A TECHNIQUE FOR IMPROVING DETECTION AND ESTIMATION OF SIGNALS

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A Technique for Improving Detection and Estimation of Signals Contaminated by Under Ice Noise

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Preface

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Title: A Technique for Improving Detection and Estimation of Signals Contaminated by Under Ice Noise

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Abstract:
Recent analyses of FRAM II arctic data have shown that under ice ambient noise can be at times highly impulsive and non-Gaussian. The analyses included time domain statistical measurements which were consistent with previously reported results of experiments made within the Canadian Arctic Archipelago. New findings of frequency domain estimates of complex skew and kurtosis and cumulative distribution functions, measured in 2, 6, and 10 Hz resolution cells at the output of a discrete Fourier transform, also indicate...
the existence of strong non-Gaussian noise. It is known that the ability to
detect and estimate signals contaminated with non-Gaussian noise using
conventional processing is degraded compared with optimum techniques which
utilize knowledge of the noise statistics. Results comparing the performance
of conventional and nearly optimum signal processing methods are presented
using the FRAM II data.
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A TECHNIQUE FOR IMPROVING DETECTION AND ESTIMATION OF SIGNALS CONTAMINATED BY UNDER ICE NOISE

INTRODUCTION

The recent analyses of FRAM II data have shown that strong non-Gaussian noise exists in the frequency domain. This technical document presents the results of processing FRAM II data in the frequency domain using data partitioning. We also present the frequency domain envelope distribution at three frequencies. These results suggest that data modeling should also be considered as a signal processing technique. However, here we shall concentrate on the data partitioning results and leave to a later time the discussion of the results of data modeling.

MATHEMATICAL PRELIMINARIES

Let the received data \( x(i,q) = x[(i*(q-1)M)]h \), \( i=0,1,2,...,M-1 \), \( q=1,2,...,n \) where \( h \) is the interval between successive samples, be composed of an additive mixture of signal, interference, and Gaussian noise of the form

\[
x(i,q) = m(i,q)s(i,q,a_1) + c(i,q)I(i,q,a_2) + N(i,q),
\]

(1)

where

\( s(i,q,a_1) \) is the transmitted or radiated signal,
\( I(i,q,a_2) \) is a narrowband interference (ice noise),
\( N(i,q) \) is independent Gaussian noise,
\( m(i,q) \) represents fading or multipath effects of the signal,
\( c(i,q) \) represents fading or multipath effects of the interference, and
\( a_t \) is a random variable taking on two values, \( a_t=0, a_t=1 \) with probabilities \( P(a_t=0)=1-L_t, P(a_t=1)=L_t \) respectively, for \( t=1,2 \).

The random variables \( a_1, a_2 \) model transient or frequency modulation components of the signal and interference, respectively, in the sense that \( L_1 \) and \( L_2 \) represent the probability of the signal or interference being in a particular frequency location measured over the observation interval. The parameters \( a_1, a_2, m, \) and \( c \) render the data non-Gaussian in the frequency domain. In general these parameters are also functions of frequency; however, for the sake of notational simplicity, we shall not explicitly show this dependence. The medium can also cause the received signal to be frequency modulated; we will assume that the frequency modulation caused by the medium is incorporated in the parameters \( a_1 \) and \( a_2 \).

The discrete fourier transform (DFT) is defined as

\[
X(q,F_p) = \sqrt{\frac{h}{M}} \sum_{i=0}^{M-1} x(i,q) \exp(-jF_p i)
\]

\[= X^r(q,F_p) - j X^i(q,F_p), \]

(2)
where \( j = \sqrt{-1} \), \( F_p = 2 \pi f_p h \), and \( f_p = \frac{p}{M} \) Hz. The components \( X_r(q,F_p) \) and \( X_I(q,F_p) \) signify real and imaginary parts, respectively. Temporal weighting may also be included but is not treated here.

If we assume that the components of equation (2) are mutually independent, the corresponding spectrum estimate is

\[
P(F_p) = L_1 E(m^2(q)) S_S(F_p) + L_2 E(c^2(q)) S_I(F_p) + R_N,
\]

where we have assumed slow fading with respect to the length of the DFT so that \( m(i,q) = m(q) \) and \( c(i,q) = c(q) \), and \( S_S(F_p) \) is the spectrum estimate at the \( p \)-th frequency of the signal, \( S_I(F_p) \) corresponds to the \( p \)-th frequency spectrum estimate of the interference, and \( R_N \) represents the independent Gaussian noise spectrum estimate. The components \( L_1 \) and \( L_2 \) were assumed to be multiplicative in the frequency domain.

The problem to be addressed here concerns processing the data in the frequency domain (as given by equation (2)) in the presence of strong non-Gaussian interference. This problem arose as a consequence of the FRAM II data analysis study, although it was addressed on a theoretical level in references 1 and 2.

In this paper we will present under ice ambient noise data which are dominated by non-Gaussian interference in both the time and frequency domains. Then we will show that by processing the data using partitioning techniques, significant performance improvements in terms of signal-to-noise ratio (SNR) are possible in the under ice environment, assuming that the signal level is much smaller than the interference. This assumption is not always met in practice. Reference 3 discusses a frequency domain processing method that treats the interference-free case.

The theory of data partitioning methods was introduced by Ching and Kurz. Here we will present only the FRAM II data partitioning results. Anyone interested in the theoretical aspects of partitioning should consult references 4 and 5.

Another approach to optimum processing is data modeling. The frequency domain envelope distribution of FRAM II data is also presented for three frequencies at three different resolutions.

FRAM II DATA RESULTS

Since the FRAM II data results of under ice ambient noise have been reported in detail in reference 6, we will only summarize the results needed in this technical document.

The data analyses are composed of time and frequency domain statistical measurements. The time domain data were filtered, sampled, and grouped into records of 1024 samples each. The mean, variance, skew, and kurtosis were then estimated for each record. Over time intervals consisting of hundreds of records, the cumulative distribution function (CDF) of the energy (square of the data samples) was estimated and was shown, for the most part, to be non-Gaussian but with nonstationary behavior over successive intervals. The time domain data were then transformed into the frequency domain via a 1024 point fast Fourier transform (FFT). Frequency domain statistics were then compiled for each frequency cell for both the real and imaginary parts.
Figure 1* shows the statistical moments for the time domain data, which were filtered through a 2500 Hz lowpass filter and then sampled at a 1000 Hz rate. Therefore each record represents a time interval of about 0.1 second, giving an overall data length of 10 minutes. Some important observations about these data are the variability of the variance over time and the significant deviation from the Gaussian assumption based on the skew and kurtosis estimates. We found, by filtering the data in bands, that the variability in the variance was due to higher frequency (greater than 750 Hz) components. The kurtosis is especially important because it indicates deviations from the Gaussian distribution by values greater or less than 3. The values greater than 3 pertain (in many cases) to distributions that are more peaked than the Gaussian distributions whereas values less than 3 correspond (in many cases) to distributions that are less peaked.* For example, a purely sinusoidal signal with uniformly distributed phase has a kurtosis of 1.5. None of the records in figure 1 has a kurtosis value of 1.5 although some are near 2. The additive Gaussian noise is most likely contributing to this result since it is known that kurtosis is SNR dependent (reference 3).

Figure 2 compares the power spectral density (PSD) (top curve), real skew (middle curve), and the real kurtosis for the under ice data which have been processed with a 10 Hz resolution and averaged over 1000 consecutive FFT's. This gives a total time interval of 1.7 minutes. The data clearly indicate non-Gaussian noise based on the frequency domain skew and kurtosis. Over a relatively flat portion of the band as seen in the PSD we estimated the amplitude CDF using the 1000 consecutive FFT's for both the real and imaginary parts. The results (figure 3) show significant deviation from a Gaussian distribution (dashed curve) for both the real and imaginary parts. Later in our discussion we will compare these results with the original data after it has been processed by partitioning.

Another data set of under ice noise is given in figures 4 and 5. The data were first filtered by a 100 Hz bandpass filter centered at 350 Hz, were sampled at 2000 Hz, and were processed in records of 1024 samples, each giving an interval of approximately 0.5 second. The important observations in figure 4 are that the variability in the variance is greatly reduced in this band and that many records deviate from a Gaussian distribution based on the kurtosis estimate. The frequency domain results (figure 5) show that many frequency locations also significantly deviate from a Gaussian distribution based on the frequency domain kurtosis estimate. In addition, a 60 Hz tonal and some of its harmonics are present in the PSD estimate in figure 5. The corresponding kurtosis estimate shows values significantly less than 3. An in-depth theoretical discussion explaining the significance of these results for a signal propagating in a medium with fading or multipath effects is given in reference 3.

**FRAM II PROCESSING RESULTS**

We shall discuss the results of processing signals in the frequency domain here. Optimum techniques for processing time domain signals in non-Gaussian noise can be found in references 7-10. These frequency domain processing

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* All figures are located at the end of the report.
+ There may be pathological distributions where this interpretation is not justified (12). However, deviations from 3 pertain to non-Gaussian distributions.
techniques, which we shall discuss, represent a new methodology for extracting signals embedded in narrowband non-Gaussian noise. References 1 and 2 develop, on a theoretical level, optimum partitioning techniques in sufficient detail that we can concentrate here on the results of processing FRAM II data. First, the results of processing the 10 Hz resolution frequency domain data by partitioning the real and imaginary parts separately will be given. Then we shall present the frequency domain envelope distribution results in 2, 6, and 10 Hz resolution cells. These results suggest that noise modeling should also be considered for optimum signal processing in the frequency domain.

Figure 6 compares the amplitude CDF of the partitioned data with that of the original data for both the real and imaginary parts. The circles correspond to the partitioned results. The interesting feature is that partitioning for this data set produces nearly a Gaussian distribution. This result can be explained by noting that partitioning is equivalent to a nonlinear transformation that significantly reduces the high amplitude excursions and leaves the smaller amplitudes unchanged. It should also be pointed out that this result of a Gaussian output was predicted in reference 11. The performance improvement can be significant, as shown by the following simple example. Suppose it is desired to set the threshold so that the false alarm rate per FFT is .0001. Then, assuming a small SNR, the partitioned detector would have approximately a 15 dB processing advantage over a conventional (linear) detector, as shown in figure 6.

Another way of comparing performance is based on the asymptotic relative efficiency, which is essentially a ratio of the output SNR of two detectors as the signal approaches zero and the integration time approaches infinity. As shown in reference 9, this turns out to be a ratio of output variances for the two detectors under noise-only conditions. Figure 7 compares, for the 10 Hz resolution case, the CDF of the partitioned and original data for the real part only. The circles represent the partitioned data. Again note the close approximation to a Gaussian distribution after partitioning. We calculated the output variance for both data sets and found that partitioning improves performance in this case by about 5 dB in the sense that its variance is decreased by 5 dB over the variance of the original data.

Another approach to optimum processing of the frequency domain FRAM II data is to model the data. Figure 8 shows a plot of the envelope distribution at the output of an FFT at three frequencies and for three different resolutions. We considered FFT's with 10, 6, and 2 Hz resolutions with time-resolution products (TRP) of 1000, 1000, and 750, respectively. The vertical scale represents the envelope (normalized by its standard deviation) in dB, and the horizontal axis is the exceedance probability. In order to better visualize the tail behavior of the envelope distribution, we included a small number of adjacent bins in the estimate. These estimates followed approximately the curves shown in figure 8. In this way we were able to extend the tail region and observe its trend. The solid line in the figure represents a Rayleigh distribution. As can be seen, the data deviate from the Rayleigh distribution for all three cases considered. This suggests the possibility of modeling the envelope distribution of FRAM II data in the frequency domain.
SUMMARY

We have presented the FRAM II data analysis results and have shown that the ambient noise was highly non-Gaussian in the time and frequency domains. The frequency domain results are new, but we have confirmed them by a theoretical analysis and by comparison with other data. For the small signal case, two techniques were considered for improving detection and estimation of signals contaminated with under ice noise.

In the first technique, we employed an adaptive partitioning method which approximates the optimum nonlinearity, as derived by an expansion of the loglikelihood ratio, for the particular noise distribution prevailing at the time a decision is to be made. Partitioning transformed the CDF of the FRAM II data into a Gaussian distribution and thereby decreased the threshold needed to maintain a prescribed false alarm rate. A particular example showed that, for a false alarm rate of .0001, a 15 dB improvement in performance could be achieved. We also calculated the variance of the original and partitioned data and found that a gain of 5 dB in performance could be achieved based solely on reduction of the variance. This improvement applies to both detection and estimation of signals in the under ice noise environment.

Another approach to optimum processing was considered. We plotted the envelope distribution, at the output of the FFT for three frequencies for three different resolutions. All three cases deviated from the Rayleigh distribution which suggests that data modeling should be considered as a signal processing approach in the frequency domain.

REFERENCES


12. A. Nuttall, Personal Communication
Figure 1. Time Domain Statistical Moments for 2500 Hz Band
Frequency Domain
Full Band
4/23/80
23:32:00 Z
10 Hz Resolution

Figure 2. Frequency Domain Statistical Moments
FREQUENCY DOMAIN: AMPLITUDE DISTRIBUTION

Full Band  Data (4/23/80)
23:31:00    Z

Figure 3. Frequency Domain Amplitude Distribution
Figure 4. Time Domain Statistical Moments for 300-400 Hz Band
Figure 5. Frequency Domain Statistical Moments for 2 Hz Resolution
FREQUENCY DOMAIN: AMPLITUDE DISTRIBUTION

Full Band Data (4/23/80)
23:31:00 Z
O PARTITIONING

![Graphs showing frequency domain data and partitioned data](image)

Figure 6. Frequency Domain Data and Partitioned Data
UNDER ICE NOISE

- 10 Hz, f = 1750 Hz
  TIME = 1.67 MINUTES

- 6 Hz, f = 1487 Hz
  TIME = 2.78 MINUTES

- 2 Hz, f = 350 Hz
  TIME = 6.25 MINUTES

Figure 8. Frequency Domain Envelope Distribution
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