TRILATERAL BRIDGE RATING CRITERIA

Final Report to

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RESEARCH AND DEVELOPMENT COMMAND
FORT BELVOIR, VIRGINIA 22060

April 1982

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Arthur D. Little, Inc.

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Trilateral Bridge Rating Criteria

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The procedure for determining the probability of failure for a normal crossing consistent with the Trilateral Design and Test Code for Military Bridging and Gap Crossing Equipment is presented. Methodology for calculating caution and risk crossing bridge ratings is developed. Proposed text on bridge rating criteria to be included in the Trilateral Design and Test Code is presented.
PREFACE

This report is submitted to the U.S. Army Mobility Equipment Research and Development Command (MERADCOM), Fort Belvoir, Virginia 22060, by Arthur D. Little, Inc., 15 Acorn Park, Cambridge, Massachusetts 02140, and was prepared under Task Order No. 0025 of Contract No. DAAK-70-79-D-0036. This report was prepared under the guidance of Messrs. Richard Helmke, and John Peterson as the technical points of contact, and Mr. Kenneth Dean as the COR. Questions of a technical nature should be directed to Peter D. Hilton, (617) 864-5770, the Technical Program Manager and principal investigator of the study. The Administrative Program Manager was Roger G. Long, and other investigators included Ranganath Nayak and Bruce Lamar.
This report contains the development of methodology for rating mobile bridging in a manner consistent with the *Trilateral Design and Test Code for Military Bridging and Gap Crossing Equipment*. The rating criteria are based on estimates of probability of failure for components of the bridging. These failure probabilities are to be calculated for the bridge for normal crossing conditions—i.e., those allowable under the Trilateral Design and Test Code. Caution crossing criteria are developed by restricting the crossing conditions to allow increased vehicle weight or gap size without increasing the probability of failure beyond the code allowable levels.

Risk crossing ratings are specified using the criteria that only failure modes which would interfere with the current mission are to be considered and that the allowable probability of failure during a risk crossing may exceed that for normal and caution crossings. There will also be more severe restrictions on vehicle crossing procedures for risk crossings. Taken together, these conditions are expected to enable a substantial increase in bridge rating for risk crossings.

This report details suggested additions to the *Trilateral Design and Test Code for Military Bridging and Gap Crossing Equipment* to specify the procedures for rating mobile bridging for caution and risk crossing conditions.
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1. INTRODUCTION

This report contains the results of a program carried out in response to the following "statement of work."

1.1 Background

The U.S., U.K., and GE are participating in a trilateral effort to produce a uniform "Trilateral Design and Test Code for Military Bridge and Gap Crossing Equipment" for use in the three countries. The new code changes the way bridges are designed and will require a new criteria for Bridge Class Rating. The old bridge rating criteria is based on rating tests which have been determined from past experience. The tests specified are not expected to be compatible with the new Design and Test Code as it is now constituted.

1.2 Objective

The objective of this report is to analyze the design criteria now set forth in the Design and Test Code and develop a bridge class rating criteria which is compatible with it.

1.3 Approach

The U.S. wishes to develop a system under which all ratings (Normal, Caution and Risk) would be assigned to new bridge designs by the bridge developer. The Normal Rating would be that which is defined by the Design and Test Code. The Caution and Risk Ratings are the areas to be studied and defined. It seems reasonable to assume that by carefully specifying the bridge and vehicle crossing variables (speed, location operation, bridge support geometry, freeboard, current velocity, etc.) in a table or listing similar to the one now listed in the old rating criteria "Factors," that the developer could determine a
Load Class which would be higher than the designed for "Normal" Load Class, and still produce stresses (load) no higher than those anticipated for the Normal Design. This technique would balance increased crossing restrictions against higher Load Class. The higher the Load Class, the greater the restriction. This method can be applied to the "Caution Rating" and possibly to the Risk Rating. The Risk Rating however, can be risky and bridge failure could be an acceptable result of an overload situation in a low percentage of its applications. This percentage should not exceed 10%. The desire here is to find a Risk Criteria which would increase the Load Rating over the Caution Rating by approximately the same increment as the Caution Rating exceeds the Normal Rating. Since both Caution and Risk Loadings are to be determined the Caution Rating can be set to split the increment between Normal and Risk. A desirable Normal to Risk increment would be 20% unless found to be impractical by study of the Code. The secondary rating classifications (Caution and Risk) need not be tested for, but could be adjusted by proportion with respect to the Normal Rating. A corollary to the Load Class Rating System will relate bridge capability to span. This system will specify Normal Caution and Risk span of the bridge for the specified Design Class. The limitations will be the same as used under the Bridge Load Class Rating System. Specific tasks required to accomplish the objectives are:

Task 1. Define Normal Crossing Rating as now specified within the Design Code.

Task 2. Define Caution and Risk Crossing Rating with respect to Class/Span and detail how it is determined and specified.

Task 3. Produce Draft Section or Paragraph which can be added to the current Design Code and would require minimum (no change is highly desirable) change to other sections of the Code. If changes to other parts of the Code are required these must be detailed.
2. FAILURE MODES AND DETAILED DESIGN CRITERIA

The Trilateral Design and Test Code for Military Bridging and Gap Crossing Equipment\(^1\) (denoted as the Design Code in future references) identifies a number of failure mechanisms relevant to mobile bridging and specifies margins of safety with respect to each of these. These failure modes include yielding, fracture, buckling and fatigue for dry gap bridges and, in addition, loss of freeboard and stability for floating bridges.

Each component or member of a bridge is to be designed to prevent the relevant failure modes. This is accomplished by performing analyses to obtain stress levels in terms of the geometric and material properties of the member and of the applied loadings. The stress levels which have been determined based on assumed loadings are required to be less than design allowable levels, which are equal to critical levels divided by safety factors.

The loadings which a bridge will experience in service are not known deterministically, but their probability distribution may be obtainable. Similarly, the material strength parameters, such as yield stress, endurance limit, etc., are actually random variables with probability distributions. Thus, at best, one can determine the likelihood (probability) that the stress level in a component of the bridge will exceed the corresponding strength measure of the material in terms of the probabilistic distributions of the stress and strength variables.

It is implicitly assumed that failure of an individual component or member of the bridge corresponds to failure of the bridge. This approach is conservative as it does not take into account the possible redundancies in the bridge design which may enable the bridge to continue to function after failure of one or more members. On the other hand, this approach is expected to be reasonably accurate because a goal in mobile bridge design minimizing weight and size is
likely to lead to bridge configurations which lack substantial redundancy.

We will make use of the structural reliability approach to estimate the probability of failure for a bridge built to the Design Code specifications when subjected to a "normal" crossing. The term normal crossing as used here refers to a crossing of the bridge by a vehicle of the maximum class for which the bridge was designed over a gap of maximum size. The "caution" and "risk" crossing ratings will then be set to be consistent with the normal crossing reliability.

The stress analysis used to verify a bridge design necessarily contains some degree of approximation. Thus the design stress which is to be compared to the material strength to determine the margin of safety is not the true stress. The safety factor set in the design code is meant to account for these and other approximations as well as the consequences of exceeding the critical condition. For example, the safety factor for yielding is less than that for other failure modes (1.33 vs. 1.5) because the consequence of local yielding at points of stress concentration is small. Therefore, inaccuracies in the predictions for local stress concentration relative to yielding will not have serious consequence on the performance of the bridge. On the other hand, the stress analysis should be sufficiently detailed to give accurate estimates of net section stress in each member.

The absolute predictions for bridge reliability during normal crossings are likely to be in error because of the approximations in the stress analysis and because of uncertainties in the probability distributions of applied loadings. However, by developing caution and risk criteria based on reliability estimates which are consistent with those for the normal crossing, we will be able to assure consistent performance expectations. In the case of the caution crossing, the same reliability level as for the normal crossing will be set.
We have chosen not to take the influence of prior damage on present performance or the reduction in fatigue life which may result from caution and/or risk crossings into account. The reason for not explicitly addressing the fatigue failure mode follows. Only the estimate of probability of failure in one crossing is required to define the normal crossing and to set caution and risk crossing levels because these crossings will be approved on an individual basis during field service. The probability of failure due to fatigue in an individual crossing is extremely low. The increase in this failure probability for caution and risk crossings is also extremely low.

Mobile bridges must be designed to support loads associated with the launching process as well as those it will see in-service. However, caution and risk crossing criteria refer to vehicle crossing conditions only. Therefore, we are able to ignore the analysis of the launch process in defining the normal crossing and setting criteria for caution and risk crossings.
3. DETERMINATION OF PROBABILITY OF FAILURE

In this section, we develop a general methodology for determining the probability that the strength of the bridges will not sustain the stresses applied to it. To do this, we first present a generalized stress measure for a bridge component and then illustrate the application of this measure through several specific examples. We next discuss procedures for determining the probabilistic distribution of stress measure. Then, by comparing the stress distribution with the distribution of component strength, we evaluate the probability of bridge failure.

3.1 Generalized Stress Measure

A bridge failure may, in fact, occur by any of several modes. For instance, the effective bending stress may exceed the yield strength of a component, or the shear stress may be longer than the shear buckling stress, or a stress may exceed its endurance limit, etc. To evaluate the likelihood of a bridge failure, it is necessary to evaluate the stress measure associated with each of these failure modes.

For a given component of known geometry, the stress measure for each failure mode will be a function of the loads applied to the bridge. In general, we can express this relationship in the form

\[ S_j = f_j(D,V,G,M,X,I,B,W,F,T,Q) \quad j = 1, 2, \ldots \]  

(3.1)

where

- \( S_j \) is the stress measure associated with \( j \)-th failure mode
- \( D \) is the bridge weight per unit length
- \( V \) is the vehicle weight
G is the gap length

M is the mud load per unit length

X is the vehicle eccentricity

I is the vehicle impact factor

B is the vehicle braking factor

W is the wind speed

F is the footpath load

T is the snow or ice load per unit bridge length

Q is the hydrodynamic load due to current, it is only applicable to floating bridging.

To illustrate the use of Equation (3.1) consider the first failure mode (i.e. j = 1) in which the tensile stress due to bending exceeds the material yield stress of a component. In this case, we must determine the tensile stress generated during a vehicle crossing at the point of maximum stress—namely, at the bottom chord of the treadway at the mid-span connector of the bridge. The explicit form of Equation (3.1) for tensile stress at this point may be expressed as

\[ S_1 = \frac{c}{8f} \left[ \frac{4VG}{h} X + 2VG(1+I) + 4yVB + G^2(D+M) + 2ayVG^2 \right] \]  

(3.2)

where

\( c \) is the distance from the neutral axis to the bottom chord of the
bridge

i is the section moment of inertia

h is the bridge width

y is the vertical distance from the bridge centroid to the crossing vehicle centroid

a is the surface area of the vehicle exposed to wind pressure multiplied by the wind speed/pressure conversion factor

and the loading variables D, V, G, M, X, I, B, W are as defined in Equation (3.1). We have assumed that the footpath, snow, and ice loads will be negligible and so they do not appear in Equation (3.2). We note that since the constants c, i, h, y, and a depend on the cross-sectional geometry of the bridge, and the dimensions of the crossing vehicle, they will be specific to the particular bridge under consideration. Equation (3.2) is used as an example to illustrate our methodology. In fact, the dependence of tensile stress on eccentricity and wind load is also related to the torsional characteristics of the bridge. Therefore, the form of stress may be somewhat different than expressed in Equation (3.2). The stress due to braking used in Equation (3.2) assumes that the equivalent braking force is through the centroid of the vehicle rather than at the bridge deck level as specified in Section 5.3.8 in the Design Code.

As a second application of Equation (3.1), consider the second failure mode (i.e., j = 2) in which the shear buckling strength cannot support the shear stress. Here, the appropriate stress measure is the shear stress produced in a treadway web during a vehicle crossing. This stress may be expressed as

\[ S_2 = \frac{G}{16\pi t} \left[ \frac{2V}{h} X + V(1+I) + \frac{4yV}{G} B + G(D+M) \right] \] (3.3)
where

\( q \) is the structural moment of the portion of the cross section about the neutral axis.

\( t \) is the web thickness

and the other values are as defined in Equations (3.1) and (3.2). Once again, Equation (3.3) is intended for illustrative purposes only since this representation of shear stress is bridge and location dependent and may differ from the form expressed here.

3.2 Distribution of Stress

Naturally, the loading variables such as vehicle load, eccentricity, gap width, etc. will vary from location to location and from crossing to crossing. This implies that the stress measures will vary as well; and, in order to evaluate the probability of failure by mode \( j \), we must determine the probability distribution of the stress measure \( S_j \).

In our analysis, we will consider the bridge weight to be constant and we will make parametric changes in the vehicle load and gap width. Thus, in Equations (3.2) and (3.3), the stress measure can be expressed as a series of additive terms. Each term consists of a constant multiplied by a loading variable. Symbolically, Equation (3.2) can be rewritten as:

\[
S_1 = a_0 + a_1X + a_2I + a_3B + a_4M + a_5W^2
\]  

(3.4)

where

\[
a_0 = \left( \frac{c}{8t} \right) (2VG + G^2D)
\]

\[
a_1 = \left( \frac{c}{8t} \right) \frac{4VG}{h}
\]

\[
a_2 = \left( \frac{c}{8t} \right) 2VG
\]

\[
a_3 = \left( \frac{c}{8t} \right) 4yV
\]
\[ a_4 = \left( \frac{c}{\delta I} \right) G^2 \]

\[ a_5 = \left( \frac{c}{\delta I} \right) 2aYVG \]

and the loading variables \( X, I, B, M, W \) are as defined in Equation (3.1). Similarly, Equation (3.2) can be expressed as

\[ S_2 = b_0 + b_1X + b_2I + b_3B + b_4M \quad (3.5) \]

where

\[ b_0 = \left( \frac{q}{10I^2} \right) (V + GD) \]

\[ b_1 = \left( \frac{q}{10I^2} \right) \frac{2V}{h} \]

\[ b_2 = \left( \frac{q}{10I^2} \right) V \]

\[ b_3 = \left( \frac{q}{10I^2} \right) \frac{4gV}{G} \]

\[ b_4 = \left( \frac{q}{10I^2} \right) G \]

and the loading variables \( X, I, B, \) and \( M \) are as defined in Equation (3.1).

Equations (3.4) and (3.5) highlight the additive form of the equations and suggest the probability distribution of the stress measures. Specifically, since realizations of the loading variables results from a large number of factors which are (to a large extent) independent, the distribution of the variables may approach a Gaussian distribution. Under this scenario, the stress measures will be the sum of Gaussian distributed random variables and so will themselves be
Gaussian distributed. This form of the distribution for the bending stress $S$, will be approximate at best since the stress contribution from wind varies as the square of the wind load. However, the wind stress component will typically be small compared to stress contributions of the other loading variables. Thus, for Equations (3.4) and (3.5) if the loading variables are Gaussian distributed, then it is reasonable to assume that the stress measures are Gaussian distributed.

On the other hand, even if the distribution loading variable is not Gaussian, the distribution of the stress measures in Equations (3.4) and (3.5) may still approximate a Gaussian distribution. This approximation relies on the linear combination of random (loading) variables in stress equations in the form of equations (3.4) and (3.5). Since the loading variables have finite means and variances, and if they are not highly correlated, then the distribution of stress will approach a Gaussian distribution regardless of the form of the distributions of the loading variable.

In short then, it is reasonable to assume a Gaussian distribution for the stress measures given in Equations (3.4) and (3.5). We emphasize, however, that the Gaussian assumption cannot be applied to the generalized stress measure given in Equation (3.1) without establishing that the stress is a linear combination of the loading variables.

The Gaussian assumption for the distribution of $S_1$ and $S_2$ simplifies our analysis since now we need only to determine the mean and variance of $S_1$ and $S_2$ to completely specify their distribution. Taking the mean and variance of $S_1$ as expressed in Equation (3.4) we obtain

$$\mu_{S_1} = a_0 + a_1 \mu_x + a_2 \mu_I + a_3 \mu_B + a_4 \mu_M + a_5 \mu_w^2 \quad (3.6)$$
\[ \sigma_{S_1}^2 = a_1^2 \sigma_X^2 + a_2^2 \sigma_I^2 + a_3^2 \sigma_B^2 + a_4^2 \sigma_M^2 + a_5^2 \sigma_{P_x}^2 + 2a_1 a_2 \sigma_X \sigma_I \rho_{xI} + 2a_1 a_3 \sigma_X \sigma_B \rho_{xB} + 2a_2 a_3 \sigma_I \sigma_B \rho_{IB} \]  

(3.7)

\[ \mu_{S_1} \text{ is the mean of } S_1 \]

\[ \mu_X \text{ is the mean of } X \]

\[ \sigma_{S_1}^2 \text{ is the variance of } S_1 \]

\[ \sigma_X^2 \text{ is the variance of } X \]

\[ \sigma_X = \sqrt{\sigma_X^2} \text{ is the standard deviation of } X \]

\[ \rho_{xI} \text{ is the correlation between } X \text{ and } I \]

and the other terms are similarly defined. The variance given in Equation (3.7) includes the correlation between the vehicle eccentricity, impact, and braking. However, it implicitly assumes that the mud load and wind load are independent of all other variables. In other words, we assume that all correlations involving mud and/or wind loads are zero.

In a similar manner, using Equation (3.5), we may express the mean and variance of \( S^2 \) as

\[ \mu_{S_2} = b_0 + b_1 \mu_X + b_2 \mu_I + b_3 \mu_B + b_4 \mu_M \]  

(3.8)

\[ \sigma_{S_2}^2 = b_1^2 \sigma_X^2 + b_2^2 \sigma_I^2 + b_3^2 \sigma_B^2 + b_4^2 \sigma_M^2 + 2b_1 b_2 \sigma_X \sigma_I \rho_{xI} + 2b_1 b_3 \sigma_X \sigma_B \rho_{xB} + 2b_2 b_3 \sigma_I \sigma_B \rho_{IB} \]  

(3.9)
where the terms used here are defined in a manner similar to those defined in Equations (3.6) and (3.7). Once again, we assume that the mud load is independent of the other variables so that its correlation is zero.

To complete our specification of the distribution of the stress measures, we must evaluate the means and standard deviations of the loading variables and the correlations between them. As described above, it is reasonable to assume that the loading variables are approximately Gaussian distributed. Accordingly, the mean and standard deviation parameters of each of the loading variables may be estimated by equating these parameters with a set of prescribed extreme values. For example, Figure 3.1 shows the distribution of the mud load \( M \) which typically ranges between zero and a given extreme value \( M_{\text{ext}} \). By symmetry, we take the mean of \( M \) to be \( \mu_M = M_{\text{ext}}/2 \). In general, though, it may be appropriate to locate the mean at a point other than the mid-point between zero and \( M_{\text{ext}} \). However, using this symmetrical representation of the mean, we may assume that the extreme value \( M_{\text{ext}} \) is \( K \) standard deviations from the mean.

That is

\[
M_{\text{ext}} = \frac{M_{\text{ext}}}{2} + K \sigma_M
\]

Thus, the standard deviation of \( M \) is

\[
\sigma_M = \frac{M_{\text{ext}}}{2K}
\]
Figure 3.1 Distribution of Mud Load
Typically, $K$ will take on the value of 2 or 3 indicating that the mudload $M$ will exceed $M_{\text{ext}}$ less than 5 percent or 0.5 percent of the time, respectively.

Since the wind velocity $W$ appears as a squared term in Equation (3.4), we must also determine the mean and standard deviation of $W^2$. If $W$ is Gaussian distributed with mean $\mu_W$ and standard deviation $\sigma_W$, then $U = W^2$ will have mean and standard deviation.

\[
\mu_{W^2} = \mu_W^2 + \sigma_W^2
\]
\[
\sigma_{W^2} = 2\sigma_W \sqrt{\mu_W^2 + \frac{\sigma_W^2}{2}}
\]

Substituting $\mu_W = \frac{W_{\text{ext}}}{2}$ and $\sigma_W = \frac{W_{\text{ext}}}{2K}$ we obtain

\[
\mu_{W^2} = \left(\frac{W_{\text{ext}}}{2K}\right)^2(1+K^2)
\]
\[
\sigma_{W^2} = \frac{W_{\text{ext}}^2}{2K} \sqrt{1 + \frac{1}{2K}}
\]
Table 3.1 summarizes the information relevant for the
distributions of the load variables.

We do not have sufficient information available to determine the
level of correlation between vehicle eccentricity, impact, and braking.
It is reasonable to presume, though, that all three of these loading
variables are positively correlated with vehicle speed. Thus as the
speed of the crossing vehicle increases so does the vehicle's impact
factor and eccentricity; and the stress produced by braking would also
be more pronounced. In other words, we presume that the correlation
coefficients $P_{XI}$, $P_{XB}$, and $P_{IB}$ will be non-negative. Then by
parametrically varying these correlation coefficients between zero and
one we can determine their effects on the probability of a bridge
failure.

3.3 Probability of Bridge Failure

In general, failure will occur by mode $j$ if

$$\text{Prob}(S_j > R_j) \text{ for any } j = 1, 2, \ldots \quad (3.10)$$

Assume that $R_j$ has a Gaussian distribution with mean $\mu_{R_j}$ and standard
deviation $\sigma_{R_j}$. Then we can also evaluate the deterministic case by
setting $\sigma_{R_j} = 0$. Let $Z$ be standard Gaussian random variable, i.e.,
$\mu_Z = 0, \sigma_Z = 1$. Then we evaluate Equation (3.10) as
TABLE 3.1
SUMMARY OF DISTRIBUTION OF THE LOADING VARIABLES

<table>
<thead>
<tr>
<th>Component</th>
<th>Term</th>
<th>Assumed Distribution</th>
<th>Extreme Value*</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
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<tr>
<td>Excentricity</td>
<td>X</td>
<td>Gaussian</td>
<td>$X_{\text{EXT}}$</td>
<td>$\mu_X = X_{\text{EXT}}/2$</td>
<td>$\sigma_Y = X_{\text{EXT}}/2K$</td>
</tr>
<tr>
<td>Impact Factor</td>
<td>I</td>
<td>Gaussian</td>
<td>$I_{\text{EXT}}$</td>
<td>$\mu_I = I_{\text{EXT}}/2$</td>
<td>$\sigma_I = I_{\text{EXT}}/2K$</td>
</tr>
<tr>
<td>Mud Load</td>
<td>M</td>
<td>Gaussian</td>
<td>$M_{\text{EXT}}$</td>
<td>$\mu_M = M_{\text{EXT}}/2$</td>
<td>$\sigma_M = M_{\text{EXT}}/2K$</td>
</tr>
<tr>
<td>Vehicle Braking</td>
<td>B</td>
<td>Gaussian</td>
<td>$B_{\text{EXT}}$</td>
<td>$\mu_B = B_{\text{EXT}}/2$</td>
<td>$\sigma_B = B_{\text{EXT}}/2K$</td>
</tr>
<tr>
<td>Wind Velocity</td>
<td>W</td>
<td>Gaussian</td>
<td>$W_{\text{EXT}}$</td>
<td>$\mu_W = W_{\text{EXT}}/2$</td>
<td>$\sigma_W = W_{\text{EXT}}/2K$</td>
</tr>
<tr>
<td>Squared Wind Velocity</td>
<td>$W^2$</td>
<td>Squared Gaussian</td>
<td></td>
<td>$\mu_{W^2} = (\frac{W_{\text{EXT}}}{2K})^2(1+k^2)$</td>
<td>$\sigma_{W^2} = \frac{W_{\text{EXT}}^2}{2K} \cdot \frac{1}{1+\frac{1}{2K}}$</td>
</tr>
</tbody>
</table>

*The extreme value is defined as $K$ standard deviation greater than the mean.
\[ \text{Prob}(S_j > R_j) = \text{Prob}(-R_j + S_j > 0) \]

\[ = \text{Prob} \left( \frac{(-R_j + S_j) + (\mu_{RJ} - \mu_{SJ})}{\sqrt{\sigma_{RJ}^2 + \sigma_{SJ}^2}} > \frac{(\mu_{RJ} - \mu_{SJ})}{\sqrt{\sigma_{RJ}^2 + \sigma_{SJ}^2}} \right) \]

\[ = \text{Prob} \left( Z > \beta_j \right) \]

where \( \beta_j = \frac{(\mu_{RJ} - \mu_{SJ})}{\sqrt{\sigma_{RJ}^2 + \sigma_{SJ}^2}} \) \( j = 1, 2 \) \( (3.11) \)

The implication of Equations (3.11) and (3.12) is that the parameter \( \beta \) can be used to assess the probability of failure, in particular

\[ P_f = \frac{1}{2} \text{erfc} \left( \frac{\beta}{\sqrt{2}} \right) \]

Figure 3.2 relates \( P_f \) to \( \beta \).
4. EVALUATION OF A NORMAL CROSSING

We use the term "normal" crossing to refer to a bridge crossing by a vehicle of the weight class for which the bridge has been designed over a gap of the design length. For each failure mode $j$, the stress measure for a normal crossing, $S_j^N$, will be a function of the load variables mud, vehicle impact, wind, etc. in which the extreme values of these variables are set at the design values specified in the Design Code. Thus, the probability of failure by mode $j$ in a normal crossing will be specified as

$$\text{Prob} \left( S_j^N > R_j \right) \quad j = 1, 2, ... \quad (4.1)$$

Equation (4.1) provides a basis for evaluating "caution" and "risk" crossings. In the caution crossing scenario we increase the vehicle weight or the gap width by controlling a subset of the other loading variables in order to maintain the same probability of failure as specified in Equation (4.1). In the risk crossing scenario we allow the probability of failure to increase beyond that specified in Equation (4.1) in order to accommodate an increased vehicle weight or gap width. These two scenarios are evaluated in succeeding sections of this report.

In this section we evaluate Equation (4.1) for two failure modes—(1) where the tensile stress due to bending exceeds the yield strength, and (2) where the shear stress exceeds the shear buckling strength. For each mode, we first determine specific values for the non-random terms of the stress measure for each failure mode. This determination is performed with the aid of a safety margin constraint equation. We then evaluate the failure probabilities for a normal
crossing by assigning probability distributions to the stress and strength measures.

4.1 First Failure Mode

Consider the terms set forth in Table 4.1. The numeric values for these terms were taken from Design Code where possible or typical in-service values were assigned. The material property characteristics $\bar{r}_1$, $\mu_{R_1}$, and $\sigma_{R_1}$ were derived from the literature as described in our previous report. This information is not sufficient to directly evaluate the stress measure for the first failure mode as specified in Equations (3.2) or (3.4). We introduce the safety factor constraint equation to evaluate the term $c/b_i$. This equation relates the safety factor of a failure mode to the exceedance points of the stress and strength measures for that mode. Symbolically,

$$\gamma_j = \frac{\bar{r}_j}{s_j} \quad j = 1, 2, \ldots$$

(4.2)

where

- $\gamma_j$ is the safety factor for mode $j$
- $\bar{r}_j$ is the $100(1-p)$ percent exceedance point of the strength measure $R_j$
- $s_j$ is the $100p$ percent exceedance point of the stress measure $S_j$

A pictorial representation of Equation 4.2 is given in Figure 4.1. For $p = 0.01$, Table 4.1 gives $\bar{r}_1$, which is also referred to as the nominal material yield stress, as 38 KSI. The Design Code stipulates that the stresses due to yielding should not exceed three-fourths of the nominal material yield strength. Thus $\gamma_1 = 4/3$. The actual safety margin for
Figure 4.1 Representation of the Constraint Equation
service conditions will depend on the level of inaccuracy in the stress and strength measures. We consider $S_1$ to be the stress evaluated in the MERADCOM bridge design. Specifically, we take the design values for vehicle weight, gap width, bridge dead load, and vehicle impact factor (i.e., $V = V_{DES}$, $G = G_{DES}$, $D = d_{DES}$, $I = i_{DES}$). We take the mud load as 0.8 times the design values (i.e., $M = 0.8 M_{DES}$) and set all other load variables to zero. Applying these values to Equation (3.2) yields

$$S_1 = \frac{C}{81} \left[ 2V_{DES}g_{DES}(1 + i_{DES}) + g_{DES}^2 (d_{DES} + 0.8M_{DES}) \right]$$

(4.3)

More generally, we can evaluate $S_j$ by using the generalized stress Equation 3.1 and substituting a weighted set of design values for the load variables. Equating Equations 4.2 and 4.3 we can solve explicitly for the constant term $\frac{C}{81}$, i.e.,

$$\frac{C}{81} = \frac{\bar{F}_1/\gamma_1}{2V_{DES}g_{DES}(1 + i_{DES}) + g_{DES}^2 (d_{DES} + 0.8M_{DES})} = 0.132 \text{ cubic feet}^{-1}$$

(4.4)

Using Equation (4.4) and the design values for vehicle weight and gap width given in Table 4.1 we may easily calculate the coefficients given
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>V&lt;sub&gt;DES&lt;/sub&gt;</td>
<td>Design code vehicle weight</td>
<td>60 tons</td>
</tr>
<tr>
<td>g&lt;sub&gt;DES&lt;/sub&gt;</td>
<td>Design code gap width</td>
<td>100 feet</td>
</tr>
<tr>
<td>x&lt;sub&gt;DES&lt;/sub&gt;</td>
<td>Design code eccentricity</td>
<td>15 inches</td>
</tr>
<tr>
<td>t&lt;sub&gt;DES&lt;/sub&gt;</td>
<td>Design code impact factor</td>
<td>0.15</td>
</tr>
<tr>
<td>b&lt;sub&gt;DES&lt;/sub&gt;</td>
<td>Design code braking factor</td>
<td>0.50</td>
</tr>
<tr>
<td>d&lt;sub&gt;DES&lt;/sub&gt;</td>
<td>Design code dead load</td>
<td>250 lbs/foot</td>
</tr>
<tr>
<td>m&lt;sub&gt;DES&lt;/sub&gt;</td>
<td>Design code mud load</td>
<td>120 lbs/foot</td>
</tr>
<tr>
<td>W&lt;sub&gt;DES&lt;/sub&gt;</td>
<td>Design code wind speed</td>
<td>38.4 knots</td>
</tr>
<tr>
<td>h</td>
<td>Bridge width</td>
<td>128 inches</td>
</tr>
<tr>
<td>y</td>
<td>Bridge to vehicle distance</td>
<td>5 feet</td>
</tr>
<tr>
<td>a</td>
<td>Vehicle surface area</td>
<td>0</td>
</tr>
<tr>
<td>γ&lt;sub&gt;1&lt;/sub&gt;</td>
<td>Safety factor for failure mode 1</td>
<td>4/3</td>
</tr>
<tr>
<td>γ&lt;sub&gt;2&lt;/sub&gt;</td>
<td>Safety factor for failure mode 2</td>
<td>3/2</td>
</tr>
<tr>
<td>R&lt;sub&gt;1&lt;/sub&gt;</td>
<td>99 percent exceedance point for material strength R&lt;sub&gt;1&lt;/sub&gt;</td>
<td>38 KSI</td>
</tr>
<tr>
<td>μ&lt;sub&gt;R1&lt;/sub&gt;</td>
<td>Mean of material strength R&lt;sub&gt;1&lt;/sub&gt;</td>
<td>42 KSI</td>
</tr>
<tr>
<td>σ&lt;sub&gt;R1&lt;/sub&gt;</td>
<td>Standard deviation material strength R&lt;sub&gt;1&lt;/sub&gt;</td>
<td>1.6 KSI</td>
</tr>
</tbody>
</table>
in Equation (3.4) for the normal crossing stress measure $S_1^N$. This information is summarized in Table 4.2.

To evaluate the probability of failure using Equation (4.1), we assume that $S_1$ and $R_1$ are Gaussian distributed as justified in Section 3.

Based on this Gaussian assumption, the first failure mode reliability factor for a normal crossing, $\beta_1^N$, is given by

$$\beta_1^N = \frac{\mu_{R_1} - \mu_{S_1}^N}{\sqrt{\sigma_{R_1}^2 + \sigma_{S_1}^2}}$$  \hspace{1cm} (4.5)

The material strength parameters, $\mu_{R_1}$ and $\sigma_{R_1}$, are given in Table 4.1. The tensile stress parameters for a normal crossing, $\mu_{S_1}^N$ and $\sigma_{S_1}^N$, are obtained from Equations (3.6) and (3.7) using the coefficient values given in Table 4.2. To estimate the means of the loading variables, we equated the extreme values with the design values (i.e., $X_{EXT} = X_{DES}$, $I_{EXT} = I_{DES}$, etc.) and applied the equations given in Table 3.1. To estimate the standard deviation of the loading variables, we used Table 3.1 and assumed that the extreme value was two standard deviations from the mean ($K = 2$). Table 4.3 summarizes the first mode probability of failure for correlations between the vehicle eccentricity, impact factor, and braking factor which are simultaneously varied between zero and one. We found that the probability of failure for the first failure mode during a single normal crossing can range between $0.08 \times 10^{-7}$ and $27.4 \times 10^{-7}$.

The estimates of probability of failure are also expected to be sensitive to the choice of the number of standard deviations ($K$) between the means value and the design value of the loading parameters. To
### TABLE 4.2

**COEFFICIENTS FOR FIRST FAILURE MODE IN NORMAL CROSSING**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>3,501,481 lbs/ft$^2$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>598,591 lbs/ft$^3$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>3,177,153 lbs/ft$^2$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>158,558 lbs/ft$^2$</td>
</tr>
<tr>
<td>$a_4$</td>
<td>1,320 /ft</td>
</tr>
<tr>
<td>$a_5$</td>
<td>0</td>
</tr>
</tbody>
</table>
### TABLE 4.3

**PROBABILITY OF FAILURE FOR FIRST FAILURE MODE IN NORMAL CROSSING**

<table>
<thead>
<tr>
<th>CORR. COEFF.*</th>
<th>$K$</th>
<th>$\mu_{SN}$ (KSI)</th>
<th>$\sigma_{SN}$ (KSI)</th>
<th>$\beta_{SN}$</th>
<th>PROBABILITY OF FAILURE ($10^{-7}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>2</td>
<td>29.374</td>
<td>1.563</td>
<td>5.646</td>
<td>0.082</td>
</tr>
<tr>
<td>0.1</td>
<td>2</td>
<td>29.374</td>
<td>1.647</td>
<td>5.498</td>
<td>0.192</td>
</tr>
<tr>
<td>0.2</td>
<td>2</td>
<td>29.374</td>
<td>1.728</td>
<td>5.362</td>
<td>0.411</td>
</tr>
<tr>
<td>0.3</td>
<td>2</td>
<td>29.374</td>
<td>1.884</td>
<td>5.236</td>
<td>0.823</td>
</tr>
<tr>
<td>0.4</td>
<td>2</td>
<td>29.374</td>
<td>1.878</td>
<td>5.118</td>
<td>1.548</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>29.374</td>
<td>1.949</td>
<td>5.007</td>
<td>2.763</td>
</tr>
<tr>
<td>0.6</td>
<td>2</td>
<td>29.374</td>
<td>2.017</td>
<td>4.904</td>
<td>4.706</td>
</tr>
<tr>
<td>0.7</td>
<td>2</td>
<td>29.374</td>
<td>2.084</td>
<td>4.806</td>
<td>7.692</td>
</tr>
<tr>
<td>0.8</td>
<td>2</td>
<td>29.374</td>
<td>2.148</td>
<td>4.714</td>
<td>12.122</td>
</tr>
<tr>
<td>0.9</td>
<td>2</td>
<td>29.374</td>
<td>2.210</td>
<td>4.628</td>
<td>18.491</td>
</tr>
<tr>
<td>1.0</td>
<td>2</td>
<td>29.374</td>
<td>2.271</td>
<td>4.546</td>
<td>27.400</td>
</tr>
<tr>
<td>0.0</td>
<td>3</td>
<td>29.374</td>
<td>1.563</td>
<td>6.61</td>
<td>0.0002</td>
</tr>
<tr>
<td>1.0</td>
<td>3</td>
<td>29.374</td>
<td>2.271</td>
<td>5.73</td>
<td>0.006</td>
</tr>
</tbody>
</table>

*Correlation between vehicle eccentricity, impact, and braking.
determine the magnitude of this influence we repeated the calculations for the extreme values of the correlation coefficient ($p = 0, 1.0$) at $K = 3$. The results, also shown in Table 4.3, indicate that if the design loadings are further from the mean loadings, there is a reduction in the calculated probability of failure.
4.2 Second Failure Mode

Once again, consider the values given in Table 4.1. In this table, we have even less material property data available for the second failure mode than we had for the first. Specifically, the mean \( \mu_R \) standard deviation \( \sigma_R \) and 99 percent exceedance point, \( \bar{F}_2 \) for the buckling strength \( R_2 \) are unavailable. This means that we cannot evaluate the constant term \( \left( \frac{q}{16 \alpha} \right) \) in Equation (3.3) by the direct method that was utilized in Section 4.1. However, we may still determine the probability of failure for the second failure mode by defining the unitless strength and stress measures.

\[
\tilde{R}_2 = \frac{R_2}{\bar{R}_2} \quad (4.6a)
\]

\[
\tilde{S}_2 = \frac{S_2}{\bar{S}_2} \quad (4.6b)
\]

Then, using the MERADCOM bridge design values for vehicle weight, gap width, bridge dead load, vehicle impact factor, and eight-tenths of the design mud load, we can evaluate the one percent exceedance point for \( \tilde{S}_2 \) as

\[
\tilde{S}_2 = \frac{q}{T671} \left[ V_{DES} (1 + \gamma_{DES}) + g_{DES} (d_{DES} + 0.8 m_{DES}) \right] / \bar{F}_2 \quad (4.7)
\]

Equating Equation (4.7) with Equation (4.2) we can express the constant term as

\[
\frac{q}{T671} = \frac{1/\gamma_2}{V_{DES} (1 + \gamma_{DES}) + g_{DES} (d_{DES} + 0.8 m_{DES})} \quad (4.8)
\]
Using the specification in the Design Code for a shear strength-to-stress safety factor of $\gamma_2 = 3/2$ we can evaluate Equation (4.8) numerically as

\[
\frac{q}{16t} = 3.86 \times 10^{-6}
\]

Using this constant, the coefficient in Equation (3.5) can be evaluated as shown in Table 4.4.

In order to determine the second mode probability of failure for a normal crossing we need to evaluate the distribution of $R_2$. Since the value of $R_2$ depends on material constants such as Young's modulus and on bridge dimensions such as the ratio of the web thickness to its length, the variability of $R_2$ will be quite small for a particular bridge design. This implies that $\sigma_{R_2}$ will be close to zero and $\bar{R}_2$ will be close to $\mu_{R_2}$. Therefore,

\[
\hat{\mu}_{R_2} = \frac{\mu_{R_2}}{\bar{R}_2} \approx 1 \quad (4.9a)
\]

\[
\hat{\sigma}_{R_2} \approx 0 \quad (4.9b)
\]

Thus, the specific form of the normal crossing reliability factor for the second failure mode $\beta_2$ is simplified

\[
\beta_2^N = \frac{1 - \frac{\hat{\sigma}_N}{\sigma_N}}{\hat{\mu}_N}
\]
TABLE 4.4

COEFFICIENT FOR SECOND FAILURE MODE
IN NORMAL CROSSING

<table>
<thead>
<tr>
<th>COEFFICIENT</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_0$</td>
<td>0.560062</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.0809061</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.463499</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.0463499</td>
</tr>
<tr>
<td>$b_4$</td>
<td>0.00038625</td>
</tr>
</tbody>
</table>
As with the first failure mode, we took the extreme values of the loading variables as the design values. Table 4.5 summarizes the probability of failure for the second failure mode for correlations of the vehicle eccentricity impact, and braking ranging between zero and one, assuming that the design loads are two standard deviations above the mean values ($K = 2$). Results for $K = 3$ at correlation coefficients of 0.0 and 1.0 are also included in Table 4.5.
### TABLE 4.5

**PROBABILITY OF FAILURE FOR SECOND FAILURE MODE**

**IN NORMAL CROSSING**

<table>
<thead>
<tr>
<th>CORR. COEFF.</th>
<th>K</th>
<th>$\mu^N_{S_2}$</th>
<th>$\sigma^N_{S_2}$</th>
<th>$\beta^N_2$</th>
<th>PROBABILITY OF FAILURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>2</td>
<td>0.6839</td>
<td>0.03475</td>
<td>9.097</td>
<td>&lt;10^{-16}</td>
</tr>
<tr>
<td>0.1</td>
<td>2</td>
<td>0.6839</td>
<td>0.03679</td>
<td>8.592</td>
<td>&lt;10^{-16}</td>
</tr>
<tr>
<td>0.2</td>
<td>2</td>
<td>0.6839</td>
<td>0.03872</td>
<td>8.163</td>
<td>1.665 x 10^{-16}</td>
</tr>
<tr>
<td>0.3</td>
<td>2</td>
<td>0.6839</td>
<td>0.04057</td>
<td>7.792</td>
<td>3.289 x 10^{-15}</td>
</tr>
<tr>
<td>0.4</td>
<td>2</td>
<td>0.6839</td>
<td>0.04237</td>
<td>7.468</td>
<td>4.073 x 10^{-14}</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>0.6839</td>
<td>0.04402</td>
<td>7.181</td>
<td>3.459 x 10^{-13}</td>
</tr>
<tr>
<td>0.6</td>
<td>2</td>
<td>0.6839</td>
<td>0.04565</td>
<td>6.925</td>
<td>2.182 x 10^{-12}</td>
</tr>
<tr>
<td>0.7</td>
<td>2</td>
<td>0.6839</td>
<td>0.04722</td>
<td>6.694</td>
<td>1.084 x 10^{-11}</td>
</tr>
<tr>
<td>0.8</td>
<td>2</td>
<td>0.6839</td>
<td>0.04874</td>
<td>6.485</td>
<td>4.428 x 10^{-11}</td>
</tr>
<tr>
<td>0.9</td>
<td>2</td>
<td>0.6839</td>
<td>0.05022</td>
<td>6.295</td>
<td>1.540 x 10^{-10}</td>
</tr>
<tr>
<td>1.0</td>
<td>2</td>
<td>0.6839</td>
<td>0.05165</td>
<td>6.120</td>
<td>4.768 x 10^{-10}</td>
</tr>
<tr>
<td>0.0</td>
<td>3</td>
<td>0.6839</td>
<td>0.03475</td>
<td>13.64</td>
<td>&lt;10^{-16}</td>
</tr>
<tr>
<td>1.0</td>
<td>3</td>
<td>0.6839</td>
<td>0.05165</td>
<td>9.18</td>
<td>&lt;10^{-16}</td>
</tr>
</tbody>
</table>

*Correlation between vehicle eccentricity, impact, and braking.*
5. EVALUATION OF CAUTION CROSSING

A mobile bridge is designed to support a specified vehicle class over a set gap size. In field service, it may be advantageous to use the bridge for conditions outside this specification. Therefore, it is important for the field officer to have information on the performance limitations of the bridge. The caution and risk ratings for mobile bridging serve this purpose; they provide the field officer with information on the capability of the bridge to support additional loading for an individual crossing and on the added precautions that need to be used during such a crossing.

It is possible to either specify the desired increase in vehicle weight or gap size and determine the crossing restrictions required to achieve the allowable reliability or to specify the crossing conditions and determine the possible increases in vehicle weight or span at the same reliability level. The methodology that must be used to trade-off increased loading against crossing restrictions is independent of which approach is ultimately used to specify caution crossing conditions to the field commander.

The caution crossing rating for mobile bridging is to be set so that the bridge will perform with the same degree of reliability during a caution crossing as is attained in a normal crossing. This is to be achieved by trading off increased vehicle weight or gap size against restrictions on the crossing conditions such as eccentricity, impact, and braking. For each failure mode $j$ we have chosen to use the parameters normal crossing reliability factor $\beta_j^N$ as the measure of bridge reliability. For a caution crossing, $\beta_j$ associated with failure mode $j$ cannot exceed the allowable value for a normal crossing $\beta_j^N$ as defined by equations (4.5) and (4.6). This restriction serves as the basis for specifying caution crossing conditions.
5.1 First Failure Mode

To evaluate the level of control on the crossing conditions required for the first failure mode in a caution crossing we first parametrically increase in vehicle weight or gap size, by

\[ V = (1+pv) V_{DES} \]  
\[ G = (1+pg) g_{DES} \]

where

\( V \) is the vehicle weight
\( G \) is the gap width
\( V_{DES} \) is the vehicle weight specified in the Design Code
\( g_{DES} \) is the gap width specified in the Design Code
\( pv \) proportion change in the vehicle weight
\( pg \) proportion change in the gap width

An increase in \( V \) or \( G \) will increase the mean and variance of \( S_1 \) by increasing the coefficient \( a_0 \) as given in Equation (3.4). This, in turn, will increase the probability of failure by decreasing \( \beta_1 \) as
defined in Equation (3.12). In order to restore $\beta_1$ to its normal crossing value at $\beta_1^N$ we need to decrease the mean and/or variance of one or more of the loading variables. We have chosen to simultaneously reduce the variability of loading variables directly associated with the vehicle: the eccentricity, impact factor, and braking factor. These variables may be most directly controlled by the field commander through restrictions on the speed of the crossing vehicle. We assume that all other loading variables will remain unaltered from their normal crossing values.

Vehicle eccentricity, impact, and braking are simultaneously controlled by reducing the extreme value for each of these variables below their design values. That is,

\[ x_{\text{EXT}} = (1 + q)x_{\text{DES}} \]  
\[ i_{\text{EXT}} = (1 + q)i_{\text{DES}} \]  
\[ b_{\text{EXT}} = (1 + q)b_{\text{DES}} \]

where

- $x_{\text{EXT}}$ is the extreme value for vehicle eccentricity
- $x_{\text{DES}}$ is the design value for vehicle eccentricity
- $q$ is the proportion change

and the other variables are similarly defined. We assume that the distribution of $X$, $I$, and $B$ will remain Gaussian (and that $K=2$) so that the formulas given in Table 3.1 may be used to obtain their means and
variances. We note, however, that this assumption will probably underestimate the amount of control required since we would expect the loading variables to be skewed toward their extreme value in a caution crossing scenario. To represent the interaction between vehicle eccentricity, impact, and braking we take a correlation coefficient of 0.5. Then, from Table 4.3 we see that the normal crossing reliability factor for the first failure mode is $B_1^N = 5.00$.

Table 5.1 gives the extreme values required on vehicle eccentricity, impact, and braking to maintain a reliability factor of $B_1 = 5.00$ given vehicle load increases up to 20 percent above the design value of 60 tons. In a similar manner, Table 5.2 gives the extreme value restrictions for increases in the gap width beyond the 100 foot design value. These tables bring out three features about the caution crossing for the first failure mode. Table 5.3 shows the influence of the choices of the numbers of standard deviations ($K$) between the mean and extreme values for the load parameters and the correlation coefficient ($p$) on the required restrictions on loading conditions for caution crossings.

First, as we would expect, the mean of the tensile stress $S_1$ is greater for a caution crossing than for a normal crossing whereas the standard deviation of $S_1$ is less. This is because we are trading off an increase in value of some loading variables for a decrease in variability in others. Second, since the gap width appears as a squared term in Equation (3.2), a 20 percent increase in gap width requires greater control of the vehicle loading variables than does a 20 percent increase in vehicle load. Third, and most interesting, for the range of values considered, $q$, the change in the extreme value for the controlled load variables, is approximately linearly related to $pv$ and $pg$, the change in the vehicle load and gap width, respectively. For example, if $pv$ is 0.1 then $q$ is -0.26; and if $pv$ is doubled, then $q$ is approximately doubled. Thus for the range of vehicle loads considers in Table 4.1 we may write

\[ q \approx -2.6 pv \quad (5.3a) \]
### TABLE 5.1

**VARYING VEHICLE LOAD IN CAUTION CROSSING**

*FOR FIRST FAILURE MODE* *\(^*\)

<table>
<thead>
<tr>
<th>( \frac{V}{V_{DES}} )</th>
<th>( \frac{x_{EXT}}{x_{DES}} )</th>
<th>( \frac{f_{EXT}}{f_{DES}} )</th>
<th>( \frac{b_{EXT}}{b_{DES}} )</th>
<th>( \mu_{S1} ) (KSI)</th>
<th>( \sigma_{S1} ) (KSI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
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<td>1.00</td>
<td>29.37</td>
<td>1.95</td>
</tr>
<tr>
<td>1.02</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
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<td>1.87</td>
</tr>
<tr>
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<td>0.89</td>
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<td>0.84</td>
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<td>0.79</td>
<td>0.79</td>
<td>30.47</td>
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</tr>
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<td>0.74</td>
<td>0.74</td>
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</tr>
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</tr>
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<td>0.69</td>
<td>0.69</td>
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</tr>
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<td>0.55</td>
<td>0.55</td>
<td>31.76</td>
<td>1.28</td>
</tr>
<tr>
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<td>0.50</td>
<td>0.50</td>
<td>31.97</td>
<td>1.19</td>
</tr>
</tbody>
</table>

*Assumes correlation is 0.5. Design values given in Table 4.1.*
TABLE 5.2
VARYING GAP WIDTH IN CAUTION CROSSING
FOR FIRST FAILURE MODE*

<table>
<thead>
<tr>
<th>( \frac{G}{g_{DES}} )</th>
<th>( \frac{x_{EXT}}{x_{DES}} )</th>
<th>( \frac{y_{EXT}}{y_{DES}} )</th>
<th>( \frac{b_{EXT}}{b_{DES}} )</th>
<th>( \mu s_1 ) (KSI)</th>
<th>( \sigma s_1 ) (KSI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
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<td>1.95</td>
</tr>
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</tr>
<tr>
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<td></td>
<td>30.05</td>
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<td>0.69</td>
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</tr>
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</table>

*Assumes correlation of 0.5. Design values given in Table 4.1.
TABLE 5.3

THE INFLUENCES OF THE CORRELATION COEFFICIENTS \((p)\) AND
THE NUMBER OF STANDARD DEVIATIONS \((K)\) ON THE CROSSING
RESTRICTIONS FOR CAUTION CROSSINGS

<table>
<thead>
<tr>
<th>(p_g)</th>
<th>(p_g)</th>
<th>(p)</th>
<th>(K)</th>
<th>(\frac{x_{\text{EXT}}}{x_{\text{DES}}})</th>
<th>(\frac{t_{\text{EXT}}}{t_{\text{DES}}})</th>
<th>(\frac{b_{\text{EXT}}}{b_{\text{DES}}})</th>
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</thead>
<tbody>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>.55</td>
<td></td>
</tr>
<tr>
<td>0</td>
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<td>0</td>
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<td></td>
<td>.47</td>
<td></td>
</tr>
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<td></td>
<td>.50</td>
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</tr>
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<td>.76</td>
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<td>.5</td>
<td>3</td>
<td></td>
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</tr>
</tbody>
</table>

Arthur D Little, Inc.
Similarly, for the range of gap width considered in Table 4.2, we have

\[ q \hat{=} -3.0 \, pg \]  
\hspace{1cm} (5.3b)

We note that Equation (5.3) cannot be extrapolated beyond a 20 percent increase of the vehicle load and gap width since, in general, \( q \) is not a linear function of \( pv \) and \( pg \).

5.2 Second Failure Mode

We may apply the same approach for the second failure mode as was used for the first failure mode. That is, we increase the vehicle load or gap width using Equation (5.1) and control the extreme values of vehicle eccentricity, impact, and braking using Equation (5.2) in order to maintain a reliability factor of \( \beta_2^n = 7.18 \) for a correlation of 0.5. The results are shown in Tables 5.4 and 5.5. Again, for the second failure mode, there is an approximate linear relationship between \( q \) and \( pv \) as well as between \( q \) and \( pg \). Specifically,

\[ q \hat{=} -1.75 \, pv \]  
\hspace{1cm} (5.4a)

and

\[ q \hat{=} -0.3 \, pg \]  
\hspace{1cm} (5.4b)

Note that the coefficient in Equation (5.4) are smaller in magnitude than those in Equation (5.3). In fact gap width changes in the second failure mode can be controlled by only marginal changes in the vehicle eccentricity, impact, and braking.

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**TABLE 5.4**

**VARYING VEHICLE LOAD IN CAUTION CROSSING**

**FOR SECOND FAILURE MODE***

<table>
<thead>
<tr>
<th>$\frac{V}{V_{DES}}$</th>
<th>$\frac{x_{EXT}}{x_{DES}}$</th>
<th>$\frac{t_{EXT}}{t_{DES}}$</th>
<th>$\frac{b_{EXT}}{b_{DES}}$</th>
<th>$\mu_{S2}$</th>
<th>$\sigma_{S2}$</th>
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</thead>
<tbody>
<tr>
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<td></td>
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<td>0.0440</td>
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<tr>
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<td></td>
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</tr>
</tbody>
</table>

*Assumes correlation is 0.5. Design values given in Table 4.1.*
### TABLE 5.5

**VARYING GAP WIDTH IN CAUTION CROSSING**

*FOR SECOND FAILURE MODE*

<table>
<thead>
<tr>
<th>$G$</th>
<th>$\frac{g_{\text{DES}}}{g_{\text{DES}}}$</th>
<th>$\frac{x_{\text{EXT}}}{x_{\text{DES}}}$</th>
<th>$\frac{i_{\text{EXT}}}{i_{\text{DES}}}$</th>
<th>$\frac{b_{\text{EXT}}}{b_{\text{DES}}}$</th>
<th>$\mu_{s2}$</th>
<th>$\sigma_{s2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.6839</td>
<td>0.0440</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.02</td>
<td>0.99</td>
<td>0.99</td>
<td>0.6851</td>
<td>0.0436</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.04</td>
<td>0.99</td>
<td>0.99</td>
<td>0.6851</td>
<td>0.0436</td>
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<td></td>
</tr>
<tr>
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<td>0.98</td>
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<td></td>
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<td>0.97</td>
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<td>0.7000</td>
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</table>

*Assumes correlation is 0.5. Design values given in Table 4.1.*
This illustrates that in a caution crossing, the failure modes cannot be evaluated independently. Rather, the most severely limiting failure mode will determine the control required for each failure mode. For the two failure modes we have considered, the first mode is the more critical.

In general, the designer of a mobile bridge calculates stress components at a number of critical locations and compares the resulting values to corresponding material strength measures to determine if the bridge meets the Design Code requirements on standard performance. The design code specifies the ratio of allowable stress-to-strength for each failure mode, i.e., .75-yield, .67-ultimate, .67-fatigue, .67-buckling.

Then, for each failure mode $j$, it is possible to determine the reliability factor $\beta_j$. The design code specification for the normal crossing reliability is then defined by calculating the $\beta_j^N$'s for the design load (vehicle class) and the full gap assuming that the design precisely meets the code requirements, i.e., $S_j = r_j / \nu_j$. For the actual bridge design $\beta_j > \beta_j^N$ because the bridge design at least meets the code requirements. In order to treat various failure modes in a consistent manner, we scale the $\beta_j$'s by the code required values to obtain $a_j = \beta_j / \beta_j^N$. This approach recognizes that the judgement of the relative importance or consequence of each failure mode has previously been set in the design code. The necessary conditions for caution crossing are now set based on the assumption that the caution crossings should have no higher probability of failure than the normal crossing, i.e., that $a_j > 1$ for each failure mode and location.

In practice, the designer would calculate $\beta_j^N$ and $\beta_j$ for each critical condition in the bridge at the time that he performs the corresponding stress calculation. The methodology for carrying out these calculations is shown by example in the case of the tensile bending stress in the bottom chord and for shear stress in the bridge.
web. To determine the caution crossing conditions, the vehicle weight is increased and the eccentricity, impact, and braking factors are reduced simultaneously until each of the $a_j$'s reaches the condition $a_j \geq 1$. The calculation can be repeated at reduced gap widths to obtain less severe caution crossing operating conditions. Similarly, the designer may define caution crossing criteria for the design class vehicle weight but increased gap using the conditions $a_j \geq 1$ to prescribe operating condition limitations. As the caution crossing criteria only apply to in-service conditions for the bridge, it is not necessary to review launching conditions to set the caution crossing ratings.
6. RISK CROSSING DEFINITIONS

The risk crossing is distinct from the normal and caution crossings in two areas. The allowable probability of failure is to be larger than for the normal and caution crossings. For normal and caution crossings the definition of failure includes damage to the bridge which may prevent retrieval or subsequent use; while, for risk crossings failure modes to be considered only include those which would prevent completion of the crossing.

In specifying the risk crossing rating criteria, we recommend that procedures similar to those described for setting caution crossing ratings be used. Failure modes identified in the design of the bridging and employed to assess the probability of failure for normal crossings through the $\beta_j$ should be separated into two categories—those which will cause collapse of the bridge and those which result in permanent damage but not collapse. Only the former failure modes should be addressed in the determination of the risk rating for a bridge. For example, the ultimate stress rather than the yield stress criteria should be imposed.

For those failure modes which may cause collapse of the bridging probabilities of failure are to be determined as a function of the vehicle weight and of the crossing restrictions using the parameters $\beta_j$ much as they were for the caution crossing conditions. It is now necessary to address the issue of how to best define an allowable probability of failure for a risk crossing. This may be done relative to the normal crossing results, e.g., the allowable probability of failure for a risk crossing is $x$ times that for a normal crossing. Alternatively, an absolute probability of failure may be allowed for a risk crossing, e.g., 10%.
The issue is of concern because the calculation of probability of failure will include some error. The error sources are the estimation errors of the probability distributions for the loading variables \( I, M, X, B, ... \) and the approximations involved in performing the stress analysis, i.e., in the equation \( S_j = f_j (D, G, X, I, M, B, ...). \)

Increased confidence in the risk ratings can be gained by using overload tests to verify aspects of this rating. Section 8 contains further discussion concerning the specification of such a testing program.

The two example failure modes analyzed for normal and caution crossings are considered here again. Numerical calculations are performed which give reliability factors and failure probabilities for each mode in terms of increased vehicle weight or gap length. These calculations assume three levels of crossing restrictions, and therefore, reduced impact, eccentricity and braking. In this manner, they simulate normal, caution, and risk crossing conditions. The results are useful to obtain an understanding of the relationship of increased loading to increased probability of failure and then to set reasonable risk rating criteria.

Figures 6.1-6.4 contain plots of \( \beta \) or probability of failure against increased vehicle weight and gap size for failures modes 1 and 2, respectively. Each plot includes three curves representing \( q = 0, .5, \) and \( .8 \) where \( q \) measures the amount of crossing restrictions, i.e.,

\[
I_{\text{EXT}}^\text{NORMAL} = (1-q) I_{\text{EXT}}
\]
Figure 6.2 Varying Gap Width For First Failure Mode in Risk Crossing
Figure G.3 Varying Vehicle Weight For Second Failure Mode In Risk Crossing
The $\beta_1$ value calculated for the normal crossing and reported in section 4 was $\beta_1^N = 5.0$. For risk crossing conditions ($\epsilon = 0.8$) this $\beta$ value corresponds to a 34 percent increase in vehicle weight (Figure 6.1) or a 27 percent increase in gap (Figure 6.2). If we were to allow $\beta$ for the risk crossing to decrease approximately 20% from the normal crossing value, then, according to Figures 6.1 and 6.2, we could accept a 40 percent increase in vehicle weight or a 31 percent increase in gap size for risk crossings. The 20% decrease in $\beta$ from the normal crossing value in Figures 6.1 and 6.2 corresponds to a two order-of-magnitude increase in the probability of failure.

Figures 6.3 and 6.4 refer to the second failure mode. The increase in vehicle weight which would be associated with a 20 percent decrease in $\beta_2$ from $\beta_2^N = 7.2$ to $\beta_2^R = 5.8$ would allow more than a 50 percent increase in vehicle weight. Thus, mode 1 limits vehicle weight increases for risk crossings according to these results. Similar reasoning will also show that mode 1 is the limiting mode for increases in gap size associated with risk crossing conditions.

We made use of these results to recommend the risk rating criteria which are specified in section 8 of this report. In particular we suggest that a 20% decrease in $\beta$ or a one hundred fold increase in probability of failure beyond the normal crossing level is reasonable for risk crossings. The relationship between $\beta$ and the probability of failure is highly nonlinear (see Figure 3.2), therefore, the criterion which is controlling (20% decrease in $\beta$ or 100 fold change in $p_f$) is dependent on $\beta$. In particular for large $\beta$, the limit to a 20% change in $\beta$ is more restrictive than a 100 fold change in $p_f$ while for small $\beta$ the reverse is the case. Thus, it is sensible to require that the risk crossing meet both conditions.
7. TESTS TO VERIFY THE RISK RATING CRITERION

The risk rating criterion specifies vehicle loads which approach the bridge carrying capacity. These loads are set so as to allow increased probability of bridge collapse over normal operating conditions. The methodology described earlier for the bridge rating determination includes approximations and is subject to omissions or errors. For example, failure may occur by a mode not anticipated by the bridge designer. Alterations in the Design Code are recommended to address these uncertainties. The revised testing plan will accomplish two added goals. It will include measurements to check the stress estimate for the quantities associated with the most critical failure modes. Second it will determine if there exist failure modes which precede those identified by the designer and have been inadvertently omitted from the analysis.

The risk rating verification test program should simulate the risk crossing conditions. The bridging should be set across a full length gap under the most degrading conditions which are permitted for a risk crossing, e.g., there should be opposite transverse slopes at the two supporting banks and mud loading on the bridge.

The bridging should be strain gauged in order to verify the relationship between stress and vehicle load for the mode (or modes) of failure which are anticipated to be critical. Vehicles of increasing weight from the design class, to the caution class, and then to the risk class should cross the bridge applying the risk crossing conditions on speed, eccentricity, and braking. For each crossing, strain gauge readings should be taken and checked against stress predictions. In the event of (1) significant discrepancies (10%) between the measured and calculated stresses, or (2) failure indications by the expected mode or, (3) failure indications by a different mechanism, the tests should be interrupted. These events indicate either
(1) inadequate or incorrect stress analysis, or

(2) incorrect estimates of the resistance to failure (strength), or

(3) omission of a significant failure mode in the design analysis considerations.

In this situation, appropriate corrections need to be made to the analysis which served as a basis for setting the risk rating and a new risk rating vehicle class must be set.

Satisfactory bridge performance for a sequence of instrumented tests as envisioned here will give assurance that the aspects of the design analysis on which the risk rating depends are not in error and that they are sufficiently accurate to enable a meaningful risk rating specification.
8. DETERMINATION OF CAUTION AND RISK RATINGS

This chapter presents recommended additions to the text of the Trilateral Design Code to enable determination of caution and risk ratings for mobile bridging. These include a section on procedures, and a section describing additions to the testing requirements in the code.

8.1 Determination of Caution and Risk Ratings

Mobile bridging is designed to support a specified vehicle weight class over a gap of specified maximum length. During field service it may be advantageous to use the bridging beyond its design specification. The field officer requires information on expected bridge performance for this purpose. The caution and risk crossing ratings provide guidelines on the load carrying capacity of the bridge under restricted operating conditions. The bridge design should include specification of the vehicle class and/or gap size permissible for caution and risk crossing conditions.

The caution and risk crossing conditions are specified relative to the normal crossing conditions. The crossing conditions follow.
<table>
<thead>
<tr>
<th></th>
<th>Normal</th>
<th>Caution</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>vehicle speed</td>
<td>25 mph</td>
<td>10 mph</td>
<td>2.5 mph</td>
</tr>
<tr>
<td></td>
<td>(40% of normal)</td>
<td>(10% of normal)</td>
<td></td>
</tr>
<tr>
<td>vehicle eccentricity</td>
<td>limited</td>
<td>limited</td>
<td>on bridge centerline</td>
</tr>
<tr>
<td></td>
<td>(40% of maximum)</td>
<td>(10% of maximum)</td>
<td></td>
</tr>
<tr>
<td>braking and</td>
<td>limited</td>
<td>limited</td>
<td>no braking or acceleration</td>
</tr>
<tr>
<td>acceleration</td>
<td>(40% of maximum)</td>
<td>(10% of maximum)</td>
<td></td>
</tr>
<tr>
<td>vehicle spacing</td>
<td>100 ft.</td>
<td>150 ft.</td>
<td>one vehicle on bridge at a time</td>
</tr>
</tbody>
</table>

The caution crossing rating is to be set such that a caution crossing will produce no greater risk of bridge failure than is calculated for normal crossings using this design code. The risk crossing criteria permits increased probability of failure.

To determine the probability of failure for a crossing, it is necessary to identify the possible failure modes and the likelihood that each will occur. The failure modes and locations are implicitly identified during the design analysis. Stress levels are calculated for these modes and compared to the corresponding strength quantities. Section 6 specifies the required safety margins for the failure modes.
The probability of occurrence for failure mode $j$ is the probability that the stress measure $S_j$ will exceed the strength (resistance) measure $R_j$.

The probability of occurrence of failure mode $j$ is

$$P_f = \frac{1}{2} \text{erfc} \left( \frac{\beta}{\sqrt{2}} \right) \text{ where } \beta = \frac{(\mu_R - \mu_S)}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$  \hspace{1cm} (8.1)

provided the distributions of stress and strength can be approximated as Gaussian.

The Table 8.1 gives numerical values for the failure probability in terms of

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>1</th>
<th>1.28</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_f$</td>
<td>0.158</td>
<td>0.1</td>
<td>2.3(10)^{-2}</td>
<td>1.4(10)^{-3}</td>
<td>3.2(10)^{-5}</td>
<td>3.(10)^{-7}</td>
</tr>
</tbody>
</table>

The symbols $\mu_R$, $\sigma_R$, $\mu_S$, $\sigma_S$ refer to the means and standard deviations of $R$ and $S$ respectively.

The stress $S_j$ is a function of the generalized loading parameters of the form (see Section 5.4).

$$S_j = f_j(D, V, G, M, X, I, B, W, F, T, Q)$$

For a known bridge weight, vehicle weight and gap size, the mean and standard deviation for $S_j$ are to be determined from the means and standard deviation of the variables $M, X, I, \ldots$.

In general the means and variance (standard deviation squared) of
the stress can be calculated in terms of the means and standard
deviations of the loadings; however, if the stress depends on the
loading variables in an additive manner, i.e.,

\[ S = C_0 + C_1 X + C_2 I + C_3 B \]

then the explicit expression for the variance is

\[ \mu_S = C_0 + C_1 \mu_X + C_2 \mu_I + C_3 \mu_B \]
\[ \sigma_S^2 = C_1^2 \sigma_X^2 + C_2^2 \sigma_I^2 + C_3^2 \sigma_B^2 \]
\[ + 2C_1C_2 \rho_{XI} \sigma_X \sigma_I \]
\[ + 2C_2C_3 \rho_{IB} \sigma_I \sigma_B \]
\[ + 2C_3C_1 \rho_{BX} \sigma_B \sigma_X \]

where \( \rho_{XI} \) is the correlation coefficient between the variables \( X \) and \( I \). Further, if the variables, e.g., \( X \) and \( I \), are independent, this correlation coefficient is zero, i.e., \( \rho_{XI} = 0 \).

The mean and standard deviation for the strength parameters are themselves material properties. Material strength measures are normally given as lower bound values from which the mean and standard deviation can be determined

\[ \mu_R = \bar{R} + K \sigma_R \]

For base metal, the nominal or lower bound values of yield or ultimate stress reported in Appendix A are the 1% values for U.S. grade
aluminum and steel alloys, i.e.,

The standard deviations of the yield stress for 7075 and 7005 alloys range between 7 and 17 N/mm² (1. and 2.5 ksi).

For the caution crossing, the probability of failure must not exceed that allowable during a normal crossing. To set the caution crossing rating, calculate the $\beta_j^N$ allowable for each failure mode $j$, limit the crossing conditions and determine the largest increased weight or gap size which yields $\beta_j^C \geq \beta_j^N$ for all $j$.

The parameter

$$\beta_j^N = \frac{\mu_{R_j} - \mu_{S_j}^N}{\sqrt{\sigma_{R_j}^2 + \sigma_{S_j}^2}}$$

where

$$S_j^N = f_j(D, V \text{ Design, G Design, M, X, I, B, W, F, T, Q})$$

and $M, X, I, \ldots$ are the random loading variables for a normal crossing. The calculation of the mean and standard deviation of the stress measure $S_j$ require a knowledge of the means and standard deviations of the loading variables $M, X, I, B, W, F, T,$ and $Q$. For convenience, these variables may be assumed to be Gaussian with mean equal to half the extreme value and standard deviation equal to the extreme value divided by $K$, e.g.,

$$\mu_M = \frac{M\text{EXT}}{2}$$

$$\sigma_M = \frac{M\text{EXT}}{2K} \quad (8.6)$$
For a caution crossing

\[
\beta_j^C = \left( \frac{\nu_{Rj} - \nu_{Sj}^C}{\sqrt{\sigma_{Rj}^2 + \sigma_{Sj}^2}} \right)
\]

where

\[
s_j^C = f, (D, V_c, G_c, M, x, \overline{x}, \overline{I}, \overline{B}, \overline{W}, \overline{F}, \overline{T}, \overline{Q})
\]

and \( \overline{M}, \overline{x}, \overline{I}, \ldots \) are the random loading variables for the caution (restricted) crossing conditions. The random variables \( \overline{x}, \overline{I}, \) and \( \overline{B} \) have extreme values \( \overline{x}_{\text{EXT}}, \overline{I}_{\text{EXT}} \) and \( \overline{B}_{\text{EXT}} \) reduced by 60% from the normal crossing condition to reflect caution crossing restrictions. The means and standard deviations follow as

\[
\mu_x = \frac{\overline{x}_{\text{EXT}}}{2}, \quad \sigma_x = \frac{\overline{x}_{\text{EXT}}}{2K}
\]

The quantities \( V_c \) and \( G_c \) are to be determined by the condition

\[
\beta_j^C \geq \beta_j^N \quad \text{for all } j
\]

For caution crossing, the bridging design documentation should include a table of the percent increase allowable in vehicle weight, \( V/V_{\text{DES}} \) for various gap sizes \( G/G_{\text{DES}} \) and of allowable gap sizes, \( G/G_{\text{DES}} \) for given vehicle weights, \( V/V_{\text{DES}} \) in the form

<table>
<thead>
<tr>
<th>( V/V_{\text{DES}} )</th>
<th>( G/G_{\text{DES}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>1.0</td>
</tr>
<tr>
<td>*</td>
<td>0.8</td>
</tr>
<tr>
<td>1.0</td>
<td>*</td>
</tr>
<tr>
<td>0.8</td>
<td>*</td>
</tr>
</tbody>
</table>

where the values \( * \) are to be determined by the designer.
To set the risk crossing rating, it is only necessary to consider those failure modes j which will prevent completion of the mission. One need not consider failure modes which will prevent retrieval and/or future use of the bridging. For each failure mode j which is considered, the risk crossing specification is required to meet two conditions:

1. \( \beta_j^R \geq (0.80) \) _j^N
2. \( p_{f_j}^R \leq (100) p_{f_j}^N \)

where \( p_{f_j}^R \) is the probability of failure by mode j for (R) risk crossing conditions. This constraint is to be used in conjunction with the equation.

\[ S_j^R = f_j(D, V_R, G_R, M, X, I, B, W, F, T, Q) \]

to determine the \( V_R \) and \( G_R \) ratings for the random loading variables corresponding to risk crossing conditions, \( X, I, \) and \( B \) with extreme values set at 10% of those for normal crossing conditions.
8.2 Recommended Changes to Bridge Testing Requirements in Support of Risk Crossing Ratings

Section 8.5.10 Overload Test - Preceding 8.5.10.1

The overload tests will be strain gauged so as to verify the stress analysis for the most likely failure modes during risk crossings. The designer will specify gauge locations and expected readings.

Section 8.6 Trafficking Tests

8.6.3 Risk rating tests will be performed to verify that the bridging can support the rated loading. The bridging will be strain gauged in order to verify the relationship between stress and vehicle load for the mode (or modes) of failure which are anticipated to be critical. Vehicles of increasing weight from the design class, to the caution class, and then to the risk class will cross the bridge applying the risk crossing conditions on speed, eccentricity, and braking. For each crossing strain gauge readings will be taken and checked against stress predictions. In the event of (1) significant discrepancies (10%) between the measured and calculated stresses, or (2) failure indications by the expected mode or (3) failure indications by a different mechanism, the tests will be interrupted. These events indicate either

(1) inadequate or incorrect stress analysis, or

(2) incorrect estimates of the resistance to failure (strength), or

(3) omission of a significant failure mode in the design analysis considerations.

In this situation, appropriate corrections need to be made to the analysis which served as a basis for setting the risk rating and a new risk rating vehicle class must be set.
REFERENCES


LIST OF SYMBOLS

a is the surface area of the vehicle exposed to wind multiplied by the wind speed/pressure conversion factor

B is the vehicle braking factor

C is the distance from the neutral axis to the bottom chord

D is the bridge weight per unit length

F is the footpath load

G is the gap length

h is the bridge width

I is the impact factor

i is the moment of inertia of the bridge section

K is the number of standard deviations between the mean and extreme values

M is the mud load per unit length

\( P_{XI} \) is the correlation coefficient between X and I

Q is the hydrodynamic load due to the current

q is the moment of the portion of the cross section about the neutral axis

\( R_j \) is the strength measure for the \( j \)th failure mode

\( S_j \) is the stress measure for the \( j \)th failure mode

\( t \) is the web thickness

T is the snow or ice load per unit length

V is the vehicle weight

W is the wind speed

x is the vehicle eccentricity

y is the vertical distance from the neutral axis of the bridge to the centroid of the vehicle
LIST OF SYMBOLS (cont.)

\( \beta \) is the reliability factor
\( \gamma \) is the safety factor
\( \mu_S \) is the mean of \( S \)
\( \sigma_S \) is the standard deviation of \( S \)
<table>
<thead>
<tr>
<th>No. of Copies</th>
<th>Address</th>
</tr>
</thead>
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