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ABSTRACT

In determining policies for the acquisition and management of repairable spares for the Space Shuttle, two objectives are paramount. First is the optimization of some measure of system performance such as the expected number of shuttles launched on time per year. Second, since the cost of a spares mix can run into the hundreds of millions of dollars, we would like to minimize the cost of achieving a certain performance level. Both requirements suggest a need for mathematical models of the supply system.

The high cost, low demand rate items found on the shuttle are usually controlled via an (s-1,s) inventory system. An (s-1,s) policy involves sending an item to a repair depot immediately upon failure. Using an assumed (s-1,s) repair policy, this thesis will examine ways of choosing a spares mix according to three different mathematical models of system performance. Developed by Muckstadt [9], these models are specifically adapted to the operating characteristics of the Space Shuttle. Features of these models include a nonstationary Poisson demand rate whose parameter depends upon the pre-launch maintenance schedule and a variable weight of backorders over the pre-launch cycle. Items are maintained at predetermined points in time, and we expect more demands on the supply of repairable spares for an item whenever it undergoes maintenance. The interlaunch cycles are probabilistic replicas of one another and so form a convenient time span over which to evaluate system performance. Near the end of such cycles, the backorder cost increases sharply, and so the models allow for changing weights of their respective objective functions.

Each model generates spares mixes at various budget levels, and the performance of each mix is evaluated and compared with the performance of more elementary models. The models used for comparison include the one in use by NASA when our study was begun and a Lagrange multiplier technique based on backorders. We use these models to demonstrate that nonstationary demand rates are important only for long interlaunch cycles and for short repair times.

Another issue in the minimization of delayed shuttles is the shipment policy used by the serviceable spares supply system. Since the shuttles will some day operate out of two geographically separated locations, planners have the option of building spares facilities at one or both of the sites. They must also decide whether to initiate a lateral resupply capability in order to allow base to base shipments when desired. A computer program based on the need and reluctance formulas proposed by Miller [7] is implemented to investigate these questions. Without regard to the costs of the various shipment policies, the results indicate that the best performance is attained when both bases have spares facilities and lateral resupply capability. The next best expected performance comes from prepositioning spares facilities at both bases without the lateral resupply option. Least desir~~e~~able is a system with only one fully equipped base and lateral resupply capability.

ANALYSIS OF REPAIRABLE SPARE PARTS

STOCKAGE POLICIES FOR THE SPACE SHUTTLE

A Thesis

Presented to the Faculty of the Graduate School

of Cornell University

in Partial Fulfillment for the Degree of

Master of Science

by

Kathleen Marie Conley

January 1982

Biographical Sketch

The author was born [redacted] on [redacted] She attended elementary and secondary schools [redacted] In 1976, upon graduating from high school in [redacted] she accepted an appointment to the United States Air Force Academy. While there, she majored in operations research, humanities and management, and received the Bachelor of Science Degree in May, 1980. Honors upon graduation included the Outstanding Cadet in Operations Research and the title of Distinguished Graduate. As a Second Lieutenant in the United States Air Force, she began work in September 1980 toward the Master of Science degree in operations research at Cornell University.

Dedication

To my family.

Acknowledgements

My sincere appreciation goes to Professor John A. Muckstadt and Lt. Col. Jon Reynolds for their continued interest and participation in this effort. I would also like to thank Jim Cogliano for contributing some of the thought processes and computer methodologies used in my research.

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Introduction

The Space Shuttle, a space transportation system introduced in 1981, presents planners with several unique features in the area of spare parts provisioning for repairable items. Since the system is in some respects very like an airplane, we might initially concern ourselves with adapting the models used by military and civilian air operations. Common elements of all these systems include a number of very expensive components with low failure rates, a requirement that all components must be operating in order to take off, and a certain penalty for any delay. Thus many such systems employ what is known as an (s-1,s) inventory system, whereby if the stock level s of any spare decreases by one unit, the failed unit is immediately sent to a depot for repairs. Since the repair times are not nearly as variable as the inter-failure times, repair times often are fixed while failures are assumed to occur at a Poisson rate.

If the (s-1,s) policy is pursued for shuttle spares at a given launch site, there will be two important departures from the typical methods of setting spares levels. First, the simple or compound Poisson demand rate for failures will be replaced with a nonstationary Poisson demand rate reflecting higher failure rates at certain stages in the pre-launch countdown. Since most models employ demand rates over repair time as a measure of how many spares are needed, we must find some way of incorporating the time-dependent demand patterns into our model. Secondly, the penalty for backorders will also depend upon time, due to the nature of the pre-launch sequence. This is especially true near the launch date, when delays may cause some very costly preparations to be extended or reaccomplished.

A second critical option available to planners involves the projected management of spares assets when more than one launch site is operating. Since spares must be shipped to a storage location upon completion of repair, a good shipment rule should account for the proximity to launch at each site as well as the spares stock and expected demand at each site. More important perhaps is the evaluation of the expected performance of an initial investment in two stocking locations versus only one, and of the option for a lateral resupply capability which would allow one site to ship spares to another site when necessary.

The first problem, that of setting spares levels for repairable items, is solved by developing analytical algorithms to optimize a performance measure subject to budget constraints. Each base will be considered separately although extensions are easily made, and the depot will be seen as having infinite capacity. These and other assumptions will permit us to focus on a single inter-launch cycle and to analyze both nonstationary demands over that cycle and increasing backorder penalties near the launch date. We will present details of the models' implementations and examine the patterns of spares mixes selected by each. These patterns will be contrasted with two simpler algorithms for setting spares levels. From this comparison we will show that as the cycle length shortens, the nonstationarity in the demand rate may be ignored.

In addressing the decision of how best to design the spares transportation and location system, we will consider two sites operating at different launch rates, and employ a simulation to compare the performance of the major types of shipment disciplines. We will continue to assume nonstationary demand rates, although we will show that this assumption may be relaxed in certain cases. The simulation operates much as a continuous review inventory policy would function in real time. The number of spares in the entire system, however, is fixed, and so the simulation is not directly capable of suggesting both a good spares mix and a corresponding spares distribution system.

Some synthesis of the spares mix decisions and the transportation and stocking location problem is appropriate. Suggestions as to practical applications of the methods will be presented, as well as some insight into how they might be used interactively as planning tools.

CHAPTER 1

The number of spares required for a given item used by the shuttle depends largely upon the failure pattern it experiences and upon its importance to the timely launch of the shuttle. We make the following assumptions about the units in question and their operating environment:

1. Each item undergoes maintenance during one or more predetermined periods prior to launch.
2. An (s-1,s) inventory policy is followed.
3. The numbers of failures for the different items are independent and have a nonstationary Poisson distribution whose parameter declines as new items are substituted for failed ones.
4. Each launch cycle is of fixed length and is a probabilistic replica of all other cycles.
5. The repair facility has unlimited capacity: each item i has fixed repair time T_i which includes transportation time to and from the repair depot.
6. An unfilled demand results in a backorder, and backorders are more critical late in the cycle. Backorders do not substantially increase the length of a launch cycle, however.
7. Since resupply time between bases is small compared to repair time, a single location is examined.

These assumptions are discussed in detail in this chapter, and a general outline of shuttle pre-launch operations is presented. A discussion of models previously developed for use in this problem environment is also included.

As a first step in the analysis of shuttle operations, let us consider the maintenance cycle preceding the launch of an individual shuttle. Two components of the space shuttle, the solid rocket boosters and the orbiter are designed to be reused many times. Beginning some time after the shuttle returns from a space flight, a pre-launch cycle takes place during which maintenance crews ready the shuttle for launch according to a predetermined schedule. Detection of a failed component immediately leads to a demand on the spare parts stock and to the initiation of repair on the failed component. The number of failures experienced during the cycle is directly related to the length of the flight just completed, and the components' failure rates are expressed in terms of failures per flying hour. We may assume that flights completed prior to the most recent flight do not contribute substantially to failures in the present cycle, because items are thoroughly tested and maintained before each launch. Although those items which are replaced during a cycle have a failure rate somewhat lower than those which have undergone a space flight, the difference should be small and is not of great concern.[1]

The periods of increased maintenance activity are important in that they are often accompanied by an increased number of failures, bringing

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[1] The "new item" failure rate can appear in another context. The shuttle is composed of three main units; the orbiter, the external tanks, and the solid rocket booster (SRB). Because the solid rocket boosters are completely overhauled before the launch cycle begins, and because a new external tank is used for each launch, components of these two units have failure rates independent of flying time.

about higher demands on the stock of repairable spares. This is true for several reasons. First, the item may have failed during flight or earlier in the cycle but escaped detection (or remained inaccessible to the crews) until the crews actually had contact with it. Second, many of the tests performed to insure that an item is working properly place higher than usual stress upon the item and may contribute to a failure. Lastly, there is the possibility that the test equipment is not working properly and mistakenly indicates that the item is broken. This is equivalent, from a spares standpoint, to an actual breakdown of the item, because the item must be sent to the depot for tests. The shipment and testing time may take nearly as much time as a normal repair. The increased failure rate during these maintenance periods has been shown to be significant, so that it is necessary to treat the number of demands at a given time as a random variable with a nonstationary Poisson distribution. This is true regardless of whether the item was on the shuttle during flight or was recently installed.

A random variable is said to have a nonstationary Poisson demand rate if its failure rate varies as a function of time. Its distribution is specified by the failure rate function $\lambda_0(t)$. Through it we determine the failure rate over an interval of length Δt as follows:

$$\lambda(t,t+\Delta t) = \int_t^{t+\Delta t} \lambda_0(s)\, ds.$$

Since much of our analysis is in terms of expected demand during resupply, we define $\Lambda(i,j)$, the approximate lead time (resupply time) demand rate for item i evaluated on day j, as follows:

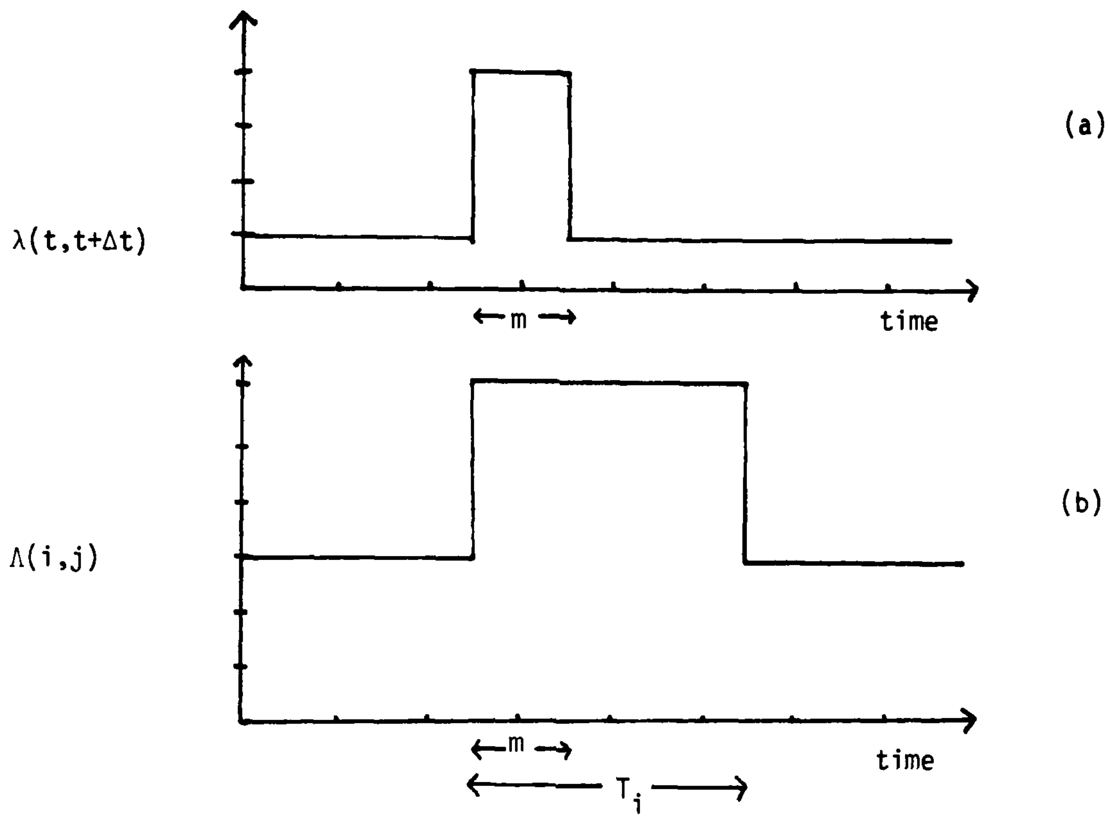


Figure 1.1 Failure Rates and Lead Time Demand Rates as a Function of Time.

$$\Lambda(i,j) = \sum_{t=j-T_i+1}^{i} \lambda(t,t+1)$$

where T_i is the resupply time for item i. Figure 1.1 illustrates how $\lambda(t,t+\Delta t)$ and $\Lambda(i,j)$ depend upon the particular time interval being examined, where m is the day on which maintenance is performed, and L is the time between launches for a single shuttle. We will refer to the sharp increase in demands on day m as a demand spike.

We will assume later that failure detection does not lengthen the maintenance period devoted to other units of the same item type, and that the length of the maintenance cycle is always fixed at L, and cannot be increased by item backorders. These assumptions enable us to assert that each cycle is identical to every other cycle in demand patterns and in length. Lastly, we assume that the cycles are continuously repeated, so that the history of demands prior to a given cycle is identical in probability to that of every other cycle.

A failed item entering repair is returned to the serviceable spares stock upon completion of repair. The repair time T is a random variable and is independent of the number of items already in repair. For our purposes it is acceptable to assume that T is fixed, and so items are returned to serviceable spares stock T time units after they are removed from a shuttle. The maintenance crews will replace a failed unit as quickly as possible, and if there are no spares on hand when a demand occurs, a backorder results. There may be some time period d(t) during which no penalty is incurred on such a backorder, but in general d(t) is quite brief (or zero) due to precedence relationships in the maintenance schedule.

While backorders on some days may carry only a small penalty, those occurring on other days, especially just before a launch, may be significantly more detrimental to shuttle operations. This is because preparations which are made just prior to launch are often more involved and costly than earlier activities. These last-minute activities range from temperature control of the liquid fuel to assembly of a launch control team consisting of many specialists and technicians. They are generally not part of the maintenance cycle as we have described it, but are rather more closely linked to the operational activity of the shuttle. A delay during this phase will be very expensive, even though it is not likely to be long enough to violate our earlier assumption of a fixed cycle length. Since cycles are expected to be on the order of a week or more, while backorders (as will be seen later) last no more than a day, backorders will probably not be the cause of prolonged launch delay. If w(t) is the severity or weight of a backorder at time t, we can assume that w(t) increases with time as shown in Figure 1.2.

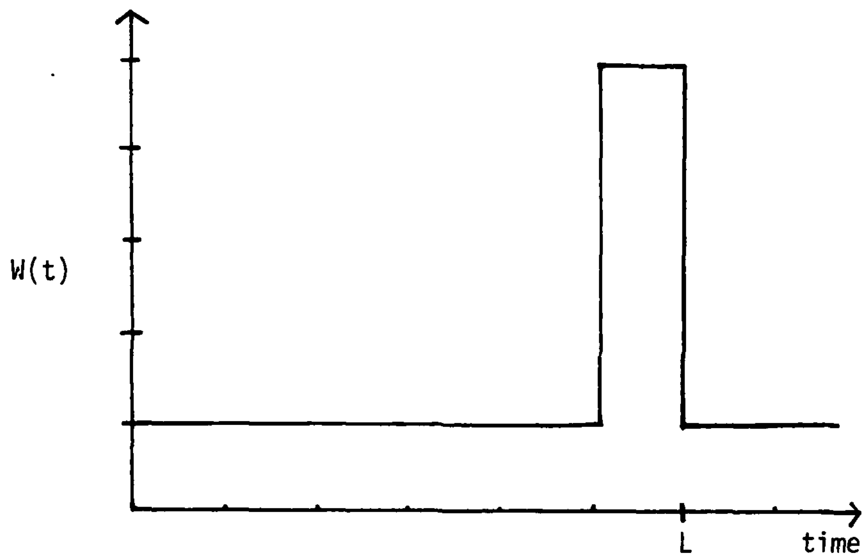


Figure 1.2 Severity of a backorder (w(t)).

After time L, we assume that the shuttle will be ready for launch. The actual launch takes place at some time during a launch window, which is limited by such factors as crew rest requirements for the astronauts, ice accumulation on fuel tanks, weather conditions, and daylight availability at both the launch and landing sites. If the countdown is interrupted and the launch window closes before the shuttle can launch, a minimum of 48 hours must elapse before the next launch attempt, resulting in another setup and incurring a large penalty cost. This sort of postponement would probably be due to a system failure, not a backordered spare. Thus our assumptions about $w(t)$ should be reasonable. In addition, since there is only a small possibility of an item failure after all pre-launch maintenance is completed, we may assume that a shuttle will make no more demands on the supply system after time L. If a shuttle should experience a significant ground delay after time L, its mission will be completed late, but there is sufficient slack in the schedule to allow that shuttle's next pre-launch maintenance to begin on time. Thus, variations in the schedule due to backorders or any other problem do not disturb the pattern of pre-launch maintenance cycles we have described. All demands for a given shuttle take place during its maintenance cycle, and each cycle is identical in probability to the next.

Current plans for the shuttle program call for one or more shuttles operating out of two launch sites. A shuttle will, in general, return to the base from which it was launched, and so at each base the shuttles will progress through pre-launch maintenance in a certain order. There will be some overlap in the launch cycles, so that the demand distribution for a given item on a given day is the sum of the demand rates for items on the shuttles being maintained on that day. The number of demand spikes experienced during any one period L is exactly equal to the number of shuttles based at the launch site, assuming that items are maintained once per cycle on each vehicle. The demand distribution for each item is still nonstationary Poisson, but with a parameter $\lambda_0(t) = \sum_{i=1}^{M} \lambda_i(t)$, where $\lambda_i(t)$

is the demand rate of the i^{th} shuttle on day t. This superposition of demand rates is illustrated in Figure 1.3 for a two-shuttle system with no time between pre-launch cycles.

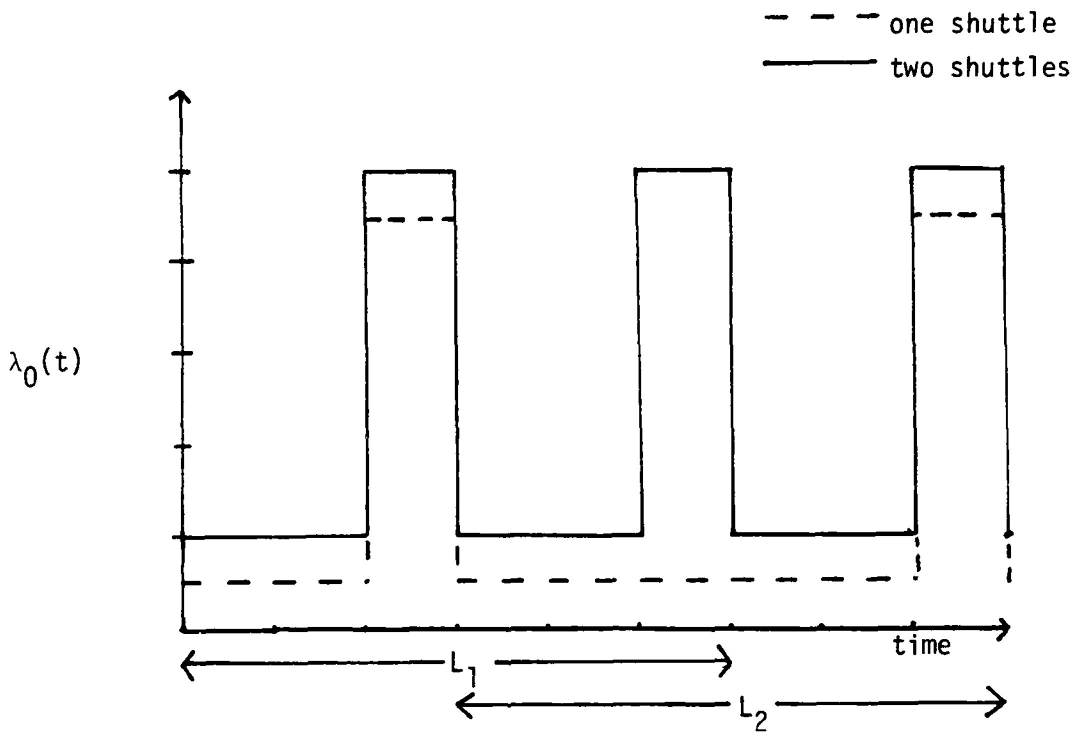


Figure 1.3 Superimposed failure rates for a two-shuttle system.

Here the length of the maintenance cycle is constant but the expected number of demands per cycle is twice as great. Note that if we redefine a launch cycle to be the time between any two launches, we experience successive periods of length L/2 whose expected number of demands is equal to the expected number of demands for one shuttle over a cycle of length L. If the time between launches is not fixed, we may not redefine the cycle in this way, and so we find it convenient to assume that the time between launches is fixed.

It is useful to point out some similarities between the system described above and a system in which there is only one shuttle launched at twice the original rate. If the launch cycle is defined as the time between launches of that shuttle, then we may generate the new demand rate by considering the demand spike to occur at the same relative position in the cycle (a proportional transformation) and then increasing the demand rate on each day by the demand rate of a day in the original cycle on which there was no spike. This will result in the cycle illustrated in Figure 1.4.

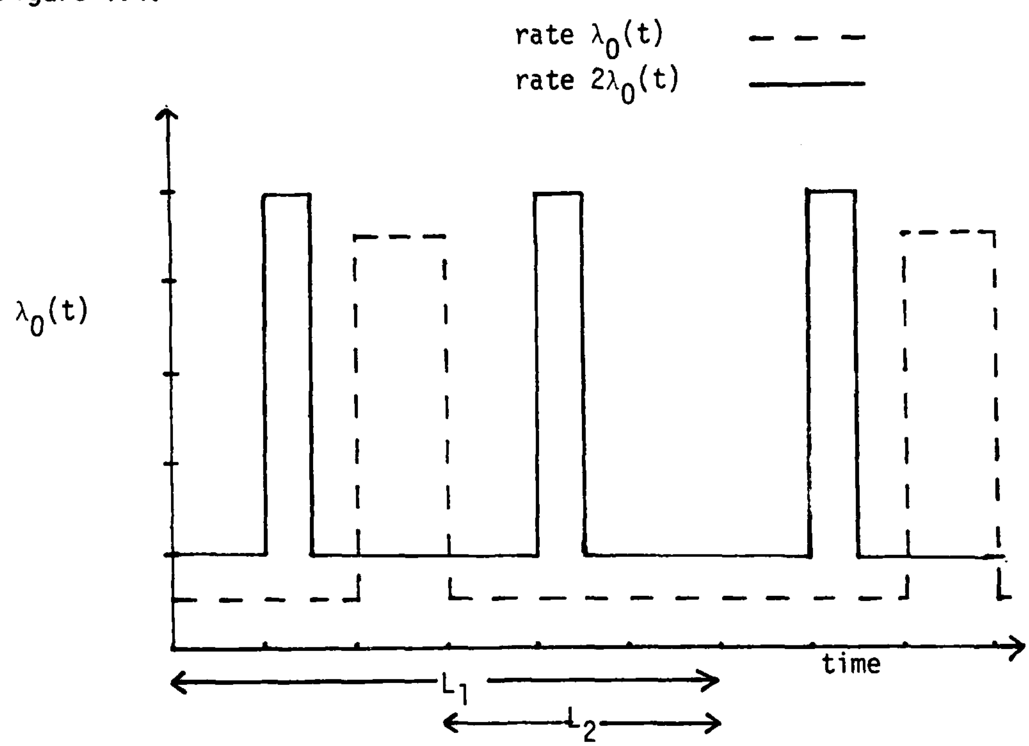


Figure 1.4 Compressed demand in a one-shuttle system.

This pattern has some important similarities to the launch cycle shown in Figure 1.3. Both cycles experience an equal number of expected backorders per cycle of length L/2, demand spikes of the same magnitude at equally spaced intervals, and interlaunch cycles which are probabilistic replicas of one another. In fact, the only important difference between the two demand patterns is the change in the position of the demand spike within the shortened cycle. Each time the cycle is compressed, the maintenance day will move closer to the beginning of the cycle for the single shuttle, whereas this will not be the case for many shuttles with overlapping launch cycles. Since the weight of a backorder near the end of a cycle is great, items with later maintenance dates will be relatively more critical than items maintained earlier in the cycle. As the cycles become shorter in the one shuttle case, certain items will consistently become more important than others. This is not the case if there is more than one shuttle operating: in that scenario an item's importance will depend upon how many shuttles there are.

The multi-shuttle case is the more realistic of the two, but one shuttle with an increasing launch rate is simpler to model, and was sufficient for our analysis when the actual day of the demand spike was unavailable. It also represents a worst case analysis, with certain items becoming more and more important as cycles shortened. The results of this effect will be discussed in greater detail later. Lastly, as we will demonstrate, the effect of performing maintenance on one day in the cycle diminishes as the launch cycle shortens. In other words, the distribution begins to approximate a stationary Poisson distribution, rendering the position of the demand spike extraneous to the spares determination problem.

The above discussion of pre-launch maintenance operations may conveniently be translated into a single location model concerned with the inter-launch maintenance for a single shuttle. If we allow the shuttle's launch rate to vary, we simulate the effect of changing the number of shuttles when each has a constant launch rate. We are interested in identifying a spares mix for serviceable items maintained during the cycle. These stock levels should give the best system performance at a given level of investment. We require a method of evaluating the expected performance of the system which recognizes the special demand distribution we have described and which allows for a changing backorders penalty over time.

Review of the Literature

This problem is similar to one encountered in setting spares levels for a variety of operations involving high cost, low demand items. Feeney and Sherbrooke [5] developed models for systems experiencing compound Poisson demand and developed measures of supply performance that could be used to minimize backorders for a single item and arbitrary resupply distribution. These same authors later optimized supply system performance under budget limitations through use of Lagrange multipliers, still for compound Poisson demand [4]. They obtained results when system performance was in terms of proportion of demands filled by on-hand inventory.

The model developed by NASA for its own use in spares provisioning had a different objective. NASA's model computed stock levels so that each item had at least a .95 probability of being filled by on-hand inventory. It assumed stationary Poisson demand rates. The NASA model contained no analysis of an item's contribution to system performance relative to its cost [12].

Mitchell [8] made changes in the NASA computer program so that it implemented marginal analysis and then computed the overall probability that demands would be met by the supply system using on-hand inventory. By examining the results for a group of items selected from the shuttle's avionics subsystems, he identified those items that accounted for the top 80% of the group's spares costs. Mitchell's model did not consider the nonstationarity we have discussed in demand patterns, nor did it account for an increased backorder penalty near the end of the cycle. Neither the NASA model nor Mitchell's model recognized that the distribution of item failures is often independent of the number of flying hours in the previous mission.

The launch cycle described here was fully developed by Muckstadt [9]. He defined criteria by which system performance could be evaluated and described algorithms by which these objective functions could be optimized. These criteria include, among others, the total weighted probability that demands are filled with on-hand inventory, the total weighted expected number of backorders, and the total weighted expected number of backorder days. These algorithms employed marginal analysis or Lagrange multiplier techniques to maximize system performance relative to the given objective function over a range of budgets. The models had not yet been implemented or compared with the methods of setting spares levels described above, and so our objective here is to determine which spares stocking model one should use in a given situation.

We now have a specific set of assumptions with which we can model the spares stocking problem. The models developed by Muckstadt seem to most nearly approximate shuttle operations as we have described them. In Chapter 2 we will continue to develop our statement of the problem and introduce the algorithms proposed by Muckstadt for determining an optimal spares mix.

CHAPTER 2

Given a set $S = (s_1, s_2, \ldots, s_N)$, where s_i represents the number of spares of type i in the spares mix S, we are first faced with the question of how to evaluate its expected performance.

The model in use by NASA gives the following rule: choose a spares mix which sets the minimum probability that any one item's demand will be met with on-hand inventory greater than or equal to a constant, denoted by PCNST. That is, choose S as follows:

$$\sum_{x=0}^{s_i} P\{R_i = x\} \geq \text{PCNST}, \quad \text{all } s_i \in S$$

where R_i equals the demand over resupply time for an item, and is assumed to be a Poisson-distributed random variable. PCNST may presumably be varied to produce different sets S. Note that we have no way of evaluating system performance, except by stating the N values of $\sum_{x=0}^{s_i} P\{R_i = x\}$.

If we also consider that each item i has a cost c_i associated with it, we may use a Lagrange multiplier technique which begins with the product of each item's probability of sufficiency and attempts to maximize this quantity for a given budget level. It chooses a spares mix S which may be evaluated by using the following function:

$$\text{POS}_s = \prod_{i=1}^{n} \left[\sum_{x=0}^{s_i} P\{R_i = x\} \right], \quad \text{all } s_i \in S$$

where POS_s is called the system probability of sufficiency.

In this section we will present three different objective functions, including total weighted probability of sufficiency, total expected weighted backorders, and total expected weighted backorder-days. Each of the three models will then treat only one of these measures of performance, and hence we expect that they will result in different spares mixes S.

For example, we may set stock levels for items A and B in Table 2.1 using the NASA model, the Lagrange multiplier technique, and the total weighted probability of sufficiency model, which we will derive later. Without detailing the actual computations, we present the resulting spares mixes $S = (s_A, s_B)$ in Table 2.2 when the spares budget is \$1,300,000 and the shuttles are launched every 35 days. We can see that the NASA model makes a different choice of S than the other two. In this case the Lagrangian technique and the weighted POS model give the same stock levels, but as can be seen from Table 2.2, they give different objective function values. For example, the Lagrange multiplier technique evaluates its objective function, POS_s, at .621. In general, we will use the objective functions presented in this chapter to evaluate the spares mixes given by all other models.

Table 2.2 illustrates how different assumptions about the spares system can lead to the use of a variety of models which can in turn give different values of $s_1, s_2, \ldots, s_N$ to the decision maker. We now present those assumptions which were used to derive the models developed here, and explain why they are reasonable in the context of the system described in Chapter 1. We will also discuss the formulations of the three models, the algorithms used to solve them, and their computer implementations.

We will present three alternative mathematical formulations of the serviceable spares mix problem. They relate to the first, second and

Table 2.1 Item Descriptions

Item	Repair Time	Cost	Daily Demand Rate	Maintenance Day
A	60 days	$415,000	.0364	34
B	60 days	$230,000	.0075	3

Table 2.2 Spares Mixes

Item	NASA	Lagrange Multiplier	Weighted POS
A	3	2	2
B	0	2	2
Objective function value	(.638, .628)	.621	.529

fourth of five models discussed by Muckstadt, and we will keep our terminology consistent with his by denoting them as models A, B and D, respectively.[1] Although the problems were initially formulated using both continuous and discrete objective functions, we will state them as discrete models with the basic time period of one day. Model A seeks to maximize the probability, weighted for each day in a vehicle's pre-launch cycle, that no item will experience more demands during its resupply time than there are spare parts. This is a weighted measure of the total

[1] Muckstadt also introduced a model to minimize the wieghted sum of shortage incidents (model C) and a model to minimize the maximum expected weighted delay days for any item (model E). Detailed discussions of both appear in reference [9].

probability of sufficiency. Model B's objective is to minimize the expected weighted number of unit backorders for all items and each day during a vehicle's maintenance cycle. Lastly, in model D we minimize the expected weighted number of days a unit is backordered, totaled over all items and all days in the vehicle's maintenance cycle.

In all three cases, the launch cycle may be assumed to last exactly L days, with $w(j)$ corresponding to the weight applied to the objective function for the given model on day j (probability of sufficiency, expected backorders, or expected backorder days). All models use the constant C to represent the amount of investment available for serviceable item spares.

The following information is required for each of the N items that compete for the limited spares budget:

n_i = number of identical units of item i aboard one shuttle,

c_i = total procurement cost for one unit of item i,

T_i = a constant resupply time for item i, including transportation time,

$\nu_i(t)$ = failure rate (failures per day) for each unit of item i, assuming one mission has occurred since its last maintenance period, evaluated at time t,

m_i = day on which maintenance crews prepare all units of item i for launch.

In all of models A, B, and D, the following assumptions apply:

1. The group of n_i identical units of item type i may be considered to be a single item whose failures have a nonstationary Poisson distribution with parameter $\lambda_i(t) = n_i\nu_i(t)$.

2. Successive cycles of varying length L are treated; there is no overlap between cycles.
3. All items are due on the day they fail. This assumption serves only to simplify computations. It may easily be relaxed within the framework of the algorithms we present.
4. The failure distribution is not significantly different for items on board the shuttle when it lands versus items replaced during the launch cycle.
5. There is no slack time between a launch and the beginning of the next maintenance cycle (i.e., flying time is assumed to be zero).
6. Since the days on which maintenance is performed on the items are unknown, the values of m_i, which are identical throughout all the models for a given cycle length L, were sampled from uniformly distributed random variables over the range (0,L).

Assumption 1 follows from the fact that the n_i units of item type i have independent identically distributed nonstationary Poisson failure distributions. So for every day j during the cycle, the sum of the n_i demand rates gives the demand rate for all units of type i. We showed in Chapter 1 that there is a similarity between a system having decreasing cycle length for one shuttle and one having an increasing number of shuttles operating simultaneously, their demand distributions effectively superimposed to give an overall demand distribution. Since we do not actually know the day of scheduled maintenance for each item, we may choose either of the two systems as a model, and we will take the first. This has the result that given a cycle length of L days and maintenance day m_i for item i, a second cycle of length αL will schedule item i for maintenance at around time αm_i (this is inexact due to the fact that αm_i may not be an integer).

The backorder penalty will begin immediately when an item is backordered. This reflects the time-critical nature of shuttle operations as well as our lack of complete information about the system and, consequently, our preference for conservative analysis. Another area where information is lacking is that of failure rates for newly installed items. We have noted that our information on failures is tied to the execution of one mission since the previous overhaul, and so the distribution of more than one failure in a given vehicle has a parameter slightly lower than $\lambda_i(t)$. Table 2.3 shows the probability for various numbers of failures over a 60-day lead time for an item having a failure rate over the lead time of 0.3. This corresponds to the highest shuttle activity rate and the highest item failure rate $n_i \nu_i$ contained in the data.

Table 2.3 Probabilities of f Failures

f	"Used Part"	"New Part"
0	.7408	.8607
1	.2222	.1291
2	.0333	.0097
3	.0033	.0004

Next, suppose that an item fails and is replaced with a part having a resupply time failure rate of .15. The individual terms for the probability that 0, 1, 2, or 3 of these new items are demanded over the entire 60-day lead time are also given in Table 2.3. But the "new" part will only be "new" for the rest of the cycle during which it is installed; since the cycle length is 8 days, this corresponds to a small fraction of the total lead time. The analysis shows that the probabilities for "new" parts are different from those for "old" parts, but that as the number of failures increases the magnitude of the difference drops off sharply. Thus we will assume that the expected failure rate for a "new" item is little different from that of a "used" item.

Assumption 5 states that slack time will be brief between the launch and the next maintenance cycle; if an actual one-shuttle facility were under study we would lengthen the launch cycle by the length of an average flight and allow zero expected demand on those days. The last assumption specifies our method of randomly assigning maintenance days. We emphasize that the maintenance schedules are identical between models for each cycle length and that if $L' = \alpha L$, then $m_i' = \alpha m_i$ for all i.

For each of the items we consider, the models' output consists of a recommended integer value for the spares level s_i, where there are N item types in the system. If each spare for repairable item i costs c_i dollars, and the investment limit is C, then we have the following constraints:

$$\sum_{i=1}^{N} c_i s_i \leq C$$

$$s_i \in \{0,1,\ldots\} \qquad i = 1,\ldots,N.$$

We are not actually given the nonstationary distribution for $\nu_i(t)$, but are instead given the failure (removal) rate per flight, r_i. Arbitrarily choosing to place all of the demand on the item's maintenance day m_i leads to the following definition of $\nu_i(t)$:

$$\nu_i(t) = \begin{cases} 0 & t \neq m_i \\ r_i & t = m_i . \end{cases}$$

In our discussion of model D we will explain the variation of this distribution used in that model. Once $\nu_i(t)$ is known, it is easy to compute $\lambda_i(t)$ using assumption 1.

All three models also involve an expression for demands over a resupply time beginning T_i-1 days before day j. The lead time failure rate $\Lambda(i,j)$ was introduced in Chapter 1. Recall that:

$$\Lambda(i,j) = \sum_{t=j-T_i+1}^{j} \lambda_i(j)$$

is an approximation of this failure rate. If we then wish to calculate $P[R_i(j)=k]$, the probability that there were k demands for item type i over the interval $[j-T_i+1,j]$, we may use the following equation:

$$P[R_i(j)=k] = \frac{e^{-\Lambda(i,j)} \cdot \Lambda(i,j)^k}{k!} .$$

Again, implicit in this equation is the assumption that all items of type i ("old" and "new") have indistinguishable failure distributions. It further assumes the following:

$P[R_i(j) > n_i+s_i] \cong 0.$ [2]

Model A

We might wish to maximize the sum of weighted probabilities of zero backorders over all days in the cycle. The probability that an item experiences no backorders is also known as probability of sufficiency. This objective function is:

$$A(s_1,\ldots,s_N) = \sum_{j=1}^{L} w(j) \prod_{i=1}^{N} P[R_i(j) \leq s_i].$$

System probability of sufficiency is a generally accepted measure of supply system performance, and the above expression extends its definition to cases of nonstationary demand over unevenly weighted cycles.

A solution procedure for this problem was discussed by Muckstadt [9]. As the objective function is nonseparable when there is more than one day j with nonzero weight w(j), we cannot employ a Lagrange multiplier technique. A workable method first computes an initial solution which gives a very low investment level and a low value of $A(s_1,\ldots,s_N)$. The algorithm sequentially selects the spares whose contributions to the objective function are greatest relative to their costs. The value of $A(s_1,\ldots,s_N)$ is improved every time a spare is added to the mix. This

[2] We justify this notion because the items we analyze do have very low failure rates, and because the higher an item's failure rate, the higher its stock level. For $\Lambda(i,j) = .3$, a high rate for our analysis, and $s_i = 4$, $n_i = 2$, $P[R_i(j) = 7] = 3.2 \times 10^{-8}$. Thus we may avoid the mechanical complexity of using a truncated nonstationary Poisson distribution. To be strictly correct, the probability of more than s_i+n_i demands should be exactly zero for the one-shuttle case.

is basically a marginal analysis technique and yields good, if not optimal, results for many applications.

The problem is formally stated in Figure 2.1 and a diagram outlining the solution procedure appears in Figure 2.2. For additional information, see Muckstadt's paper [9].

$$\text{Maximize} \sum_{j=1}^{L} w(j) \prod_{i=1}^{N} P[\hat{R}_i(j) \leq s_i]$$

$$\text{subject to} \sum_{i=1}^{N} c_i s_i \leq C$$

$$s_i \geq 0 \text{ and integer, } i = 1,\ldots,N.$$

Figure 2.1 Problem Statement for Model A

Model B

In this model we are interested in minimizing the total expected weighted number of backorders for all items throughout the launch cycle. As before we will be evaluating the objective function for each day and then summing across days and items, rather than trying to measure expected backorders continuously through time. For a single item i, the expected number of backorders on day j is given by the following:

$$E[B_i(j)] = \sum_{x>s_i} (x-s_i)\ P(R_i(j) = x),$$

assuming that items incur a penalty from the first moment they are

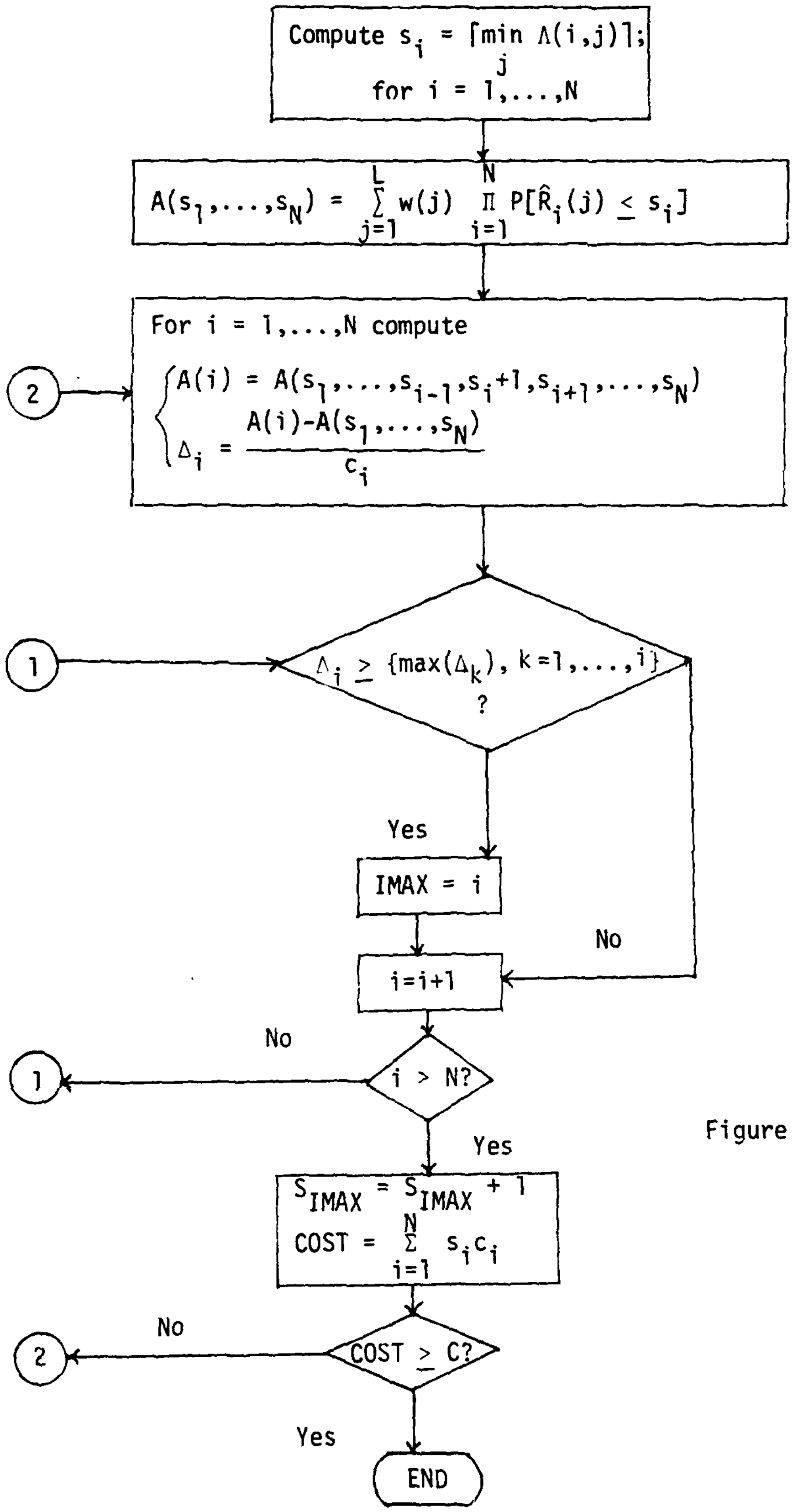


Figure 2.2 Flowchart for Model A.

backordered, where $R_i(j)$ has the same meaning here as in Model A. The problem is formally stated in Figure 2.3.

Model B

$$\text{Minimize} \sum_{i=1}^{N} \sum_{j=1}^{L} w(j) \sum_{x>s_i} (x-s_i)*P[R_i(j)=x]$$

$$\text{subject to} \quad \sum_{i=1}^{N} c_i s_i \le C$$

$$s_i \in \{0,1,\ldots\}, \quad i = 1,\ldots,N$$

Figure 2.3 Problem Statement for Model B

Muckstadt [9] points out that this is a separable problem and rewrites the objective function using a Lagrange multiplier θ. Thus, for each i, the object is to minimize the following for a given nonnegative θ:

$$F_i(s_i) = \left[\sum_{j=1}^{L} w_i(j) \sum_{x>s_i} (x-s_i)\ P[R_i(j)=x] + \theta c_i s_i \right],$$

$$s_i \in \{0,1,\ldots\}.$$

Since F_i is convex in s_i [9], we may minimize its value by taking first differences, and identifying the smallest s_i for which adding an additional spare will cause $F_i(s_i)$ to decrease. In other words, we are finding:

$$\min\{s_i : F_i(s_i) - F_i(s_i+1) \le 0\}$$

$$= \min\{s_i : \sum_{j=1}^{L} w(j)[(\sum_{x>s_i} (x-s_i)P\{R_i(j)=x\} + \theta c_i s_i)$$

$$- (\sum_{x>s_i+1} (x-(s_i+1))P[R_i(j)=x] + \theta c_i s_i + \theta c_i)] \le 0\}$$

$$= \min\{s_i : \sum_{j=1}^{L} w(j)(1 - \sum_{x=0}^{s_i} P[R_i(j)=x]) \le \theta c_i\}.$$

Examining the above equation, we see that it involves the probability of one or more backorders: [3]

$$1 - \sum_{x=0}^{s_i} P[R_i(j)=x].$$

This quantity provides some insight for choosing θ. Denote the maximum acceptable probability of a backorder by $(1-\alpha)$. Then we must choose θ so that $\theta \cdot c_i$ will always give an acceptable upper bound to the weighted probability of a backorder. If k is the costliest spare, θc_k is the greatest upper bound we will create, and we must insure that the following holds:

$$\sum_{j=1}^{L} w(j) \cdot (1-\alpha) = \theta c_k.$$

The result is to choose θ so that

$$\theta = \frac{\sum_{j=1}^{L} w(j) \cdot (1-\alpha)}{c_k}.$$

[3] As before, we note that the probability of more than n_i+s_i backorders is close to zero. When only one shuttle is involved, the actual probability of more than n_i+s_i backorders is exactly equal to zero.

If we then allow θ to decrease, the upper bounds will all decrease, and we will need higher stock levels to bring down the weighted backorder probabilities. The outcome of decreasing acceptable weighted backorder probabilities is to increase spares costs. Muckstadt [9] proposed computing spares levels for all items at decreasing levels of θ. This will result in steadily increasing spares costs until we reach the desired budget level C. The successive values of θ and the corresponding investment required are related in Figure 2.4.

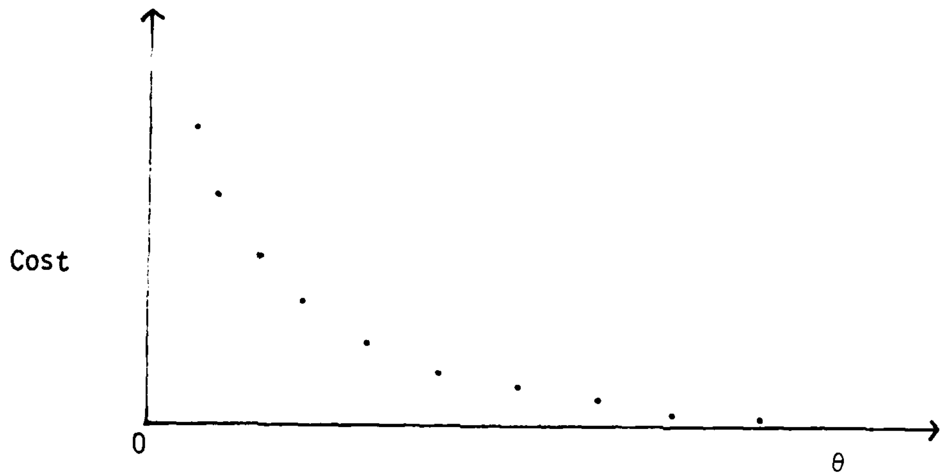


Figure 2.4 Investment in Spares vs. Multiplier θ.

The algorithm used to implement the above ideas is shown in Figure 2.5. Note that the original objective function for Model B is not directly minimized. However, if any c_m is "close to" C, then the $\{s_i: (i=1,\ldots,N)\}$

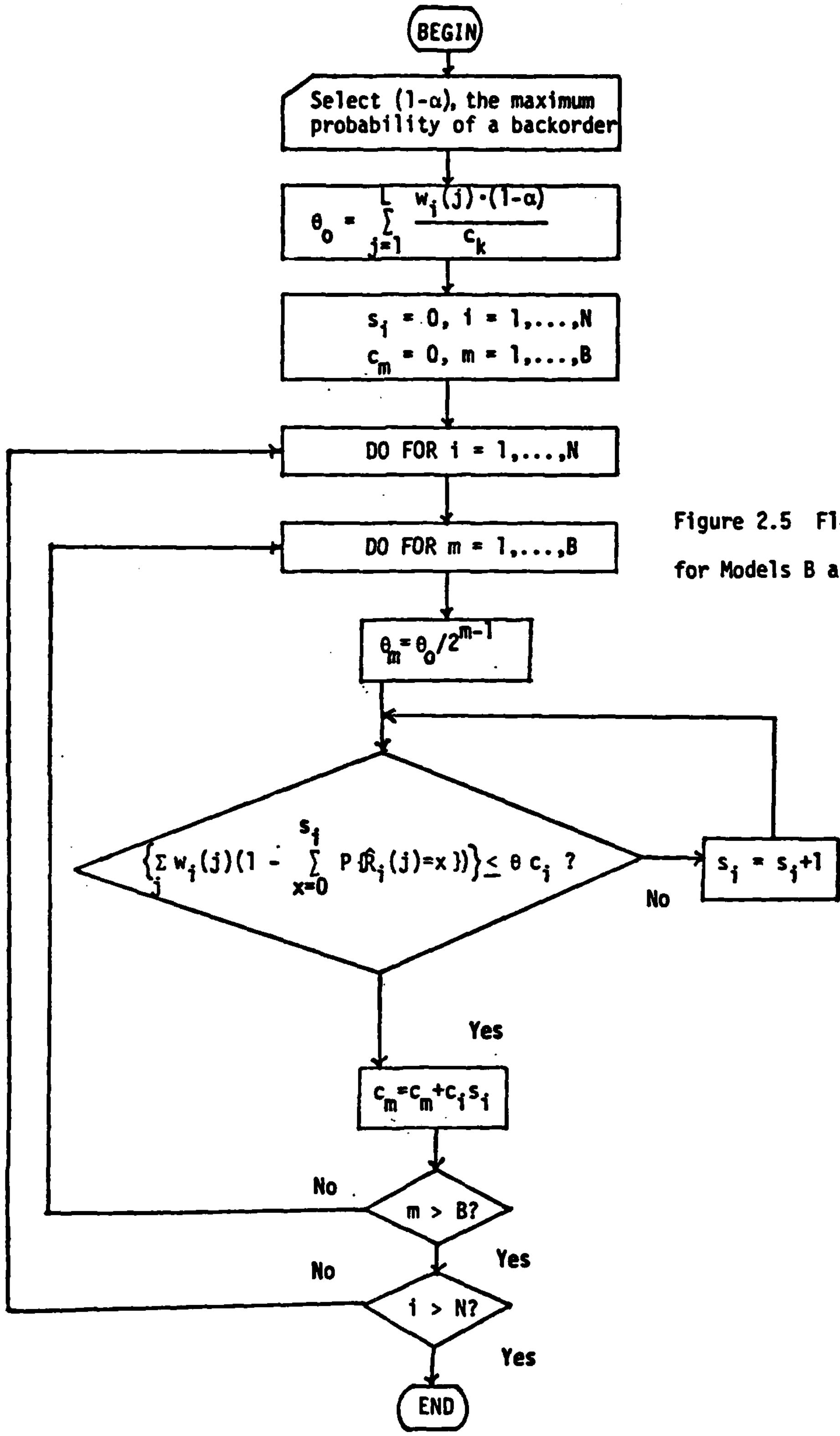


Figure 2.5 Flowchart for Models B and D

which relate to c_m will give a near-optimal solution. By changing the increments of θ, we can generate any number of pairs (c,θ). The algorithm presented here produces B such pairs by setting $\theta_m = \theta_o/2^m$, $m = 1,\ldots,B$.

Model D

A third model considered by Muckstadt [9] involved minimizing the total weighted expected backorder-days across the cycle. The expected waiting time per backorder, which is just the length of time that a shuttle is delayed on the average, may be expressed as $E[D_i(j)]$. Here we assume that the length of a backorder is independent of the number of backorders outstanding. This is in keeping with the earlier assumptions of fixed resupply time and low expected numbers of backorders.

We now require an expression for the length of time which a backorder lasts, $E[D_i(j)]$. The following relationship holds in the case of stationary Poisson demands:[4]

$$E[D_i(j)] = \frac{E[B_i(j)]}{\frac{1}{T_i} \sum\limits_{t=j-T_i+1} \lambda_i(t)} .$$

First, we note that were we to use this approximation in Model D, there is a possibility of having a resupply time demand rate equal to zero should the interval $(j-T_i+1,j)$ contain no demand spikes. In order to avoid this situation, we may redefine the unit failure rate $\nu_i(t)$ as follows:

[4] This equation follows from $W = L/\lambda$ where W is the waiting time for all backorders, L is the number of backorders, and λ is the arrival rate.

$$\nu_i(t) = \begin{cases} \dfrac{r_i}{3L} & t \neq m_i \\[2ex] \dfrac{2r_i}{3} + \dfrac{r_i}{3L} & t = m_i. \end{cases}$$

Thus the resupply time failure rate is less variable than in the case when all demand is concentrated on one day in each cycle. We continue to calculate this value as the sum of the daily failure rates over the resupply time.

A second important observation regarding this approximation is that it is independent of the stock level s_i and so may be included in the constant term w(j) used in Model B. We define $w_i(j)$, the weighted number of days that a backorder on day j of item i will wait, as follows:

$$w_i(j) = \frac{w(j)}{\frac{1}{T_i} \sum_{t=j-T_i+1}^{j} \lambda_i(t)},$$

where w(j) has the same meaning here as it did in Model B. Using $w_i(j)$ instead of w(j) in Model B will give stock levels which minimize the expected weighted backorder-days for all items.

An important change to Model B will be required when appropriate values for θ are sought, however. Since the sum of the weights may be different for each item, θ_m will now be given by the following:

$$\theta_m = \min_i \left[\frac{(1-\alpha) \sum_{j=1}^{L} w_i(j)}{c_i} \right].$$

We noted earlier that the approximation discussed above is generally used only in cases where the demand distribution is stationary Poisson.

In order to test the robustness of the equation when $\lambda_i(t)$ varies over time, a test was devised using a computer simulation. The simulation will be discussed in detail later, but for now a simple description of the system will suffice.

Figure 2.6 depicts the simulated system, consisting of two bases with six and eight shuttles, and a single depot with a repair time of 60 days. Lateral resupply between bases is not permitted. There is one unit aboard each shuttle with a failure rate r_i = .0166, and turnaround time is 50 days. Four days prior to each launch, the failure rate for the items in all eleven shuttles increases for one day. This is a very rapid activity rate which, as we will later discuss, detracts from the effects of nonstationarity. However, computer run time is a limiting factor, and so to observe many backorders it is necessary to have a high activity rate. Seven spares are initially provided to each base, and subsequent spares shipments from the depot are made on the basis of greatest need.[5]

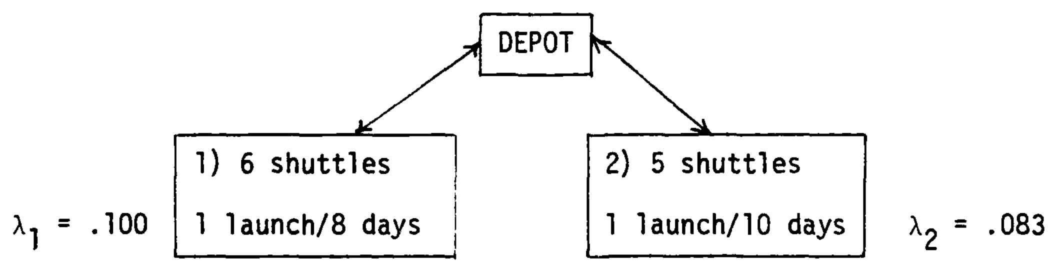


Figure 2.6 System Used for Simulation

[5] A quantitative definition of base need will be given in Chapter 4 and is a function of the base's inventory position and of the time remaining until the launch.

In tabulating the results of the simulation, which ran for 500 "days", we first record the average number of shuttles grounded, $\overline{L}$. Then, on every day, we observe a value of the lead time demand rate multiplied by the actual waiting time experienced by a backorder on that day (if any). These values yield some average value over 500 days of $\overline{\Lambda W}$. The comparisons of the values of $\overline{L}$ and $\overline{\Lambda W}$ for an increasing nonstationarity factor ρ are shown in Table 2.4 and are plotted in Figure 2.7. (ρ = 2.0 indicates that the demand rate doubled once before each launch.)

Table 2.4 Validation of Waiting Time Approximation
$\overline{L} = \overline{\Lambda W}$ with Nonstationarity Factor ρ.

ρ	$\overline{L}$	$\overline{\Lambda W}$
1.0	0	0
2.0	0	0
2.5	.022	.025
3.0	.048	.062
3.5	.082	.106
4.0	.126	.182

Note that as nonstationarity increases, backorders are overestimated to a greater and greater extent, at least in the simulation we performed. We caution that the simulation parameters do not approximate the system we describe here with any great degree of accuracy. The results of the simulation do suggest, however, that as nonstationarity increases, the

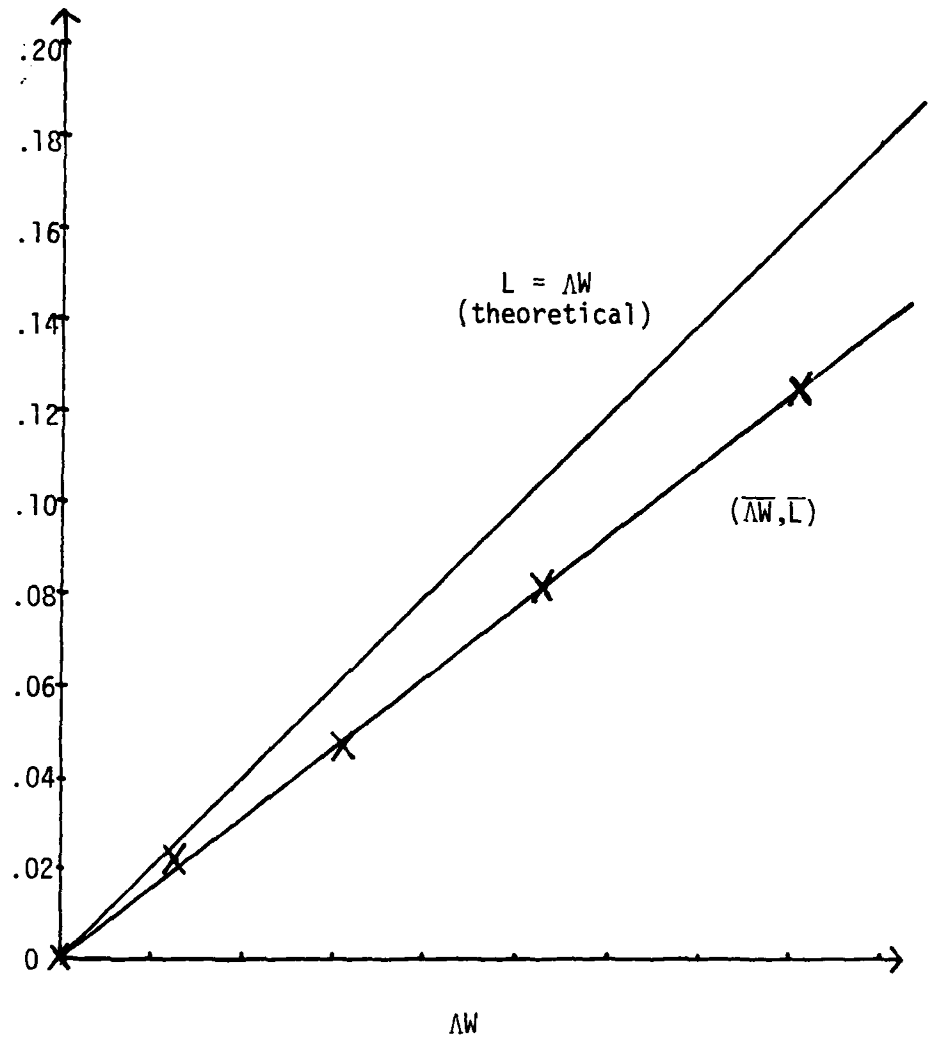


Figure 2.7 Comparison of actual backorders ($\overline{L}$) and estimated ($\overline{\Lambda W}$) using simulation.

approximation tends to overestimate expected backorders in this case.

Sherbrooke [11] notes that for simple Poisson demand with parameter λ_i and fixed resupply time T_i, the probability distribution of waiting time D_i for item i, is given by the following relation:

$$P[D_i \leq d] = \begin{cases} \sum_{m=0}^{s_i-1} e^{-x} x^m/m!, & d < T_i \\ 1 & , d \geq T_i \end{cases}$$

where $x = \lambda_i*(T_i-d)$. In the case of nonstationary Poisson demands, the demand rate over time period $[j-(T_i-d)+1,j]$, would be required for all values of d, between 0 and T_i, for each day j and item i. Denoting this value as $\Lambda(i,j,d)$, we have the following expression for expected waiting time;

$$E[D_i(j)] = \sum_{d=1}^{T_i} (1 - \sum_{k=0}^{s_i-1} \frac{(e^{-\Lambda(i,j,d)})(\Lambda(i,j,d))^k}{k!})$$

The expression for expected backorder days over the cycle,

$$\sum_{i=1}^{N} \sum_{j=1}^{L} E[D_i(j)] ,$$

is a separable relation and is convex in $(s_1,s_2,\ldots,s_m)$. We now have two approximations, and choose to implement the approximation introduced earlier by Muckstadt.

Using the approximation for expected waiting time given by Muckstadt, we may write an objective function for Model D. The integer program for Model D appears in Figure 2.8.

Model D

$$\text{Minimize} \quad \sum_{i=1}^{N} \sum_{j=1}^{L} w_i(j) \sum_{x>s_i} (x-s_i)*P[\hat{R}_i(j)=x]$$

$$\text{subject to} \quad \sum_{i=1}^{N} c_i s_i \leq C$$

$$s_i \geq 0 \text{ and integer, } i = 1,\ldots,N.$$

Figure 2.8 Problem Statement for Model D

With the exception that w(j) is now replaced with $w_i(j)$ and that θ_m is derived differently (as outlined above), the algorithm for Model B given in Figure 2.5 applies to Model D as well.

The Computer Models

The three models just described were each implemented using FORTRAN programs on an IBM 370/168 computer running with VM operating system at Cornell University. The programs each have essentially the same structure, and were modifications of programs written earlier by Cogliano [2]. The program structure is shown in Figure 2.9. Element 4 in Figure 2.9 represents the core of each program. Computer listings of these three subroutines may be found in Appendix A. NASA has provided failure and cost data on items in the avionics subsystem [12]. These items are reduced to those 24 that NASA computed as being the most expensive from

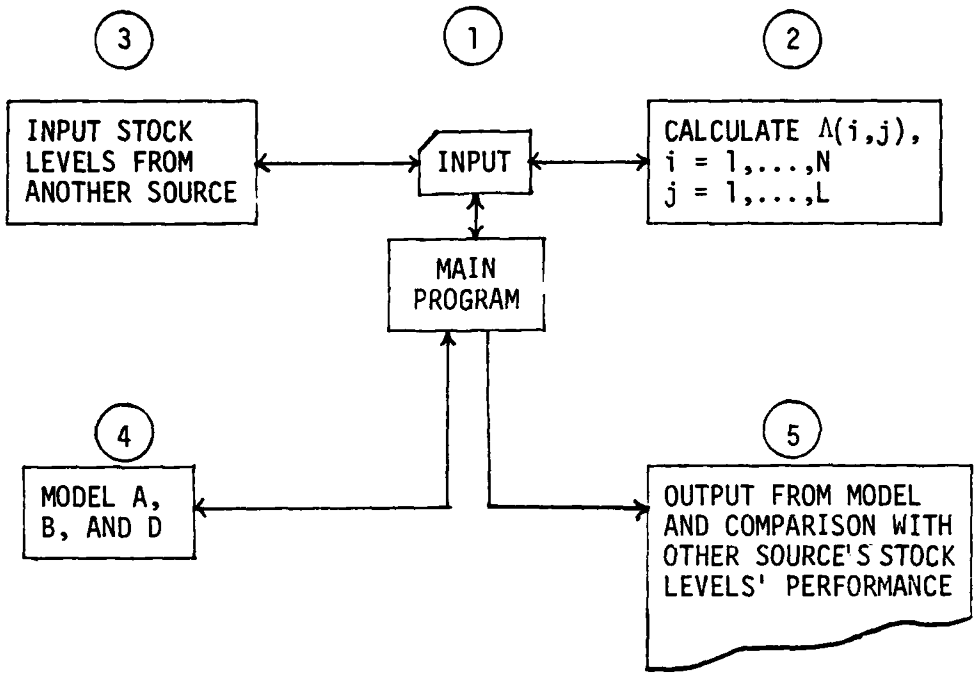


Figure 2.9 General Outline of Computer Models[a]

a spares standpoint. Together they represent 80% of the spares cost when NASA's probability of sufficiency model is used and each item is computed to a .95 probability of sufficiency (i.e., PCNST = .95).

Using the program that NASA had developed we generated stock levels based on probability of sufficiency and compared them with the solutions given by Models A, B, and D. With stock levels from the NASA program as input, the programs for models A, B, and D computed their expected performance using the objective function for each model. We were thus able to establish a common measure for comparison purposes.

[a] Circled numbers indicate the order of operation.

Similarly, another set of stock levels was produced by a computer program by use of a Lagrange multiplier technique to minimize backorders when demand was assumed to be stationary and there was no increased backorder penalty [1]. These stock levels are also used as input for models A, B, and D in order to evaluate the performance of Lagrangian analysis with respect to the objective functions developed in this section.

Information concerning costs, failure rates, and resupply times are not varied throughout the experimental procedure. In order to retain comparability with the NASA model, we assume that two shuttles make simultaneous demands on the supply system. (Note that this is the worst case of a two-base system in which a common depot is used and the distance between the bases is ignored, as no two shuttles could be launched simultaneously from the same launch complex.) Shuttle planners feel that a reasonable range of launch cycle lengths would include cycles of 4, 8, 16, 32, and 50 days, and so runs were performed for each of these activity levels. The last important variable we have identified is the weight of the launch day as compared with all other days in the cycle. To reflect a possible increased backorder penalty on this day, we let the weight be equal to either one, corresponding to an equal weighting, or five, on the last day of every cycle.

Another key element for the three models involves the shape of the failure rate function. If this rate is nonzero only on the maintenance day for item i during a vehicle's launch cycle, the resulting nonstationarity will be as severe as possible. This is why we chose earlier to define the failure rate function in this manner, except for Model D, which, for reasons discussed earlier, must experience some minimal failure rate on every day during the cycle. We will now discuss the impact of this assumption in some detail.

We have the following values for $\lambda_i(t)$:

$$\lambda_i(t) = \begin{cases} L\cdot r_i & t = m_i \\ 0 & \text{otherwise.} \end{cases}$$

In Model D, this equality is only an approximation. Therefore, the lead time demand $\Lambda(i,j)$ will be some multiple of $L\cdot r_i$. In Figure 2.10(a) the lead time is slightly less than L, and so $\Lambda(i,t_1)$ is equal to 0. At $j = t_2$, however, we have $\Lambda(i,t_2) = L\cdot r_i$. Similarly, for the longer lead time T_i' in Figure 2.10(b) $\Lambda(i,t_1) = L\cdot r_i$ while $\Lambda(i,t_2) = 2(L\cdot r_i)$. Since $\Lambda(i,t_1)$ and $\Lambda(i,t_2)$ can differ from one another by at most $L\cdot r_i$ no matter how t_1, t_2 and T_i are chosen, we have the following inequality:

$$0 \leq \frac{|\Lambda(i,t_1) - \Lambda(i,t_2)|}{\Lambda(i,t_k)} \leq 1 , \qquad k = 1 \text{ or } 2, \quad \Lambda(i,t_k) > 0.$$

It is easy to see that as T_i increases, the variation in lead time demand represents a diminishing proportion of the constantly increasing values of $\Lambda(i,t_k)$. As we will show later, however, as few as 2 or 3 demand spikes during the lead time are enough to erase the effects of nonstationarity in the spares systems we examined. The argument may be formalized as follows: we recognize that $T_i \equiv nL + \Delta t$ where $\Delta t < L$ and $n \geq 0$ and integer. It can easily be shown that the average value of $\Lambda(i,j)$ over the cycle has the following relationship to T_i:

$$\frac{\frac{1}{L}\sum_{j=1}^{L}\Lambda(i,j)}{T_i} = \frac{r_i(\Delta t(n+1)+(L-\Delta t)n)}{nL + \Delta t} .$$

Denoting this average ratio as U_i, we see that:

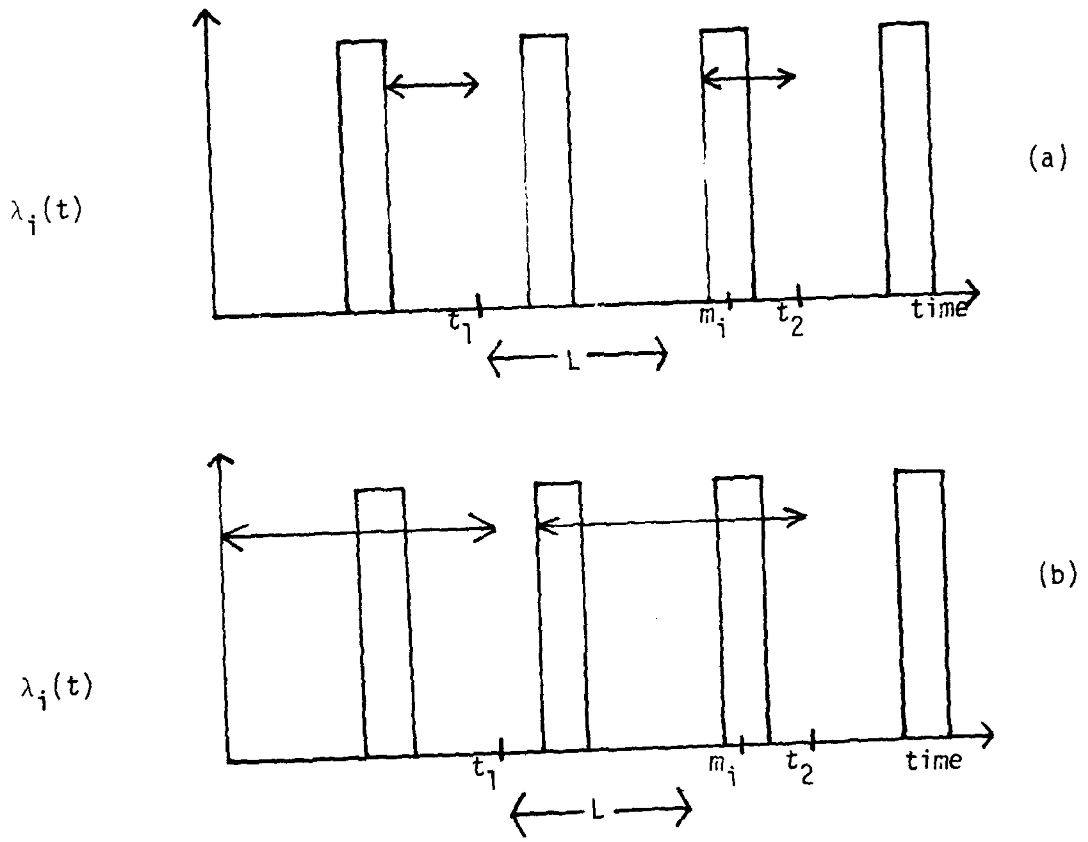


Figure 2.10 Variability in Lead-Time Demand

$$U_i = r_i.$$

Further, if we define the variability of U_i as follows:

$$V(U_i) \equiv \frac{1}{L} \sum_{j=1}^{L} (U_i - \frac{\Lambda(i,j)}{T_i})^2$$

we may also show that the following holds:

$$\lim_{n\to\infty} V(U_i) = 0.$$

Since the asymptotic properties of V will result from either fixing T_i and decreasing L or fixing L and increasing T_i, we note that nonstationarity will become less important $(V(U_i) \to 0)$ if either the resupply time increases or the launch cycle becomes shorter. U_i can thus be seen as a measure of nonstationarity with variability converging to 0 as n gets large. We will see that as U_i becomes less variable for all items i as a result of steadily decreasing L, the results of Models A, B and D more closely approximate those of models where stationarity is assumed. Finally, we note that these results support our earlier claim that a single shuttle with steadily increasing launch rate will require the same spares support as an increasing number of shuttles. This is because both result in the same values for L which will in turn produce equal values of U_i for each item i.

A sample table of the input data is given in Table 2.5. Computational results are discussed in Chapter 3. The programs' output includes, for a range of investment levels, the performance which the algorithm achieves in terms of its objective function, as well as the spares mix it identifies. The output may also include the value relative to the objective function achieved by spares mixes supplied externally. The sources of these stock levels are the NASA program and the marginal analysis program discussed earlier.

NUMBER OF ITEMS 24
NUMBER OF SHUTTLES 2
FIRST LAUNCH ON DAY 35
WEIGHT OF SPIKES= 5.
TOTAL DEMAND PER CYCLE 12.75796

INPUT DATA

ITEM	DAY OF DEMAND SPIKE	DAILY DEMAND RATE	COST	REPAIR TIME (HOURS)
1	34	0.03643	$229500.00	1440.
2	10	0.03522	$228000.00	1440.
3	27	0.01218	$402800.00	1440.
4	20	0.00850	$706000.00	648.
5	30	0.00748	$437500.00	1440.
6	3	0.00968	$415140.00	1440.
7	35	0.00360	$527000.00	1440.
8	21	0.00360	$527000.00	1440.
9	31	0.01494	$450000.00	360.
10	14	0.00713	$289000.00	1440.
11	2	0.00374	$343750.00	1440.
12	21	0.00279	$598700.00	240.
13	18	0.00279	$598700.00	240.
14	7	0.00374	$281250.00	1440.
15	8	0.00143	$547000.00	1440.
16	8	0.00090	$527000.00	1440.
17	4	0.00180	$527000.00	1440.
18	14	0.00180	$516400.00	1440.
19	13	0.00775	$257000.00	648.
20	6	0.00151	$451000.00	1440.
21	20	0.00643	$222000.00	648.
22	32	0.00628	$221000.00	1440.
23	11	0.00048	$437500.00	1440.
24	32	0.00207	$435000.00	1440.

Table 2.5. Input Data

CHAPTER 3

Each model's performance is measured and evaluated by use of three analytical approaches. The output of each program is first compared with the NASA and Lagrange multiplier techniques discussed earlier. Thus we may determine the effects of nonstationarity and weighted back-orders with respect to these two baselines. If Models A, B, and D provide the same levels of the objective function for less cost than the other models, then we have effectively exploited our assumptions about the system. Next, each model is analyzed to determine its behavior in terms of the objective function as well as the spares mix when back-order weights increase and inter-launch cycles lengthen. In this section, we will refer to the cycle as the time between consecutive launches. Finally, we contrast the spares mixes at comparable levels of investment for each of the three models. In this manner we may draw inferences as to the consequences to the spares mix of selecting one objective function over another.

The computer codes for each of models A, B and D's main subroutines appear in Appendix A. The subroutines not shown are essentially the same for each model, and relate to input and output functions. A comment in Model A's listing reveals the procedure used to determine lead-time demand rates unique to Model D, and so the code for the computations of lead-time demand rates is omitted from listings of the other two models. In terms of their own objective functions, the models performed better than, or as well as, both the NASA and the simple Lagrange multiplier models. For cycle lengths of 4, 16, and 50 days, the results appear in graphical form in Figures 3.1-3.18. A table of values for a 35-day cycle is given in Table 3.1 and sample output for each program appear in Tables 3.2, 3.3 and 3.4.

Table 3.1 Performance of Models for a 35-Day Cycle:

SPARES BUDGET	LAGRANGE POS(S)	SPARES BUDGET	MODEL A POS(S)	SPARES BUDGET	NASA POS(S)
4699440.	0.29019	9938780.	0.70345	9205780.	0.63102
5552240.	0.32797	10833780.	0.76034	10203780.	0.68576
6606240.	0.43380	11637780.	0.80632	10870780.	0.72495
6835740.	0.44005	12789580.	0.85227	11387780.	0.77347
8840880.	0.56348	13528580.	0.87660	12135780.	0.81561
9129880.	0.60244	14722580.	0.90753	12538580.	0.83192
11061780.	0.76431	15937830.	0.93465	17881088.	0.91871
11838280.	0.79978	16808576.	0.95082	18797600.	0.95258
14952720.	0.89015	17882192.	0.96341	27862400.	0.99616
		18801392.	0.97240		
		19915872.	0.98187		
		20911856.	0.98667		
		21668336.	0.98906		
		22825328.	0.99197		

SPARES BUDGET	LAGRANGE E(BO)	SPARES BUDGET	NASA E(BO)	SPARES BUDGET	MODEL B E(BO)
		9205780.	1.1561	4074440.	2.9783
4699440.	2.5893	10203780.	0.9926	6835740.	1.7145
5552240.	2.2029	10870780.	0.8197	11061780.	0.6912
6606240.	1.8057	11387780.	0.6721	13703080.	0.3783
6835740.	1.7145	12135780.	0.5606	15632220.	0.2525
8840880.	1.1364	12538580.	0.5159	19765072.	0.0838
9129880.	1.0611	17881088.	0.1808	22026784.	0.0463
11061780.	0.6912	18797600.	0.1245	24727168.	0.0246
11838280.	0.5762	27862400.	0.0133	28638688.	0.0094
14952720.	0.3145				

SPARES BUDGET	LAGRANGE E(BO DAYS)	SPARES BUDGET	MODEL D E(BO DAYS)	SPARES BUDGET	NASA E(BO DAYS)
4699440.	9.8615	8279340.	2.9900	9205780.	4.9967
5552240.	9.4755	10355640.	1.9000	10203780.	3.3543
6606240.	7.9368	13938180.	0.8187	10870780.	3.1798
6835740.	7.9070	16766480.	0.4379	11387780.	3.0145
8840880.	6.4390	20438992.	0.1764	12135780.	2.0523
9129880.	6.2991	22748496.	0.0794	12538580.	2.0062
11061780.	3.7093	24005792.	0.0553	17881088.	0.4438
11838280.	2.8763	26691936.	0.0276	18797600.	0.3556
14952720.	0.8821	28949904.	0.0161	27862400.	0.0335
144155440.	0.0	32819488.	0.0050		

SUMMARY OF TOTAL ASSETS AND THEIR DISTRIBUTION

ITEM	SPARES STOCK LEVELS	PROBABILITY OF SUFFICIENCY
1	4	0.88443
2	3	0.96333
3	2	0.94474
4	1	0.96360
5	1	0.90250
6	1	0.95409
7	1	0.97313
8	1	0.97313
9	1	0.90274
10	2	0.98567
11	1	0.99214
12	0	1.00000
13	0	1.00000
14	1	0.99214
15	0	0.95131
16	0	0.96900
17	1	0.99810
18	1	0.99270
19	1	0.96926
20	0	0.94857
21	1	0.97814
22	2	0.98980
23	0	0.98325
24	1	0.99043

TOTAL SPARES INVESTMENT 9072640.00

SYSTEM POS 0.64623320

Table 3.2 Sample Output for Model A

SUMMARY OF TOTAL STOCK LEVELS AND EXPECTED BACKORDERS

ITEM	SPARES STOCK LEVELS	TOTAL EXPECTED AVERAGE SHORTAGES
1	8	0.10
2	8	0.07
3	4	0.13
4	2	0.24
5	3	0.14
6	3	0.32
7	2	0.15
8	2	0.15
9	3	0.09
10	3	0.12
11	2	0.15
12	1	0.10
13	1	0.10
14	2	0.15
15	1	0.28
16	1	0.11
17	1	0.45
18	2	0.02
19	2	0.19
20	1	0.32
21	2	0.11
22	3	0.07
23	1	0.04
24	2	0.03

TOTAL EXPECTED WEIGHTED BACKORDERS 0.0463
TOTAL SPARES INVESTMENT 22026784.0

Table 3.3 Sample Output for Model B

SUMMARY OF TOTAL STOCK LEVELS AND EXPECTED BACKORDER DAYS

ITEM	SPARES STOCK LEVELS	TOTAL EXPECTED BACKORDER DAYS
1	4	0.05
2	4	0.04
3	2	0.06
4	1	0.04
5	2	0.02
6	2	0.04
7	1	0.09
8	1	0.09
9	1	0.02
10	2	0.02
11	2	0.01
12	1	0.00
13	1	0.00
14	2	0.01
15	1	0.03
16	1	0.02
17	1	0.04
18	1	0.05
19	1	0.04
20	1	0.04
21	1	0.03
22	2	0.02
23	1	0.01
24	1	0.05

TOTAL EXPECTED WEIGHTED BACKORDER DAYS 0.818674
TOTAL SPARES INVESTMENT 13938180.0

Table 3.4 Sample Output for Model D

Looking at Figure 3.1, we note that in a four-day cycle, Model A does perceptibly better than both the simple Lagrangian technique and the NASA probability of sufficiency model. For a given budget level, the improvement over the Lagrangian method is about 1%, measured in terms of weighted probability of sufficiency. When the weight increases from one (an equal weighting) to five, we see in Figure 3.2 that little change in the models' relative performance is evident. Moving to a 16-day cycle in Figure 3.3, we see an increased differential in performance level, to about 2%. This relfects the increased nonstationarity effects inherent to a longer cycle. A similar increase is evident in Figures 3.5-3.6 when the launch days are increased to 50 days apart, leading to a performance increase of about 2-4%. The curves for Model A are derived by executing the algorithm presented in Chapter 2 and stopping every time $1 million is added to the total spares cost. Since the data points for the NASA program and the Lagrange multiplier technique are also discrete, due to the integer nature of the decision variables, it would be misleading to fit a curve through them. We are limited to comparing points which are more or less adjacent to one another and drawing inferences from their costs and performances relative to the objective function. Thus one conclusion we may draw from the data is that as the launch cycle for a single shuttle becomes shorter, the effects of the nonstationarity we expressed in the formulation of Model A are less pronounced.

Secondly, it appears that the backorder weight on the launch day does not affect the choice of a spares mix as much as does the non-stationary nature of lead-time demand. It should be the case, however,

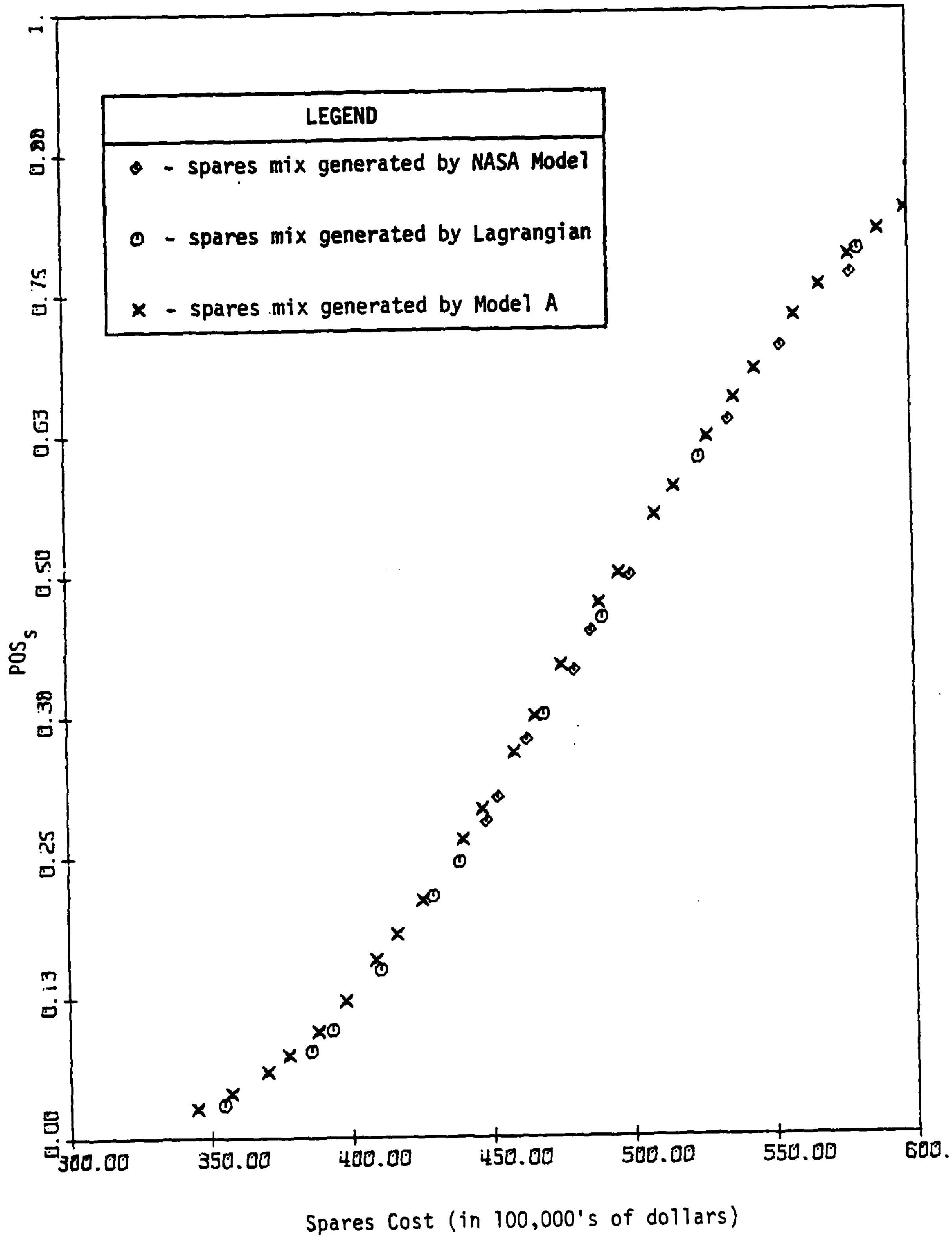


Fig. 3.1 Spares Mix Comparison for Four Day Unweighted Cycle - Model A.

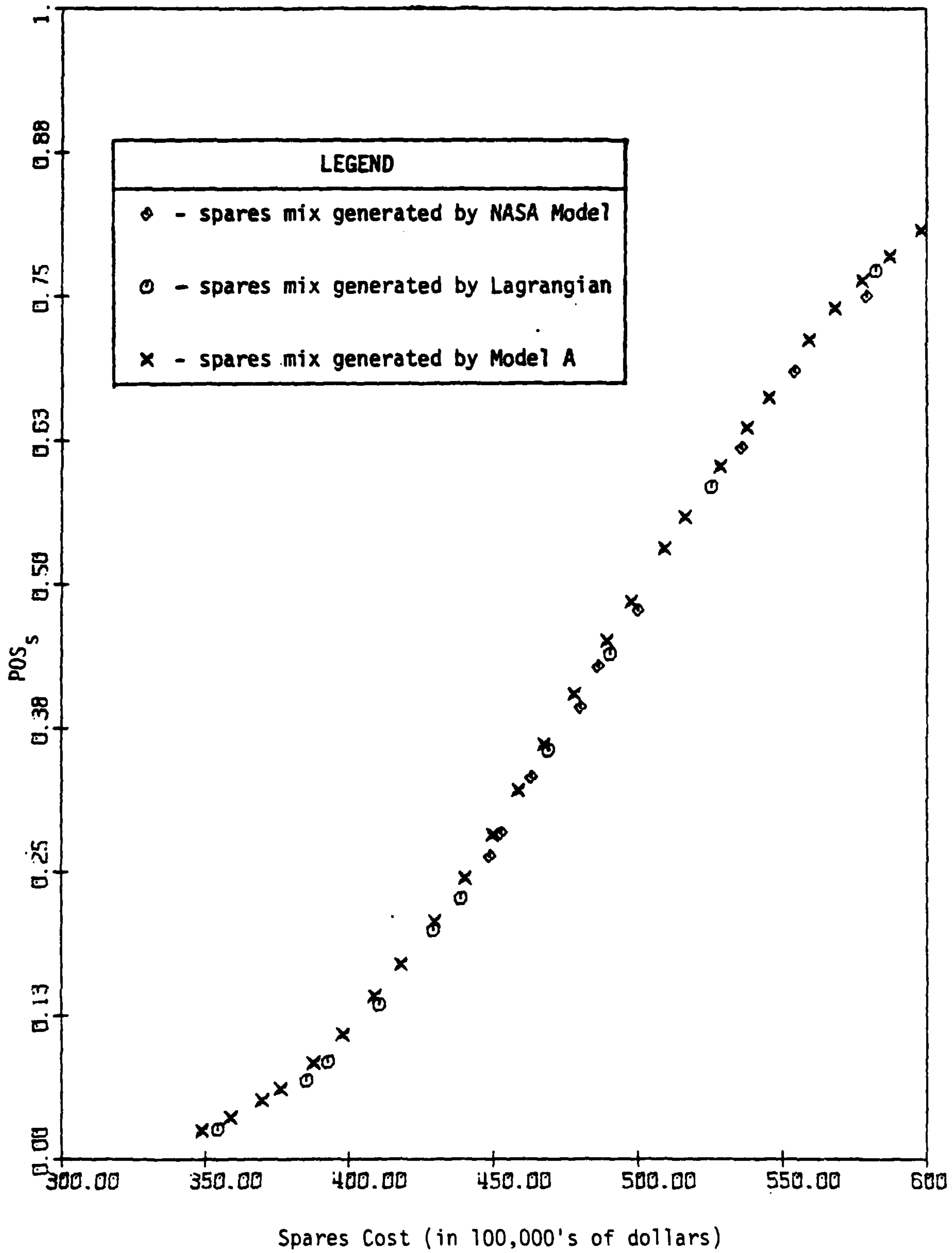


Fig. 3.2 Spares Mix Comparison for Four Day Weighted Cycle - Model A.

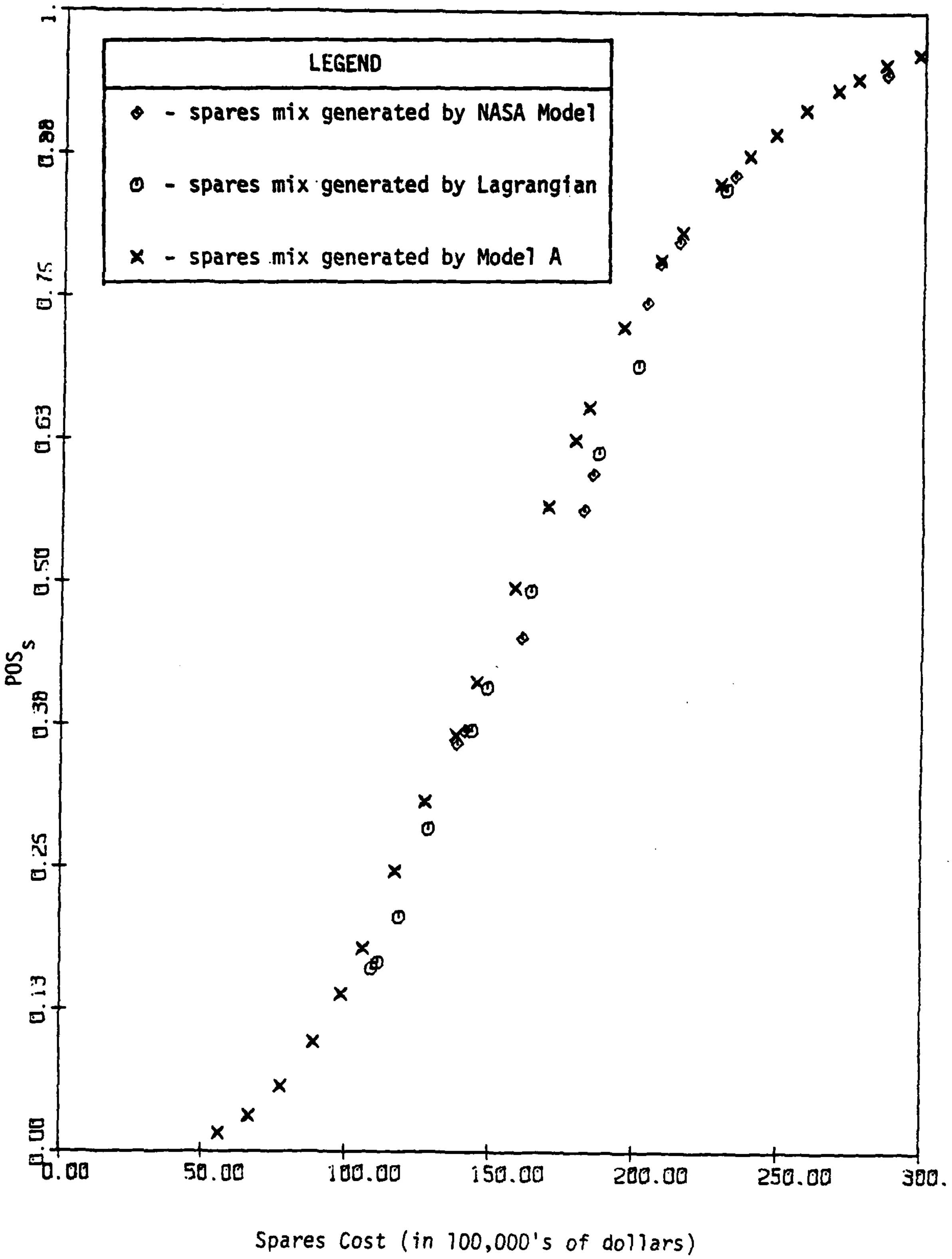


Fig. 3.3 Spares Mix Comparison for Sixteen Day Unweighted Cycle - Model A.

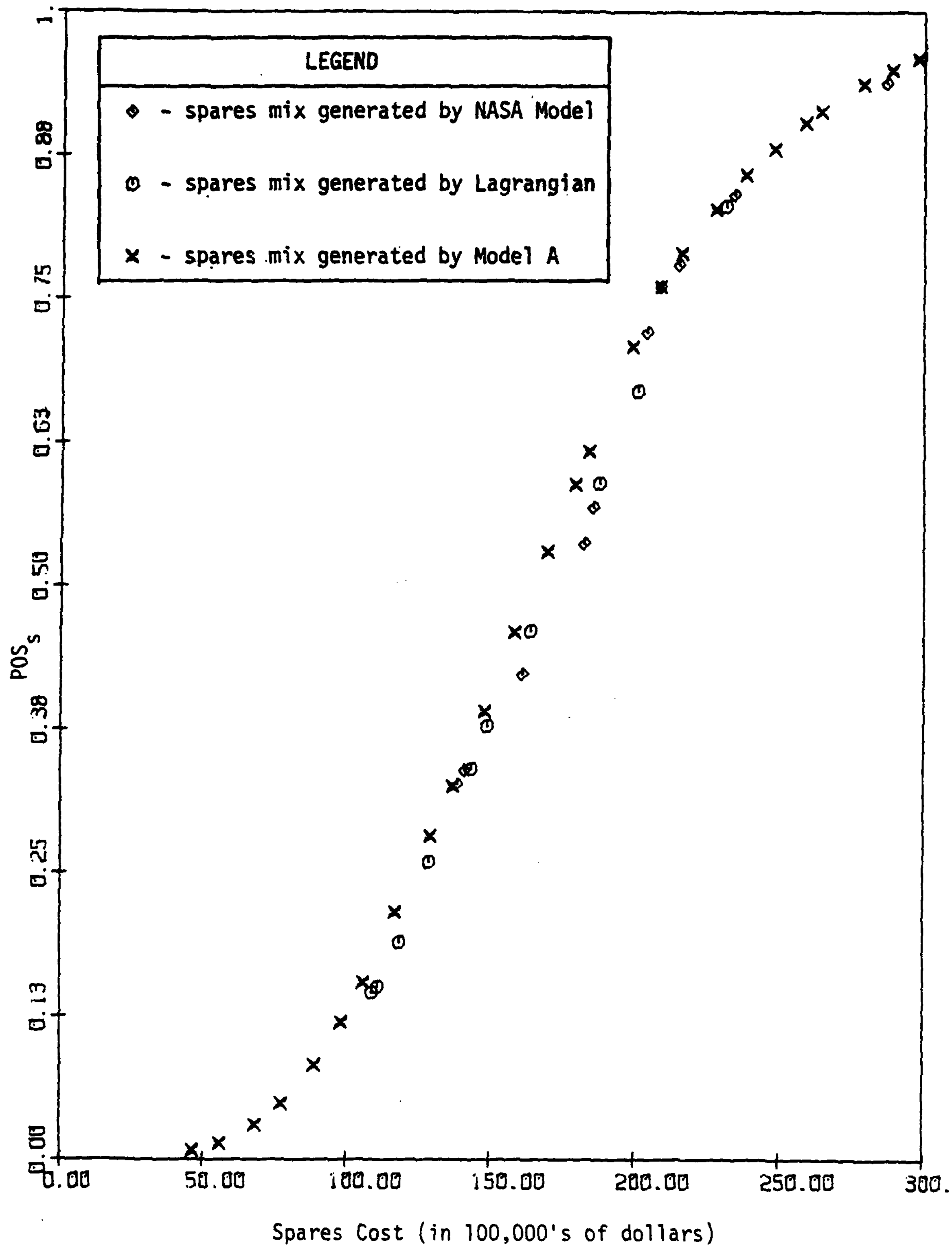
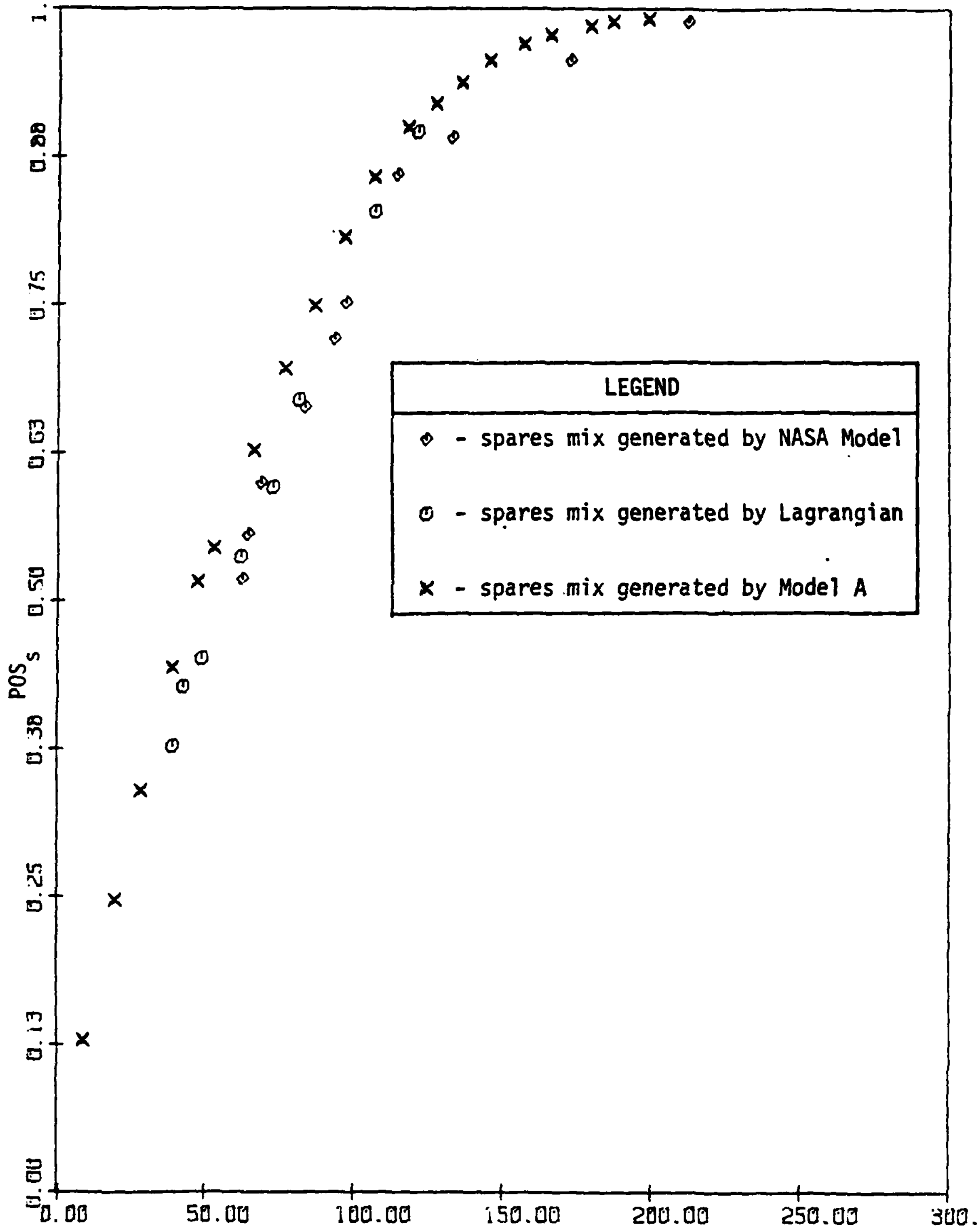


Fig. 3.4 Spares Mix Comparison for Sixteen Day Weighted Cycle - Model A.



Spares Cost (in 100,000's of dollars)

Fig. 3.5 Spares Mix Comparison for Fifty Day Unweighted Cycle - Model A.

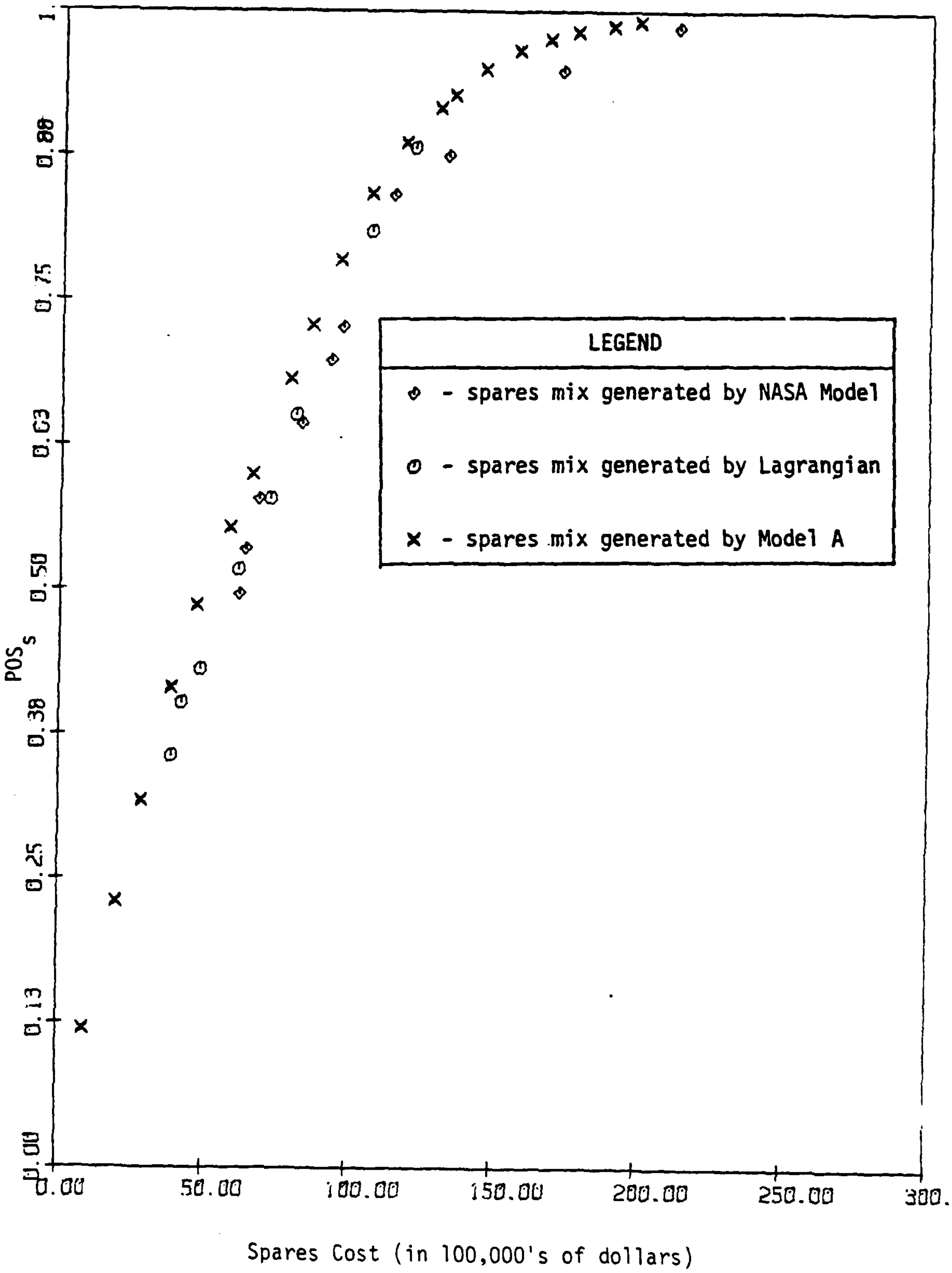


Fig. 3.6 Spares Mix Comparison for Fifty Day Weighted Cycle - Model A.

that if only certain items were given increased backorder weight and if the increase lasted longer than one day, those items would necessarily be stocked to a greater depth in a spares mix produced by Model A. Model A would then be more responsive to weights than it is under the environment we assumed for our experiment.

The output for Model B, assuming a four-day cycle and equal weighting of the launch day, is shown in Figure 3.7. Here we see a difference of about 2-4% between the performances of Model B and the NASA model, but no apparent difference between it and the Lagrange multiplier method. The same pattern is observed in Figure 3.8 for a launch day weight of 5, only the performance of all the methods has dropped. In fact, Model B actually chose the same stock levels as it did for a launch weight of 1, suggesting that it is perhaps not possible to increase system performance by modifying stock levels if a model considers only the increased weight of the launch day relative to cost. When the cycle length increases to 16 days, in Figures 3.9-3.10, we again observe that Model B gives generally lower backorders than does the NASA technique, but achieves close to the same performance as a Lagrangian technique. Even where the cycle between the launches is 50 days, in Figures 3.11-3.12, we fail to distinguish an improvement over the simple Lagrangian technique when we use Model B. Since the Lagrange technique has a measure of backorders as its objective function, we would expect that of the three models, the results of Model B should be most closely approximated by the Lagrange multiplier method. Therefore, we recognize that any improvement obtainable using Model B might not be evident among the small number of spares we examined.

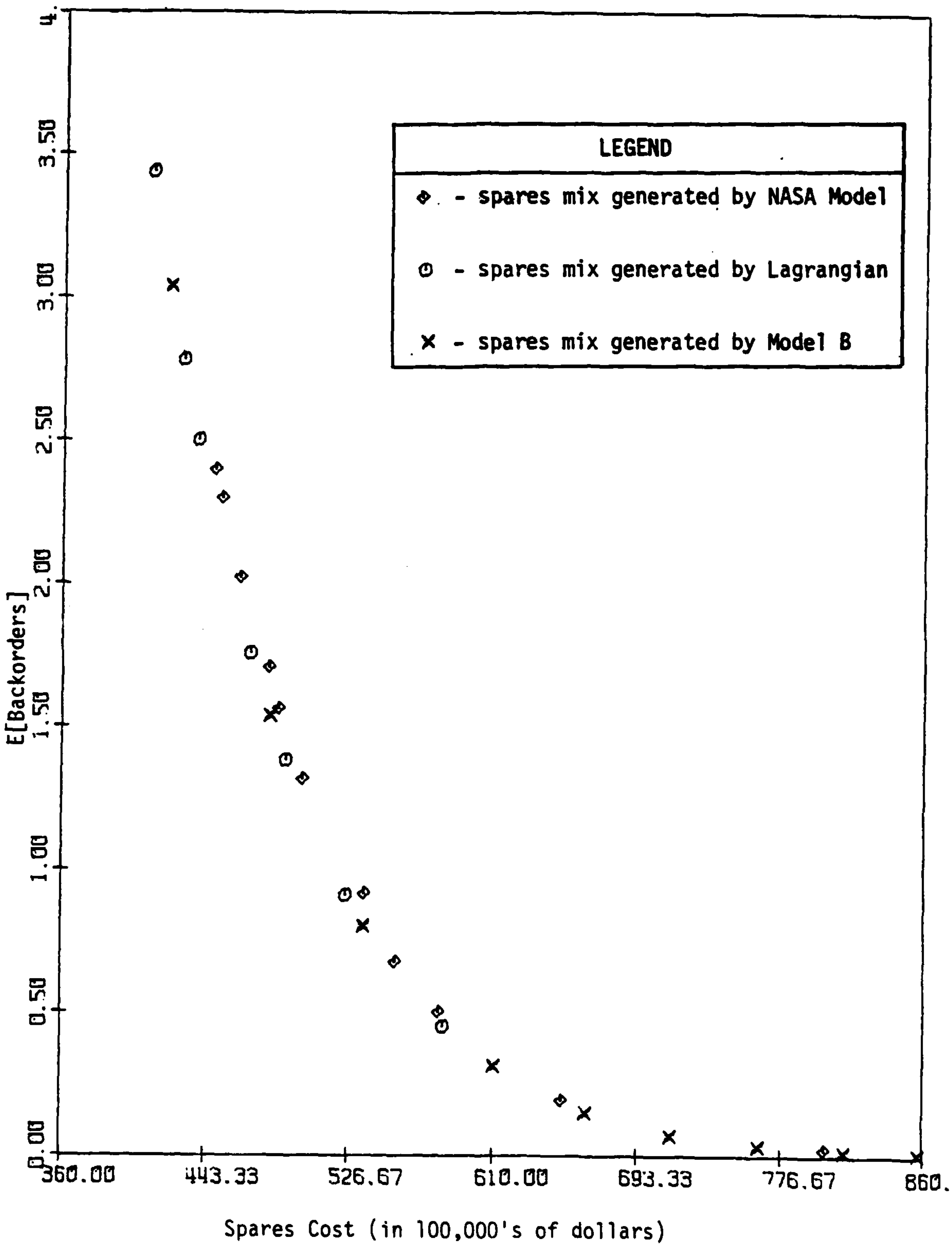


Fig. 3.7 Spares Mix Comparison for Four Day Unweighted Cycle - Model B.

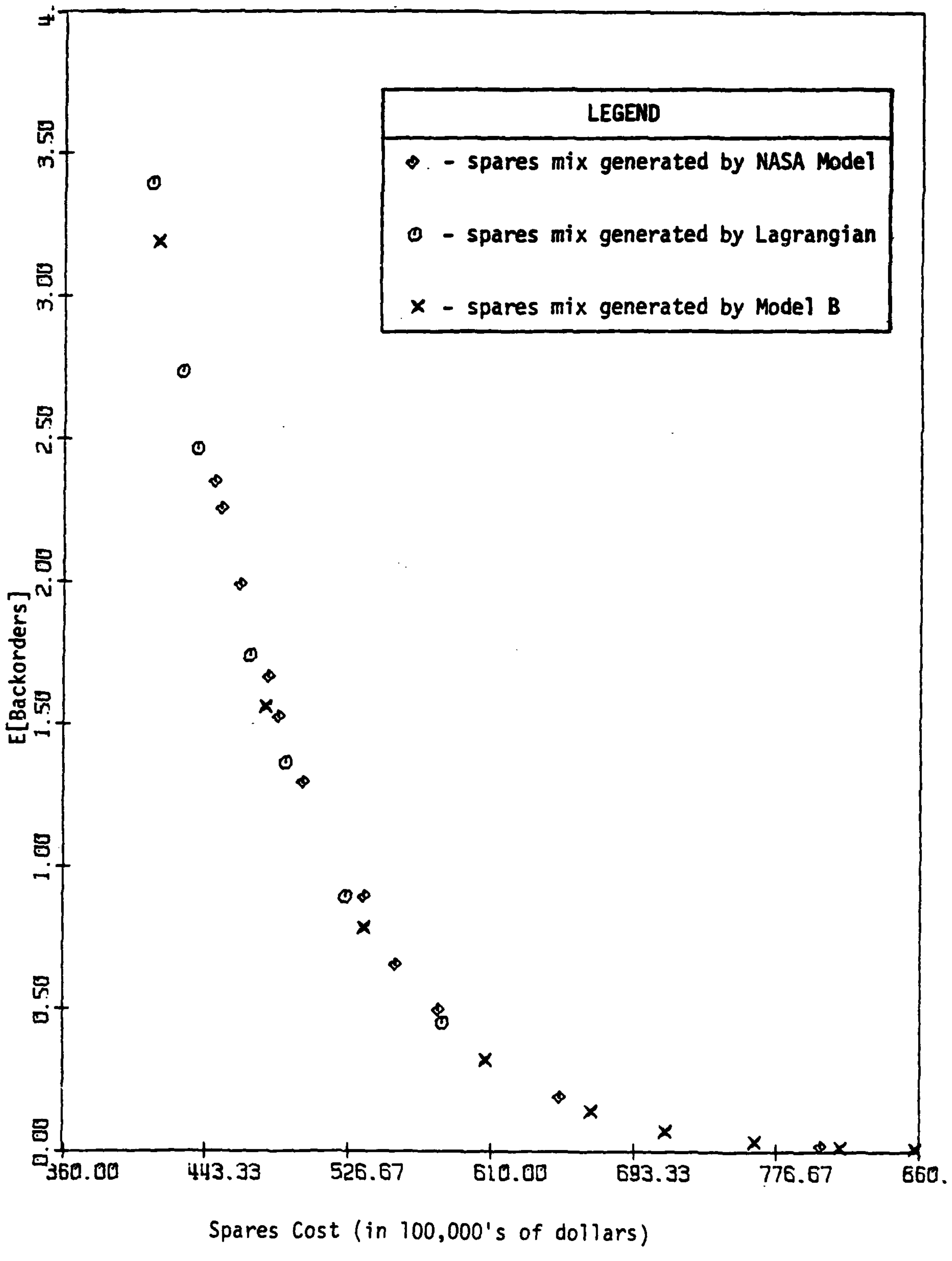


Fig. 3.8 Spares Mix Comparison for Four Day Weighted Cycle - Model B.

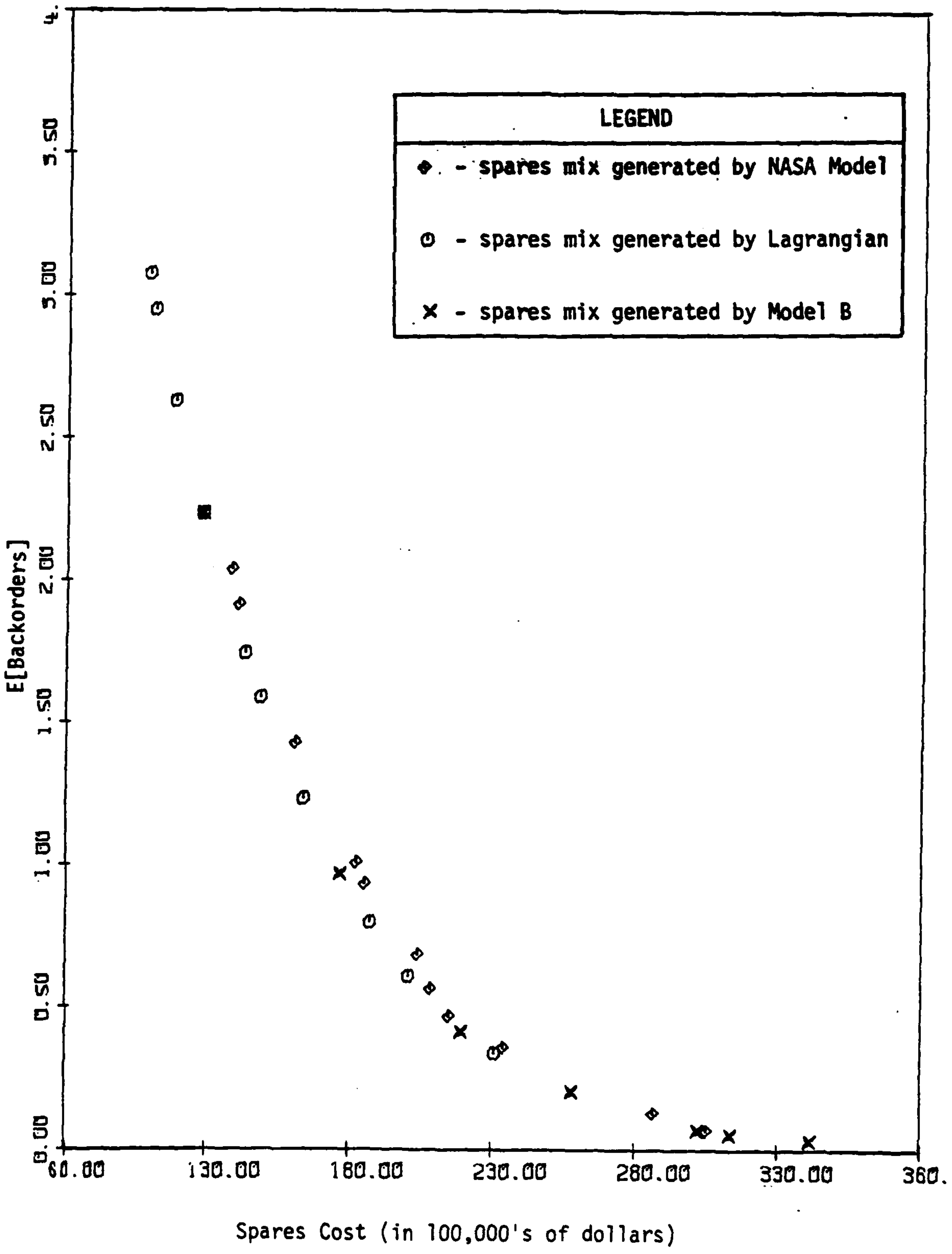
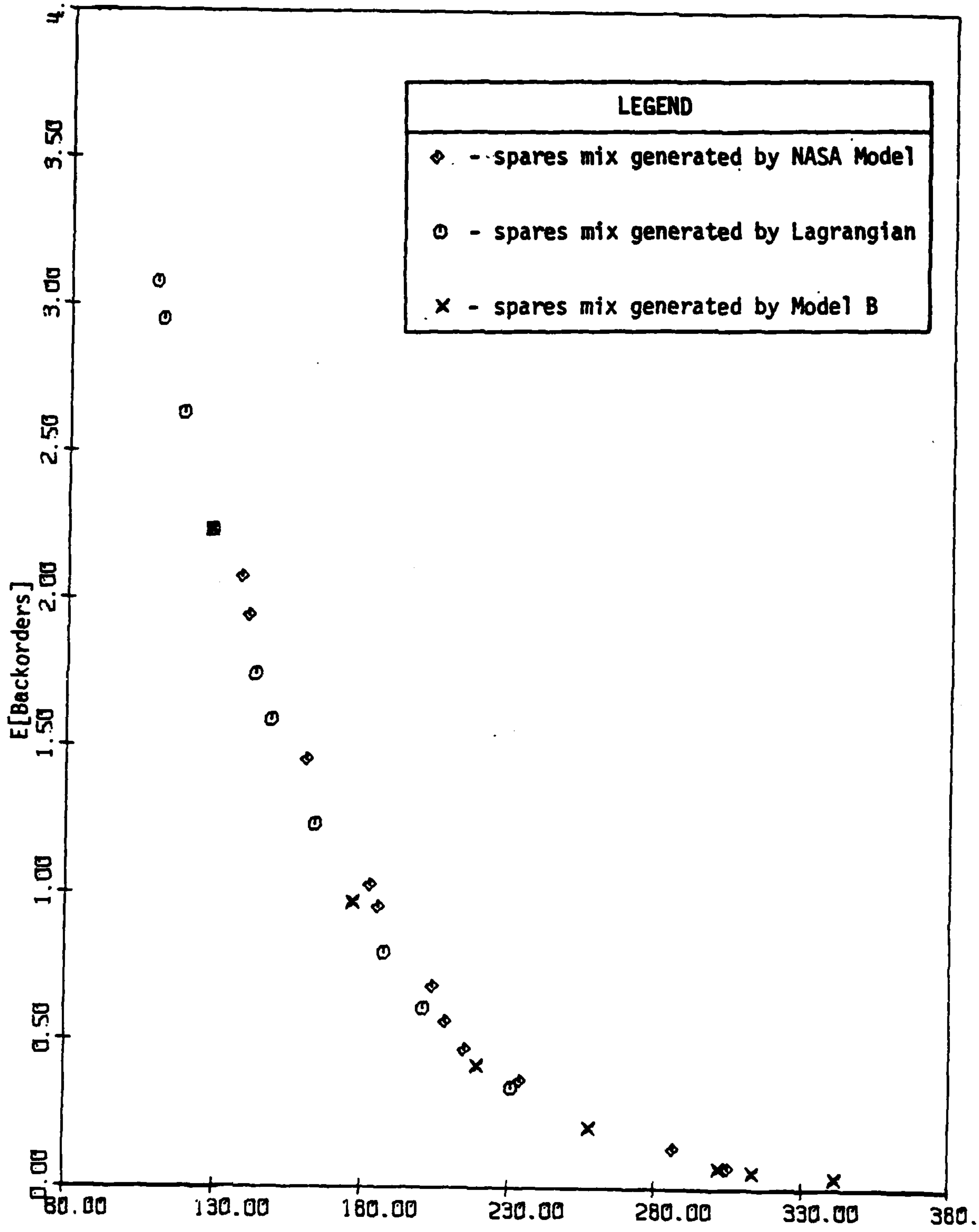
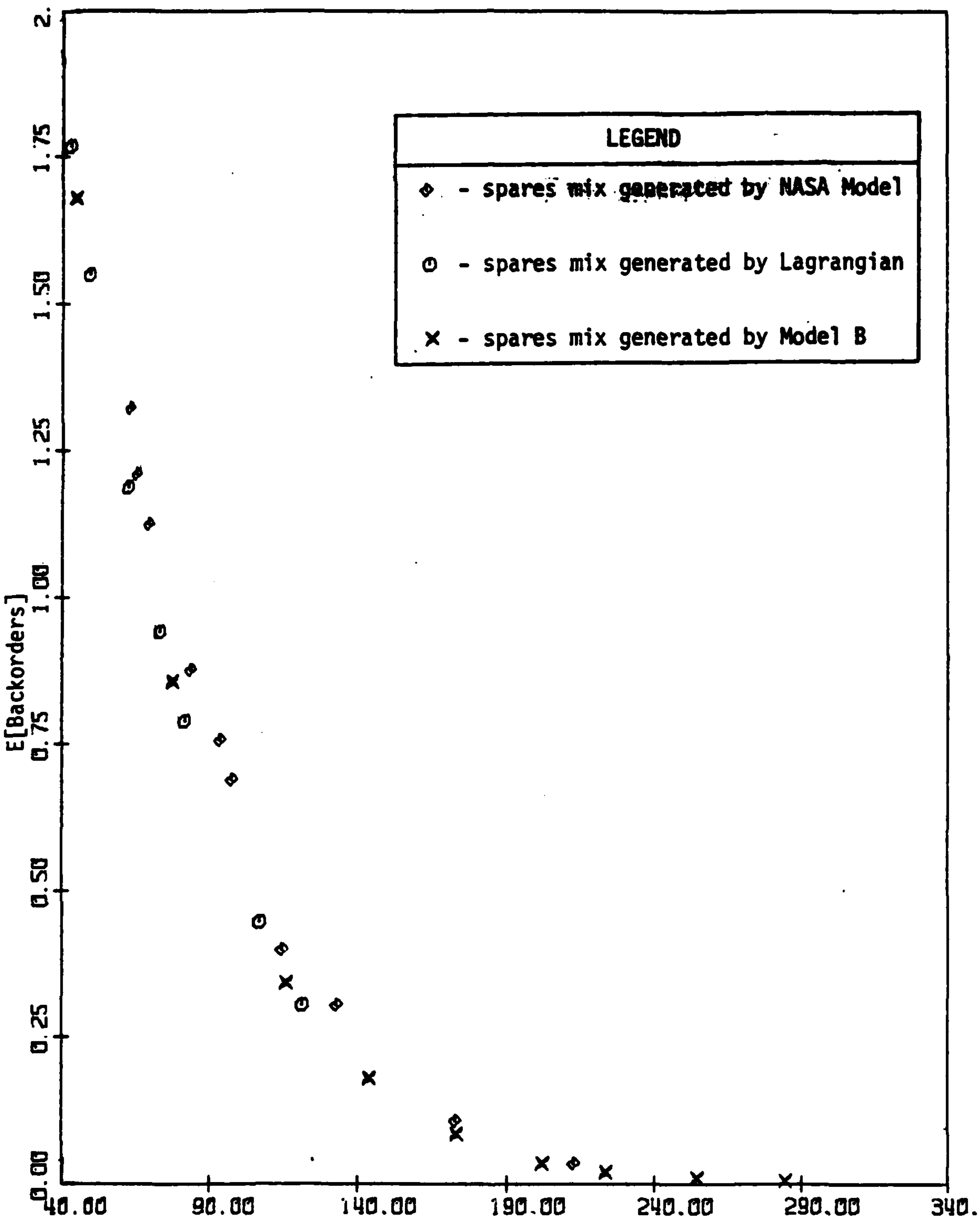


Fig. 3.9 Spares Mix Comparison for Sixteen Day Unweighted Cycle - Model B.



Spares Cost (in 100,000's of dollars)

Fig. 3.10 Spares Mix Comparison for Sixteen Day Weighted Cycle - Model B.



Spares Cost (in 100,000's of dollars)

Fig. 3.11 Spares Mix Comparison for Fifty Day Unweighted Cycle - Model B.

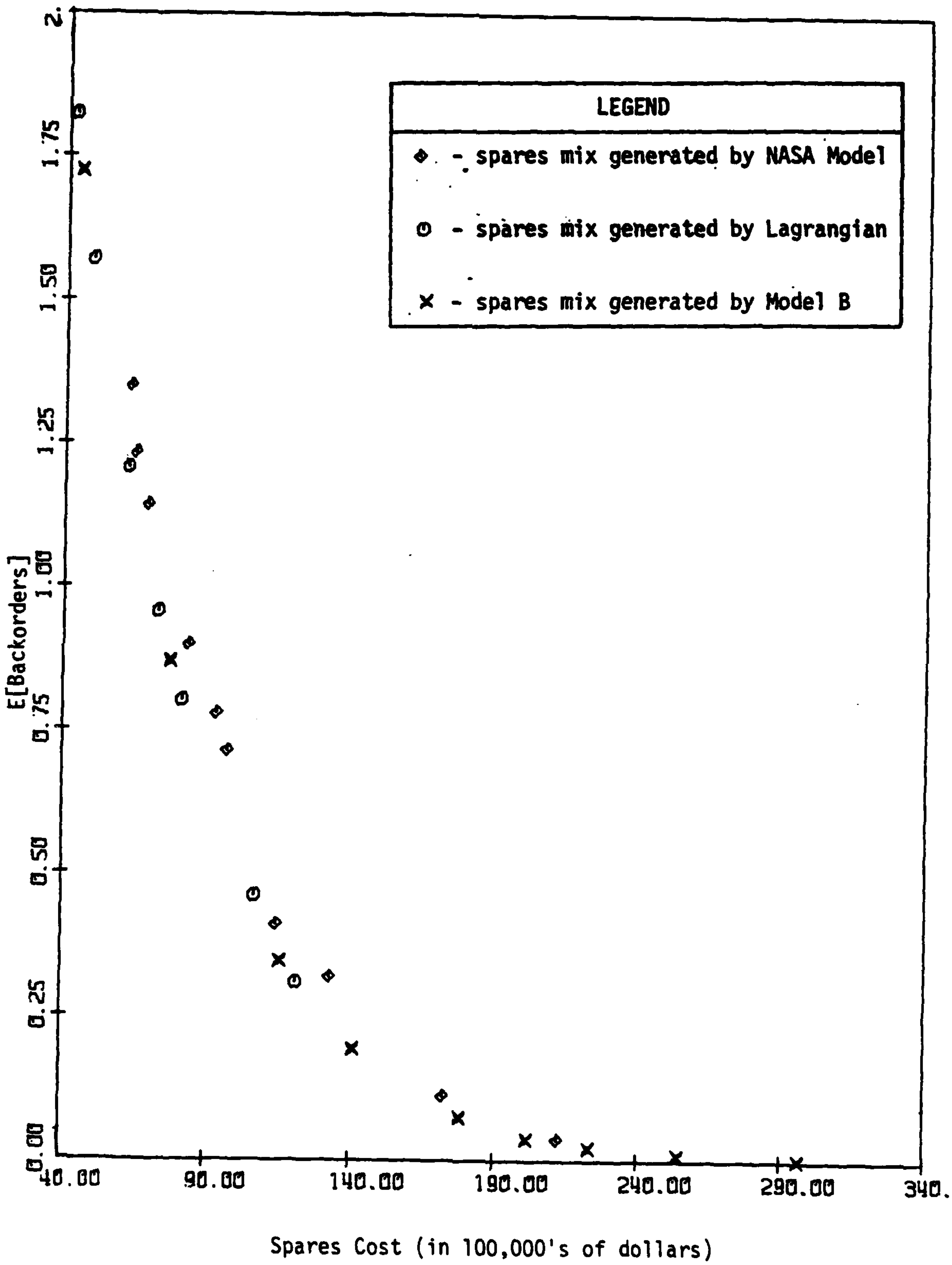


Fig. 3.12 Spares Mix Comparison for Fifty Day Weighted Cycle - Model B.

Finally, in Figures 3.13-3.14, the expected waiting time over the cycle is shown for Model D, the NASA model, and the Lagrangian model. Even for the four-day evenly weighted launch cycle, there are significant advantages to be gained by choosing one spares mix over another. As much as a 50% reduction in expected waiting time for backorders will accrue from using Model D to set the stock levels. Most of the reduction seems to stem from a single item which has such a low lead time demand that few spares would ever be in resupply and a long delay would result were it ever backordered. This item is bypassed for low budget levels in other models because of its low failure rate. As investment increases, Model D and the other models begin returning about the same levels of performance with respect to expected weighted backorder days. As was noted in the analyses of Models A and B, the introduction of longer cycles has the result of both decreasing the required budget for all levels of performance and increasing the model's sensitivity to nonstationary demand rates which increase instability in expected lead-time demand. The result is that in Figures 3.17-3.18 for a 50-day cycle there can be as much as a 70% reduction in expected weighted backorder-days for low budget levels when Model D stock levels are employed.

We next turn to an analysis of how the various models allocate spares as the budget limits increase while the cycle length is held constant at 35 days. The launch day has five times the weight of other days in the cycle. For each of the models we tried to find three spares mixes priced at $10 million, $15 million, and $20 million, although some of the budgets were lower or higher than others. Budgets which were higher by $1 million could include from two to five more spares than

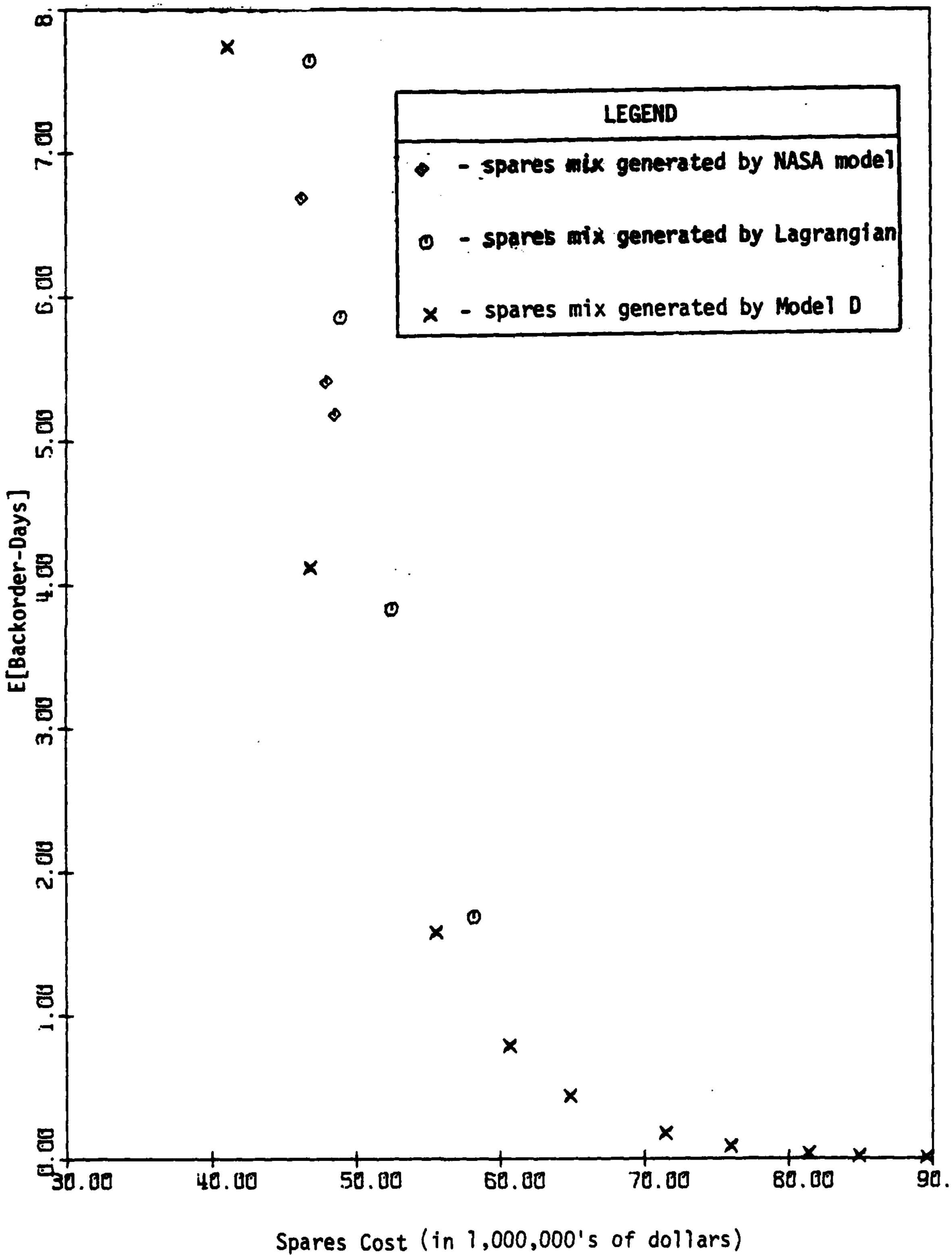


Fig. 3.13 Spares Mix Comparison for Four Day Unweighted Cycle - Model D.

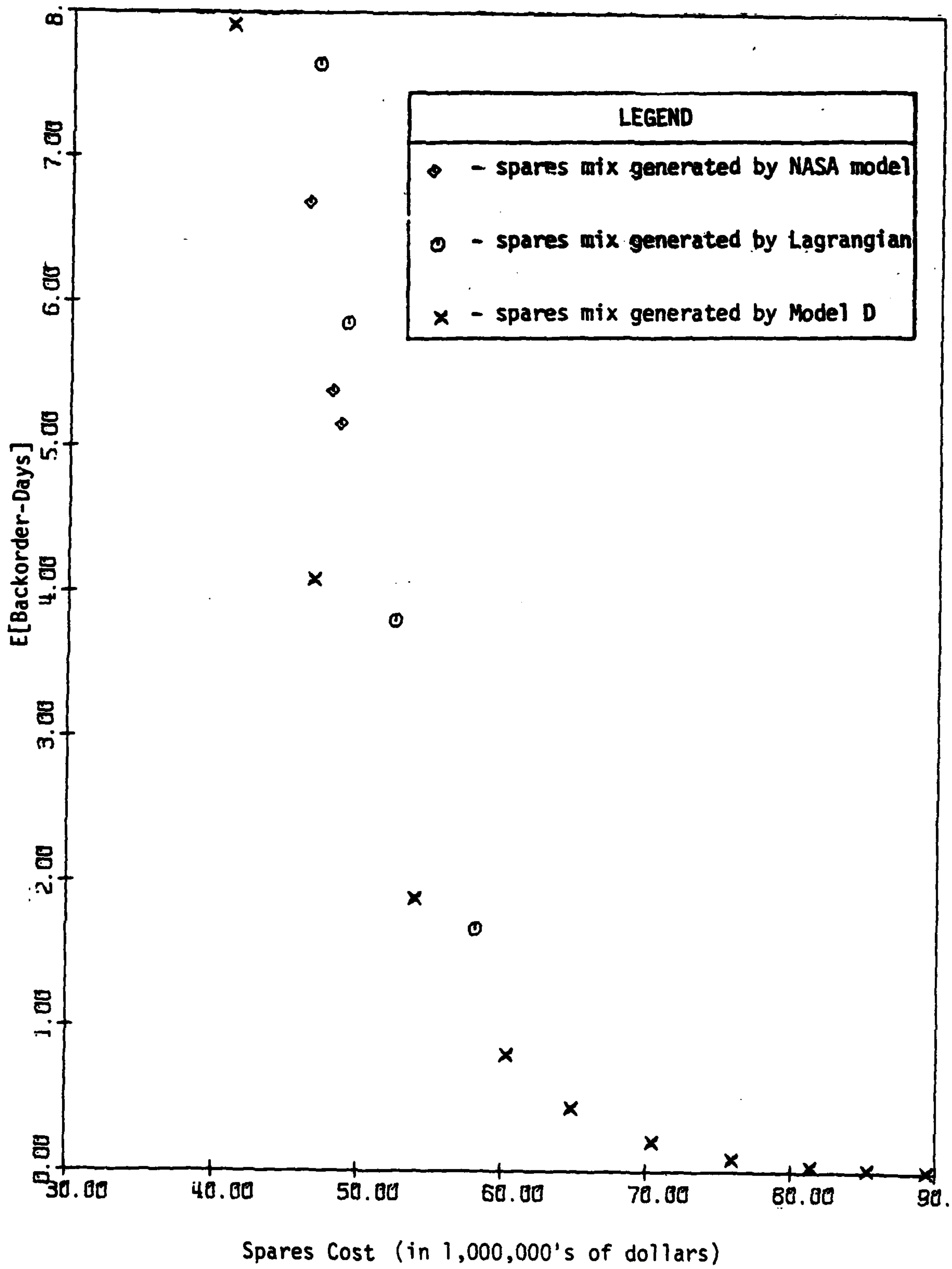


Fig. 3.14 Spares Mix Comparison for Four Day Weighted Cycle - Model D.

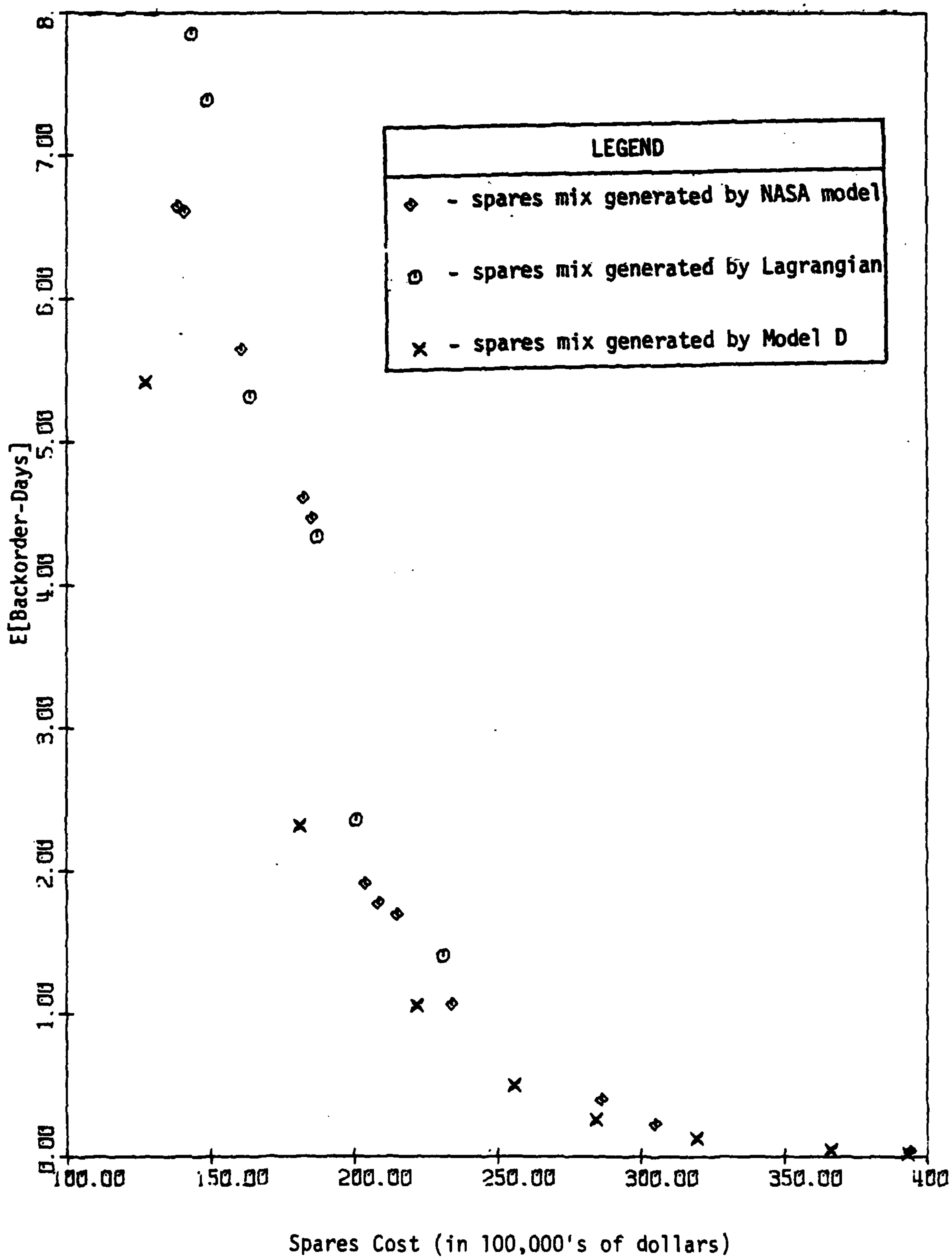
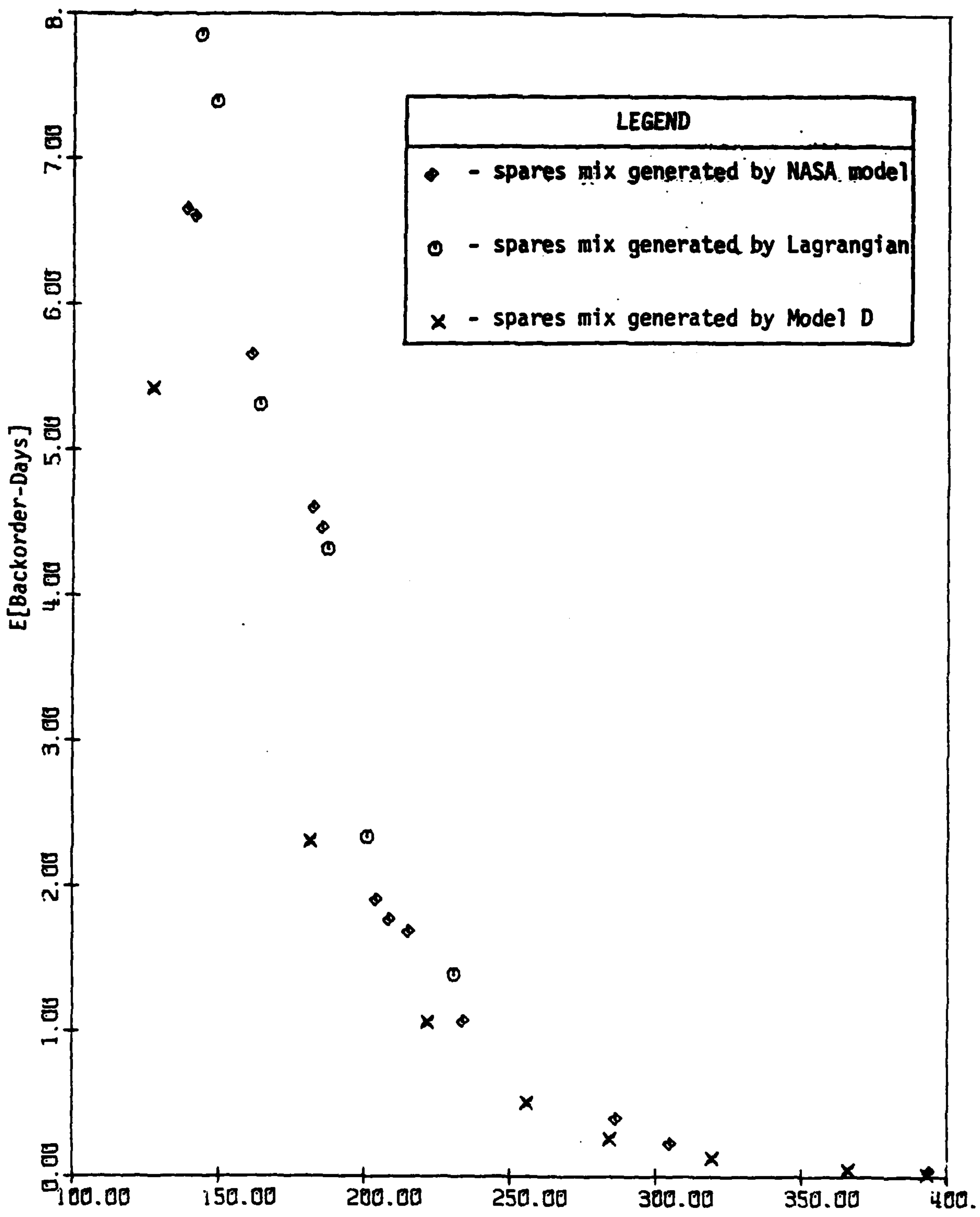


Fig. 3.15 Spares Mix Comparison for Sixteen Day Unweighted Cycle - Model D.



Spares Cost (in 100,000's of dollars)

Fig. 3.16 Spares Mix Comparison for Sixteen Day Weighted Cycle - Model D.

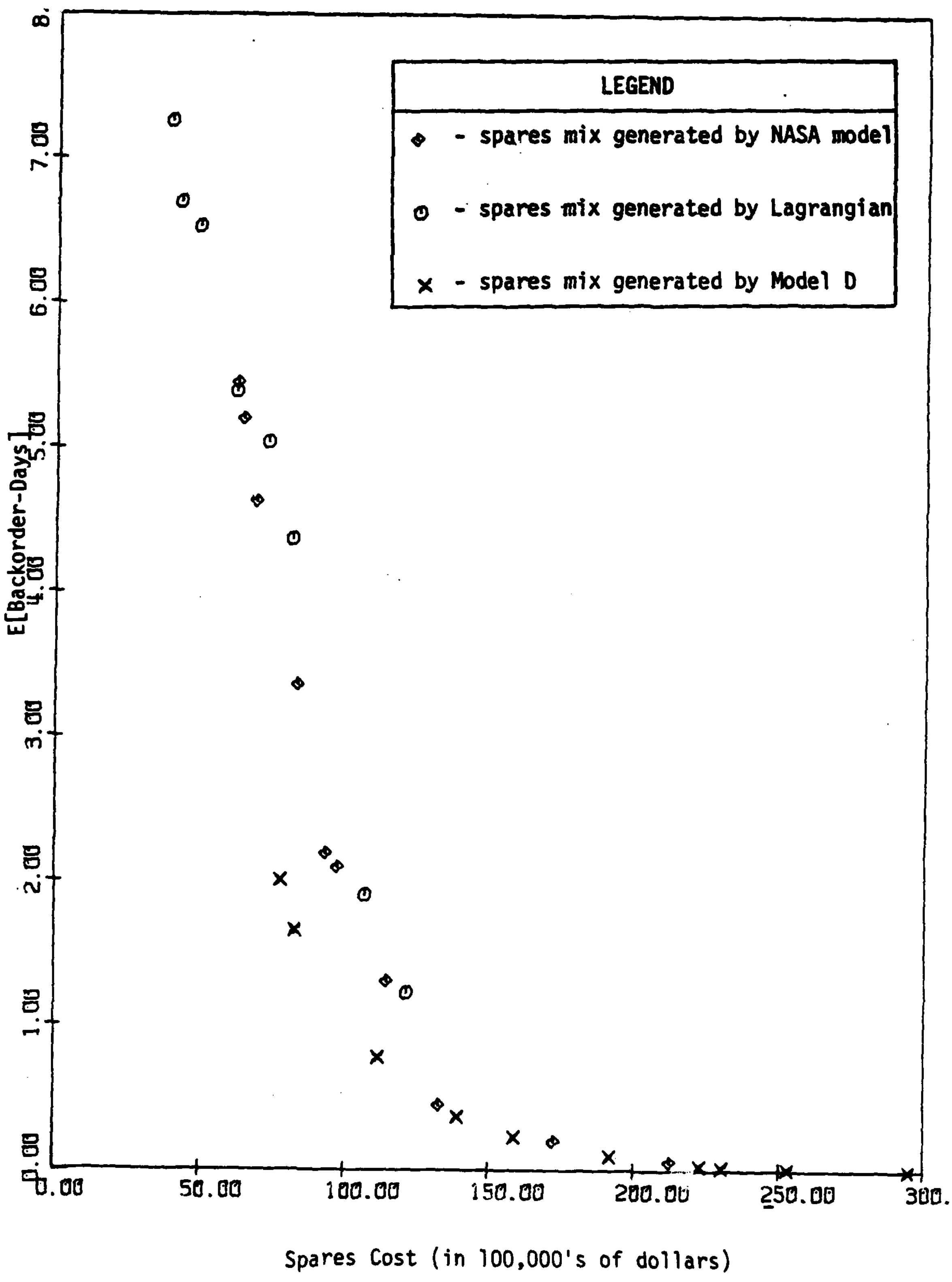


Fig. 3.17 Spares Mix Comparison for Fifty Day Unweighted Cycle - Model D.

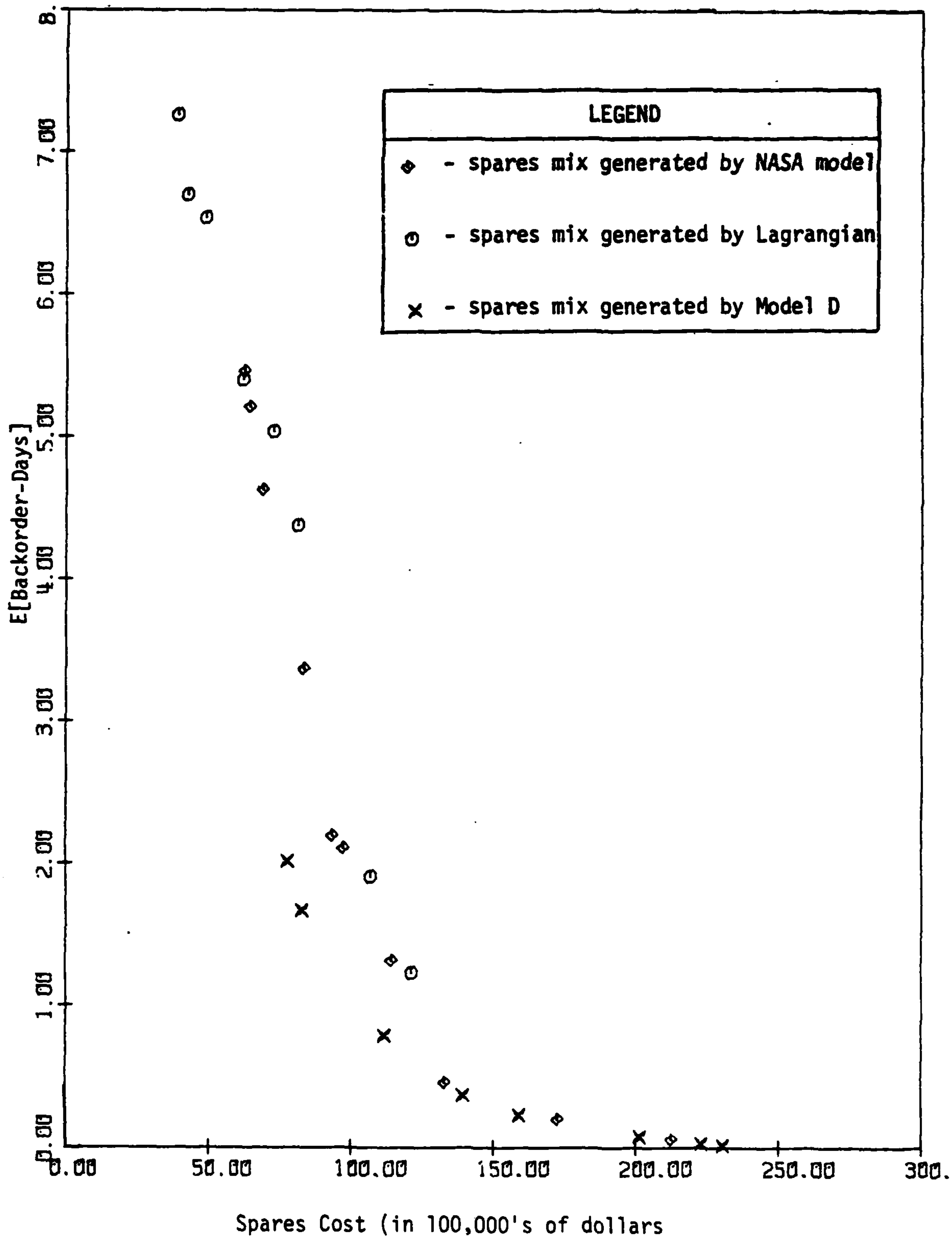


Fig. 3.18 Spares Mix Comparison for Fifty Day Weighted Cycle - Model D.

more austere budgets, and so the fluctuations in performance with respect to an objective function could appear to be greater than they actually are. The budgets and performance levels are shown in Tables 3.5 and 3.6 respectively, while the item stock levels for each of the twelve spares mixes are given in Table 3.7.

Close examination of Table 3.7 shows that of the Models A, B, and D, Model B gives priority to the high-demand, low-cost items (1 and 2) sooner than Model A or Model D. Model A tends to stock the moderate demand, high cost items (7 and 8) sooner than either of the other two models; but, as demand decreases slightly and cost rises, and lead time shortens (items 12 and 13) the opposite effect is apparent. The lead time proves to be an important factor for Model D as well, for as it lengthens, as in the cases of items 15 and 16, Model D invests in the high cost, low demand items more quickly. Conversely, for a short lead time (27 days for item 21) Model D buys less of an inexpensive, moderately demanded item than either of the other two models.

The overall trends in stocking policy seem to be very similar for Models A and B, whereas Model D seems to stock at least one item of each type by the time it reaches budget level 2. This could be because it overestimates backorders for the low demand items under the assumption of nonstationary demand, as discussed earlier. Another intuitive result is that Model B closely follows the stock levels set by the Lagrangian model, because both are based on a measure of backorders due to lead-time demand. The items used here evidently do not vary significantly in terms of expected numbers of backorders, even when the demand has a nonstationary Poisson distribution. By far, the most unusual spares mixes are produced by Model D, which gives the same spares mixes as Model A in any one budget only 57% of the time, and with Model B only 64% of

Table 3.5 Budgets Used for Comparison

Budget	Actual Spares Mix Cost, millions				
	Model A	Model B	Model D	NASA	Lagrange
1	10.1	11.1	10.3	10.2	11.1
2	15.4	15.6	14.0	12.5	15.0
3	20.4	19.8	20.4	18.8	-

Table 3.6 Performance Relative to Models A, B, and D of Different Spares Mixes

(a)

Spares Mix Computed By	Expected Weighted Probability of Sufficiency		
	Budget 1	Budget 2	Budget 3
Model A	.719	.923	.985
NASA	.685	.831	.952
Lagrange	.764	.890	-

(b)

Spares Mix Computed By	Expected Weighted Backorders		
	Budget 1	Budget 2	Budget 3
Model B	.691	.253	.084
NASA	.993	.516	.125
Lagrange	.691	.315	-

(c)

Spares Mix Computed By	Expected Weighted Backorder Days		
	Budget 1	Budget 2	Budget 3
Model D	1.90	0.82	0.18
NASA	3.35	2.01	-
Lagrange	3.70	0.88	-

Table 3.7 Stock Levels for Comparison

SOURCE	Model A			Model B			Model D			NASA			Lagrange		
BUDGET	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
ITEM															
1	4	6	7	5	7	7	3	4	5	4	5	6	5	6	
2	4	4	6	5	6	7	3	4	5	4	5	6	5	6	
3	2	3	4	2	3	3	2	2	3	2	3	3	2	3	
4	1	2	2	1	1	2	1	1	2	1	1	2	1	1	
5	1	2	3	2	2	3	1	2	2	1	2	3	2	2	
6	2	2	3	2	3	3	1	2	3	2	2	3	2	3	
7	1	1	2	1	1	2	1	1	2	1	1	2	1	1	
8	1	2	2	1	1	2	1	1	2	1	1	2	1	1	
9	1	2	3	1	2	2	0	1	1	1	1	2	1	1	
10	2	3	3	2	2	3	2	2	3	1	2	2	2	2	
11	1	1	2	1	2	2	1	2	2	1	1	2	1	2	
12	0	0	0	0	0	1	0	1	1	0	0	1	0	0	
13	0	0	0	0	0	1	0	1	1	0	0	1	0	0	
14	1	1	2	1	2	2	1	2	2	1	1	2	1	2	
15	0	1	1	0	1	1	1	1	2	1	1	1	0	1	
16	0	1	1	0	1	1	1	1	1	0	1	1	0	1	
17	1	1	1	1	1	1	1	1	2	1	1	1	1	1	
18	1	1	2	1	1	1	1	1	2	1	1	1	1	1	
19	1	2	3	1	2	2	1	1	2	1	1	2	1	2	
20	1	1	1	1	1	1	1	1	2	1	1	1	1	1	
21	1	2	2	1	2	2	1	1	2	1	1	2	1	2	
22	2	2	3	2	2	3	2	2	3	1	2	2	2	2	
23	0	1	1	0	1	1	1	1	1	0	0	1	0	1	
24	1	1	2	1	1	1	1	1	2	1	1	1	1	1	

the time. By contrast, models A and B returned the same stock levels 73% of the time.

The choice of an objective function and hence one of the models given above has a major impact on the spares mix that one would purchase and upon the performance that the serviceable spares system can provide relative to any one measure. The final system performance cannot be predicted, however, until the number of shuttles, their launch frequency, and their maintenance schedules are known. Yet another important system characteristic from a repairable spares perspective will be the presence of two launch sites which will have different activity rates associated with them. It will be important to know how the launch periods at the two bases correspond to one another, because when the two time-dependent demand distributions are combined, the basic period for the system will depend upon their interactions. In some cases, the number of days to be examined could be as much as one year; if, for example, Base A were to launch every 4 weeks while Base B launched every 13 weeks. In the next chapter, we examine the two base problem as it relates to the location problem for spares, as well as its effect upon the choice of a spares mix.

CHAPTER 4

The choice of an optimal mix of serviceable spares is not entirely separable from that of where the spares storage facilities should be located. The most obvious interdependence is through the increased number of backorders (and backorder-days) we would expect at bases where no spares are permanently stocked. Alternatively, if facilities exist to store spare parts at both bases, a model must recognize that only one of the two bases will receive a given spare after it completes repair at the depot. However, especially in the more likely case where lateral (base to base) resupply is allowed, the increased transportation time will only be on the order of a day or two, or only between two and five percent of the transportation times we encountered in Models A, B, and D. Even without a lateral resupply capability, this means that if demand is low, failures at each base will be rare enough to permit most repaired units to be returned to the base at which they originally failed and so each base may be treated as a single location.

While the choice of spares basing locations may not overly influence the optimal number of each item to be procured, it could still have significant impact on the real time efficiency of the system. In the event that a backorder occurs, the expected waiting time could be significantly altered by the existence of a spares facility at each base and/or lateral resupply capabilities. The time-critical nature of shuttle operations will probably result in the use of a real time inventory monitoring system. Such a system could be used to track the inventory position of those spares which could adversely affect the launch date, and to direct routine shipments from the depot to the bases as well as emergency or precautionary shipments from one base to another. The number and duration of backorders could thus be held to a minimum, but much depends upon where the spares facilities are located and upon the conditions which will permit or prevent a shipment. One interesting kind of spares shipment would take place as the launch date for one of the bases approaches. If no demands occur and the launch takes place, the shipment will normally be returned to the sending base. Such a shipment will be the result of what we will call a launch critical event.

Given the many interrelationships between the supply system, the launch rates at the different bases, and the interbase shipment discipline, it is extremely difficult to develop analytical models which would provide some optimal level of fills or expected backorders. Miller [7] presents a model which can be used as a real time decision-making mechanism for repairable spares allocation. This model, which he terms Real Time Metric (RTM), compares the need of each base with the reluctance of the depot whenever a supply event such as an item failure or a repair completion takes place. The RTM generates quantitative values for base need and depot reluctance as functions of the state of the system. If depot reluctance is smaller than at least one base's need, a spare is shipped to the base with the greatest need.

This model is easily extended to incorporate a nonstationary demand distribution and launch critical events. Miller's model has been programmed in FORTRAN on an IBM 370/168 at Cornell University as an experimental simulation by Cogliano [2].

Using this simulation as the basis for our model of the shuttle's repairable spares supply system, we restate some of Miller's assumptions and extend them as follows:

1. The system consists of a depot and two bases. Any units removed from the shuttle due to failure or suspected failure are sent to the depot for repair.
2. A single item is examined.
3. Repair time is fixed. Transportation time from base to depot is considered part of the repair time, and is fixed at t_d days from the depot to a base and at t_b days between bases. (These assumptions are easily modified.)
4. The item experiences a nonstationary Poisson demand distribution with a demand spike m_b days before the launch at each base.
5. There is a fixed number of spares.
6. The depot reluctance is zero; thus repaired units are shipped immediately to the base with the greatest need.
7. Base need is a function only of its inventory position and the length of time before its next scheduled launch.
8. Backorders are assumed not to prolong the launch cycle so that the launches at a given base are evenly spaced, although this assumption may be relaxed.
9. The launch rate at a base is directly proportional to the number of shuttles at the base and does not change over the simulation period.

Assumption 1 ascribes to the depot those functions carried out by the contractor in the real system. NASA has no control over the contractor's operations, and, as discussed in Chapter 2, the shipment and testing is so time-consuming that any removal must be treated as a demand on the supply system. In Assumption 2, we limit the scope to one item, although the simulation may easily be expanded. We may allow resupply times and travel times to be exponential, but the fixed quantities are acceptable as well. Next, although demand is assumed to be nonstationary, the number of shuttles in the system is so high that the effects of nonstationarity become unimportant as we noted in Chapter 3. We use high demand rates because otherwise it would be very expensive to run the simulation long enough to collect significant observations of backorders.

Each of the fixed number of spares is either in stock, en route to or from a base, or in depot repair. As Miller points out, the number in repair is beyond our direct control, and so the model seeks to maximize the number which are in stock where the need is greatest, and places limits upon the number of shipments and hence the en route inventory. In defining base need, we make use of the following definition of inventory position (IP):

$$\text{IP} = \text{\# on-hand} + \text{\# on order} - \text{\# backordered}.$$

We also would like to increase a base's need substantially if inventory position is negative or if there is an impending launch at that base. When a lateral resupply is considered, the need of the supplying base is computed and compared with the need of the potential recipient.

Lastly, we make some assumptions about the launch cycle. It is again necessary to assume that the launch cycles are fixed in length, although it is not unreasonable when considering many shuttles at a base to allow some latitude in the launch date. The simulation is much more flexible than the analytic models, and could easily be modified were this a crucial assumption. The assumption of a fixed number of shuttles at a given base, however, may be very important. It could be the case that a shuttle will take off from one base and land at another, due to either landing site weather conditions or an emergency landing. The subsequent change in the launch pattern, although only temporary, could cause some changes to supply system performance, especially if it is a rigid system which stocks at only one location. The computer simulation could be used to analyze these transient effects; however, that is not our primary objective here.

In modeling the above system with a simulation, several events must first be identified. The relevant events are listed in Table 4.1. In addition to the actions which are produced automatically by these events, there are several actions which may be produced by some events, depending upon the state of the system; specifically, the relevant variable is the base need. We now describe in some detail how that quantity is calculated.

Miller defines base need to be a function of the expected backorders over the travel time from the depot to the base. He approximates the discrete conditional distribution of expected backorders given the on hand inventory by a normal distribution. This technique is given in reference [6]. Then the mean number of backorders is given by the following:

Table 4.1 Events To Be Simulated

EVENT	ACTION
1. Failure	- decrement on hand inventory (-1) - increment depot rapair inventory (+1) - schedule a repair completion - compute new base need - schedule another failure event
2. Repair completion	- decrement depot inventory (-1) - choose a destination for the repaired unit - increment en route inventory to that base (+1) (conditional event) - compute new base need at destination
3. Arrival of a spare at base	- decrement en route inventory to the base (-1) - increment on hand inventory (+1)
4. Launch critical (Begin/End)	- recompute base need at the base - schedule next launch critical (Begin/End)
5. Maintenance	- reschedule the next failure event (reflecting increased failure rate) - schedule next maintenance date

$$\mu_B = \textit{On hand} + \textit{En route} - \lambda t_d$$ [1]

where λ is the demand rate over the travel time t_d. Further, the variance of the number of backorders is given by the following:

$$\sigma_B^2 = \lambda t_d.$$

[1] If μ_B is positive, it represents expected net inventory with no backorders.

Miller then makes a correction for the fact that the true distribution is not continuous, as follows:

$$\mu_B' = \begin{cases} \dfrac{\mu_B}{2\sigma_B} + \left(\sqrt{\sigma_B^2 + \mu_B} - \sigma_B\right) & \mu_B > 0 \\ \dfrac{\mu_B}{2\sigma_B} - \left(\sqrt{\sigma_B^2 - \mu_B} - \sigma_B\right) & \mu_B < 0 \end{cases}$$

where $\sigma_B = \sqrt{\sigma_B^2}$ is the standard deviation. The final step is the calculation of the mean of the backorders distribution, using the following equation:

$$E[B] = \sigma_B \cdot \frac{1}{2\pi} e^{-1/2(\mu_B'^2)} - \sigma_B \mu_B' \int_{-\infty}^{-\mu_B'} \frac{1}{2\pi} e^{-(1/2)\omega^2} d\omega.$$

We are now able to define base need in a variety of circumstances. If an item fails at a base, then we are concerned with the desirability of immediately initiating a resupply to that base from the other base. However, if the sending base's need will increase past that of the receiving base's during the time it would take to ship the part back and forth, then the lateral shipment should not take place. In effect, we are minimizing the maximum base need over a short time horizon. We will thus carry out a shipment from base 1 to base 2 at time t if the following holds:

$$N_{B_2}(t) \geq \max_{t,t+2t_b} [N_{B_1}(t)]$$

where $N_{B_i}(t)$ is the need of base i at time t. The difference between this model and Miller's model is that we must now look forward in time towards fluctuations in need which take place at predetermined times. The other events for which similar comparisons are performed are summarized in Table 4.2.

Table 4.2 Conditional Events for Shipments

EVENT	CONDITION	ACTION
1. Failure (base i)	$N_{B_i}(t) \geq \max\limits_{s\epsilon(t,t+2t_b)} [N_{B_{\bar{i}}}(s)]$	- decrement on-hand inventory, base $\bar{i}$ - increment en route inventory, base i - compute new base needs
2. Repair completion	$\max\limits_{s\epsilon(t,t+t_b+t_d)} [N_{B_i}(s)]$ $\geq \max\limits_{s\epsilon(t,t+t_b+t_d)} [N_{B_{\bar{i}}}(s)]$	- ship to base i
3. Launch critical (base i)	Same as for failure, base i	Same as for failure, base i

The overall computation of base need is derived only partly from the expected number of backorders at the base. Other important considerations include the presence of backorders and the proximity to launch at the base. In addition, at some maximum stock level, IP_{max}, the base has no need at all. Thus we may define the need at base i at time t as follows:

$$N_{B_i}(t) = \begin{cases} \infty & \text{if } IP < 0 \\ E[B]+\eta y_i & \text{if } 0 \leq IP < IP_{max} \\ 0 & \text{if } IP \geq IP_{max} \end{cases}$$

where η is a constant representing the penalty for backorders as the launch nears, and y_i is a variable taking value 1 if a base is launch critical and 0 otherwise. Thus base need might be represented graphically as in Fig. 4.1. We note that expected backorders E[B] as well as y_i will depend not only on inventory position but also on the time period over which $N_{B_i}(t)$ is evaluated.

The choice of the constant η must be made carefully, for a system which ships back and forth with abandon can be just as inefficient as one which ships only when a backorder occurs. The best value for η will depend upon the activity rates at the bases, the expected backorders, the nonstationarity of the demand distribution, and the number of spares in the system. For our purposes it is acceptable to choose η by using trial and error, but in any general use of the method, a more precise formulation would be required. It could be that a good value for η is obtainable in the same way that Miller calculates depot reluctance: as an exponential function which decreases in the number of spares on hand at the location in question.

The above model relates to the case where there are two stocking locations for repairable spares, and there is an unlimited capacity for lateral resupply between bases. In our analysis we consider three separate cases. Case I allows shipments to either base upon repair completion, but does not allow lateral resupply. Case II allows shipment to the second base only if its stock of spares is low and it is close to the launch date at that base, so that limited lateral resupplies are permitted. Lastly, Case III corresponds to the stiuation where two bases, each with complete stocking facilities, are allowed to perform lateral resupplies as often as is indicated by the comparison of base needs.

The Computer Implementation

The model presented here was coded in FORTRAN and run on an IBM 370/168 at Cornell University with a VM operating system. Most of the simulation structure is drawn from similar work done by Cogliano [2]. Significant changes include the addition of launch critical and maintenance events, the extension of the time frame for evaluating expected backorders, the introduction of nonstationary demand rates, and the change in shipment discipline to incorporate the three cases detailed above.

The simulation has an event-scheduling format, and is modularly designed so that each subroutine relates to a specific event. The key launch critical subroutine is given in Appendix B. The typical program input is shown in Figure 4.2, with a flowchart representation of the simulation given in Figure 4.3. The item which was used for analysis is like item 3 from the group of items we examined in Chapter 3. The item experiences .300 failures per day on the average in the simulation whereas the average failure rate for item 3 in models A, B, and D was .318. The system is assumed to consist of seven shuttles at base 1 and five shuttles at base 2. Since shuttle turn-around time is fifty days, there is a

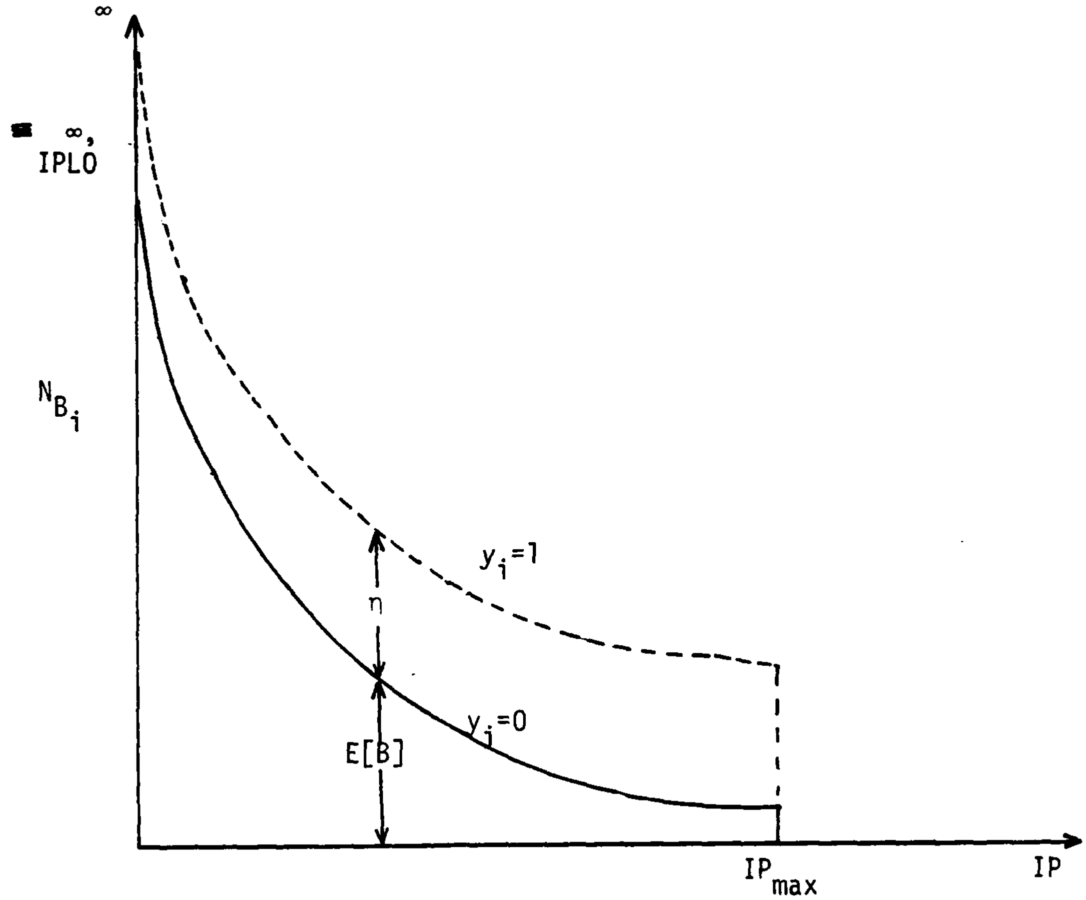


Fig. 4.1 Base Need as a Function of Inventory Position.

seven day interlaunch cycle at base 1 compared with a ten day cycle at base 2. There is one item on each shuttle, each with a failure rate of 1/40 per day if stationary demand is assumed, and a resupply time of 60 days. There are 25 spares in the system, 15 of which are initially at base 1, and the remaining ten are at base 2. Depot to base travel time is 1.0 days, while base to base travel time is 2.0 days. The simulations were "warmed up" for a set period of time before a series of daily observations were begun. The output, which reports the average

Figure 4.2 Input for Simulation

```
OEACH INPUT FORMAT IS EITHER 10I4 OR 10F4.0
ONUMBER OF BASES
         2
 NUMBER OF ITEM TYPES
         1
 NUMBER OF TYPE-1, TYPE-2, ... UNITS AT BASE  1
         7
 NUMBER OF TYPE-1, TYPE-2, ... SPARES AT BASE  1
        15
 NUMBER OF TYPE-1, TYPE-2, ... UNITS AT BASE  2
         5
 NUMBER OF TYPE-1, TYPE-2, ... SPARES AT BASE  2
        10
 NUMBER OF TYPE-1, TYPE-2, ... SPARES AT THE DEPOT
         0
 AVERAGE TIME-TO-FAILURE FOR TYPE-1, TYPE-2, ... ITEMS
        40.000
 AVERAGE BASE REPAIR TIME FOR TYPE-1, TYPE-2,... ITEMS
         0.0
 AVERAGE DEPOT REPAIR TIME FOR THESE SAME ITEMS
        60.000
 PROBABILITY OF A BASE REPAIR FOR TYPE-1, TYPE-2, ...
         0.0
 TRAVEL TIME FROM BASE-1, BASE-2, ... TO DEPOT
         0.0     0.0
 TRAVEL TIME FROM DEPOT TO BASE-1, BASE-2, ...
 AND FROM BASE 1 TO 2, 2 TO 1, ..., WITH LATERAL RESUPPLY
         1.000   1.000   2.000   2.000
 PROCESSING TIME FOR ORDERS FROM BASE-1, BASE-2, ...)
         0.0     0.0
OLENGTH OF THE WARM-UP PERIOD
       150.000
 TIME BETWEEN SUCCESSIVE OBSERVATIONS
         1.000
 NUMBER OF OBSERVATIONS TO TAKE (FORMAT I6)
         500
 ENTER 1 FOR AN ESTIMATION OF VARIANCES
         0
 ENTER 1 IF FAILURE TIMES ARE EXPONENTIAL
         1
 ENTER 1 FOR A TRACE OF EVENTS
         0
 SEEDS FOR FAILURE TIME AND PLACE (2F10.0)
       745623964.  235187469.
 SEEDS FOR REPAIR TIME AND PLACE (2F10.0)
       254768137.  647629632.
 SEED FOR INITIAL CONDITIONS (F10.0)
       645734621.
OBASE ORDERING POLICY:
         (1)  ORDER TO MATCH THE UNIT THAT FAILED
         (2)  ORDER TO REPLACE THE PART JUST INSTALLED
         (3)  ORDER TO REPLACE THE PART THAT FAILED
         (0)  DOES NOT APPLY; SHIP USING NEED & RELUCTANCE
         0
```

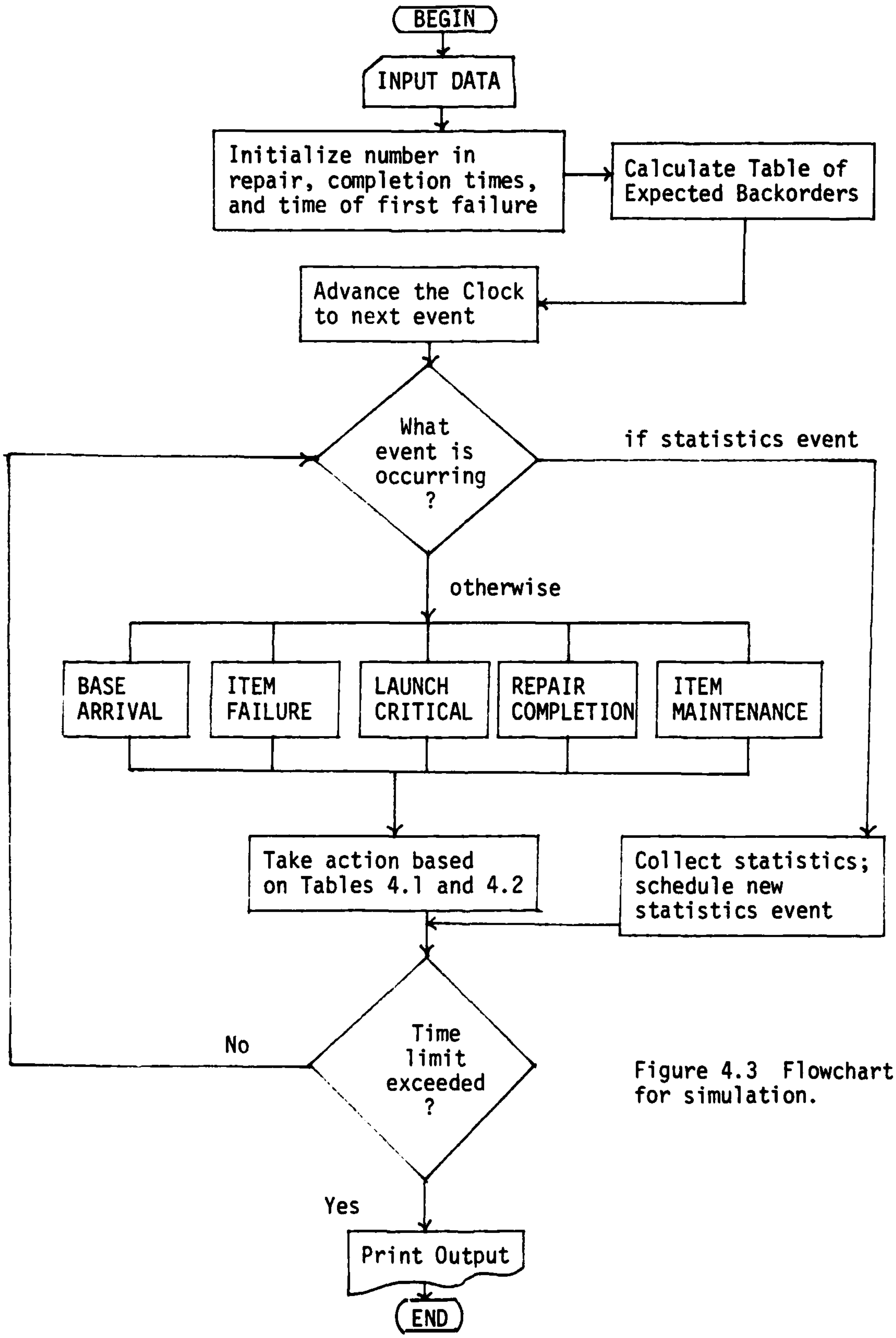


Figure 4.3 Flowchart for simulation.

number of shuttles grounded, is discussed in the next section.

Simulation Results

Using the hypothetical item developed above, the computer simulation is used to test the expected performance of supply systems under the three cases we identified. Although the system's configuration and shipment limitations are varied, the failure rates and random number seeds are identical for all runs. This results in nearly identical patterns of failures during each of the simulation trials.

Our primary results concern the case where there is no nonstationarity in the demand distribution. As we are dealing with an activity rate of one launch every 4.4 days, the results of Chapter 3 would suggest that incorporating nonstationarity is unnecessary. The simulation output is shown in Table 4.3 for the three cases defined earlier. We see that the best performance is returned by a system allowing unlimited lateral resupply. In addition, the output would suggest that in this case it is preferable to have both basing locations stocked with spares than to stock only one location. This fact is demonstrated by the 70% reduction in backorders which Case I achieves relative to Case II. Table 4.4 gives information on the performance of the three decision rules with respect to the number of shuttles grounded during launch critical periods. It is possible to achieve close to zero backorders if Case III is implemented.

Taken together, Tables 4.3 and 4.4 suggest than an optimal policy in all respects might be the use of Case III. We note, however, that even when the launch critical constant is zero, we will make close to 100 shipments of our hypothetical item in a 500-day time period. Thus Case I might be a better choice if the cost of a lateral resupply capability is high.

Table 4.3 Average Number of Shuttles Delayed

Launch critical constant, η	Cases I	II	III
0	.066	.222	.008
.01	.066	.222	.014
.05	.066	.222	.012

Table 4.4 Average Number of Shuttles Delayed During Launch Critical Period

Launch critical constant, η	Cases I	II	III
0	.076	.208	0.0
.01	.076	.208	.004
.05	.076	.208	0.0

Table 4.5 Average Number of Shuttles Delayed with Nonstationary Demand

Launch critical constant, η	Cases I	II	III
0	.400	.500	.398
.05	.400	.500	.386

Table 4.6 Average Number of Shuttles Delayed During Launch Critical Period with Nonstationary Demand

Launch critical constant, η	Cases I	II	III
0.0	.002	.184	0
.05	.002	.176	0

Nonstationarity in the lead time demand rates may easily be integrated into the framework of Miller's RTM. Since the simulation kept a current value of the next failure time, the demands in a given day could effectively be doubled by simply combining the expected number of demands for two days to produce the demand for a spike day. We chose to do this on the day the shuttles become launch critical, but it could occur on an arbitrary day or days in the launch cycle. With a failure rate of 1/60 per day on non-peak days and 1/30 per day on spike days, we see in Table 4.5 that Case III is able to completely avoid backorders for η=.05. When we examine the performance of the three cases during the launch critical period, in Table 4.6, we note that Case II's performance worsens during the launch critical phase. This is due to the fact that we may ship a spare to base 2 only rarely for lack of prepositioned spares facilities at that location. Thus many backorders at location 2 will last at least a day or two. In this case we note that increasing the launch critical constant in Table 4.6 may remedy the situation for Case II. It has no effect on either Case I or Case III. It is interesting to note that the precautionary shipments which we allow in Case III are able to eliminate all backorders during the launch critical phase. However, we must caution that these data may only be considered as preliminary results since they are based on single simulation runs of 500 days. Much more investigation is necessary before we can actually identify a good value for η in all cases and before we can state that Case III is always superior to the other cases with the proper choice of η.

Once the desireability of lateral resupply capability and multiple location spares prepositioning is determined for all items, decisions as to the shipment discipline for the overall system may be made.

This determination will have to depend upon the probable utilization of the lateral resupply system relative to its cost.

There is thus no single relationship that determines which basing concept is best for all spares in the sense that it simultaneously minimizes grounded shuttles on the average as well as during launch critical periods. The launch critical constant as well as the shipment discipline must be carefully chosen using the best information available about each item. Methods such as those outlined in this chapter can be invaluable in performing the necessary analysis and in implementing the results. However, one might well attempt to formulate some combination of Models A, B, and D with the simulation introduced above. It may be that for the price of lateral resupply capability we could substantially enrich the spares mix and surpass the performance of Case III. The objective of such a formulation would be to minimize the combination of spares investment with the expected ongoing costs of transportation and spares facilities. In addition, the flexibility of a multi-base capability in case of a real emergency merits additional consideration.

CONCLUSION

The results of Models A, B, and D point out the potential differences in spares mixes and performance levels which result from the choice of any one model. They indicate that, at least for the items we considered, nonstationary Poisson demand rates may not yield significantly different spares mixes than stationary rates. As lead time demand variability increases, however, nonstationarity could be an important factor in setting spares levels.

The simulation results indicate that a good real time shipment policy should respond to nonstationary demand rates and to launch critical events. Further, systems with both lateral resupply and prepositioning of assets at both bases seem to return the best performance. The final judgments may not be made, however, until the cost of each of these capabilities are included in the analysis.

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APPENDIX A - MODELS A,B, AND D

```
C
C     SPARES MIX DETERMINATION ALGORITHM (MAIN)
C       BY KATHLEEN CONLEY, DECEMBER 1981
C
C*****************************************************************
C     DESCRIPTION:
C
C     THIS IS THE MAIN SUBROUTINE FOR MODEL A.  SIMI-
C     LAR SUBROUTINES EXECUTE MODELS B AND D.  MANY
C     PARAMETERS AND FUNCTIONAL VALUES ARE PASSED
C     THROUGH THIS SUBROUTINE.  FIRST THE SYSTEM CHAR-
C     ACTERISTICS ARE READ IN (INPUT), FOLLOWED BY
C     MIXES FROM AN EXTERNAL SOURCE.  THESE SPARES
C     MIXES ARE EVALUATED BY MODEL A (COMP) AND  SPARES
C     MIXES FROM MODEL A ARE GENERATED (AA).  FINALLY,
C     THE STOCK LEVELS AND PERFORMANCE LEVELS FOR ALL
C     SPARES MIXES ARE DISPLAYED (OUT).  VARIABLE
C     DEFINITIONS ARE AS FOLLOWS:
C
C     BDDR(I)--THE AVERAGE DAILY DEMAND FOR ITEM I (INCLUDING
C         SPIKES.)
C     SDX(I)--A MEASURE OF EXPECTED SHORTAGES FOR STATIONARY
C          DEMAND.
C     RDCRST(I)--FIXED RESUPPLY TIME FOR ITEM I (IN HOURS)
C     LAMBDA(I)--MINIMUM ACCEPTABLE STOCK LEVEL FOR ITEM I.
C     W(J)--THE WEIGHT OF A BACKORDER ON DAY J.
C     ECOST(I)--ITEM I COST (IN DOLLARS)
C     SHORT--A MEASURE OF TOTAL EXPECTED SHORTAGES.
C     BAC(I)--PROBABILITY OF A BACKORDER OF ITEM I
C     COST--CURRENT COST OF A SPARES MIX.
C     POS(I)--PROBABILITY OF SUFFICIENCY FOR ITEM I
C          ON THE LAST DAY OF THE CYCLE.
C     DDELAY--TIME BETWEEN ITEM FAILURE AND DATE NEEDED
C          (ASSUME =0)
C     BUDGET--MAXIMUM BUDGET LEVEL FOR SPARES MIX CURRENTLY
C          BEING COMPUTED.
C     TSHORT--A MEASURE OF TOTAL EXPECTED AVERAGE SHORTAGES.
C     TOCEIL--ABSOLUTE CEILING ON BUDGET FOR ANY SPARES MIX.
C     OPTION--SIGNAL FOR WHETHER COMPARISON IS BEING MADE
C          OR WHETHER MODEL A IS COMPUTING SPARES MIXES.
C     TOTIND--TOTAL EXPECTED DEMAND PER CYCLE (ASSUMING
C          STATIONARITY)
C     BBI--SIGNAL FOR WHETHER LEAD TIME DEMAND HAS BEEN CALCU-
C          LATED.
C     WEIGHT--WEIGHT OF BACKORDERS ON LAUNCH DATE.
C     EBO--SUM OF BACKORDER PROBABILITIES (NOT A PERFORMANCE
C          MEASURE.)
C     SYSPOS--POS(I) FOR A GIVEN SPARES MIX
C     NDAY--TWICE THE NUMBER OF DAYS BETWEEN LAUNCHES.
C     NS--TWICE THE NUMBER OF DEMAND SPIKES IN A LAUNCH CYCLE.
C     LDX(I)--CURRENT STOCK LEVEL OF ITEM I.
C     ALAM(I,J)--LEAD TIME DEMAND RATE FOR ITEM I ON DAY J.
C     NITEMS--THE TOTAL NUMBER OF ITEMS BEING STOCKED
C     NSHUT--NUMBER OF SHUTTLES IN EQUAL LAUNCH CYCLES
C          (ASSUMED =2)
```

```
C     SPIKE(J)--TAKES ON VALUE +1 IF DEMAND SPIKE ON DAY J--
C          OTHERWISE =0.
C     ISP--DATE OF FIRST LAUNCH (ASSUMED =NDAY/2)
C
C**************************************************************
C
C
      COMMON BDDR(100),SEX(100),RDCRST(100),LAMBDA(100),
     *      W(100),BCOST(100),SHORT,BAC(100),
     *COST,POS(100)
      COMMON DUEDAY,BUDGET,TSHORT,TOOBIG,
     *OPTION,TOTDMD,EPT,WEIGHT,EBO,SYSPOS,NDAY,NS
      COMMON IEX(100)
      COMMON ALAM(30,100)
      COMMON NITEMS,NSHUT,SPIKE(100),ISP
C
C ***** BEGIN EXECUTION
C
      CALL INPUT
      CALL COMB
      CALL AA
      CALL OUTPUT
      RETURN
      END
C
      SUBROUTINE CADRE(II)
C
      INTEGER II
C
C*************************************************************
C
C     THIS SUBROUTINE CALCULATES THE LEAD TIME DEMAND FOR ITEM
C     II ON ALL DAYS DURING THE CYCLE OF LENGTH NDAY.
C
C*************************************************************
C
      COMMON BDDR(100),SEX(100),RDCRST(100),LAMBDA(100),
     *      W(100),BCOST(100),SHORT,BAC(100),
     *COST,POS(100)
      COMMON DUEDAY,BUDGET,TSHORT,TOOBIG,
     *OPTION,TOTDMD,EPT,WEIGHT,EBO,SYSPOS,NDAY,NS
      COMMON IEX(100)
      COMMON ALAM(30,100)
      COMMON NITEMS,NSHUT,SPIKE(100),ISP
      REAL IIJ
C
      T1=RDCRST(II)/24.
      SMINL=10.
      DIJ=DUEDAY
      DDL=BDDR(II)
      J=0.
      ISK=ISP
C
C ***** BEGIN CALCULATIONS FOR LEAD TIME DEMAND
CC
```

```
      DO 50 JJ=1,NDAY
            J=J+1.
            IF (ISK.GE.J)GO TO 6
            ISK=ISK+NDAY/2.
 6          IA=J-TI+(ISK-J)+1.
            IB=J
 7          IF (IA.GE.C) GO TO 8
              IA=IA+NDAY
              IB=IB+NDAY
              GO TO 7
 8          CONTINUE
            A=0.
            KK=IA
            DO 10 K=IA,IB
                 KK=KK+1
                 IF (KK.GT.NDAY) KK=1
                 A=A+SPIKE(KK)
 10         CONTINUE
            ALAM(II,J)=A*(DDB*NDAY)/2.
C
C*************************************************************************
C
C     IN MODEL D,
C          ALAM(II,J)=A*(DDB*NDAY)/3 + DDB/3*TI
C
C*************************************************************************
C
C ***** COMPUTE MINIMAL ACCEPTABLE STOCK LEVEL (MODEL A ONLY)
C
            LIJ=ALAM(II,J)
            IF (LIJ.GT.SMINL)GO TO 45
              SMINL=LIJ
 45         CONTINUE
 50   CONTINUE
            LAMBDA(II)=INT(SMINL)
      RETURN
      END
C
      SUBROUTINE INPUT
C
C*************************************************************************
C
C     AN ABBREVIATED VERSION OF THE INPUT SUBROUTINE IS SHOWN
C     BELOW
C
C*************************************************************************
C
      COMMON DDDB(100),SBX(100),BDCEST(100),LAMBDA(100),
     *       W(100),BCOST(100),SHOPT,BAC(100),
     *COST,POS(100)
      COMMON DUEDAY,BUDGET,ISHORT,TOORIG,
     *OPTION,TOTDAD,BEL,WEIGHT,IBC,SYSPOS,NDAY,NS
      COMMON IDX(100)
      COMMON ALAM(30,100)
      COMMON NITEMS,NSHOT,SPIKE(100),ISK
```

```
      INTEGER FIRSTI,LASTI,FIRSTJ,LASTJ,QUAN(30)
C
C ***** QUAN(I) IS THE QUANTITY OF ITEM I PER VEHICLE
C
C ***** READ IN INPUT DATA
C
      READ (5,601) NITEMS,NSHUT,OPTION
      READ (5,602) BUDGET,NDAY
      READ (5,606) NS,ISE
      READ (5,603) DDELAY,ALPHA
      READ (5,604) NREPS,WEIGHT
  601 FORMAT (2I10,F10.0)
  602 FORMAT (F10.0,I10)
  606 FORMAT (10I10)
  603 FORMAT (2F10.2)
  604 FORMAT (I10,F10.2)
      TOOBIG=BUDGET+25000000.
      DO 610 I=1,NITEMS
            READ (5,605)  BCOST(I),BDCRST(I),BDDR(I),QUAN(I)
  605       FORMAT (3F10.3,I10)
C
C ***** CORRECT ISSUE RT FOR FAILURES PER HOUR GROUND TIME
C ***** FLIGHT TIME PER FLIGHT IS 96.0 HOURS
C
            BDDR(I)=BDDR(I)*96.0*NSHUT*QUAN(I)/(NDAY/2)
  610 CONTINUE
      TOTDMD=0.0
      WT=WEIGHT
      DO 611 J=1,NDAY
            SPIKE(J)=0
            W(J)=1.
 611  CONTINUE
      ISK=ISE
C
C ***** INCREASE BACKORDER WEIGHT ON LAUNCH DATES
C
      DO 612 I=1,NS
            ISK=ISK+(I-1)*NDAY/NS
            W(ISK)=WT*2./NS
 612  CONTINUE
      DO 617 I=1,NITEMS
            TOTDMD=TOTDMD+BDDR(I)
  617 CONTINUE
C
      RETURN
      END
C
C
      SUBROUTINE COMP
C
C*****************************************************************
C
C    THIS SUBROUTINE READS IN STOCK LEVELS FROM AN EXTERNAL
C    SOURCE AND CALLS THE SUBROUTINE FOR MODEL A IN ORDER TO
C    EVALUATE THEM.
```

```
C
C***********************************************************************
C
      COMMON BDDB(100),SBX(100),BDCBST(100),LAMBDA(100),
     *     W(100),BCCST(100),SHORT,BAC(100),
     *CCST,POS(100)
      COMMON DUEDAY,BUDGET,ISHORT,TOOBIG,
     *OPTION,TOTDMD,PFI,WEIGHT,EBO,SYSPOS,NDAY,NS
      COMMON LBX(100)
      COMMON ALAM(30,100)
      COMMON NITEMS,NSHOT,SPIKE(100),ISP
C
      WRITE(6,3)
 3    FORMAT(1H ,'COMPARISON WITH BASE CASE')
      WRITE(6,4)
 4    FORMAT(1H /,3X,'POS',5X,'EBO',5X,'INVESTMENT')
      READ(2,1) NREC
 1    FORMAT(I3)
      OPTION=2
      PFI=0.
      DO 10 I=1,NREC
          READ(2,2) CCST,(LBX(J),J=1,NITEMS)
  2       FORMAT(F15.0,30I3)
          CALL AA
          CALL OUTPUT
          PFI=1.
          WRITE(6,5) SYSPOS,EBC,CCST
          WRITE(16,20) COST,SYSPOS
 20       FORMAT  (F9.0,F6.5)
 5        FORMAT(F10.6,F10.6,F15.0)
 10   CONTINUE
      OPTION=1.
      RETURN
      END
C
      SUBROUTINE OUTPUT
C
C
C***********************************************************************
C
C     THE OUTPUT SUBROUTINE IS NOT INCLUDED.  FOR EACH BUDGET
C     GENERATED OR EVALUATED, THIS SUBROUTINE OUTPUTS THE ITEM
C     STOCK LEVELS, THE VALUE OF POS(I) ON THE LAST DAY IN THE
C     CYCLE FOR ITEM I, AND THE EXPECTED PERFORMANCE POS(S)
C     OF THE ASSOCIATED SPARES MIX.
C
C***********************************************************************
C
      COMMON BDDB(100),SBX(100),BDCBST(100),LAMBDA(100),
     *     W(100),BCCST(100),SHORT,BAC(100),
     *CCST,POS(100)
      COMMON DUEDAY,BUDGET,ISHORT,TOOBIG,
     *OPTION,TOTDMD,PFI,WEIGHT,EBO,SYSPOS,NDAY,NS
      COMMON LBX(100)
      COMMON ALAM(30,100)
```

```
      COMMON NITEMS,NSHUT,SPIKE(100),ISP
      COMMON /STATE/ MDAY(30)
C
      WRITE(6,5) SYSPCS
5     FORMAT('PCS(S)= ',F5.4)
      PEC=0.
      RETURN
      END
```

```
C
C     SUBROUTINE TO IMPLEMENT ALGORITHM AND EVALUATE
C     SPARES MIXES FOR MODEL A
C       BY KATHLEEN CONLEY, DECEMBER 1981
C
      SUBROUTINE AA
C
C*****************************************************************
C
C     VARIABLES UNIQUE TO THIS SUBROUTINE ARE DEFINED BELOW:
C
C     DSEED--SEED FOR RANDOM NUMBER GENERATOR
C     A2(I)--A(S1,S2,...,S(I)+1,...,S(N))
C     RHAT--PRODUCT OF INDIVIDUAL ITEMS' POS(I) ON ONE DAY
C          WHEN ONE ITEM'S STOCK LEVEL INCREASES BY +1.
C     PLUS(I,J)--INCREASED POS FOR ITEM I WHEN ITS STOCK
C          LEVEL INCREASES BY +1.
C     RHAT1(J)--RHAT ON DAY J
C     WPOS-- WEIGHTED VALUE OF RHAT FOR ONE DAY
C     A--SUM OF WEIGHTED POS(S) OVER ALL NDAY DAYS
C     DEL--INCREMENTAL INCREASE IN A FOR ONE MORE
C     UNIT OF ONE ITEM.
C     DELMAX--MAXIMUM VALUE OF DEL
C     CUR(I,J)--INDIVIDUAL PROBABILITY OF S(I) DEMANDS
C          OVER THE LEAD TIME FOR DAY J.
C     CUM(I,J)--SUM OF PROBABILITIES OF <=S(I) DEMANDS
C          OVER THE LEAD TIME FOR DAY J
C     PX--VALUE OF CUR(I,J) FOR ONE DAY AND ONE ITEM
C     PCUM--VALUE OF CUM(I,J) FOR ONE DAY AND ONE ITEM
C
C*****************************************************************
C
      COMMON BDDR(100),SBX(100),BDCRST(100),LAMBDA(100),
     *     W(100),BCOST(100),SHORT,BAC(100),
     *COST,POS(100)
      COMMON DUEDAY,BUDGET,ISHORT,IOCBIG,
     *OPTION,TOTDMD,BRT,WEIGHT,EBO,SYSPOS,NDAY,NS
      COMMON IDX(100)
      COMMON ALAM(30,100)
      COMMON NITEMS,NSHOT,SPIKE(100),ISP
      REAL F
      DOUBLE PRECISION DSEED,A2(100),RHAT2,PLUS(30,100)
      DOUBLE PRECISION RHAT,WPOS,DEL,DELMAX,A,DEI,DELMAX
      DOUBLE PRECISION CUR(30,100),CUM(30,100),RHAT1(100)
      DOUBLE PRECISION PX,PCUM
      COMMON /STATE/ NDAY(30)
C
      DSEED=123457.D0
      STOCK=0.
      COST=0.
      A=0.
C
C ***** INITIALIZATION
C
      DO 50 I=1,NITEMS
```

```
              A2(I)=0.
              COST=COST+BCOST(I)*LBX(I)
              IF (BRT.EQ.1) GO TO 52
C
C *****RANDOMLY DETERMINE  MAINTENANCE DAY FOR ITEM I
C
                R=GGUBFS(DSEED)
                JSP=INT(R*(NDAY/2.-1)+.5)+1
                MDAY(I)=JSP
                SPIKE(JSP)=1.
                KSP=JSP+INT(NDAY/2.)
                SPIKE(KSP)=1.
                CALL CADPE(I)
                SPIKE(JSP)=0.
                SPIKE(KSP)=0.
 52           CONTINUE
              PCS(I)=0.
              SBX(I)=0.
              EAC(I)=0.
  50   CONTINUE
       TSHORT=0.
       EBO=0.
C
C ***** OPTION =2 WHEN EVALUATING EXTERNALLY GENERATED
C    SPARES MIX
C
       IF (OPTION.EQ.2) GO TO 105
         COST=0.
         DO 100 I=1,NITEMS
         LBX(I)=LAMBDA(I)
              COST=COST + BCOST(I)*LBX(I)
 100          CONTINUE
 105   CONTINUE
       FLAG=0.
C
C ***** BEGIN ALGORITHM/EVALUATION
C
 110   DO 300 J=1,NDAY
              IF (FLAG.EQ.1.)GO TO 265
                KHAT=1.
C
C ***** FOR EACH ITEM ON DAY J COMPUTE A PCS
C
                DO 250 I=1,NITEMS
                     T=ALAM(I,J)
                     BAC(I)=BAC(I)+BDDR(I)*HDCOST(I)/24./NDAY
                     PX=EXP(-T)
                     PCUM=PX
                     STOCK=0.
                     IF (LBX(I).EQ.0) GO TO 270
                       L=LBX(I)
                       STOCK=0.
                       DO 260 K=1,L
                            STOCK=STOCK+1.
                            PX=PX*T/STOCK
```

```
                        PCUM=PCUM+PX
                        BAC(I)=BAC(I)-(1.-PCUM)/NDAY
  260                CONTINUE
  270              CONTINUE
C
C ***** CALCULATE PRODUCT OF INDIVIDUAL ITEMS'
C    POS ON DAY J
C
                  RHAT=PCUM*RHAT
                  PLUS(I,J)=(PCUM+(PX*T)/(STOCK+1.))/PCUM
                  CUR(I,J)=PX
                  CUM(I,J)=PCUM
                  POS(I)=PCUM
                  SHORT=(1.-PCUM)*EDDR(I)
                  SBX(I)=SBX(I)+SHORT
 250          CONTINUE
              RHAT1(J)=RHAT
C
C ***** CALCULATE WEIGHTED POS(S) ON DAY J
C
              WPOS=W(J)*RHAT
C
C ***** SUM WEIGHTED POS FOR ALL DAYS
C
              A=WPOS + A
              IF (OPTION.EQ.2) GO TO 281
 265          DELMAX=0.0
              DO 280 I=1,NITEMS
C
C ***** CALCULATE A(S1,S2,...,S(I)+1,...,S(N))
C
                  RHAT2=RHAT1(J)*PLUS(I,J)
                  A2(I)=A2(I)+RHAT2*W(J)
                  IF (J.LT.NDAY) GO TO 285
C
C ***** FIND IMPROVEMENT OVER CURRENT A(S)
C
                  DEL=(A2(I)-A)/BCOST(I)
C
C ***** FIND MAX IMPROVEMENT
C
                  IF (DEL.LT.DELMAX) GO TO 285
                    IBEST=I
                    DELMAX=DEL
 285              CONTINUE
 280          CONTINUE
 281        CONTINUE
 200        CONTINUE
      IF(FLAG.EQ.1) GO TO 245
      DO 240 I=1,NITEMS
           TSHORT=TSHORT+SBX(I)/NDAY
 240  CONTINUE
  245 CONTINUE
      IF (OPTION.EQ.2) GO TO 545
C
```

```
C ***** ADD BEST ITEM TO SPARES COST IF < BUDGET
C
        TCOST=COST + BCOST(IBEST)
 295    IF (TCOST.GT.BUDGET) GO TO 530
          COST=TCOST
          A=A2(IBEST)
          DO 300 I=1,NITEMS
               A2(I)=0.
 300      CONTINUE
C
C ***** ADD BEST ITEM TO SPARES MIX
C
          LBX(IBEST)=LBX(IBEST)+1.
          SHORT=0.
C
C ***** RECOMPUTE WEIGHTED PCS(S) ON DAY J FOR
C     NEW SPARES MIX AND BACKORDER PROBABILITIES
C     FOR NEWLY ADDED ITEM
C
        DO 500 J=1,NDAY
               I=IBEST
               CUR(I,J)=CUR(I,J)*ALAM(I,J)/FLOAT(LBX(I))
               RHAT1(J)=RHAT1(J)/CUM(I,J)
               CUM(I,J)=CUM(I,J)+CUR(I,J)
               BAC(I)=BAC(I)-(1.-CUM(I,J))/NDAY
               PCS(I)=CUM(I,J)
               RHAT1(J)=RHAT1(J)*CUM(I,J)
               PLUS(I,J)=(CUM(I,J)+CUR(I,J)*ALAM(I,J)
     */FLOAT(LBX(I)+1))/CUM(I,J)
               SHORT=CUR(I,J)*EDDR(I)+SHORT
 500      CONTINUE
          SBX(I)=SBX(I)-SHORT/NDAY
          TSHORT=TSHORT-SHORT/NDAY
           CUMWT=NDAY-2+(WEIGHT*2.)
C
C ***** CALCULATE PCS(S)
C
           SYSPCS=A/CUMWT
          FLAG=1.
C
C ***** FLAG PREVENTS RECALCULTION OF NEW BACKORDER
C     PROBABILITIES.
          GO TO 110
 530    IF (SYSPCS.GT..99) GO TO 540
C
C ***** INCREMENT BUDGET CEILING
C
        IF(BUDGET.GT.TCCEIG) GO TO 540
        BUDGET=BUDGET+1000000.
        DO 531 I=1,NITEMS
            BBC=BBC+BAC(I)
 531    CONTINUE
C
C ***** DISPLAY OUTPUT
C
```

```
          CALL OUTPUT
          GO TO 295
545      CUMWT=NDAY-2.+WEIGHT*2
         SYSPOS=A/CUMWT
540   CONTINUE
      DO 546 I=1,NITEMS
           EPO=EPO+PAC(I)
546   CONTINUE
      RETURN
      END
```

```
C
C     SUBROUTINE TO IMPLEMENT ALGORITHM AND EVALUATE SPARES
C     MIXES FOR MODEL B
C       BY KATHLEEN CONLEY, DECEMBER 1981
C
      SUBROUTINE BB
C
C*************************************************************
C
C     THE FOLLOWING VARIABLES ARE UNIQUE TO MODEL B
C
C     CBO(M)--CUMULATIVE WEIGHTED PROBABILITY OF A
C          BACKORDER FOR ITERATION M
C     ALPHA--MAXIMUM ACCEPTABLE PROBABILITY OF A
C          BACKORDER FOR ANY ITEM
C     NREPS--NUMBER OF ITERATIONS DESIRED
C     MAX--MAXIMUM ITEM COST
C     CUMWT--CUMULATIVE WEIGHTS FOR BACKORDERS ON ALL DAYS
C     COMPAR--INDICATES SUBROUTINE IS BEING USED TO
C          EVALUATE EXTERNALLY GENERATED SPARES MIX
C     EXBO(I,M)--EXPECTED WEIGHTED BACKORDERS OF ITEM I AT
C          ITERATION M
C     LSTOCK(I,M)--STOCK LEVEL FOR ITEM I AT ITERATION M.
C     COSTM(M)--SPARES MIX COST FOR ITERATION M
C
C*************************************************************
C
      COMMON BDDF(100),BDCRST(100),
     *       W(100),BCOST(100),BO(30,15),CBO(15),
     *COST(3)
      COMMON PX,DUEDAY,ALPHA,NREPS,BUDGET,MAX,CUMWT,
     *OPTION,TOTDMD,COMPAR,WEIGHT,DDF,NDAY,NS
      COMMON L5X(100),CUMBO(20),EXBO(30,20)
      COMMON ALAM(100,100),LSTOCK(30,20),COSTM(20)
      COMMON NITEMS,NSHOT,SPIKE(100),ISP
      REAL PX,PCUM,THETA,THETAM
      DIMENSION P(100)
C
C ***** FIND MAX COST
C
      IF(OPTION.NE.3) GO TO 315
      MAX=0.
      DO 300 I=1,NITEMS
            IF(BCOST(I).LE.MAX) GO TO 250
            MAX=BCOST(I)
 250  CONTINUE
            WRITE(60,305)  (ALAM(I,J),J=1,NDAY)
 305        FORMAT(15(/7F10.5))
 300  CONTINUE
C
C ***** FIND SUM OF WEIGHTS
C
      CUMWT=0.
      DO 350 J=1,NDAY
            CUMWT=CUMWT+W(J)
```

```
 350  CONTINUE
 315  CONTINUE
      THETA=(1.-ALPHA)*CUMWT/MAX
      DO 355 MM=1,NREPS
           CUMBO(MM)=0.
           COSTM(MM)=0.
           CBO(MM)=0
 355  CONTINUE
      DO 500 I=1,NITEMS
           CUMSUM=0.
           STOCK=0.
           N=NREPS
           IF (COMPAR.EQ.1) N=1
           DO 450 M=1,N
C
C ***** COMPUTE MULTIPLIER FOR ITERATION M AND CRITICAL
C          VALUE FOR ITEM I.
C
                THETAM=THETA/2.**(M-1.)
                RHS=CUMWT-THETAM*BCOST(I)
                IF (STOCK.GT.0) GO TO 380
                  CUMSUM=0.
C
C ***** SUM PROBABILITIES OF ZERO LEAD TIME DEMANDS
C
                  DO 370 J=1,NDAY
                       T=ALAM(I,J)
                       P(J)=EXP(-T)
                       PCUM=P(J)
                       CUMSUM=CUMSUM+W(J)*PCUM
 370              CONTINUE
 380            CONTINUE
                IF(COMPAR.EQ.0.) GO TO 390
                  RHS=C.
                  STOCK=LBX(I)-1.
                  GO TO 395
 390            CONTINUE
                IF (CUMSUM.GE.RHS) GO TO 400
 395              CUMSUM=0.
                  STOCK=STOCK+1.
                  DO 375 J=1,NDAY
                       T=ALAM(I,J)
                       P(J)=EXP(-T)
                       PCUM=P(J)
                       IF(STOCK.EQ.0.) GO TO 377
                         ISTOCK=INT(STOCK)
                         DO 376 K=1,ISTOCK
                              P(J)=P(J)*T/FLOAT(K)
                              PCUM=P(J)+PCUM
 376                     CONTINUE
 377                   CONTINUE
C
C ***** SUM WEIGHTED PROBABILITIES OF ZERO BACKORDERS
C
                       CUMSUM=CUMSUM+W(J)*PCUM
```

```
 375              CONTINUE
C
C ***** REPEAT UNTIL STOCK CAUSES FUNCTION TO ECXCEED
C          CRITICAL VALUE.
C
                  IF (COMPAR.EQ.0.) GO TO 380
 400             CONTINUE
                 EXBC(I,M)=0.
                 LSTOCK(I,M)=STOCK
                 COSTM(M)=COSTM(M)+LSTOCK(I,M)*BCOST(I)
C
C ***** EVALUATE EXPECTED WEIGHTED BACKORDERS FOR
C          EACH ITEM
C
                 DO 460 J=1,NDAY
                   SUM=0
                   STK=STOCK
                   PR=P(J)
                   T=ALAM(I,J)
  462              STK=STK+1.
                   PR=PR*T/STK
                   SUM=SUM+(STK-STOCK)*PR
                   IF(PR.LT.1E-5) GO TO 465
                     GO TO 462
  465              CONTINUE
                   EXBO(I,M)=EXBO(I,M)+W(J)*SUM
  460              CONTINUE
C
C ***** EVALUATE CUMULATIVE E[WT BC]
C
                 CBO(M)=CBO(M)+EXBC(I,M)/CUMWT
  450            CONTINUE
  500  CONTINUE
     CALL OUT
     RETURN
     END
```

```
C
C     SUBROUTINE TO IMPLEMENT ALGORITHM AND EVALUATE SPARES
C     MIXES FOR MODEL D
C       BY KATHLEEN CONLEY, DECEMBER 1981
C
      SUBROUTINE DD
C
C*****************************************************************
C
C     THE FOLLOWING VARIABLES ARE DEFINED FOR NEITHER MODEL
C          A NOR MODEL B:
C
C     WBAR(I,J)--WEIGHT IF A BACKORDER DAY TIMES EXPECTED
C          WAITING TIME FOR A BACKORDER ON DAY J FOR ITEM
C          I.
C     CWT(I)--CUMULATIVE WBAR(I,J) FOR ALL DAYS FOR ITEM I.
C
C*****************************************************************
C
      COMMON BDDR(100),RDCRST(100),
     *     W(100),ECOST(100),BO(30,15),CBO(15),
     *COST(3)
      COMMON PX,DUEDAY,ALPHA,NREPS,BUDGET,MAX1,MAX,
     *OPTION,TOTDMD,COMPAR,WEIGHT,DDR,NDAY,NS
      COMMON LBM(100),CUMBC(20),EXBO(30,20)
      COMMON ALAM(100,100),ISTOCK(30,20),COSTM(20)
      COMMON NITEMS,NSHUT,SPIKE(100),ISP,WBAR(30,100),CWT(30)
      REAL PX,PCUM,THETA,THETAM
      DIMENSION P(100)
C
C
      IF (OPTION.NE.3) GO TO 315
C
C ***** FIND SUM OF WEIGHTS
C
      CUMWT=0.
      DO 350 J=1,NDAY
         CUMWT=CUMWT+W(J)
 350  CONTINUE
      DO 300 I=1,NITEMS
C
C ***** COMPUTE WBAR(I,J)
C
         CWT(I)=0.
           DO 310 J=1,NDAY
                WBAR(I,J)=W(J)*RDCRST(I)/24./ALAM(I,J)/CUMWT
             CWT(I)=CWT(I)+WBAR(I,J)
 310       CONTINUE
          WRITE(60,100) CWT(I),I,(WBAR(I,J),J=1,NDAY)
 100      FORMAT (F10.0,I4,15(/7F10.5))
 300  CONTINUE
C
      THETA=100000000.
      DO 110 I=1,NITEMS
C
```

```
C ***** FIND MAXIMUM THETA FOR ALL ITEMS
C
          THETAI=(1-ALPHA)*CWT(I)/BCOST(I)
          IF(THETA.GT.THETAI) THETA=THETAI
110    CONTINUE
       WRITE(60,105) THETA
105    FORMAT('THETA= ',F10.6)
315    CONTINUE
       DO 355 MM=1,NREPS
            CUMBO(MM)=0.
            CCSTM(MM)=0.
            CBO(MM)=0
 355   CONTINUE
       DO 500 I=1,NITEMS
            CUMSUM=0.
            STOCK=0.
            N=NREPS
            IF (COMPAR.EQ.1) N=1
            DO 450 M=1,N
                 THETAM=THETA/2.**(M-1.)
                 RHS=CWT(I)-THETAM*BCOST(I)
                 IF (STOCK.GT.0) GO TO 380
                   CUMSUM=0.
                   DO 370 J=1,NDAY
C
C ***** SUM PROBABILITIES OF ZERO DEMANDS OVER LEAD TIME
C
                          T=ALAM(I,J)
                          P(J)=EXP(-T)
                          PCUM=P(J)
                          CUMSUM=CUMSUM+WBAR(I,J)*PCUM
C
C ***** SUM WEIGHTED PROBABILITIES OF 0 LEAD TIME DEMANDS
C
 370               CONTINUE
 380             CONTINUE
                 IF(COMPAR.EQ.0.) GO TO 390
                   RHS=0.
                   STOCK=LBX(I)-1.
                   GO TO 395
 390             CONTINUE
                 IF (CUMSUM.GE.RHS) GO TO 400
C
C ***** CONTINUE UNLESS FUNCTION EXCEEDS CRITICAL VALUES
C
 395               CUMSUM=0.
                   STOCK=STOCK+1.
                   DO 375 J=1,NDAY
                        T=ALAM(I,J)
                        P(J)=EXP(-T)
                        PCUM=P(J)
                        IF(STOCK.EQ.0.) GO TO 377
                          ISTOCK=INT(STOCK)
                          DO 376 K=1,ISTOCK
                               P(J)=P(J)*T/FLOAT(K)
```

```
                              PCUM=P(J)+PCUM
 376                        CONTINUE
 377                      CONTINUE
                          CUMSUM=CUMSUM+WBAR(I,J)*PCUM
 375                 CONTINUE
                     IF (COMPAR.EQ.0.) GO TO 380
C
C ***** REPEAT UNTIL STOCK CAUSES FUNCTION TO EXCEED CRITICAL
C          VALUE
C
 400               CONTINUE
                   ISTOCK(I,M)=STOCK
                   COSTM(M)=COSTM(M)+ISTOCK(I,M)*PCOST(I)
                   EXBO(I,M)=0.
C
C ***** EVALUATE EXPECTED WEIGHTED BACKORDER DAYS FOR EACH ITEM
C
                   DO 460 J=1,NDAY
                     SUM=0
                     STK=STOCK
                     T=ALAM(I,J)
                     PR=P(J)
  462                STK=STK+1.
                     PR=PR*T/STK
                     SUM=SUM+(STK-STOCK)*PR
                     IF(PR.LT.1E-8) GO TO 465
                       GO TO 462
  465                CONTINUE
                     EXBO(I,M)=EXBO(I,M)+WBAR(I,J)*SUM/NDAY
  460                CONTINUE
C
C ***** EVALUATE CUMULATIVE EXPECTED WEIGHTED BACKORDER DAYS
C
                   CUMBO(M)=CUMBO(M)+EXBO(I,M)
                   CBO(M)=CUMBO(M)
  450              CONTINUE
  500   CONTINUE
       CALL OUT
       RETURN
       END
```

APPENDIX B - SIMULATION PROGRAM

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C
C    SIMULATION FOR BASING STUDY FOR SHUTTLE SPARES
C      BY KATHLEEN CONLEY, DECEMBER 1981
C
C    NOTE:  THE MAIN PROGRAM IS ALMOST ENTIRELY TAKEN FROM THE
C          REFERENCE GIVEN BELOW.  IT IS INCLUDED FOR PURPOSES
C          OF VARIABLE DEFINITION ONLY.
C
C        REFERENCE:  SIMULATION OF A TWO-ECHELON INVENTORY SYSTEM
C                    BY JIM COGLIANO, NOVEMBER 1980
C
C************************************************************
C
C  LIST OF GLOBAL VARIABLES
C
C /PARAM/       SYSTEM PARAMETERS
C  LMAX         NUMBER OF BASES
C  IMAX         NUMBER OF ITEM TYPES
C  NUNITS(L,I)  NO. OF UNITS OF TYPE-I AT BASE-L
C  NSPARE(L,I)  INITIAL NO. OF SPARE PARTS OF TYPE-I AT LOCATION-L
C  AFAIL(L,I,J) AVG. TIME-TO-FAILURE FOR TYPE-I PARTS INSTALLED IN
C               TYPE-J UNITS AT BASE-L
C  AREP(L,I)    AVG. REPAIR TIME FOR TYPE-I PARTS REPAIRED AT LOCATION-L
C  PREP(L,I)    PROB. OF A BASE REPAIR FOR TYPE-I PARTS FAILING AT BASE-L
C  ABD(L)       AVG. TRANSIT TIME FROM BASE-L TO THE DEPOT
C  ADB(L)       AVG. TRANSIT TIME FROM THE DEPOT TO BASE-L
C  AORD(L)      AVG. PROCESSING TIME FOR AN ORDER FROM BASE-L
C  LDEPOT       INDEX FOR THE DEPOT (ALWAYS EQUALS LMAX+1)
C
C /STATE/       STATE VARIABLES
C  NR(L,I)      NO. OF TYPE-I PARTS IN REPAIR AT LOCATION-L
C  NS(L,I)      NO. OF TYPE-I PARTS IN SPARE STOCK AT LOCATION-L
C  NBD(L,I)     NO. OF TYPE-I PARTS IN TRANSIT FROM BASE-L TO THE DEPOT
C  NDB(L,I)     NO. OF TYPE-I PARTS IN TRANSIT FROM THE DEPOT TO BASE-L
C  NU(L,I,J)    NO. OF TYPE-I PARTS INSTALLED IN TYPE-J UNITS AT BASE-L
C  NG(L,I)      NO. OF TYPE-I UNITS AT BASE-L GROUNDED FOR LACK OF PARTS
C  NGXX         NO. GROUNDED FOR LACK OF PARTS OVER ALL UNITS AND BASES
C
C /STATS/       STATISTICS:  CUMULATIVE SUMS OF THE STATE VARIABLES
C
C /CLOCKS/      SIMULATION TIMING VARIABLES
C  CLOCK        CURRENT TIME
C  CSTART       LENGTH OF WARM-UP PERIOD
C  CSTOP        TIME TO STOP SIMULATION
C  CINTER       LENGTH OF TIME BETWEEN STATISTICS OBSERVATIONS
C  NOBS         NUMBER OF OBSERVATIONS TO TAKE
C
C /OPTION/      OPTIONAL FEATURES
C
C  IVAR         1 FOR VARIANCE ESTIMATION
C  IEXP         1 FOR EXPONENTIAL FAILURE TIMES (FASTER EXECUTION)
C  ITRACE       1 FOR TRACE OF EVENTS
C
C /POLICY/      POLICY SELECTION VARIABLES
C  IBORD        BASE ORDERING POLICY
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C  IBINST      BASE INSTALLATION POLICY
C  IDSHIP      DEPOT SHIPPING POLICY
C
C /ORDERS/     DEPOT BACK-ORDER LIST (ARRAY OF FORWARD-LINKED LISTS)
C /EVENTS/     FUTURE EVENTS LIST (HEAP-SORTED)
C /SEEDS/      RANDOM NUMBER SEEDS (DOUBLE PRECISION FOR IMSL)
C /CURREV/     CURRENT EVENT CODE AND ATTRIBUTES
C /NR/         NEED-RELUCTANCE VARIABLES
C /FILES/      INPUT AND OUTPUT FILE UNIT NUMBERS
C
C*************************************************************
C
      COMMON /PARAM/ IMAX, IMAX, NUNITS(5,5), NSPARE(6,5),
     &               AFAIL(5,5,5), AREP(6,5), PREP(5,5),
     &               AED(5), ADE(5), AORD(5), IDEPOT
      COMMON /STATE/ NR(6,5), NS(6,5), NBD(5,5), NDB(5,5), NU(5,5,5),
     &               NG(5,5), NGXX
      COMMON /STATS/ SR(6,5), SS(6,5), SBD(5,5), SDB(5,5), SU(5,5,5),
     &               SG(5,5), SGXX, SSGXX,
     &               SSR(6,5), SSS(6,5), SSBD(5,5), SSDB(5,5),
     &               SSU(5,5,5), SSG(5,5)
      COMMON /CLOCKS/ CLOCK, CSTART, CSTOP, CINTER, NOBS
      COMMON /OPTION/ IVAR, IEXP, ITRACE
      COMMON /POLICY/ IBORD, IBINST, IDSHIP
      COMMON /ORDERS/ ITPRH(5,2),ITPRT(5,2),LOC(100),NXTLOC(100),NXTLF
      COMMON /EVENTS/ NXTEV(1000), TIME(1000), INFO(1000,4), NXTEVF,
     &                FTIME, FRATE
      COMMON /CURREV/ KODE, K1, K2, K3
      COMMON /NR/ RNMAX(5), LRNMAX(5), RN(5,5),
     &            BTABLE(5,5,21,21), DTABLE(21), NIMAX, NSMAX,
     &            ICB(5,8,4,4,4,4,4),IDMAX, IBMAX
      COMMON /FILES/ JIN, JOUT
     COMMON /LATS/   S1,S2
C
C
C *****
C  THE MAIN PROGRAM CONTROLS THE EXECUTION OF THE SIMULATION.
C  THERE ARE THREE PHASES:
C     (1)  READ THE INPUT PARAMETERS AND INITIALIZE THE STATE VARIABLES.
C     (2)  PROCESS THE EVENTS AS THEY OCCUR.  ADVANCE THE CLOCK, COLLECT
C          STATISTICS, AND CALL A ROUTINE TO CARRY OUT THE DETAILS.
C     (3)  PRINT THE SUMMARY STATISTICS.
C *****
C
      JIN = 1
      JOUT = 2
C
C
      CALL INPUT
      CALL INIT
      CALL STAT (1)
C
10    CALL EVENT (T,KODE,K1,K2,K3)
      CLOCK = T
C
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      IF (KODE.EQ.1) CALL ARRIVE (K1,K2,K3)
      IF (KODE.EQ.6) CALL FAIL (K1,K2,K3)
C
C ***** LAUNCH CRITICAL EVENT
C
      IF (KODE.EQ.12) CALL LCRIT (K1,K2)
      IF (KODE.EQ.15) CALL CBDEB (K1,K2,K3)
      IF (KODE.EQ.18) CALL REPAIR (K1,K2)
      IF (KODE.EQ.19) CALL STAT (2)
      IF (KODE.NE.19.OR.CLOCK.LT.CSTOP-CINTER/2) GOTO 10
C
      CALL STAT (3)
C
      STOP
      END
C
C ***** COMPUTER CODE FOR ALL BUT THE LAUNCH CRITICAL
C    SUBROUTINE IS OMITTED--HOWEVER, TO RUN THE PROGRAM,
C    ALL THE EVENTS MUST CORRESPOND TO SUBROUTINES
C
C
C***********************************************************
C
C    LAUNCH CRITICAL AND NONSTATIONARY DEMAND SUBROUTINE:
C
C***********************************************************
C
      SUBROUTINE LCRIT (L,K)
C
      COMMON /CLOCKS/ CLOCK, CSTART, CSTOP, CINTER, NOBS
      COMMON /EVENTS/ NXTEV(1000), TIME(1000), INFO(1000,4), NXTEVF,
     &                FTIME, FRATE
      COMMON /PARAM/ LMAX, IMAX, NUNITS(5,5), NSPARE(6,5),
     &               AFAIL(5,5,5), AREP(6,5), PREP(5,5),
     &               ABD(5), ADE(5), AORD(5), LDEPOT
      COMMON /STATE/ NR(6,5), NS(6,5), NBD(5,5), NDB(5,5), NU(5,5,5),
     &               NG(5,5), NGXX
      COMMON /POLICY/ IBORD, IBINST, IDSHIP
      COMMON /CURREV/ KODE,K1,K2,K3
C
C ***** K IS 1 WHEN LAUNCH CRITICAL, RESET TO 0 UPON LAUNCH,
C    AND IS 2 WHEN LAUNCH CRITICAL IS APPROACHING
C
C ***** DETERMINE BASE TO BASE TRAVEL TIME
C
     LL=L+2
     RST=ADB(LL) * 2
     IF(ADB(L).GT.RST)  RST=ADB(L)
C
     IF(K.EQ.1) GO TO 100
C
C ***** QUERY USER FOR NONSTATIONARITY IN DEMAND RATE
C
     IF(TFLAG.GT.0) GO TO11
     WRITE(7,9)
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9      FORMAT('TIMES? F3.0')
       READ(8,10) TIMES
10     FORMAT(F3.0)
11     TFLAG=1.
C
       IF(K.EQ.2) GO TO  30
C
C ***** IF LAUNCH IS TODAY (K=0), DOUBLE
C     THE DEMAND RATE FOR TODAY ONLY
C
       FTIME=FTIME-TIMES
       IF(FTIME.LT.0) FTIME=0.
C
C ***** SCHEDULE ANOTHER LAUNCH CRITICAL
C
       T=50/NUNITS(L,IMAX) - RST
       LISHIP=L
       CALL SCHED(T,12,L,1,0)
C
C ***** SCHEDULE ANOTHER LAUNCH CRITICAL APPROACHING
C
       T2=T-ADB(LL)
       IF(T2.GE.0) GO TO 25
       WRITE(JOUT,20)
20     FORMAT('LAUNCHES TOO CLOSE FOR LATERAL RESUPPLY')
       STOP
25     CONTINUE
       CALL SCHED(T2,12,L,2,0)
C
C ***** IF LAUNCH CRITICAL APPROACHING,POSSIBLY SCHEDULE
C     SHIPMENT TO THE BASE
C
30     DO 50 I=1,IMAX
          ISHIP=I
          K3=1
          CALL DSHIP(ISHIP,ISHIP,JORD)
          CALL DACT(ISHIP,ISHIP,JORD)
50     CONTINUE
       K3=0
       GO TO 990
C
C ***** IF PRESENTLY LAUNCH CRITICAL, SCHEDULE
C     A LAUNCH (K=0) IN RST TIME UNITS
C
100    T= RST
       CALL SCHED(T,12,L,0,0)
C
990    RETURN
       END
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