DYNAMIC RESPONSE OF VERTEBRAL ELEMENTS RESPONSE OF THE INTERVER-
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UNCLASSIFIED
DYNAMIC RESPONSE OF VERTEBRAL ELEMENTS
Response of the Intervertebral Joint to Torsion

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Material in this report to be presented at the First Southern Biomedical Engineering Conference, Shreveport, Louisiana, June 7-8, 1982.

Creep response obtained from torsional stress tests of the intervertebral joint are used to develop an analytical model. A series chain of four Kelvin units was found to provide an excellent representation of the viscoelastic properties of the material.
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I. INTRODUCTION

This project is concerned with the viscoelastic properties of the intervertebral joint. Our previous studies under this Grant have quantified the response of the joint to shearing deformations. The work during this reporting period was directed toward the viscoelastic response of the joint to torsional deformations.

Knowledge of the response of the intervertebral joints to external loading is important if the forces and bending moments developed in the spine are to be related quantitatively to possible modes of injury or abnormal motions of the joint. The use of biodynamic models has gained widespread acceptance as a means of evaluating human tolerance limits and the probability of injury in environments of severe mechanical stress. Numerous investigations have studied the response of the spinal column and its constituent elements under axial tension and compression, in torsion, and in bending; however, only a limited number of investigators have considered the response of the spine to shear stress. The empirical model of the response of the disc to a step input, developed by Kelley,
Lafferty and Bowman [1], is the only attempt to provide an analytical representation of disc shear properties. The present study utilizes the same experimental and modeling techniques to investigate the response of the intervertebral joint to torsional stress.

II. MODEL ANALYSIS

Analysis of the response of the intervertebral to shear stress resulted in an analytical representation of the form

\[ \varepsilon/\sigma = E_1 + E_2 t + E_3 \left[ (1 - \exp - t_1/t_1) + (1 - \exp - t_2/t_2) \right] \]  \hspace{1cm} (1)

where \( \varepsilon/\sigma \) = strain = deformation/disc height
\( \sigma \) = stress = force/disc area
\( t \) = time
\( E_1, E_2, E_3, t_1, \) and \( t_2 \) are constants

The analysis of the shear response of an elastic isotropic material defines the shear modulus \( G \) as

\[ \varepsilon/\sigma = \frac{1}{G} \]  \hspace{1cm} (2)

The analysis of the same material subjected to torsional stress shows that

\[ \frac{\varepsilon J}{TH} = \frac{1}{G} \]  \hspace{1cm} (3)

where \( \varepsilon J/TH \) is the torsional equivalent \( \varepsilon/\sigma \) and

\( \varepsilon \) = angular deformation in radians
\( J \) = polar moment of inertia of cross sectional area (m\(^4\))
\( T \) = applied torque (N.m)
\( H \) = specimen height normal to the plane of the applied moment (m)

The shear modulus \( G \) is the same as defined in Eq. 2.

Although the intervertebral joint is a complicated structure that is neither purely elastic nor isotropic, some degree of correspondence between shear response and torsional response (as indicated by Equations 2 and 3) is
expected inasmuch as the disc structure responds to all types of stress in the same basic mode. It was, therefore, anticipated that the same Kelvin unit model (represented by Eq. 1) developed for the shear response would characterize the torsional response. One would expect, however, that the change in force components relative to the applied stress would result in different values for the constants.

III. METHODS AND MATERIALS

Test specimens, consisting of two vertebrae and the intervening disc with facets intact, were taken from fresh-frozen Rhesus monkey spines. The vertebrae were clamped in holding fixtures with the superior vertebra constrained to prevent angular displacement about the spinal axis but free to move in the longitudinal direction. The torque was applied to the inferior vertebra by a MTS Servoram controlled by a Cromemco microprocessor. The torque was applied as a step function in increments of 1.875 Newton-meters up to failure. The creep response was observed for 90 seconds at each value of torque and the specimen was allowed to relax for a minimum of 180 seconds between tests. One series of tests (8 specimens) was conducted with an axial load of 12.945N and a second series (6 specimens) employed an axial load of 27.66N. The geometrical data (disc height, disc area and volume) were obtained by sectioning each specimen in the longitudinal direction and using a Talos digitizer to record the coordinates of the disc. The procedures were the same as those reported for the shear studies.

IV. RESULTS AND DISCUSSION

Typical responses of the intervertebral joint subjected to a constant torsional stress is shown in Fig. 1 where the angular displacement (in
radians) is plotted vs. time. The plots show the effect of torque level as well as that of axial compressive load. The effect of increased axial compression is more pronounced at the higher torque levels indicating that the articular surfaces of the facet joints are forced into closer proximity, thereby increasing resistance to increased angular deformation.

Normalization of the data according to Eq. 3 should permit the results for a given axial load to be represented by a function independent of the applied stress and specimen size, i.e.,

$$\frac{\theta J}{TH} = f(k_1, k_2 \ldots k_i, C_1, C_2 \ldots C_i, t)$$

where $k_i =$ shear modulus for the $i^{th}$ elastic element (N/m$^2$)

$C_i =$ damping coefficient for the $i^{th}$ viscous element (N·S/m$^2$)

$t =$ time (s)

The average values of $\theta J/TH$ displayed a standard deviation on the order of $\pm32\%$ of the mean. We concluded that the imposition of a specific axial load to specimens obtained from subjects of widely varying size and weight contributed to the magnitude of the standard deviation. The creep modulus was, therefore, divided by the axial load expressed as a percent of body weight of the donor. This procedure reduced the maximum standard deviation to the order of 22%. The creep function is then defined as

$$K = \frac{\theta J}{THW}$$

where $W = (axial \ load/body \ weight) \times 100$

Use of the model in the form of Eq. 1 provided a representation of the results to within $\pm2\%$ of the averaged data. These results are tabulated in Table I and presented graphically in Fig. 2 for the two levels of axial preload. The analytical model (represented by the continuous curve in
Fig. 2) is

\[ K = \frac{6j}{THW} = \frac{1}{k_3} + \frac{k}{C_3} + \frac{1}{k_1} [1 - \exp (- k_1 t/C_1)] + \frac{1}{k_2} [1 - \exp (- k_2 t/C_2)] \]  

(5)

for which the values of \( k \) and \( C \) are shown in Table II for the torsional tests as well as the shear tests.

Comparison of the model parameters indicate a high degree of consistency between the shear and torsional results, i.e., the time constants \( k/C \) are identical in the two cases and in both cases \( k_1 = k_2 \). These results are not surprising when one considers the fact that the intervertebral disc resists all stress modes through tensile loading of the fibers of the annulus fibrosis. The differences in the magnitudes of the corresponding \( k \) and \( C \) reflect (1) the changes in the force component directed along the annulus fibers as the stress mode is changed and (2) the effect of the articular facets in the torsional loading.

The results obtained under this grant provide an analytical model suitable for use in computer simulations of spinal response to shear and torsional stresses. The analytical representation is not, however, unique in that other mathematical functions (and mechanical models) can also represent the temporal response of the system. The model used is not considered a good representation of the system because increased response time requires an increased number of Kelvin units to accurately represent the response. This results in a series of \( k \) and \( C \) values to represent the physical characteristics of the material. We are of the opinion that a more simple physical model with time dependent elastic and/or viscous coefficients exists. Such a model would be more representative of the actual material and would provide discrete elastic and viscous element characterization. Our initial efforts to formulate such a model are promising and its application to the general response of the system will be a major part of our future investigations.
### TABLE I

**AVERAGE CREEP FUNCTION**

<table>
<thead>
<tr>
<th>TIME (seconds)</th>
<th>Creep Function, ( K(\text{cm}^2/\text{N}) )</th>
<th>( 12.945 \text{ N Axial Load} \pm \text{S.E.} )</th>
<th>( 27.655 \text{ N Axial Load} \pm \text{S.E.} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8096 ± 0.0319</td>
<td>0.4212 ± 0.0338</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.8508 ± 0.0332</td>
<td>0.4435 ± 0.0359</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.8728 ± 0.0346</td>
<td>0.4573 ± 0.0371</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.8958 ± 0.0351</td>
<td>0.4685 ± 0.0382</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.9105 ± 0.0359</td>
<td>0.4769 ± 0.0390</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.9636 ± 0.0383</td>
<td>0.5046 ± 0.0415</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>1.0161 ± 0.0401</td>
<td>0.5365 ± 0.0448</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1.0512 ± 0.0429</td>
<td>0.5584 ± 0.0464</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>1.0908 ± 0.0422</td>
<td>0.5740 ± 0.0480</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>1.1174 ± 0.0418</td>
<td>0.5883 ± 0.0495</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>1.1376 ± 0.0444</td>
<td>0.6007 ± 0.0508</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>1.1516 ± 0.0458</td>
<td>0.6116 ± 0.0519</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>1.1675 ± 0.0461</td>
<td>0.6227 ± 0.0531</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>1.1742 ± 0.0471</td>
<td>0.6199 ± 0.0551</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 1. Torsional Creep Response as a Function of Applied Torque, Axial Load and Time.

Fig. 2. Torsional Creep Function. A Comparison of Model Response to the Experimental Results.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>12.95 N Axial Load</th>
<th>27.66 N Axial Load</th>
<th>Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>$5.714 \text{ N/mm}^2$</td>
<td>11.905</td>
<td>17.39</td>
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<tr>
<td>$k_2$</td>
<td>5.714</td>
<td>11.905</td>
<td>17.39</td>
</tr>
<tr>
<td>$k_3$</td>
<td>1.529</td>
<td>2.865</td>
<td>1.95</td>
</tr>
<tr>
<td>$C_1$</td>
<td>$4.395 \text{ N-S/mm}^2$</td>
<td>9.158</td>
<td>13.38</td>
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<tr>
<td>$C_2$</td>
<td>43.95</td>
<td>91.58</td>
<td>133.8</td>
</tr>
<tr>
<td>$C_3$</td>
<td>520.92</td>
<td>795.54</td>
<td>1000.0</td>
</tr>
<tr>
<td>$k_1/C_1$</td>
<td>$1.3 \text{ s}^{-1}$</td>
<td>1.3</td>
<td>1.3</td>
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<td>$k_2/C_2$</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
</tr>
</tbody>
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V. REFERENCE

Accepted for publication in 1982.

VI. PUBLICATIONS


VII. PROFESSIONAL PERSONNEL

1. Dr. J. F. Lafferty, Principal Investigator
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