\mathcal{X}_T + \sin\theta\,\mathcal{Y}_T + \cos\theta\cos\phi\,\mathcal{Z}_T$$

$\overrightarrow{CG} \times \overrightarrow{CZ}$ is a normal to the vertical plane containing the gun direction pointing to the right of the plane (viewed from above). So the target lies to the right or the left of this plane according as

$$\overrightarrow{CT}\cdot(\overrightarrow{CG}\times\overrightarrow{CZ}) \geq 0 \text{ or } < 0.$$

Since the turret coordinates of all vectors of the triple product are known, the scalar can in fact be computed.

$\therefore$ $\alpha_G = \alpha_T \mp \lambda \mod 2\pi$ according as the target lies to the left or right of the vertical gun plane.

The attitude data may be available from both on-board system and government-supplied gyros. Target angular position in the turret frame is available from the system's optic sight or track radar, subject to tracking errors. A government-supplied tracker, PAMS, operating in the NIR region, is available for turret tracking with considerably reduced tracking errors.

PART II. Ballistics; ideal gun-pointing direction, system gun-pointing direction errors. $\longrightarrow$ 9

BALLISTICS. The equations of motion of a projectile considered as a particle acted on by an axial drag force, horizontal wind, gravity, rotation of the earth, and subject to a drift owing to aerodynamic forces not deriving from axial drag or crosswind deflection are developed in [2]. That development is not repeated here, but a summary of the trajectory equations, the acquisition and use of metro, and the numerical integration method is set down.

The ballistic frame $F(O/\mathfrak{Z}_g, \mathfrak{X}_g, \mathcal{Y}_g)$ with generic coordinates (Z,X,Y) is defined thus: O represents the vertical projection of the attachment of the gun to its elevating axis to zero MSL, $\mathfrak{X}_g$ = downrange direction, i.e., vertical projection of the current gpd, $\mathcal{Y}_g$ = gravity vertical up direction, $\mathfrak{Z}_g$ = crossrange direction, to the right of the current gpd.

Let t=current projectile time of flight,
(Z,X,Y)=ballistic coordinates of projectile at time t,
w_X, w_Z =downrange, crossrange wind components at projectile position (Z,X,Y) (data actually obtained as a function of altitude above MSL),
ρ=air density at (Z,X,Y) (data source as for wind),
$K_D(M)$=drag coefficient expressed as function of current Mach number M,
C=ballistic coefficient,
$E = \frac{\rho K_D}{C} \times$ projectile speed relative to air (retardation),
g =nominal gravitational acceleration,
K =drift constant,
$\lambda_1, \lambda_2, \lambda_3$ =coriolis terms,
V_0 =muzzle velocity,
ε =gpd elevation angle off the level plane,
s =MSL altitude of gun attachment,
ℓ =barrel length.

In the ballistic frame, the equations of motion of the projectile read

$$\ddot{X} = -E(\dot{X} - w_X) + \lambda_1 \dot{Y}$$
$$\ddot{Y} = -E\dot{Y} - g - \lambda_1 \dot{X}$$
$$\ddot{Z} = -E(\dot{Z} - w_Z - 2Kt\cos\varepsilon) + 2K\cos\varepsilon + \lambda_2 \dot{X} + \lambda_3 \dot{Y},$$

with initial conditions $X(0) = \ell\cos\varepsilon,\ Y(0) = s + \ell\sin\varepsilon,\ Z(0) = 0$
$\dot{X}(0) = V_0\cos\varepsilon,\ \dot{Y}(0) = V_0\sin\varepsilon,\ \dot{Z}(0) = 0.$

For the situation here, the reference origin=the gun attachment; if the gpd has azimuth α wrt the reference frame, the change of coordinates relating ballistic and reference coordinates is given by

$$\begin{pmatrix} Z \\ X \\ Y \end{pmatrix} = \begin{pmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z+s \end{pmatrix}.$$

Acquisition and use of metro data. All data is obtained and used as function of altitude; details concerning units, conversion of temperature, pressure, and humidity to density are omitted.

Surface metro. Here wind speed, windward, temperature, pressure, and humidity are acquired at the surface near the time of testing. Wind data is converted to ballistic coordinates in the current frame; ground air temperature T_s and ground air density ρ_s are used to convert the temperature and density values that the ICAO standard [2] assigns to various altitudes to ICAO adjusted values by additive scaling with T_s and multiplicative with ρ_s, respectively.

Metro aloft. For firing passes, winds aloft radiosonde metro is used when available. Here wind speed, windward, temperature, density are obtained at prescribed altitudes $Y_1 < \cdots < Y_m$ near the time of testing. Wind components in the current ballistic frame, temperature, and pressure for a required altitude are obtained by linear interpolation on altitude.

Numerical integration of the trajectory equations. A numerical solution of the trajectory equations is based on the following rationale. Let Δt = a prescribed small time increment; currently $\Delta t = .1\,sec$. Consider the elapsed times of projectile flight $t_i = i\Delta t, i = 0, 1, 2, \ldots$. Ballistic coordinates and their derivatives are computed only at the discrete times t_i; so known values of $Z, \dot{Z}, X, \dot{X}, Y, \dot{Y}$ at t_i and the metro data associated to altitude $Y(t_i)$ determine $E(t_i)$ and so $\ddot{X}, \ddot{Y}, \ddot{Z}$ at t_i. The assumption that acceleration remains constant throughout the interval $[t_i, t_i + \Delta t = t_{i+1})$ permits the immediate integration of $\ddot{X}, \ddot{Y}, \ddot{Z}$ there, yielding values of $\dot{Z}, Z, \dot{X}, X, \dot{Y}, Y$ at t_{i+1}. The initial conditions start the procedure, and from what has just been said, the procedure can be continued until stopped by some prescribed termination condition. Projectile position and velocity at non-discrete times are obtained by linear interpolation on time. Note that at termination projectile time of flight, velocity components are automatically available.

Ballistic coefficients, drag coefficients, drift constants, and nominal muzzle velocities for the rounds used during testing are supplied by the ballistics section. Actual muzzle velocities during firing may be obtained from a government-supplied muzzle velocity radar.

IDEAL GUN-POINTING DIRECTION.
An ideal intercept algorithm providing gpd azimuth and elevation wrt the reference frame required to intercept any target on the flight path not too near the end is now explicated. Based on the capability of generating the gpd required to intercept a stationary point target by a point round with miss distance $<$ prescribed small tolerance, the premier idea in intercepting a moving target by a projectile-firing weapon can readily be implemented: find the future position on the flight path where the target time of flight from its present position matches the projectile time of flight resulting from the gpd required to intercept that future position. Currently, this algorithm produces dynamic gpd resulting in $|$(target time of flight) - (projectile time-of-flight)$|$ $< 10^{-10}$ sec and miss at intercept $<$ .4 inch.

Stationary intercept algorithm. Given a stationary target with reference coordinates $\mathcal{C}^* = (x^*, y^*, z^*)$, it is required to generate gun aim angles in the reference frame producing a trajectory terminated at point $\mathcal{P} = (x, y, z)$ according to the stop condition ground range of $\mathcal{P} \approx$ ground range of $\mathcal{C}^*$, satisfying the near intercept condition (I) $\min(|x^*-x|, |y^*-y|, |z^*-z|)$ < prescribed tolerance ϵ. Currently, $\epsilon = .01m$. Aim azimuth, elevation angles $(\alpha_i, \varepsilon_i)$, $i = 0, 1, 2, \ldots$ are generated successively as follows. The initial angles are just $\alpha_0 = \alpha^*$ = azimuth of $\mathcal{C}^*$, ε_0 = elevation of $\mathcal{C}^*$ + superelevation (provided by the statistics section, based on a LS curve fit to trajectory angle of fall, as determined by the ballistics of the round under standard conditions, vs slant range). For the iteration, suppose that $(\alpha_i, \varepsilon_i)$ produces a terminal projectile position with reference coordinates $\mathcal{P}_i = (x_i, y_i, z_i)$. If the coordinates satisfy (I) stop and deliver $(\alpha_i, \varepsilon_i)$ as the required angles. Otherwise, either (1) $\min(|x^*-x_i|, |y^*-y_i|) \geq \epsilon$ or (2) $|z^*-z_i| \geq \epsilon$ and the improving aim angles $(\alpha_{i+1}, \varepsilon_{i+1})$ are obtained as follows. If (1) does not hold take $\alpha_{i+1} = \alpha_i$; if (1) does hold, the azimuth $\tilde{\alpha}$ of $\mathcal{P}_i$ differs enough from the azimuth α^* to make an azimuth correction to α_i by simply adding the miss azimuth $\alpha^* - \tilde{\alpha}$ to the old az - formally α_{i+1} is given by $\alpha_{i+1} \equiv \alpha_i + \alpha^* - \tilde{\alpha} \mod 2\pi, 0 \leq \alpha_{i+1} < 2\pi$. If (2) does not hold, take $\varepsilon_{i+1} = \varepsilon_i$; if (2) holds, likewise make an elevation correction to ε_i, by adding on the miss elevation $\varepsilon^* - \tilde{\varepsilon}$, to obtain ε_{i+1}, where $\tilde{\varepsilon}$ is the elevation of $\mathcal{P}_i$ (& ε^* the el of $\mathcal{C}^*$).

Dynamic intercept algorithm. Target position is available every Δt second (currently $\Delta t = .1$). Given a present target position (in reference coordinates) $\mathcal{C}_j$ corresponding to a present discrete clock time, it is required to intercept the future target flight path by firing at the present instant. Target position $\mathcal{C}_{j+\nu}$, $\nu = 1, 2, 3, \ldots$, is available $\nu\Delta t$ sec ahead of the present instant; the future target flight path - parametrized by target time of flight from the present position - is taken to be the polygonal train joining $\mathcal{C}_j, \mathcal{C}_{j+1}, \mathcal{C}_{j+2}, \ldots$, with constant velocity in the interior of the segment $\langle \mathcal{C}_{j+\nu}, \mathcal{C}_{j+\nu+1} \rangle$. So $\mathcal{C}_{\nu,\lambda} = \mathcal{C}_{j+\nu} + \lambda(\mathcal{C}_{j+\nu+1} - \mathcal{C}_{j+\nu})$, $0 \le \lambda \le 1$ on $\langle \mathcal{C}_{j+\nu}, \mathcal{C}_{j+\nu+1} \rangle$ has target time of flight $\nu\Delta t + \lambda\Delta t$. Let $\tau_{\nu,\lambda}$ = projectile time of flight to an intercept of $\mathcal{C}_{\nu,\lambda}$ for a trajectory initiated at the present instant (available from the static algorithm). Ideally we would require the determination of $\mathcal{C}_{\nu,\lambda}$ so that $\tau_{\nu,\lambda} = (\nu+\lambda)\Delta t$; but this is certainly not realistic numerically, so we relax the intercept condition to (D) $|\tau_{\nu,\lambda} - (\nu+\lambda)\Delta t| <$ small prescribed tolerance δ — currently $\delta = 10^{-10}$ sec.

The rationale for the algorithm is based on the following considerations. Suppose that 2 successive positions $\mathcal{C}_{\bar\nu,\lambda_1}$ and $\mathcal{C}_{\bar\nu,\lambda_2}$, $0 \le \lambda_1 < \lambda_2 \le 1$ on the same segment have been obtained admitting an undershoot (projectile arrives at location after target) $\tau_{\bar\nu,\lambda_1} < (\bar\nu+\lambda_1)\Delta t$ at $\mathcal{C}_{\bar\nu,\lambda_1}$, and an overshoot (projectile arrives at location before target) $\tau_{\bar\nu,\lambda_2} > (\bar\nu+\lambda_2)\Delta t$ at $\mathcal{C}_{\bar\nu,\lambda_2}$. If either time of flight difference satisfies (D), stop and deliver the aim angles $(\alpha_j, \varepsilon_j)$ of the trajectory as the so-called ideal angles producing the desired intercept. Otherwise, it is a reasonable presumption to expect an intercept somewhere between $\mathcal{C}_{\bar\nu,\lambda_1}$ and $\mathcal{C}_{\bar\nu,\lambda_2}$. We know that the target time of flight to positions

$\mathcal{C}_{\bar\nu,\lambda_1} + \mu(\mathcal{C}_{\bar\nu,\lambda_2} - \mathcal{C}_{\bar\nu,\lambda_1})$, $0 \le \mu \le 1$, on the segment $\langle \mathcal{C}_{\bar\nu,\lambda_1}, \mathcal{C}_{\bar\nu,\lambda_2} \rangle$

is $\bar\nu\Delta t + [\mu(\lambda_2-\lambda_1)+\lambda_1]\Delta t = (\bar\nu\Delta t + \lambda_1\Delta t) + \mu[(\bar\nu\Delta t + \lambda_2\Delta t) - (\bar\nu\Delta t + \lambda_1\Delta t)]$, where the barycentric combination of the end-point times of flight is the same as that giving the intermediate point's coordinates as a combination of the end-point coordinates. Proceeding under the temporary assumption that the projectile time of flight to an intermediate point is likewise the same barycentric combination of end-point projectile times of flight, get a unique solution $\bar\mu$, $0 < \bar\mu < 1$, of the equation $\tau_{\bar\nu,\lambda_1} + \mu(\tau_{\bar\nu,\lambda_2} - \tau_{\bar\nu,\lambda_1}) = \bar\nu\Delta t + [\mu(\lambda_2-\lambda_1)+\lambda_1]\Delta t$ determining where target time of flight = projectile time of flight under the tentative assumption. Note that the alternating under/overshoot conditions ensures $0 < \bar\mu < 1$; the corresponding point on $\langle \mathcal{C}_{\bar\nu,\lambda_1}, \mathcal{C}_{\bar\nu,\lambda_2} \rangle$ is $\mathcal{C}_{\bar\nu,\lambda_3}$, where $\lambda_3 = \bar\mu(\lambda_2-\lambda_1) + \lambda_1$. Now drop the temporary assumption and regard $\mathcal{C}_{\bar\nu,\lambda_3}$ as a candidate intercept point. Compute $\tau_{\bar\nu,\lambda_3}$ and test the time of flight difference $(\bar\nu+\lambda_3)\Delta t - \tau_{\bar\nu,\lambda_3}$ against condition (D).

If (D) is satisfied, stop and deliver the aim angles $(\alpha_j, \varepsilon_j)$ of the trajectory yielding $\tau_{\bar{\nu},\lambda_3}$ as the ideal intercept angles. If (D) is not satisfied, we have an under-or overshoot at $\mathscr{C}_{\bar{\nu},\lambda_3}$; taking that one of $\mathscr{C}_{\bar{\nu},\lambda_1}, \mathscr{C}_{\bar{\nu},\lambda_2}$ with the opposite under/overshoot condition, we have reproduced the initial state of affairs (with a smaller distance between the points). Thus the rationale and the formal procedure for iterating.

It remains only to start the algorithm by finding two successive discrete clock time target positions $\mathscr{C}_{j+\bar{\nu}}, \mathscr{C}_{j+\bar{\nu}+1}$ exhibiting the alternating under/overshoot condition. This is done by computing all $\tau_{\nu,0}, \nu = 0,1,2,\cdots$ (projectile times of flight to the present and future discrete target positions), and determining $\bar{\nu} \geq 1$ by the conditions $\tau_{\nu,0} > \nu\Delta t$ for $\nu = 0,\ldots,\bar{\nu}-1$; $\tau_{\bar{\nu},0} \leq \bar{\nu}\Delta t$. Clearly $\tau_{\nu,0} > \nu\Delta t, \nu = 0$; if $\tau_{\nu,0} > \nu\Delta t$ for all $\nu = 0,1,2,\ldots$ no interception is possible, so this state of affairs is used as the criterion for terminating ideal intercept angle calculation.

Gun-pointing angle errors. Ideal intercept angles (α,ε) and actual gun angles $(\alpha_G, \varepsilon_G)$ wrt the reference frame as derived in Part I are compared to yield signed errors as follows. The signed elevation error is just $\varepsilon_G - \varepsilon$; so positive if the gun actually points above the ideal elevation, negative if the gun points below. Azimuth error is signed so that positive means the actual is leading the ideal, negative means a lag, provided that the target flight path admits a determination of crossing.

PART III. Miss Distance and Related Parameters.

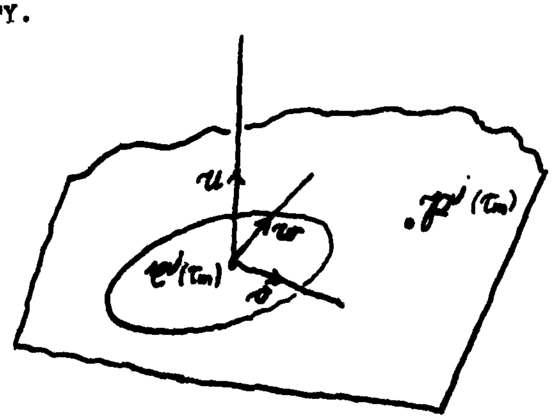
At discrete clock times separated by Δt sec (currently $\Delta t = .1$) target position in the reference frame is available to a maximum of n points (currently $n \leq 1000$: target flight time considered $\leq$ 100 sec); so we have n target coordinates $\mathscr{C}_1,\cdots,\mathscr{C}_n$ at clock times $c_1,\ldots,c_n$, where $c_i = c_{i-1} + \Delta t$. Consider a projectile trajectory, with system gun angles $(\alpha_G, \varepsilon_G)$ wrt the reference frame, initiated at the clock time c_j. Suppose that we have continuous, rectifiable arcs $\mathscr{C}^j, \mathscr{P}^j$, parametrized by elapsed time of flight $t, 0 \leq t \leq (n-j)\Delta t$, reckoned from the present instant c_j, representing target, projectile flight paths, respectively. At elapsed time t, the target-to-projectile miss vector is just $m(t) = \mathscr{P}^j(t) - \mathscr{C}^j(t)$. So theoretically, there is a smallest τ_m at which the miss distance $\|m(t)\|$ assumes its minimum value: $\|m(\tau_m)\| = \min_{0 \leq t \leq (n-j)\Delta t} \|m(t)\| =$ the minimum miss distance. Further assuming differentiability of both arcs we can obtain target and projectile velocity at minimum miss time τ_m. If τ_m is an interior point (as expected), we also have $\frac{d}{dt}\|m\|^2 = m \cdot \dot{m} = 0$: this leads directly to the introduction of the so-called normal plane and a single shot hit probability set-up in the normal plane.

MINIMUM MISS VECTOR. The construction of $\mathcal{C}^j, \mathcal{P}^j$, and the numerical procedures used to obtain τ_m, $\mathfrak{m}(\tau_m)$, velocities follow. Introduce the distinguished elapsed times $t_\nu = \nu\Delta t, \nu = 0, \dots, n-j$. As before, $\mathcal{C}^j$ is taken as the polygonal train joining $\mathcal{C}_j, \dots, \mathcal{C}_n$ with intervalwise parametrization $\mathcal{C}^j(t_\nu + \lambda\Delta t) = \mathcal{C}_{j+\nu} + \lambda(\mathcal{C}_{j+\nu+1} - \mathcal{C}_{j+\nu}), 0 \le \lambda \le 1$, in $[t_\nu, t_{\nu+1}], \nu = 0, \dots, n-j$. Velocity at time t_ν is just the velocity $\dot{\mathcal{C}}_{j+\nu}$ already numerically obtained, as described in Part I; for interior points $\mathcal{C}_{j+\nu} + \lambda(\mathcal{C}_{j+\nu+1} - \mathcal{C}_{j+\nu})$ velocity is obtained by linear interpolation on the relative position barycenter λ, if needed. From the numerical trajectory computations, projectile position $\mathcal{P}_\nu$ and velocity $\dot{\mathcal{P}}_\nu$ are available in reference coordinates at the discrete times t_ν. Just as for the target flight path, the projectile trajectory $\mathcal{P}^j$ is taken as the polygonal train joining $\mathcal{P}_j, \dots, \mathcal{P}_n$, parametrized as for the target flight path, with velocity obtained at interior points of a segment $\langle \mathcal{P}_{j+\nu}, \mathcal{P}_{j+\nu+1} \rangle$ as for target velocity.

$$\|\mathfrak{m}(t_k + \lambda\Delta t)\|^2 = \|\mathcal{P}_k - \mathcal{C}_k + \lambda[(\mathcal{P}_{k+1} - \mathcal{C}_{k+1}) - (\mathcal{P}_k - \mathcal{C}_k)]\|^2$$

is just a quadratic function of $\lambda, 0 \le \lambda \le 1$, so the minimum value m_k and the point $t_k + \lambda_k\Delta t$ of attainment where $\mathfrak{m}$ assumes its minimum value over the interval $[t_k, t_{k+1}]$ are readily computed. The discrete minimization problem: find the smallest index ℓ such that $m_\ell = \min\limits_{k=0,1,\dots,n-j-1} m_k$, is trivially solvable (by computer). So we have minimum miss time τ_m, the minimum miss vector $\mathfrak{m}(\tau_m)$ (in reference coordinates), and other required values $\mathcal{C}^j(\tau_m), \mathcal{P}^j(\tau_m), \|\dot{\mathcal{C}}^j(\tau_m)\|$.

NORMAL PLANE. HIT PROBABILITY.



Let $\mathbf{u} = \dot{m}(\tau_m)^0$ As we have seen at the beginning, in theory $m(\tau_m)\cdot\mathbf{u} = 0$. The numerical procedures used do not strictly guarantee this; however, for minimum miss distances $\|m(\tau_m)\| \leq 30m$, it has been observed that usually $|m(\tau_m)\cdot\mathbf{u}|$ computes out $<.01m$, invariably $|m(\tau_m)\cdot\mathbf{u}| < .1m$; for minimum miss distances $>30m$, the magnitude is unpredictable. We will proceed assuming $m(\tau_m)\cdot\mathbf{u} = 0$. This simply says that at minimum miss the projectile lies in the plane passing through the target perpendicular to the relative velocity vector $\dot{m}(\tau_m) = \dot{\mathbf{p}}^J(\tau_m) - \dot{\mathbf{t}}^J(\tau_m)$, the so-called normal plane. A two-dimensional coordinate frame $F(\mathbf{t}^J(\tau_m)/\mathbf{v}, \mathbf{w})$ in the normal plane is introduced as follows. With $\mathbf{u} = u_1\mathcal{X} + u_2\mathcal{Y} + u_3\mathcal{Z}$, set $\mathbf{v} = (u_1^2 + u_2^2)^{-\frac{1}{2}}(-u_2\mathcal{X} + u_1\mathcal{Y})$ in the free normal plane and parallel to the ground, $\mathbf{w} = (\mathbf{u}\times\mathbf{v})^0$; generic coordinates, (v,w,) wrt this frame are called normal coordinates, with v being regarded as a horizontal or azimuth miss distance component, w as a vertical or elevation component. The normal coordinates of the projectile at minimum miss are $(v_0, w_0) = (m(\tau_m)\cdot\mathbf{v}, m(\tau_m)\cdot\mathbf{w})$.

To arrive at a set-up leading to a simple single shot hit probability, we assume that random projectile location in the normal plane has random normal coordinates (V, W) following a bivariate normal distribution where (1) the center of the distribution is taken as (v_0, w_0) , the computed normal coordinates of the projectile at minimum miss; (2) the standard deviations σ_V, σ_W are normal axes distance dispersions obtained from an average of normal axes mil dispersions derived by the statistics section from PINS & miss distance radar scorings - the target range $\|\mathbf{t}^J(\tau_m)\|$ at minimum miss is required for the customary mils-to-meters conversion.

A target region in the normal plane is obtained as follows. First the target is mathematically represented as the region enclosed by an ellipsoid in 3 space defined as follows:

center= $\mathbf{t}^J(\tau_m)$;

longitudinal axes in the direction $\mathbf{r}$ of target velocity at miss, with semi-axis length a: $\mathbf{r} = \dot{\mathbf{t}}^J(\tau_m)^0 = r_1\mathcal{X} + r_2\mathcal{Y} + r_3\mathcal{Z}$;

traverse axis parallel to the ground, direction $\mathbf{q} = \frac{-r_2\mathcal{X} + r_1\mathcal{Y}}{\sqrt{r_1^2 + r_2^2}}$, with semi-axis length b;

vertical axis directed by $\mathbf{s} = (\mathbf{r}\times\mathbf{q})^0$, with semi-axis length c.

The definition of $\mathbf{r}$ adequately accounts for target yaw and pitch, but in the absence of any data the definition of $\mathbf{q}$ amounts to assuming zero roll. The ellipsoid is projected in the direction $\mathbf{u}$ onto the normal plane in accordance with the notion of presented area, resulting in a target region given by the interior of an ellipse $\alpha v^2 + \beta vw + \gamma w^2 = 1$. The straightforward but tedious calculations required to produce the coefficients α, β, γ from the definition of the target ellipsoid are omitted.

e abstractly, the desired single shot hit probability is just $\beta V \cdot W + \gamma W^2 \leq 1\}$. If the covariance turns out negligible, the oduct term of the quadratic form in normal, independent random s can be removed by a rotation of axes, and the equivalent prob- is evaluated by the well known Grubbs approximation [3]. e the double integral representing the probability is readily numerically.

References

grams for Digital Signal Processing, IEEE Press 1979.

thod for Calculating Exterior Ballistic Trajectories..", ent and Proof Services Report No. DPS-416, er, 1962.

proximate Circular and Noncircular Offset Probabilities ng", Operations Research, 12, pp 51-62, F. E. Grubbs,