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FINAL REPORT ON ONR CONTRACT N00014-75-C-0586

"STATISTICAL ENGINEERING"

by

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Principal Investigator

May 28, 1982

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FINAL REPORT ON ONR CONTRACT N00014-75-C-0586

ENTITLED "STATISTICAL ENGINEERING"

This is the FINAL REPORT on ONR Contract N00014-75-C-0586 entitled "Statistical Engineering" with Dr. Robert Bechhofer as Principal Investigator. The contract was initiated on June 1, 1975 and was renewed five times through March 31, 1982. ANNUAL REPORTS were submitted to cover the following periods: FIRST REPORT (6/1/75-8/31/75), SECOND REPORT (9/1/75-8/31/76), THIRD REPORT (9/1/76-8/31/77). In addition, a SPECIAL REPORT (requested by Dr. E.J. Wegman, Director of the Statistics and Probability Program) summarizing the research accomplishments achieved on the contract was submitted on 8/14/78. Copies of these reports are attached. This FINAL REPORT describes research accomplishments subsequent to the SPECIAL REPORT. Thus collectively these reports summarize progress to date.

This FINAL REPORT is divided into three sections as follows:

1. Background for the research undertaken on ONR Contract N00014-75-C-0586.
2. Recent developments
   a. General developments in this research area
   b. Some specific research accomplishments achieved on ONR Contract N00014-75-C-0586
3. Research supported in whole or in part by ONR Contract N00014-75-C-0586
   a. Technical reports
   b. Papers published or accepted for publication
   c. Papers submitted for publication

A-1

1. Background for the research undertaken on ONR Contract N00014-75-C-0586

The major thrust of research on this contract has been on the development of statistical procedures for "selecting or ordering" populations or for making "multiple comparisons" among populations. Some background concerning the early history of research in this area, and of the Principal Investigator's role in advocating and initiating the development of such procedures (particularly ones employing the so-called "indifference-zone" approach to ranking and selection problems) is given in Part I (pages 2-5) of the SPECIAL REPORT. The following paragraphs which sketch the setting for much of the research on this contract were provided by the Principal Investigator for inclusion in the report titled "Program Summary for FY 81" (edited by Edward J. Wegman, Program Director) on the Statistics and Probability Program (ONR Code 436) dated June 1981; see page 21 of that report.

"Engineers or scientists engaged in procurement, research and development, or testing are often faced by the problem of selecting the "best" of several competing categories, e.g., the best product or type or system, or brand, or treatment. Thus, a procuring activity might be sampling the product of several competing contractors prior to the award of a production contract (in order to select the one with the smallest fraction defective) or an ordnance engineer involved in research and development might be conducting firing programs to compare the ballistic performance of different types of projectiles (in which case his objective might be to select that type which, on the average, has the deepest penetration or the longest range), or a systems engineering might be comparing the performance of several competing systems to select the one with the highest reliability, or a chemical engineer might be conducting experiments to compare the performance of different brands of measuring equipment (in which case his objective might be to select that brand which has the highest precision, i.e., the greatest reproducibility).

For each of these examples, the objective of the experiment or study might have been more comprehensive than simply to identify the "best" category; for example, it might have been to rank the categories from "best" to "worst" according to some criterion of "goodness." The selection is to be done in each case on the basis of data obtained in a controlled test or experiment. To accomplish this objective the experimenter requires a statistical decision procedure which tells how to lay out the experiment (i.e., what experimental design to use), how many observations to take, how to take these observations, and based on these observations which category to select as "best." The decision procedure is to have the property that a) the probability of a correct selection is controlled at some specified level, and b) the number of observations required to achieve this objective is minimized.

In other situations the experimenter might be interested in comparing simultaneously two or more test categories (or treatments) with a control category or a standard. Here too the experimenter may wish to control at some prespecified level the confidence coefficient associated with the joint interval estimates of the differences between the treatments and the control, and to do so with a minimum total number of observations.

Such procedures, called "statistical selection procedures" and "multiple comparisons procedures," respectively, have been studied extensively at Cornell (and elsewhere) over a period of years, and research on them is continuing."

2. Recent developments

a. General developments in this research area

Interest in the "selection and ordering" approaches and in the "multiple comparisons" approaches has continued at a high level. Since our SPECIAL REPORT in August 1978, two new books have appeared which deal specifically with such methodologies. These are Multiple Decision Procedures (Theory and Methodology of Selecting and Ranking Populations) by S.S. Gupta and S. Panchapakesan published by John Wiley, 1979 and Nonparametric Sequential Selection Procedures by H. Büringer, H. Martin and K.-H. Schriever published by Birkhäuser, 1980. In addition, The Complete Categorized Guide to Statistical Selection and Ranking Procedures by E.J. Dudewicz and J.O. Koo is in press being published by American Sciences Press, 1982. Also, a substantial Bibliography on Selection Procedures listing some 638 entries had been compiled earlier by Professor Gunnar Kulldorff, Institute of Mathematics and Statistics, University of Umea, Sweden, although this volume dated January 26, 1977 is not generally available. A comprehensive review of the book, Selecting and Ordering Populations: A New Statistical Methodology by J. Gibbons, I. Olkin and M. Sobel published by John Wiley, 1977, was undertaken by the Principal Investigator; it appears in the Journal of the American Statistical Association, 75 (1980), 751-756.

A short course on "Selecting and Ordering Populations" sponsored by the American Statistical Association was presented at the Annual Meeting of the Association, August 11-12, 1979; the participants in the course were R. Bechhofer, J. Gibbons, S.S. Gupta and I. Olkin. Videotapes of that course will be shown (as a short course) at the upcoming meeting of the Association in Cincinnati on August 14-15, 1982. R. Bechhofer and S.S. Gupta offered a 5-day seminar on "Selecting and Ranking Procedures" in Berlin, West Germany, on November 17-21, 1980 sponsored by The George Washington University and AMK Berlin. The Third Conference on Statistical Decision Theory and Related Topics, organized by S.S. Gupta and J. Berger, was held at Purdue University, June 1-5, 1981; several sessions were devoted to ranking and selection procedures, the Principal Investigator participating in one of them. On May 3, 1982 the Principal Investigator presented an invited paper in a plenary session at a meeting of the Society for Clinical Trials held in Pittsburgh, Pennsylvania in which he described how a Bernoulli statistical selection procedure, recently developed by him and Dr. Radhika Kulkarni (see below), might be used effectively in medical trials. Thus the potential of ranking and selection procedures continues to capture the imagination of both theoreticians and practitioners with more and more interest being shown in the subject. As a consequence, the literature in the field continues to grow at a great rate.

b. Some specific research accomplishments achieved on ONR Contract N00014-75-C-0586

In this section we will focus our attention on two particular research accomplishments recently achieved on this contract. These are i) The development of a new class of incomplete block designs, which we term BTIB (balanced treatment incomplete block) designs, for comparing simultaneously several test treatments with a control treatment, and ii) The development of closed adaptive sequential procedures for selecting the best of $k \geq 2$ Bernoulli populations. We shall comment on each of these below.

i) BTIB designs

In a series of technical reports (TR's 414, 425, 436, 440, 441, and 453 listed in Section 3a) of the present report) the Principal Investigator and Tamhane proposed a new class of incomplete block designs (BTIB designs) for comparing several test treatments with a control treatment; these designs generalize the notion of a balanced incomplete block (BIB) design. TR 414 described the properties that such designs must posses, including a necessary and sufficient condition concerning the structure of these designs. It was shown that for the multiple comparisons with a control problem, attention could be limited to unions of such BTIB designs, and, in particular, to a subset of such designs which was termed the "minimal complete class of generator designs." This class depends on p = (the number of test treatments being compared with the control treatment) and k = (the common block size). The determination of this minimal complete class is a difficult combinatorial problem which was considered for various (p,k)-combinations in the remaining TR's in the series; once the minimal complete class is known for any particular (p,k)-combination, it is possible to find an optimal BTIB design for any given b = (number of blocks). The general theory underlying, these BTIB designs was published in Technometrics, 1981, after which a paper giving the optimal designs for $p = 2(1)6$, $k = 2$ and $p = 3$, $k = 3$ was accepted for publication in Sankhyā. The problem of constructing the minimal complete class of generator designs for $p > 3$, $k = 3$ and $p \geq 4$, $k = 4$ aroused the interest of several combinatorial experts. Professor William I. Notz of Purdue University solved this problem for $p = 3(1)10$, $k = 3$ while T.E. Ture, a Ph.D. student at Berkeley, solved it for $p = 4(1)10$, $k = 4$ and for larger k-values with $p \geq k$, as well. Notz demonstrated additional optimal properties (in the sense of the optimal design criteria of the late Jack Kiefer) of these designs. Professor C.-F. Wu of Wisconsin is now also studying these designs. Thus, starting with a practical problem for which no good answer had heretofore been provided, the Principal Investigator and Tamhane have opened up a new area of theoretical research with interesting experimental applications. The Principal Investigator and Tamhane have prepared tables of optimal BTIB designs which are under consideration for publication in the Selected Tables in Mathematical Statistics series.

ii) Closed adaptive sequential procedures for selecting the best of $k \geq 2$ Bernoulli populations

In two technical reports (TR 510 and TR 532 listed in Section 3a) of the present report) the Principal Investigator and Kulkarni proposed closed adaptive sequential procedures for selecting the best of $k \geq 2$ Bernoulli populations. In TR 510 the authors consider two general goals: Goal I (selecting the s best of k without regard to order) and Goal II (selecting the s best of k with regard to order); here $(1 \leq s \leq k-1,\ k \geq 2)$. In TR 532 they focus on the particular special case $s = 1,\ k \geq 2$, in which case the two goals coincide. The point of departure for these procedures is the Sobel-Huyett [1957] single-stage procedure which takes exactly n observations from each of the k populations. The Bechhofer-Kulkarni (B-K) procedures (which were described in a Ph.D. dissertation by Ms. Kulkarni written under the guidance of the Principal Investigator) take no more than n observations from each of the k populations, and achieve exactly the same probability of a correct selection as does the S-H procedure, uniformly in the k unknown Bernoulli "success" probabilities. Moreover, for $k = 2$ the B-K procedure is optimal in the sense that it minimizes the expected total number of observations taken from both populations at termination within a broad class of completing procedures; for $k = 2$ it possesses additional optimality properties as well. It now appears that in certain regions of the k-dimensional cube $(0 \leq p_i \leq 1,\ 1 \leq i \leq k)$ which represents the parameter space for this problem, the procedure is also optimal for $k > 2$; these regions are very easy to describe.

The B-K procedure, which is stationary, requires no special tables for implementation, is very easy to carry out in practice, and always requires less observations (usually many less) on-the-average compared to the single-stage procedure which achieves the same probability of a correct selection. Moreover, the B-K procedure tends to sample less frequently from the Bernoulli populations with small p-values, thus making it ethically desirable for use in clinical trials where small p-values are associated with treatments with low probability of "success." A comprehensive study of the performance characteristics of the B-K procedure was carried out in TR 532; quantitative assessments of its "goodness" were obtained. This procedure has provoked considerable interest among both practitioners and theoreticians. Research on its properties, and on variations of the procedure, is continuing.

Some of the results in TR 510 were presented at the Third Conference on Statistical Decision Theory and Related Topics held at Purdue University in June 1981; the entire paper will appear shortly in the Proceedings of the Conference.

3. Research supported in whole or in part by ONR Contract N00014-75-C-0586

a. Technical reports

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Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control: general theory," TR 414 (March 1979).

Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control (II): optimal designs for $p = 2(1)6$, $k = 2$ and $p = 3$, $k = 3$," TR 425 (May 1979).

Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control (III): optimal designs for $p = 4$, $k = 3$ and $p = 5$, $k = 3$," TR 436 (October 1979).

Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control (IV): optimal designs for $p = 4$, $k = 4$," TR 440 (January 1980).

Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control (V): optimal designs for $p = 6$, $k = 3$," TR 441 (June 1980).

Bechhofer, R.E., Tamhane, A.C. and Mykytyn, S.W.: "Incomplete block designs for comparing treatments with a control (VI): conjectured minimal class of generator designs for $p = 5$, $k = 4$, and $p = 6$, $k = 4$," TR 453 (April 1980).

McCulloch, C.E.: "Conditions under which $E\{N_1\} = \infty$ for Tong's adaptive solution to ranking and selection problems," TR 480 (September 1980).

Bechhofer, R.E. and Tamhane, A.C.: "Tables of optimal allocations of observations for comparing treatments with a control," TR 489 (January 1981).

Bechhofer, R.E. and Dunnett, C.W.: "Multiple comparisons for orthogonal contrasts," TR 495 (March 1981).

Bechhofer, R.E. and Kulkarni, R.V.: "Closed adaptive sequential procedures for selecting the best of $k \geq 2$ Bernoulli populations," TR 510 (July 1981).

Faltin, F.W. and McCulloch, C.E.: "On the small-sample properties of the Olkin-Sobel-Tong estimator of the probability of correct selection," Florida State University Statistics Report No. M581 (July 1981). Based on research done while the authors were at Cornell University.

Bechhofer, R.E. and Kulkarni, R.V.: "On the performance characteristics of a closed adaptive sequential procedure for selecting the best Bernoulli population, TR 532 (April 1982).

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Barton, R.R. and Turnbull, B.W.: "A survey of covariance models for censored life data with an application to recidivism analysis," Communications in Statistics--Theory and Methods, A8(8), 1979, 723-750.

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Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control: general theory," Technometrics, 23 (1981), 45-57. Corrigendum: Technometrics, 24 (1982), 171.

Bechhofer, R.E. and Kulkarni, R.V.: "Closed adaptive sequential procedures for selecting the best of $k \geq 2$ Bernoulli populations," Statistical Decision Theory and Related Topics-III (Eds. S.S. Gupta and J. Berger), New York, Academic Press, 1982.

Bechhofer, R.E. and Dunnett, C.W.: "Multiple comparisons for orthogonal contrasts." To appear in Technometrics, 24 (1982).

Bechhofer, R.E. and Tamhane, A.C.: "Design of experiments for comparing treatments with a control: tables of optimal allocations of observations." Accepted for publication in Technometrics, subject to minor revision.

Bechhofer, R.E. and Tamhane, A.C.: "Incomplete block designs for comparing treatments with a control (II): optimal designs for $p = 2(1)6$, $k = 2$ and $p = 3$, $k = 3$." Accepted for publication in Sankhyā, subject to minor revision.

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Hooper, J.H. and Santner, T.J.: "Design of experiments for selection from ordered families of distributions," Annals of Statistics, 7 (1979), 615-643.

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Santner, T.J.: "Designing two-factor experiments for selecting iteractions," Journal of Statistical Planning and Inference, 5 (1981), 45-55.

Santner, T.J. and Snell, M.S.: "Small-sample confidence intervals for $p_1 - p_2$ and p_1/p_2 in 2×2 contingency tables," Journal of the American Statistical Association, 75 (1980), 386-394.

Tamhane, A.C. and Bechhofer, R.E.: "A two-stage minimax procedure with screening for selecting the largest normal mean (II): an improved PCS lower bound and associated tables," Communications in Statistics--Theory and Methods, A8(4), 1979, 337-358.

Turnbull, B.W. and Weiss, L.: "A likelihood ratio statistic for testing goodness of fit with randomly censored data," Biometrics, 34 (1978), 367-375.

c. Papers submitted for publication

Bechhofer, R.E. and Tamhane, A.C.: "Tables of Admissible and Optimal Balanced Treatment Incomplete Block (BTIB) Designs for Comparing Treatments with a Control. Submitted for publication in Selected Tables in Mathematical Statistics.

Faltin, F.W. and McCulloch, C.E.: "On the small-sample properties of the Olkin-Sobel-Tong estimator of the probability of correct selection." Submitted for publication in the Journal of the American Statistical Association.

SPECIAL REPORT ON ONR CONTRACT N00014-75-C-0586

"STATISTICAL ENGINEERING"

by

Robert E. Bechhofer
Principal Investigator

August 14, 1978

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SPECIAL REPORT ON ONR CONTRACT N00014-75-C-0586

This is a special report on ONR contract N00014-75-C-0586 (and its predecessor contract) with Dr. Robert Bechhofer as Principal Investigator. The report deals with the last five years of these contracts. It consists of three main parts: Part I gives some of the history of the general area of statistical "ranking and selection" procedures, and of the Principal Investigator's association with research in that area which is the one that has received major attention on this contract; a precis of the most significant research accomplishments obtained on this contract in the last five years is also included in this part. Part II lists published papers and unpublished technical reports supported by the contract during the last five years; these are grouped into three categories. Part III discusses the directions of research to be undertaken by the Principal Investigator over the next 3 to 5 years, and lists specific problems which the Principal Investigator plans to study.

PART I OF REPORT

Background

The Principal Investigator has been engaged in research in the general area of statistical multiple-decision procedures with particular emphasis on "ranking and selection" procedures and multiple comparisons procedures for many years. In an early paper [B1] he proposed the "ranking and selection" approach as an alternative to the classical "test of homogeneity" approach (using, for example, the F-test associated with the analysis of variance) since he felt, based on his experience [1/] in applied statistics, that the latter formulation of the problem does not answer the questions that truly are of interest to an experimenter. When the performances of several competing categories of test items are being compared, what the experimenter usually wishes to ascertain is which *category or categories are best, and whether or* not the best ones are good enough to select, or he may be interested in estimating the differences between the performances of items in each category. Such questions can be answered more meaningfully if the statistical problems are posed at the outset as ones involving "ranking or selection" or "multiple comparisons," and the experiment designed accordingly. Properly planned experiments with well defined objectives such as these often result in considerable savings in sample size and associated costs.

Most of the research undertaken by the Principal Investigator under ONR sporsorship has proceeded with this orientation as a central philosophy. He

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1/ Dr. Bechhofer served for four years (during World War II) as a statistician at the Aberdeen Proving Ground, Maryland; he was responsible for the design, analysis, and interpretation of tests of items of ordnance. He has participated regularly in the annual Design of Experiments Conference sponsored by the Army Research Office-Durham. Following his stay at Aberdeen, Dr. Bechhofer was employed as a statistician at the Carbide and Carbon Chemical plant in Oak Ridge, Tennessee.

along with several co-workers, research students, and other independent research workers throughout the world have been developing various integrated approaches to the meaningful formulations and solution of ranking and selection problems. The two most widely adopted formulations of these problems are the so-called "indifference-zone" approach (first proposed by the Principal Investigator in [B1][2/]), and the "subset" approach (first proposed by Dr. Shanti S. Gupta in [A7], [A8]). A "restricted subset" approach, which bridges the indifference-zone and the subset approaches was proposed by Dr. Thomas J. Santner [14]. Recently, Dr. Ajit Tamhane and the Principal Investigator proposed a procedure [B26], [B30] which provides a different blending of the two basic approaches. Several other formulations have also been set forth. Considerable interest has been shown in these formulations, and at the present time more than 400 research papers have been written on this general subject and published in the technical journals.

A brief overview of this general area of statistical research (with particular emphasis on the indifference-zone approach), and of some of the Principal Investigator's contributions to it, are contained in a expository paper [B24]. The research monograph [B13] by Bechhofer, Kiefer, and Sobel gives a fairly complete set of references up to 1968. An up to date categorized bibliography on ranking and selection procedures [A3] by Dudewicz and Koo lists more than 400 references.

The book [A2] by Dudewicz is one of the first elementary texts to introduce ranking and selection procedures as one of the fundamental forms of statistical analysis available to the applied statistician. Kleijnen in his book [A10] on

[2/] References in this report prefixed by B refer to the papers of Bechhofer listed on pp. 6-8 while references prefixed by A refer to additional papers by other investigators listed alphabetically on pp. 9-10.

statistical techniques in simulation devotes more than 200 pages to showing the substantial role that selection and ranking procedures and multiple comparisons procedures can play in the design and analysis of simulation experiments. A major step forward with respect to making the ranking and selection methodology available to the practitioner in a useable form took place with the recent publication of the book [A6] by Gibbons, Olkin, and Sobel; as noted on its dustjacket, "This book is the first of its kind to provide applied statisticians and other users of statistical methods with techniques for selecting and ordering (or ranking) populations." Many of the procedures developed by the Principal Investigator and his co-workers are described in detail in this text. Finally, a major monograph [A9] by Gupta and Panchapakesan at the research level is in final draft stage, and will shortly be published; this book surveys and integrates the significant results in the field, and contains a comprehensive bibliography.

As these new techniques take their place in the textbooks alongside the traditional methods of design and analysis of experiments, there has been increasing demand on the part of practitioners to learn more about this approach and its applicability in various areas of experimentation. In December 1977 the Principal Investigator was invited to give a three-hour tutorial on ranking and selection procedures at the Thirty-Third Annual Princeton Conference on Applied Statistics; this session was attended by more than 350 individuals most of whom expressed considerable interest in the subject; many mentioned that the procedures were very relevant to work that they were doing.

Most recently, on June 8-9, 1978, the Principal Investigator conducted a two-day tutorial seminar on "Statistical Selection and Ranking Procedures" at the Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland. This tutorial which was sponsored by the Army Research Office-Durham with the cooperation of Drs. Malcolm S. Taylor and James Richard Moore of the BRL attracted approximately fifty attendees; about two-thirds were from Aberdeen or Edgewood Arsenal with nine additional installations being represented. The response of the attendees to the subject matter presented was extremely positive; most of the group indicated a strong desire to learn more about the topics discussed so that they could employ them in their own work.

PUBLICATIONS AND RESEARCH IN PROGRESS OF PRINCIPAL INVESTIGATOR

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SPECIAL RESEARCH ACCOMPLISHMENTS

The earliest research on statistical ranking and selection procedures saw particular emphasis being placed on single-stage procedures involving single-factor experiments. (see, e.g., [B1], [B3], [B6], [A8], [A18]). Although some significant progress was made in the development of sequential procedures for single-factor experiments involving normal means ([A12]), it was not until the publication of [B13] that an integrated general theory was presented for the solution of a broad class of such problems. Single-stage procedures for multi-factor experiments concerned with normal means were considered briefly in [B1] and [A1], while single-stage procedures for multi-factor experiments concerned with normal variances were considered in [B14]. The problem of optimal allocation of observations when comparing several treatments with a control (posed by Dunnett [A4]) was first solved in [B16]. Most of the special research accomplishments of recent years represent substantial improvements on or generalizations of procedures mentioned above.

DISCUSSIONS OF SPECIFIC RESEARCH ACCOMPLISHMENTS

Single-factor experiments

A) Two-stage procedures for selecting the largest normal mean when the common variance is known

In [B26], [B30] the authors propose a two-stage procedure with screening to select the normal population with the largest population mean when the populations have a common known variance. The procedure guarantees the same probability requirement using the indifference-zone approach as does the single-stage procedure of Bechhofer [B1]. It has the highly desirable property that the expected total number of observations required by the procedure is always less than the total number of observations required by the corresponding single-stage procedure [B1], regardless of the configuration of the population means. The saving in expected total number of observations can be substantial, particularly when the configuration of the population means is favorable to the experimenter. The saving is accomplished by screening out "non-contending" populations in the first stage, and concentrating sampling only on "contending" populations in the second stage. The second paper [B30] contains new results which make possible the more efficient implementation of the two-stage procedure. Tables for this purpose are given, and the improvements achieved (which are significant) are assessed.

B) Single-stage and two-stage procedures for selecting the largest normal mean when the common variance is known or unknown, respectively, and comparisons are made with a fixed known standard

In [B29] the authors propose a single-stage and a two-stage (k+1)-decision procedure for the case of common known and common unknown variance, respectively, for the problem of comparing k normal means with a specified (absolute) standard. The procedures guarantee that i) with probability at least P_0^* (specified), no population is selected when the largest population mean is sufficiently less than the standard, and ii) with probability at least P_1^* the population with the largest population mean is selected when that mean is sufficiently greater than its closest competitor and the standard. Tables to implement the procedures are provided. Such procedures are applicable in the following type of situation: If the competing categories (normal populations) are methods of heat treating steel, then the best method may not be deemed satisfactory unless the expected tensile strength resulting from that method of treatment is at least some specified minimum value.

C) Adaptive sequential procedures for selecting the largest normal mean when the common variance is known

In the monograph [B13], sequential vector-at-a-time sampling procedures are given for selecting the largest of $k \geq 2$ normal means when the common variance is known, and the so-called "indifference-zone" formulation of the ranking problem is adopted. However, in certain types of experimental situations vector-at-a-time sampling procedures are not appropriate, e.g., in biomedical clinical trials it may be desirable to concentrate sampling on contending populations which are indicated as being superior (i.e., having large population means) and sample less frequently or eliminate from sampling non-contending populations which are indicated as being inferior (i.e., having small population means). A two-population problem in which adaptive sample was employed had earlier been studied in [A13]. In the present paper [A19] the authors consider the corresponding k-population ($k \geq 3$) problem, and employ various adaptive (non vector-at-a-time) sampling rules. They show that unlike the case $k=2$, when $k \geq 3$

substantial savings in expected total sample size can be achieved when adaptive sampling is employed for the so-called identification problem (as opposed to the ranking problem); the alternative criterion of minimizing expected number of observations on the inferior populations is also studied for the identification problem. Both theoretical and simulation results are presented. However, if these same adaptive procedures are used for the corresponding "indifference-zone" formulation of the ranking problem, the authors obtain the somewhat surprizing result that the slippage configuration is no longer necessarily "least-favorable," and thus the usual indifference-zone probability requirement for the ranking problem may not be guaranteed. Hence, adaptive sampling procedures for the ranking problem involving $k \geq 3$ populations remain to be developed.

D) A single-stage procedure for selecting the normal mean which is closest to a specified value when the common variance is known

In [B27] the authors propose a single-stage procedure for selecting the production process with the highest fraction of conforming product; if the underlying variates from each of the $k \geq 2$ populations are normally distributed with a common known variance, and if two-sided specification limits are adopted for the individual items, then the problem reduces to the one described in the title of this section. In [B27] both parametric and non-parametric procedures are described, and an abbreviated set of tables to implement the parametric procedure is given. The paper [B27] is expository. A paper giving the underlying theory and an expanded set of tables is being written up with Dr. Turnbull.

E) Optimal allocation of observations when comparing treatments with a control in incomplete blocks (normal means, common known variance)

In [B32] the authors are preparing two papers which generalize the results of [B16] to the case in which (as in many biomedical experiments) observations can only be taken in incomplete blocks. All of the necessary theory has now been developed, and the results are being written up for publication. The first paper gives exact optimal allocations for $p = 2(1)6$ "test" treatments which are to be compared to a "control" treatment in b blocks of size $k = 2,3,4$ $(k < p+1)$ where b (which depends on p and k) is roughly less than 100; the second paper gives approximate continuous (in the sense of Kiefer) optimal allocations for the same (p,k)-combinations where b is arbitrary (but usually fairly large). The computations for these papers are almost completed.

F) Exact confidence intervals for p_1-p_2 in 2×2 contingency tables

In [A15] the authors have used an exact confidence interval estimate of the "odds-ratio" $\psi = p_1(1-p_2)/p_2(1-p_1)$ in a 2×2 contingency table (where the p_i $(i = 1,2)$ are Bernoulli "success" probabilities) to obtain an exact confidence interval estimate of the difference $\Delta = p_1-p_2$ and of the relative risk $\rho = p_1/p_2$. The question of how to form exact (small-sample) confidence intervals on these latter quantities has been an open problem for many years. Tables to implement the procedure are provided.

Multi-factor experiments

I. Studies in which the experiment is designed for the purpose of ranking or selecting "interactions" (normal distributions with a common known variance)

A. Single-stage procedures for selecting the largest interaction in a 2-factor experiment

i) In [B25] the authors propose a single-stage procedure for selecting the largest positive interaction in a 2-factor experiment involving qualitative variables. The procedure would be applicable in the following type of situation: "Suppose that a medical research worker wished to plan an experiment to study the effect of several (c) different methods of treatment on a physiological response of male and female subjects. It is assumed known that the effect of the treatment on the mean response is different for men than for women, and also that it varies from treatment to treatment. It is suspected that there may be a large interaction between sex and method of treatment, and it is desired to identify the sex-treatment combination for which this interaction is largest in the hope that such information might provide some clue as to the mechanism underlying the effectiveness of the methods of treatment." The statistical problem is to design the experiment on such a scale that this largest interaction can, if it is sufficiently large to be of practical importance to the experimenter, be detected with preassigned probability. (More generally, in a 2-factor experiment there might be $r \geq 2$ levels of one qualitative factor and $c \geq 2$ levels of a second qualitative factor.) The present paper considers 2-factor experiments and concentrates on the $2 \times c$ case and the 3×3 case. The probability of a correct selection is evaluated for the given procedure. The main result of the paper is to give the least-favorable configuration for the cases under study.

ii) In [A16] the author extends the work in [B25]. The paper analyzes the least-favorable configuration based on the log-concavity of the probability of a correct selection regarded as a function of the population interactions and on the characteristics of the preference zone. The 3×4 case is examined in detail and explicit results are given for that case. Nonlinear programming algorithms for finding the extreme points associated with the least-favorable configuration are given for the general r×c case.

II. Studies in which the experiment is designed for the purpose of ranking or selecting "main effects" (normal distributions with a common unknown variance).

Note: Here a different procedure must be used according as one assumes that interaction is not or is present.

A. No interaction between factors

In [B28] single-stage and two-stage selection procedures are given for two-factor experiments in which the common variance is unknown. The objective is to rank on both factors (Factors A and B, say) simultaneously. The paper [B28] is expository; a small set of abbreviated tables to implement the procedures is given. A three-part paper giving the underlying theory is being prepared as a joint effort with Dr. Charles W. Dunnett of McMaster University, Canada; the papers will contain a comprehensive set of definitive tables necessary to implement the procedures. The computation of these tables is nearing completion.

PART II OF REPORT

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PART III OF REPORT

Directions of Future Research

It is anticipated that the investigations to be undertaken will proceed along lines similar to those followed during the earlier periods. A large number of areas of research remain to be explored. Special emphasis will be placed on solving problems which have practical relevance--many of them having been brought to the attention of the Principal Investigator during his periods as a practicing statistician in various aspects of experimental design. Every effort will be made to formulate the problems in a way that will be meaningful and useful to the practitioner, and to devise procedures which will be easy to implement.

The main objective of this contract is to make contributions to the meaningful formulation and solution of statistical problems which can be usefully viewed as ranking and selection, or related problems. These can be grouped into two broad classes: a) Problems arising from single-factor experiments, and b) Problems arising from multifactor experiments. Some problems associated with single-factor experiments have an exact analogue in multifactor experiments; however, other problems are unique to multifactor experiments because the appropriate statistical model to use for multifactor experiments is often not obvious, and each choice leads to a different definition of "interaction." We discuss some of these problems below.

A) Single-factor experiments

i) Problems concerned with normal means

a) The development of methods for dealing with unbalanced experiments when ranking means of normal populations with a common known variance using a single-stage procedure. In the basic paper [B1], and in later papers, consideration was given as to how large the common number of observations per population should be chosen in order to guarantee a prescribed probability requirement. However, practical considerations may often limit the number of observations that can be taken from particular populations and/or observations may be lost or in error (in which case one would discard them); the net result is that the final experiment is unbalanced in that different numbers of observations are taken from the various populations. If this happens then the original prescribed probability requirement is no longer guaranteed, and it is necessary to assess the magnitude of the decrease in the probability of a correct selection. The problem is particularly complicated if observations are taken in (say) randomized blocks (which here can be regarded as a second factor), and some blocks are incomplete. It is planned to develop easily-implementable procedures for coping with such situations. Conservative procedures presently exist for some of these problems, but these procedures are often very inefficient. For some cases it may be possible to devise optimal procedures.

b) The development of improved procedures for estimating the achieved probability of a correct selection (PCS) in retrospective studies. If an experiment has been conducted and the sample taken it will often be of interest to estimate the probability of making a correct selection using these data and a given selection procedure (e.g., the single-stage procedure of [B1]). This important problem was posed by Olkin, Sobel, and Tong in [A11], and their proposed solution is incorporated into the text [A6] of Gibbons, Olkin, and Sobel. Unfortunately, their procedure tends to overestimate the achieved PCS (often by a considerable amount), particularly when the number (k) of populations is large, the common sample size is small, and the k population means are all almost equal.

(In this situation the sample means behave almost as order statistics from a common population.) It is hoped to develop an estimator with improved small-sample properties.

c) The development of a procedure for optimally allocating observations to populations when ranking means of normal populations with known unequal variances using a single-stage procedure. In [B1] an optimal solution to this problem is given for the case of two populations, and a general solution (which is known to be non-optimal) is given for the case of three or more populations. It is hoped to devise a procedure which will be optimal for this latter case, or minimally one which will improve substantially on the present solution.

d) The development of a procedure for optimally allocating observations among $p \geq 2$ test treatments and a control treatment when it is desired to compare simultaneously the p test treatments with the control treatment. This problem which was originally posed by Dunnett [A4] was solved for the case of completely randomized designs in [B16] and [B20]. We propose to consider this problem for the situation in which the experimenter is restricted to taking observations in incomplete blocks (which often is the case in biomedical experiments). The Principal Investigator, working with A.C. Tamhane, has solved the one-sided problem (the analogue of [B16]); it is planned to complete the writeup of this problem and then to work on the two-sided problem (the analogue of [B20]). Successful completion of this study will result in substantial decreases in the sample sizes traditionally taken in experimental situations requiring multiple-comparisons with a control.

e) The development of sequential selection procedures which concentrate sampling on contending populations, and which eliminate or sample less frequently from non-contending populations. This problem is of particular practical interest (but of great analytical difficulty) when three or more populations are involved.

Recently Turnbull, Kaspi and Smith [A19] obtained some very interesting results for a variant of the identification problem but their solution does not carry over to the corresponding ranking problem. In an as yet unpublished paper Richard L. Smith, a graduate student at Cornell, was able to improve significantly on the Paulson-Fabian [A12]-[A5] sequential procedure for the ranking problem.

f) The development of single-stage selection procedures for means of censored normal populations when the underlying variances are known and equal. Such procedures would be very useful in certain types of reliability studies. A sequential procedure for identifying the normal population with the largest population mean, all populations having a common known variance and a common known upper point of censoring is described in [B13, p. 102]. However, no ranking procedures have thus far been devised for this problem.

g) The development of lower bounds on the probability of a correct selection when single-stage selection procedures are used. It is recognized that statistical procedures which are devised under one set of statistical assumptions may behave poorly if these assumptions are not satisfied in the situation in which the procedures are actually applied. The purpose of this study will be to assess just how much the probability of a correct selection associated with the standard procedure of [B1] is affected when the statistical assumptions are violated in certain ways. This study will throw light on the "robustness" of such procedures, and thus on their general practicability. Research on this problem is underway [B23], and promising preliminary results have been obtained for certain special cases. What is required in one of the simplist special cases is a new generalization (different from that of Olkin-Marshall) of the well-known one-sided Chebyshev inequality.

ii) Problems concerned with Bernoulli probabilities of "success"

a) The development of a two-stage procedure with elimination for selecting the Bernoulli population with the largest single-trial "success" probability.

The single-stage procedure for this problem using the indifference-zone approach was devised by Sobel and Huyett [A18]. It is proposed to use the same "measure of distance" as was employed by Sobel and Huyett [A18], and to devise two-stage procedures which are analogues for Bernoulli populations of these used for normal populations by Tamhane and Bechhofer [B26], [B30]. It is hoped, using this approach, to improve uniformly on the procedure of Sobel and Huyett.

b) The development of a procedure for obtaining exact joint confidence interval estimates of the "odds-ratio" $\psi_i = p_0(1-p_i)/p_i(1-p_0)$ $(1 \leqq i \leqq c)$ in $2\times c$ contingency tables. If this problem can be solved for $c \geqq 3$, then the result can be used to determine exact joint confidence interval estimates of the differences $\Delta_i = p_0 - p_i$ and relative risks $\rho_i = p_i/p_0$ $(1 \leqq i \leqq c)$. A method for using the solution for ψ_i to generate a solution for Δ_i and ρ_i has recently been proposed by Santner and Snell [A15] for $c = 2$. It is now planned to see whether their methods can be extended to solve the general multiple comparisons with a control problem for the Bernoulli distribution.

iii) Problems concerned with exponential populations

The development of procedures for selecting the exponential population with the largest mean when data are subject to Type I or Type II censoring. Early work on the selection problem for exponential distributions was done by Sobel [A17]; a special form of Type II censoring was considered in that paper. (See also [B13, pp. 268-269].) Results for this problem would be useful in reliability studies and in biomedical research.

B) Multifactor experiments

i) Problems concerned with normal means

a) In a two-factor experiment with both factors qualitative one can distinguish the following situations: I) It is known that there is no interaction (or negligible interaction) between the two factors, or II) It is known that there is large interaction between the two factors, or III) The magnitude of the interaction between the two factors is completely unknown (i.e., it may be negligible, intermediate or large).

If I) is the situation, and it is desired to rank on both factors simultaneously (i.e., to find the best level of Factor A and simultaneously to find the best level of Factor B), then it is known (Bawa [A1]) that the most efficient design (in the sense of minimizing the total number of observations required, subject to guaranteeing a probability requirement) for the case of common known variance is a factorial experiment. The Principal Investigator is currently studying this problem with Dr. Charles W. Dunnett of McMaster University for the case in which the populations have a common unknown variance. Most of the preliminary theoretical work has been completed (see [B28] for a sketch of the procedures), and tables to implement the procedures are being prepared.

If II) is the situation, then it is not meaningful to rank on both factors simultaneously. Either the experimenter can seek to find the combination (level of Factor A and level of Factor B) which is best (in which case procedures already exist (e.g., [B1] and [B2]) or the experimenter can seek to find the best level of Factor A, simultaneously, for each level of Factor B. For the case in which the populations have a common unknown variance, the second problem requires new tables to implement the procedures; these are being prepared with Dr. Dunnett.

If III is the situation, then the Principal Investigator and Dr. Dunnett are proposing an entirely new procedure involving a preliminary test on the interactions to assess their magnitude: if the interactions are indicated as being negligible, then the experimenter will proceed as in I) while if they are indicated as being large, then the experimenter will proceed as in II). This complicated composite procedure is presently being studied with Dr. Dunnett.

b) For situation I (of a), above) the sequential procedure of [B13] has been generalized so that the experimenter can rank simultaneously on both factors. Preliminary studies indicate that analogous savings can be achieved using the sequential procedure as were noted by Bawa [A1] for the single-stage procedure.

c) The problem of jointly estimating efficiently all of the interactions in a multifactor experiment is also being studied. The analytical work for the two-factor problem has been completed. Special-purpose tables to implement the procedure remain to be computed.

ii) Problems concerned with Bernoulli probabilities of "success."

The development of models and procedures for designing and analyzing multi-factor experiments involving Bernoulli random variables. Such methodology would be useful in devising experiments to study the dependence of (say) the fraction defective (of manufactured items) on the various "levels" of the qualtitative factors (e.g., different manufacturers or different methods of manufacture) which affect the performance of these items, or to study the dependence of (say) the survival probability of cancer patients on the "levels" of the qualitative factors, e.g., stage of the disease (early, intermediate, advanced) vs. different methods of treatment, which affect this probability. Sequential procedures such as the one described in [B13, p. 270] are currently being investigated for setups in which the p_{ij} are assumed to obey a loglinear model.

iii) Problems concerned with multinomial probabilities of "occurence."

The development of models and procedures for designing and analyzing multi-factor experiments involving multinomial random variables. Such methodology would be useful in designing certain classes of experiments which lead to data in the form of contingency tables. Single-stage and sequential procedures for single-factor experiments are given, e.g. in [B6] and [B13, pp. 121-123], respectively. It is proposed to generalize these and other procedures to multi-factor experiments, and to study their properties.

THIRD ANNUAL REPORT ON CONTRACT N00014-75-C-0586

"STATISTICAL ENGINEERING"

by

Robert E. Bechhofer
Principal Investigator

School of Operations Research and Industrial Engineering
College of Engineering
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THIRD ANNUAL REPORT ON CONTRACT N00014-75-C-0586

This is the third annual report on Contract N00014-75-C-0586 which is titled "Statistical Engineering." The contract was initiated on June 1, 1975 and continued through May 31, 1976 with a budget of $20,000; the first renewal of the contract covered the period June 1, 1976 through December 31, 1976 with a budget of $15,000; the second renewal covers the period January 1, 1977 through March 31, 1978 with a budget of $20,000. The present report covers the period September 1, 1976 through August 31, 1977. During summer 1977 the contract provided partial support to the Principal Investigator, Dr. Robert Bechhofer, to one Co-Investigator, Dr. Thomas Santner (an Assistant Professor in the School of Operations Research and Industrial Engineering at Cornell), and to Mr. Avi Vardi, a graduate research assistant.

Developments during the report period

a) Published papers

Bechhofer, R.E., Santner, T.J. and Turnbull, B.W.: "Selecting the largest interaction in a two-factor experiment," Statistical Decision Theory and Related Topics, II, Academic Press, 1977, pp. 1-18.

Bechhofer, R.E. and Turnbull, B.W.: "On selecting the process with the highest fraction of conforming product," Proceedings of the 31st Annual Technical Conference of the American Society for Quality Control, May 1977, pp. 568-573.

Turnbull, B.W.: "The empirical distribution function with arbitrarily grouped, censored, and truncated data," Journal of the Royal Statistical Society, B., Vol. 38, No. 3, 1976, pp. 290-295.

Turnbull, B.W.: "Multiple decision rules for comparing several populations with a fixed known standard," Communications in Statistics, Part A5, No. 13, 1976, pp. 1225-1244.

Dudewicz, E.J.: "Generalized maximum likelihood estimators for ranked means," Z. Wahrscheinlichkeitstheorie verw. Gebeite, Vol. 35 (1976), pp. 283-297.

b) Papers accepted for publication

Bechhofer, R.E.: "Selection in factorial experiments." To appear in the Proceedings of the 1977 Winter Simulation Conference to be held at the National Bureau of Standards, Gaithersburg, Md., December 5-7, 1977.

Bechhofer, R.E. and Tamhane, A.C.: "A two-stage minimax procedure with screening for selecting the largest normal mean." To appear in Communications in Statistics, Part A6, No. 11, 1977.

Tamhane, A.C.: "A three-stage elimination type procedure for selecting the largest normal mean (common unknown variance)." To appear in Sankhyā B.

c) Papers submitted for publication and currently being revised at the request of the editor

Bechhofer, R.E. and Turnbull, B.W.: "Two (k+1)-decision selection procedures for comparing k normal means with a specified standard." Submitted to the Journal of the American Statistical Association. (First revision of two papers which have been combined into a single paper.)

d) Papers submitted for publication

Barton, R.R. and Turnbull, B.W.: "A survey of covariance models for censored life data with an application to recidivism analysis." Submitted to Communications in Statistics.

Hooper, J.H. and Santner, T.J.: "Design of experiments for selection from ordered families of distributions." Submitted to Annals of Statistics.

Turnbull, B.W., Kaspi, H. and Smith, R.L.: "Adaptive sequential procedures for selecting the best of several normal populations." Submitted to the Journal of Statistical Computation and Simulation.

e) Technical reports

Turnbull, B.W.: "The empirical distribution function with arbitrarily grouped, censored, and truncated data," TR 305, July 1976.

Turnbull, B.W. and Weiss, L.: "A likelihood ratio statistic for testing goodness of fit with randomly censored data," TR 307, August 1976.

Jakobovits, R.H.: "Goodness of fit tests for composite hypotheses based on an increasing number of order statistics," TR 310, September 1976.

Tamhane, A.C. and Bechhofer, R.E.: "A two-stage minimax procedure with screening for selecting the largest normal mean," TR 323, January 1977.

Turnbull, B.W., Kaspi, H., and Smith, R.L.: "Adaptive sequential procedures for selecting the best of several normal populations," TR 328, April 1977.

Barton, R.R. and Turnbull, B.W.: "A survey of covariance models for censored life data with an application to recidivism analysis," TR 333, May 1977.

Vickers, M.K.: "Optimal asymptotic properties of maximum likelihood estimators of parameters of some econometric models," TR 334, May 1977.

f) Papers in preparation

Bechhofer, R.E. and Dunnett, C.W.: "Selecting the best factor-level combination associated with normal means arising from a factorial experiment when the common variance is unknown."

Bechhofer, R.E. and Santner, T.J.: "Designing experiments to select the largest interaction in a 2-factor experiment: special cases."

Bechhofer, R.E. and Tamhane, A.C.: "Optimal allocation of observations when comparing several treatments with a control (IV): Incomplete block designs."

Bechhofer, R.E. and Turnbull, B.W.: "A (k+1)-decision single-stage procedure for selecting the production process with the highest fraction of conforming product."

Santner, T.J.: "Selecting the treatment combination having the largest interaction: arbitrary $r \times c$ case."

Snell, M.K. and Santner, T.J.: "Exact small sample confidence intervals for $p_1 - p_2$ and p_1/p_2 in 2×2 contingency tables."

Professional activities of Dr. Bechhofer during the reporting period

a) Director of School of Operations Research & Industrial Engineering, Cornell University.

b) Member of the Advisory Board of the Section on Physical and Engineering Sciences, American Statistical Association.

c) Member of the Committee on Statistics in the Physical Sciences of the Bernoulli Society for Mathematical Statistics and Probability.

d) Ad Hoc Member of Computer and Biomathematical Sciences Study Section of the National Institutes of Health, Washington, D.C., November 10-12, 1976.

e) Refereed research proposals for Army Research Office, Durham, and for the National Science Foundation.

f) Panelist at Twenty-second Conference on the Design of Experiments in Army Research, Development, and Testing held at Harry Diamond Laboratories, Adelphi, Md., October 20-22, 1976.

g) Gave an invited paper "Sampling plans for testing combination drugs" before the joint meeting of the American Statistical Association and the Biometric Society, ENAR and WNAR held in Chicago, Illinois, August 15-18, 1977.

SECOND ANNUAL REPORT ON CONTRACT N00014-75-C-0586

"STATISTICAL ENGINEERING"

by

Robert E. Bechhofer
Principal Investigator

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SECOND ANNUAL REPORT ON CONTRACT N00014-75-C-0586

This is the second annual report on Contract N00014-75-C-0586 which is titled "Statistical Engineering." The contract was initiated on June 1, 1975 and continued through May 31, 1976 with a budget of $20,000; the first renewal of the contract covers the period June 1, 1976 through December 31, 1976 with a budget of $15,000. The present report covers the period September 1, 1975 through August 31, 1976. During summer 1976 the contract provided partial support to the Principal Investigator, Dr. Robert Bechhofer, to two Co-Investigators, Dr. Thomas Santner and Dr. Bruce Turnbull (both being Assistant Professors in the School of Operations Research and Industrial Engineering at Cornell), and to several graduate research assistants. In addition Miss Mary K. Vickers and Messrs. Ray Jakobovits and Yehuda Vardi were supported part time as Graduate Research Assistants during the academic year; all are Ph.D. candidates.

Developments during the reporting period

a) Published papers

Bechhofer, R.E.: "Ranking and selection procedures," Proceedings of the Twentieth Conference on the Design of Experiments in Army Research, Development, and Testing held at Fort Belvoir, Virginia, October 23-25, 1974, ARO Report 75-2, pp. 929-949.

Blumenthal, S.: "Sequential estimation of the largest normal mean when the variance is unknown," Communications in Statistics, Vol. 4, No. 7, July 1975, pp. 655-669.

b) Papers accepted for publication

Bechhofer, R.E., Santner, T.J., and Turnbull, B.W.: "Selecting the largest interaction in a two-factor experiment." To appear in the Proceedings of the Second Symposium on Statistical Decision Theory and Related Topics held at Purdue University, May 17-19, 1976.

Turnbull, B.W.: "The empirical distribution function with arbitrarily grouped, censored, and truncated data." To appear in the Journal of the Royal Statistical Society, B, Vol. 38, No. 3.

Turnbull, B.W.: "Multiple decision rules for comparing several populations with a fixed known standard." To appear in Communications in Statistics.

c) Papers submitted for publication and currently being revised at the request of the editor

Bechhofer, R.E. and Turnbull, B.W.: "A (k+1)-decision single-stage selection procedure for comparing k normal means with a fixed known standard: the case of common known variance," and "A (k+1)-decision two-stage selection procedure for comparing k normal means with a fixed known standard: the case of common unknown variance." These papers have been combined at the request of the editor, and the revised paper has been resubmitted.

d) Papers submitted for publication

Tamhane, A.C.: "On 2- and 3-stage screening procedures for selecting the population having the largest mean from k normal populations with a common unknown variance."

Turnbull, B.W. and Weiss, L.: "A likelihood ratio statistic for testing goodness of fit with randomly censored data."

e) Technical reports

Awate, P.: "Dynamic programming with negative rewards and average reward criterion," TR 251, May 1975.

Bechhofer, R.E. and Turnbull, B.W.: "A (k+1)-decision two-stage selection procedure for comparing k normal means with a fixed known standard: the case of common unknown variance," TR 256, May 1975.

Turnbull, B.W.: "Multiple decision rules for comparing several populations with a fixed known standard," TR 257, June 1975.

Tamhane, A.C.: "On minimax multistage elimination type rules for selecting the largest normal mean," TR 259, May 1975.

Tamhane, A.C.: "On minimax two-stage permanent elimination type procedure for selecting the smallest normal variance," TR 260, June 1975.

Gelber, R.D.: "A sequential goodness-of-fit test for composite hypotheses involving unknown scale and location parameters," TR 266, August 1975.

Bechhofer, R.E., Santner, T.J., and Turnbull, B.W.: "An application of majorization to the problem of selecting the largest interaction in a two-factor experiment," TR 292, May 1976.

Turnbull, B.W.: "The empirical distribution function with arbitrarily grouped, censored, and truncated data," TR 305, June 1976.

Turnbull, B.W. and Weiss, L.: "A likelihood ratio statistic for testing goodness of fit with randomly censored data," TR 307, August 1976.

Jakobovits, R.H.: "Goodness of fit tests for composite hypotheses based on an increasing number of order statistics," TR 310, September 1976.

f) Papers in preparation

Bechhofer, R.E. and Ramberg, J.S.: "A comparison of the performance characteristics of two procedures for ranking means of normal populations."

Bechhofer, R.E. and Tamhane, A.C.: "Optimal allocation of observations when comparing several treatments with a control (IV): Incomplete block designs."

Bechhofer, R.E. and Turnbull, B.W.: "Chebyshev type lower bounds for the probability of correct selection, I: the location problem with one observation from each of two populations."

Bechhofer, R.E. and Turnbull, B.W.: "A (k+1)-decision single-stage procedure for selecting the production process with the highest fraction of conforming product."

Kaspi, H. and Wong, T.: "On the performance characteristics of the Hoel-Mazumdar elimination procedure for Poisson processes, with applications to clinical trials."

Santner, T.J.: "Contributions to the problem of selecting the largest interaction in factorial experiments."

Turnbull, B.W., Kaspi, H., and Smith, R.: "Sequential allocation in clinical trials for choosing the best of several normal populations."

Tamhane, A.C. and Bechhofer, R.E.: "A minimax permanent elimination type procedure for selecting the largest normal mean (common known variance)."

Professional activities of Dr. Bechhofer during the reporting period

a) Director of School of Operations Research & Industrial Engineering, Cornell University.

b) Member of the Advisory Board of the Section on Physical and Engineering Sciences, American Statistical Association.

c) Member of the Committee on Statistics in the Physical Sciences of the Bernoulli Society for Mathematical Statistics and Probability.

d) Refereed research proposals for Army Research Office, Durham, and for National Research Council, Canada.

e) Gave an invited talk "Ranking and selection procedures" before the Washington Statistical Society Chapter of the American Statistical Association, October 3, 1975.

f) Panelist at Twenty-first Conference on the Design of Experiments in Army Research, Development, and Testing held at Walter Reed Army Medical Center, Washington, D.C., October 22-24, 1975.

g) Member of Visiting Committee on Operations Research, Tel-Aviv University, Israel. Visit took place December 8-11, 1975.

h) Gave seminars on "Ranking and selection procedures" before the Department of Statistics, the Hebrew University and the Faculty of Industrial and Management Engineering of the Technion, Israel, on December 17 and 18, 1975, respectively. Also, at the Department of Statistics, Ohio State University, January 21, 1976, Department of Mathematics, University of Toronto, Canada, March 4, 1976; Southern Ontario Chapter of the American Statisticsl Association and Department of Applied Mathematics, McMaster University, Canada, April 22, 1976.

FIRST ANNUAL REPORT ON CONTRACT N00014-75-C-0586

"STATISTICAL ENGINEERING"

by

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Principal Investigator

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FIFTH (AND FINAL) ANNUAL REPORT ON CONTRACT N00014-67-A-0077-0020

AND FIRST ANNUAL REPORT ON CONTRACT N00014-75-C-0586

This is the fifth (and final) annual report on Contract N00014-67-A-0077-0020, and first annual report on Contract N00014-75-C-0586, both contracts being titled "Statistical Engineering." The first contract was initiated on February 1, 1971 and (with its third renewal, and an extension without cost) continued through June 30, 1975; the second contract was initiated on June 1, 1975 and continues through May 31, 1976. The present report covers the one-year period September 1, 1974 through August 31, 1975. (The reporting period for the present report therefore overlaps the reporting period for the fourth annual report on the first contract.) During summer 1975 the contracts provided partial support to the Principal Investigator, Dr. Robert Bechhofer, to a Research Associate, Dr. Bruce Turnbull of Oxford University (who visited Cornell University during this period), and to several Graduate Research Assistants. In addition Messrs. Jose Kreimerman and Ajit Tamhane were supported part time during fall 1974, and Messrs. Richard Gelber and Ray Jakobovits were supported part time during spring 1975; these four Graduate Research Assistants are all Ph.D. candidates.

Developments during the reporting period

a) Published papers

A list of papers supported by the first contract and the predecessor contract) and published during the reporting period follows:

Bechhofer, R.E. and Tamhane, A.C.: "An iterated integral representation for a multivariate normal integral having block covariance structure," Biometrika, Vol. 61, No. 3, pp. 615-619, 1974.

Miller, D.R.: "Limit theorems for path-functionals of regenerative processes," Stochastic Processes and Their Applications, Vol. 2, pp. 141-161, North-Holland Publishing Co., 1974.

b) Paper accepted for publication

Bechhofer, R.E.: "Ranking and selection procedures." To appear in the Proceedings of the Twentieth Conference on the Design of Experiments in Army Research, Development, and Testing held at Fort Belvoir, Virginia, October 23-25, 1974.

c) Papers submitted for publication

Bechhofer, R.E. and Turnbull, B.W.: "A (k+1)-decision single-stage selection procedure for comparing k normal means with a fixed known standard: the case of common known variance."

Bechhofer, R.E. and Turnbull, B.W.: "A (k+1)-decision two-stage selection procedure for comparing k normal means with a fixed known standard: the case of common unknown variance."

Frischtak, R.: "Selection of subclasses of variates based on a measure of association."

Tamhane, A.C.: "On 2- and 3-stage screening procedures for selecting the population having the largest mean from k normal populations with a common unknown variance."

Turnbull, B.W.: "Multiple decision rules for comparing several populations with a fixed known standard."

d) Technical reports

Bechhofer, R.E.: "A two-sample procedure for selecting the population with the largest mean from several normal populations with unknown variances: some comments on Ofosu's paper," TR 233, October 1974.

Bechhofer, R.E. and Turnbull, B.W.: "A (k+1)-decision single-stage selection procedure for comparing k normal means with a fixed known standard: the case of common known variance," TR 242, December 1974 (revised May 1975).

Kreimerman, J.: "A bivariate test of goodness of fit based on a gradually increasing number of order statistics," TR 250, March 1975.

Awate, P.: "Dynamic programming with negative rewards and average reward criterion," TR 251, May 1975.

Bechhofer, R.E. and Turnbull, B.W.: "A (k+1)-decision two-stage selection procedure for comparing k normal means with a fixed known standard: the case of common unknown variances," TR 256, May 1975.

Turnbull, B.W.: "Multiple decision rules for comparing several populations with a fixed known standard," TR 257, June 1975.

Tamhane, A.C.: "On minimax multistage elimination type rules for selecting the largest normal mean," TR 259, May 1975.

Tamhane, A.C.: "A minimax two-stage permanent elimination type procedure for selecting the smallest normal variance," TR 260, June 1975.

Gelber, R.D.: "A sequential goodness-of-fit test for composite hypotheses involving unknown scale and location parameters," TR 266, August 1975.

e) Papers in preparation

Awate, P., Bechhofer, R.E., and Tamhane, A.C.: "A maximin procedure for ranking means of normal populations with known unequal variances."

Bechhofer, R.E.: "On designing multi-factor experiments to identify the treatment combination associated with the largest interaction."

Bechhofer, R.E. and Turnbull, B.W.: "Chebyshev type lower bounds for the probability of correct selection, I: the location problem with one observation from each of two populations."

Bechhofer, R.E. and Turnbull, B.W.: "A (k+1)-decision single-stage procedure for selecting the production process with the highest fraction of conforming product."

Bechhofer, R.E. and Turnbull, B.W.: "A (k+1)-decision single-stage procedure for selecting the normal population with mean closest to a fixed known standard."

Tamhane, A.C. and Bechhofer, R.E.: "Optimal design for comparing several treatments with a control using incomplete blocks."

Tamhane, A.C. and Bechhofer, R.E.: "A minimax 2-stage permanent elimination type procedure for selecting the largest normal mean (common known variance)."

Professional activities of Dr. Bechhofer during the reporting period

a) Member of the Committee on Summer Research Institutes of the Institute of Mathematical Statistics.

b) Member of the Advisory Board of the Section on Physical and Engineering Sciences, American Statistical Association.

c) Reviewed research proposals for the National Science Foundation and the Army Research Office, Durham.

d) Presented an invited paper, "Ranking and Selection Procedures" at the Twentieth Conference on the Design of Experiments in Army Research, Development, and Testing held at Fort Belvoir, Virginia, October 23-25, 1974.

e) Invited to give a paper at the Symposium on Statistical Decision Theory and Related Topics to be held at Purdue University on May 17-19, 1976.

f) Elected an Ordinary Member of the International Statistical Institute -- one of ten so elected from the United States.

g) Named Director of a newly-reorganized School of Operations Research and Industrial Engineering, Cornell University.

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