DISTRIBUTED SYSTEM OPTIMAL CONTROL AND PARAMETER ESTIMATION: COMPUTATIONAL TECHNIQUES USING SPLINE APPROXIMATIONS

by H. T. Banks

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ABSTRACT

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Abstract. Spline-based computational procedures for parameter estimation and
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eters in higher order models arising in elasticity.

Keywords. Parameter estimation; feedback controls; delay and distributed
systems; computational methods; splines; wind tunnels; ecology; elasticity.

INTRODUCTION

In this note we present a brief summary of some computational methods that have proven
useful in optimal control (both open loop and feedback) and parameter estimation problems
for certain distributed systems. We first outline the basic ideas which are common to our
approach whether we are dealing with functional
al differential equations (FDE) or partial
differential equations (PDE) of parabolic or


parabolic type. Roughly speaking, in each
case one views the system under consideration
as an abstract system

\[
\begin{align*}
\dot{z}(t) &= A(q)z(t) + F(q,t) \\
z(0) &= z_0
\end{align*}
\]

in an appropriately chosen Hilbert space Z.

In the event that the operator A depends on
parameters q to be estimated, one then has
data or "observations", say \(z(t_1)\), and one
attempts to choose a parameter \(q\) from an
admissible set \(Q\) so as to yield a best fit
of the model (1) to the data. For optimal
control problems, the parameters \(q\) are pre-
sumed fixed and known and \(F\) in (1) is a
control input term, say \(F(t) = Bu(t)\). Then
one has a performance measure \(J\) depending
on \(z\) and \(u\), and an admissible control set \(U\)
(either open loop or feedback). One seeks
a \(u^*\) in \(U\) that minimizes \(J\) subject to (1).

For either FDE or PDE systems, these problems
involve infinite-dimensional state systems
and hence computational schemes must be based
on some type of approximation idea. The
approach we describe here entails choosing a
sequence of finite dimensional subspaces
\(Z_N\) of \(Z\) generated by basis elements con-
isting of splines (linear, cubic, or quintic
in the examples discussed below). One then
approximates the system (1) (and the corre-
ponding control or estimation problem) in the
subspace \(Z_N\) by the system

\[
\begin{align*}
\dot{z}_N(t) &= A_N(q)z_N(t) + p_NF(t) \\
z_N(0) &= P_Nz_0,
\end{align*}
\]

where \(P_N\) is the canonical orthogonal projection of \(Z\) onto \(Z_N\). This
results in an approximate estimation or con-


control problem entailing a finite dimensional
state space to which standard computational
packages can be applied.

The fundamental convergence theory which can
be used in either control or parameter esti-


mation problems is based on semigroup approx-
imation results. Briefly the ideas are as
follows (details differ depending upon whether
one is treating FDE or PDE). One first demon-
strates that \(A(q)\) satisfies a uniform dis-


sipative inequality in \(Z\) (such as \(\langle A(q)z, z\rangle \leq \omega <z, z>\)
for \(z \in \text{Dom}(A(q))\) and \(A(q)(\omega)\)
(one maximal dissipative extension) gen-

erates a \(C_0\)-semigroup \(T(t;q)\). The approxi-
mating operators \(A_N(q)\) are defined, as we
have already indicated, by \(A_N(q) = P_NA_N(q)P_N\)
and generate a stable family of schemes such
that \(\|\exp[A_N(q)t]\| \leq M e^{\omega t}\) where \(M\) and
\(\omega\) are independent of \(q\) and \(N\). One uses
standard estimates from spline approximation
theory to argue that \(A_N(q)z = A(q)z\) in


an appropriate sense. One then employs the
Trotter-Kato theorem (a functional-analytic
version of the Lax Equivalence theorem:
stability plus consistency yield convergence)
to establish that \(\exp[A_N(q)t] = T(t;q)\)


strongly in \(Z\), and, moreover, that
FEEDBACK CONTROL FOR DELAY SYSTEMS

The problem of constructing feedback controls for hereditary or delay systems is not new and there is a rather large literature which we shall not discuss here. Our own renewed interest in this problem was motivated by problems arising in the design of controllers for a liquid nitrogen wind tunnel (the National Transonic Facility or NTF) currently under construction by NASA at its Langley Research Center in Hampton, VA. With this wind tunnel it is expected that researchers will be able to achieve an order of magnitude increase in the Reynolds number over that in existing tunnels while maintaining reasonable levels of dynamic pressure. Test chamber temperatures (the Reynolds number is roughly inversely proportional to temperature) will be maintained at cryogenic levels by injection of liquid nitrogen as a coolant into the airstream near the fan section of the tunnel. In addition to a gaseous nitrogen vent to help control pressure, motor driven fans will be used as the primary regulator of Mach number. Fine control of Mach number will be effected through changes in inlet guide vanes in the fan section.

Schematically, the tunnel can be depicted as in Fig. 1.

Fig. 1.

The basic physical model relating states such as Reynolds number, pressure, and Mach number to controls such as LN\textsubscript{2} input, GN\textsubscript{2} bleed, and fan operation involves a formidable set of PDE (the Navier-Stokes theory) to describe fluid flow in the tunnel and test chamber. This model has, not surprisingly, proved to be very unwieldy from a computational viewpoint and is difficult, if not impossible, to use directly in the design of sophisticated control laws. (Both open loop and feedback controllers are needed for efficient operation of the tunnel) - and this is a desirable goal since cost estimates for liquid nitrogen alone are \$0.5 \times 10^6 \text{ per year of operation.} In addition to the design of both open loop and closed loop controllers, parameter estimation techniques will be useful once data from the completed tunnel is available (current investigations involve use of data from a 1:50 meter scale model of the tunnel).

In view of the schematic in Fig. 1, it is not surprising that engineers (e.g. see Armstrong and Tripp, 1981) and (Gumas, 1989) have proposed design of control laws for subsystems modeled by lumped parameter models (the variables represent values of states and controllers at various discrete locations in the tunnel and test chamber) with transport delays to account for flow times in sections of the tunnel. A specific example is the model (Armstrong and Tripp, 1981) for the Mach number control loop in which variations in the Mach number in the test chamber are, in first order, controlled by variations in the inlet guide vanes angle setting (in the fan section) i.e. \[ \dot{M}_{\text{in}} = \text{sat}(r) \text{ where } r \text{ represents a transport time from the fan section to the test section.} \]

More precisely, the proposed equation...
The optimal feedback control is then given by

\[ u(t) = R^{-1} \mathcal{P} z(t) \quad (6) \]

relating the variation \( \delta M \) (from steady state operating values) in Mach no. to the variation \( \delta \theta \) in guide vane angle is

\[ \delta M(t) = \delta \theta(t) = k \delta \theta (t-r) \quad (1) \]

while the equation relating the guide vane angle variation to that \( \delta \theta \) of an actuator is

\[ \delta \theta(t) + 2 \zeta \delta \theta(t) + \omega^2 \delta \theta(t) = \omega^2 \delta \theta(t). \]

Rewriting the system in vector notation, one thus finds that the Mach no. control loop involves a regulator problem for the equation

\[ \dot{x}(t) = A \dot{x}(t) + B_0 u(t) \quad (5) \]

where \( x = (\delta M, \delta \theta, \delta \dot{\theta}) \), \( u = \delta \theta_A \). Here the control is the guide vane angle actuator input. A similar 4-vector system problem can be formulated in the case where one treats the actuator rate \( \delta \dot{\theta}_A \) as the control - see (Armstrong and Tripp, 1981), (Daniel, 1982).

Problems such as that just outlined led us to consider the spline techniques of (Banks and Kappel, 1979) for computation of feedback controls in regulator problems governed by n-vector systems

\[ \dot{x}(t) = L(x(t)) + Bu(t) \quad (4) \]

where \( x = (\delta M, \delta \theta, \delta \dot{\theta}) \), \( u = \delta \theta_A \). The cost functional is the usual integral quadratic payoff

\[ J(z(0), u) = \int_0^t (x(t) L_c x(t) + u(t) B u(t) \quad (7) \]

where \( t \rightarrow x(t) \) is the solution of (4). As is well-known (see the summary and references to previous literature in (Gibson, 1980)) the appropriate state feedback control is given in terms of a functional

\[ u(x_t) = -R^{-1} B^T [K_0 x(t) + \int_0^t K_1(s)x(t+s)ds] \quad (5) \]

where the gains \( K_0, K_1 \) satisfy certain Riccati type equations. A detailed explanation of use of the spline-based methods for computations in these problems is given in (Banks and Rosen, 1982); we only outline the procedures here and discuss our numerical findings for the NTF example.

Briefly then, one reformulates the system (4) as an abstract system (1) in the Hilbert space \( \mathcal{H} = R^n \times L_2(-r,0;R^n) \) with \( z(t) = (x(t), x_t) \).

The optimal feedback control is then given by (see the summary in §4 of (Gibson, 1980))

\[ u(t) = R^{-1} \mathcal{P} z(t) \quad (6) \]

with \( \mathcal{P} = \mathcal{P}(z_0, z_0) \) where the bounded linear operator \( \mathcal{P} \) is the solution of the Riccati algebraic equation (RAE):

\[ \mathcal{P} = \mathcal{P} - \mathcal{P} R^{-1} \mathcal{P} = p \quad (8) \]

Here \( \mathcal{P} \) and \( \mathcal{P} \) are operators on \( \mathcal{H} \), \( z \) given by

\[ \mathcal{P}(z_0, z_0) = \langle z, z, z, z \rangle \quad (9) \]

Calculations were carried out for the problem of driving \( \delta M \) from -1 to 0.0 (corresponding to \( \delta \theta \) varying from -20° to 0°) and \( \delta \theta \) from 8.55 to 0.0 (corresponding to the guide vane angle varying from 10.48° to a steady state of 1.93°). Excellent results were obtained even for low values of the approximation index \( N = 2,4,8 \). The corresponding optimal controls (7) appear to converge rapidly to an optimal control of the form (6) (of course, we
accuracy, and rate of convergence. We refer with regard to some cases superior to the averaging method generally at least as good as and in efforts. Rose, may our findings are detailed in (Banks and other delay system regulator examples and We also tested the spline method (and compared it with the averaging method.. We have successfully applied the spline methods to be adequate for the simple and Tripp, 1981). All three methods appear transport functions (such as finite difference techniques of (Armstrong consists of using field'data estimate the (e.g., see (Okubo, 1980, Kunisch, 1982) lead to different models, perhaps even density dependent) in a proposed model, one desires to estimate or identify parameters (including the transport functions in the model and quantify the success (or lack thereof) of the model in describing the data. For example, typical models might involve the general transport equation (Okubo, 1980, p.98), in one spatial variable) for population density \( u \) (here we mention only single species models but coupled equations for multiple species models could also be treated with the ideas we outline) given from mass balance considerations by

\[
\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} (u v) = \frac{\partial}{\partial x} (\alpha \frac{\partial u}{\partial x}) + f(x, u) \tag{8}
\]

where the "directed movement" or advective (convective) term \( \frac{\partial}{\partial x} (u v) \) contains a spatially varying "velocity" term \( v = v(x) \). The diffusion term is, as usual, a result of assuming Fick's first law of diffusion, while \( f \) represents a general birth/death term. In such models it is often important to allow the transport functions \( \mathcal{D} \) and \( V \) to vary spatially, temporally, or even with population density (or perhaps some combination of these). Other basic transport assumptions or hypotheses (e.g., see (Okubo, 1980, p.84-88), (Dobzhansky and colleagues, 1979)) lead to different models, but in most cases a very important problem consists of using field data to estimate the transport functions (such as \( \mathcal{D} \) and \( V \)) and perhaps birth/death parameters in \( f \).

We have successfully applied the spline methods outlined above in connection with (1) and (2) to such problems (Banks and Kareiva, 1982). In addition to (Banks and Kareiva, 1982), one may consult (Banks, 1981), (Banks, Crowley, and Kunisch, 1981) for the theory behind our efforts. Briefly, one rewrites (8) in the form (1) in the Hilbert space \( Z = L^2(0,1) \) and then uses the approximating equation (2) - in this case we employed cubic splines
for the basis elements in $\mathbb{Z}^N$ - with the data to estimate the unknown transport functions. For the resulting finite dimensional problems we employed a standard IMSL package (ZASSQ) for the Levenberg-Marquardt algorithm in our parameter search for a fixed level of approximation $N$. In the particular problems we investigated, we hypothesized equation (8) in which $V = V(x)$, $\Theta$ is constant, and $f$ contains piecewise linear (in $u$) terms with spatially dependent coefficients. We also hypothesized unknown parameters in the initial population density. Our early efforts with field data collected by P. Kareiva (the experiments involved the dispersal of flea beetles in cultivated collard patches) revealed that models such as (8) with a spatially dependent $V$ yield significantly better fits to the data than do models with $V$ vanishing or chosen as some nontrivial constant. Our more recent efforts (detailed in (Banks and Kareiva, 1982)), again using the flea beetle data, involve the particular equation (obtained from (8) after some transformations and assumptions)

$$\frac{q_2(x)}{q_3(x)} = \frac{\frac{2}{E} \frac{\partial^2 u}{\partial x^2}}{\frac{1}{C_L} \frac{\partial^2 u}{\partial x^2} + q_3(x) \frac{\partial^2 u}{\partial x^2} + \Theta(x) u} + g(t,x) \quad (9)$$

for $t > 0$, $0 \leq x \leq 1$. The function $q_2$ is assumed to have the form

$$q_2(x) = \begin{cases} \gamma, & 0 < x < \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$

where $\gamma > 0$, and $q_3$ contains an appropriate death rate term (modest death rates within the vegetation patch, high death rate outside the patch) in addition to a term involving $q_2$. The function $g$ contains terms arising from standard transformations of (8) with nonhomogeneous boundary conditions to (9) with homogeneous conditions. The cubic spline-based estimation techniques have performed extremely well in our efforts to estimate $q_1, q_2, q_3$ as well as the initial conditions from the data. The methods were very stable and rapidly convergent, yielding satisfactory estimates for low ($N = 8,16$) values of the approximation index.

There is strong evidence (Dobzhansky and colleagues, 1979; Atkinan and Hewitt, 1972) of the need in certain population studies to estimate time dependent transport coefficients. Our cubic spline methods can be developed for these problems (see Banks and Daniel, 1981b) for preliminary theoretical results) and we are currently pursuing investigations along these lines.

There are also numerous important control problems arising in the context of ecological investigations. Once an adequate model is developed (the parameter estimation problem), one might wish to estimate (calculate) the optimal vegetation density in a patch in order to hold population levels in the patch to a minimum, or at least below some given level. We believe that the methods discussed here will also prove useful in these problems.

PARAMETER ESTIMATION IN ELASTIC STRUCTURES

We turn finally to a brief discussion of use of cubic and quintic spline schemes for parameter estimation problems arising in the study of elastic and viscoelastic bodies. Our interest in such problems was motivated by discussions with NASA engineers who desired to estimate material properties for large space structures from observations of the motions structures from observations of the motions structures. Thus a basic problem, we (Banks and Crowley, 1981) considered estimation in equations such as those arising in the Euler-Bernoulli theory for transverse vibrations of a thin elastic or viscoelastic beam which is possible subject to damping. More precisely, the well-known equations for the transverse vibrations of a thin elastic beam (no damping) are

$$\mathcal{A} = E I \frac{\partial^2 u}{\partial x^2}$$

and

$$\mathcal{B} = \frac{\partial^2 u}{\partial t^2} + f(t,x)$$

where $\mathcal{A}$ is the bending moment, $m$ is the mass per unit length and $f$ is the applied load. Two types of damping are included in our formulation. The first is simply viscous damping $\gamma u$, while the second is structural damping. For a Voigt solid (the simplest viscoelastic model) one has the constitutive relationship $\mathcal{C} = c_E = c_L$. Thus the stress $\sigma$ is no longer proportional to the strain alone (as in Hooke's law) but a term proportional to the strain velocity is added. In this case the usual Euler-Bernoulli formulation becomes

$$\mathcal{A} = \int \mathcal{F} \, dA = E I \left( \frac{\partial^2 u}{\partial x^2} + c_L \frac{\partial^2 u}{\partial x^2} \right).$$

This results in the equation

$$\frac{\partial^2 u}{\partial t^2} + m \frac{\partial^2 u}{\partial x^2} \left( \frac{\partial^2 u}{\partial x^2} + c_L \frac{\partial^2 u}{\partial x^2} \right) + u_0 = f(t,x)$$

which can be rewritten in the form (11) in the Hilbert space $H^2 = L^2$. We have developed and tested estimation schemes (for estimation of parameters such as $E I / m, c_L / m$) using cubic and quintic spline approximation subspaces $\mathbb{Z}^N$ modified to treat various important boundary conditions (simply supported, cantilevered, as well as beams with applied moments at one end). The methods proved extremely efficient as the detailed presentation in (Banks and Crowley, 1981) documents.

While the Euler-Bernoulli equation (10) is applicable in many applications (especially large space structures), a somewhat more...
involved analysis is required in situations where rotatory inertia and shear effects play a significant role in the dynamics. This theory is often necessary when high frequency oscillations of the beam must be considered (e.g., in aerodynamic structures). In this event the Timoshenko formulation is more appropriate. This theory can be embodied in a single higher order equation (fourth order in t and x derivatives) where the boundary conditions for even the simply supported beam involve second order derivatives in both x and t. For our purposes it is much more desirable to treat a system of lower order equations with the corresponding boundary conditions. The equations modeling transverse vibrations of a homogeneous isotropic elastic beam, including rotatory inertia and shear effects, can be written in terms of the transverse displacement y and the angle \( \psi \) of rotation of the beam cross section from its original vertical position as

\[
\begin{align*}
y_{tt} &= a^2 y_{xx} + x \\
y_{tt} &= b^2 y_{xx} + c^2 (y_{x} - v)
\end{align*}
\]

(11)

with the boundary conditions for, say, a fixed end beam given by \( y(t,0) = y(t,1) = 0 \), \( \psi(t,0) = \psi(t,1) = 0 \). Here \( a^2 = k/A/m, b^2 = EI/m, c^2 = Aa'^2/L \) with \( A = \) cross sectional area, \( E = \) Young's modulus, \( C = \) shear modulus, \( I = \) moment of inertia, and \( k' = \) shear coefficient.

Equation (11) can be rewritten in the form (1) in the Hilbert space \( Z = H^1_0 \times L^1 \times H^1_0 \times L \), and then cubic spline schemes can be applied (the approximating equations again have the form (2)) to estimate parameters such as \( a, b, \) and \( c \). We did this (Banks and Daniel, 1981) and once again extremely efficient algorithms resulted in very accurate estimates.

CONCLUSION

We have outlined above several problems to which our spline based approximation techniques can be applied with great success. Both theoretical and numerical findings (some reported in the literature cited, some as yet unreported in manuscripts) support our claim that these methods have even wider applicability than we have indicated here. For example, we are currently successfully applying the methods for estimation of parameters in nonlinear PDE (Banks and Daniel, 1981a) to the study of models for the enzyme column reactors as discussed in (Banks, 1981). (Daniel, 1981) As one might anticipate from the elasticity examples mentioned above, both the theoretical soundness and computational feasibility of our methods have been demonstrated for hyperbolic systems. In particular, we have successfully developed the theoretical and computational packages to treat test problems in seismic inversion (see (Banks, 1981)) in which not only the parameter \( E \) in

\[
u_{tt} = (Lu)_x
\]

but also parameters \( l_1, k_1 \) in elastic \( w_1+1_0 + k_1 w_1(t) \) and absorbing \( (u_1(t,0), k_1 u_1(t,1)) \) on boundary conditions must be identified from data.

A general theory plus numerical results obtained when applying our approximation methods to nonlinear hyperbolic and parabolic PDE can be found in (Banks and Kunisch, 1981). (Banks, Crowley, and Kunisch, 1983). Other areas of application in which we have introduced or are currently using these spline based methods include estimation problems for transport of labelled substances in brain tissue, determination of static antenna configuration and shape, and estimation of porosity and permeability in porous media.

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