THE INFLUENCE OF THE TEMPERATURE COEFFICIENT OF REFRACTIVE INDEX

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The temperature coefficient of refractive index for germanium was determined in the 8-12 µm and 3-5 µm wavebands. The Strehl intensity ratio and frequency response of an optical system are related to a lens temperature change, and a simple factor for a comparison of different lens materials is included. The relative change in minimum resolvable temperature difference MRTD of a thermal imaging system is shown for a defocused optical system, where the defocus is related to a lens temperature change. It is shown that the criterion for a good optical performance may be applied to the relative change in MRTD for a thermal imager, and that the Maréchal approximation to the Strehl ratio may be used to determine a tolerance upon a change in lens temperature.
THE INFLUENCE OF THE TEMPERATURE COEFFICIENT OF REFRACTIVE INDEX ON A THERMAL IMAGER

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NOMENCLATURE

\( n \quad \text{Lens refractive index} \)
\( n' \quad \text{Image-space refractive index} \)
\( \sigma' \quad \text{Image-space ray curve} \)
\( \sigma_0' \quad \text{Gaussian focal plane} \)
\( \sigma' \quad \text{Out of focus image-plane} \)
\( \lambda \quad \text{Wavelength} \)
\( \Delta \quad \text{Focal shift} \)
\( \Omega'_{20} \quad \text{Longitudinal focal shift aberration coefficient} \)
\( E' \quad \text{Centre of optical exit-pupil} \)
\( \frac{\partial n}{\partial T} \quad \text{Temperature coefficient of refractive index} \)
\( (\frac{\partial n}{\partial T})_\lambda \quad \text{Temperature coefficient of refractive index at a particular wavelength} \)
\( df_n \quad \text{Focal shift due to a change in refractive index} \)
\( df_E \quad \text{Focal shift due to thermal expansivity} \)
\( df_L \quad \text{Focal shift following a change in lens thickness} \)
\( df_H \quad \text{Focal shift following a change in the lens housing} \)
\( t_o \quad \text{Change in the lens centre thickness} \)
\( t_o \quad \text{Lens centre thickness} \)
\( \Delta T \quad \text{Temperature change} \)
\( r \quad \text{Lens radius of curvature} \)
\( \alpha_L \quad \text{Linear expansivity of the lens} \)
\( L_o \quad \text{Length of lens holder} \)
\( \alpha_H \quad \text{Linear expansivity of the lens holder} \)
\( df \quad \text{Total focal shift with temperature} \)
\( \text{PSF} \quad \text{Point spread function} \)

\[ G(u',v') = |u_p|^2 \quad \text{Intensity of the point spread function} \]

\((x,y)\quad \text{Pupil coordinates}\)
\((u',v')\quad \text{Image-plane coordinates}\)
\(A\quad \text{Area of pupil}\)
\(W(x,y)\quad \text{Wavefront aberration in the pupil}\)
\(E\quad \text{Variance of } W(x,y) \text{ over the pupil}\)
\(f(x,y)\quad \text{Pupil function}\)
\(I\quad \text{Strehl intensity ratio}\)
1 INTRODUCTION

Germanium is an important and frequently used material for infra-red optical components (RSRE Physics Newsletter August 1976). A disadvantage however is the large temperature coefficient of refractive index (H W Icenogh et al 1976). The consequence of this applied to a lens is a change in power and hence in focal length. For a uniform change in lens temperature the position of best focus is displaced along the optical axis; this is referred to as longitudinal focal shift.

Throughout this paper the conventional representation for wavefront aberration is used, where the indices of the coefficients describe the aperture and the field dependence of a given aberration. For example in the case of defocus \( O_{20}^W \), the wavefront aberration varies only as the square of the aperture.

In some situations the wavefront aberration \( O_{20}^W \) is introduced to in part cancel the effect of spherical aberration to produce an image of a higher quality. For a random introduction of \( O_{20}^W \) the wavefront aberration of a diffraction limited optical system is increased. This gives a reduced contrast in the modulation transfer function (MTF) and a loss in image quality. It is the aim of this paper to show how (MTF) alters with a change in lens
temperature and further to show how this affects a thermal imagers performance.

An experiment was performed to measure the shift of focus $\Delta$ with temperature of a thermal radiometer lens in the 3-5 $\mu$m and in the 8-12 $\mu$m waveband to establish the temperature coefficient of refractive index ($\beta n/\beta T$)$\lambda$ for germanium.

2 THEORY

2.1 RELATIONSHIP BETWEEN LONGITUDINAL FOCAL SHIFT AND WAVEFRONT ABERRATION

Figure 1 shows any general optical system, a wavefront is drawn tangential to a reference sphere at $E'$ in the exit pupil. The coefficient $W_{20}$ measures the wavefront aberration between an emergent wavefront and a reference sphere centred upon the axial point $O'$, of the out of focus image-plane. Following (Hopkins 1955)$^3$, $\Delta = O' - O'$, where $O'$ is the true focal plane, and the coefficient of defocus is

$$0W_{20} = \frac{1}{2} n' \sin^2 \alpha' \Delta$$  \hspace{1cm} (2.1)

2.1.1 Contributions to the longitudinal focal shift

Two basic contributions that occur independently are identified.

First a change in the optical properties, this is manifested by a change in refractive index with temperature and yields the coefficient $(\beta n/\beta T)\lambda$. This occurs as a result of a shift of an absorption edge in the material, which for an increase in temperature is towards a longer wavelength.

The second contribution is a result of a thermal expansion in the lens and associated housing. Combination of the two contributions yields

$$D = (df_n + df_e)$$  \hspace{1cm} (2.2)

The case of a germanium lens in an aluminium holder is considered in Appendix A and yields the result

$$|\Delta| \sim |df_n|$$
2.2 THE STREHL RATIO, ITS USE AS A MERIT FUNCTION

Lord Rayleigh (1879) was the first to show that in an optical system a dependence existed between wavefront aberration in the exit pupil and the intensity at the Gaussian focus. The resulting tolerance criterion, which was to become used extensively is known as Rayleigh's quarter wavelength (λ/4) rule and describes the maximum amount of wavefront aberration in the exit-pupil before a significant loss in image quality is observed. However, since the distribution of light in an image is dependent upon the shape of the wavefront aberration, in addition to its maximum deformation the λ/4 rule only serves as a rough guide as to the desirable state of correction of a system. Considering this Strehl in (1902) proposed a single-number measure of image quality for use with well corrected systems. This measure, the Strehl ratio (I) is defined as follows

\[
I = \frac{\text{Intensity at the principle maximum of the point spread function (PSF) at the best diffraction focus in the presence of aberration}}{\text{Intensity of the principle maximum of the PSF at the best diffraction focus in the absence of aberration}}
\]

\[
I = \frac{G(0,0)}{G_0(0,0)} \quad (2.3)
\]

Following Hopkins (1955) the intensity at a point P in the Gaussian image-plane is given by

\[
|\bar{u}_p|^2 = G(u',v')
\]

\[
= \frac{1}{A} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \exp i2\pi(u'x + v'y) \, dx \, dy \quad (2.4)
\]

and is known as the point spread function. Where \( f(x,y) \) describes the wavefront aberration in the exit pupil and is known as the pupil function and may be written

\[
f(x,y) = \exp ikW(x,y) \quad (2.5)
\]

In the case of an unaberrated system \( W(x,y) = 0, f(x,y) = 1 \) and (2.4) yields the familiar Airy disc intensity distribution. As aberrations are introduced, \( G(u',v') \) departs from the Airy pattern, the central peak is decreased.
and energy is transferred into the side lobes, thus the Strehl ratio is reduced.

Using the definition of I equation (2.3) and with the substitution of (2.4) and (2.5) I may be written

\[
I = \left| \frac{1}{A} \int \int_A \exp(ikW(x,y)) \, dx \, dy \right|^2
\]  

The Strehl ratio (2.6) is quicker to evaluate than a full PSF (2.4), however numerical methods must still be used except in certain simple cases.

2.2.1 Maréchal's approximation

Maréchal (1947) showed that (2.6) could be approximated by

\[
I_{\text{approx}} = \left\{ 1 - \frac{2\sigma^2}{\lambda^2} E \right\}^2
\]  

where \( E \) describes the variance of the wave aberration \( W(x,y) \), and may be written

\[
E = \frac{1}{A} \int \int_A W^2(x,y) \, dx \, dy - \left\{ \frac{1}{A} \int \int_A W(x,y) \, dx \, dy \right\}^2
\]  

The approximation (2.7) is valid for \( I \geq 0.8 \), and for such well corrected systems is a very quick method of evaluating the Strehl ratio. An agreement between \( I \) and \( I_{\text{approx}} \) of within 4% still exists for \( I = 0.6 \) in the case of longitudinal focal shift. Maréchal proposed a criterion for a well corrected system, namely that the normalized intensity at the diffraction focus shall be greater than or equal to 0.8. From (2.7) the following condition is obtained

\[
E \leq \frac{\lambda^2}{180}
\]  

This study is concerned with defocus \( \omega_{20} \), therefore substituting \( W = \omega_{20}(x^2 + y^2) \) into (2.8) gives
If Maréchal's criterion is applied to (2.10) the following result is obtained

\[ W_{20} \leq \frac{\lambda}{4} \]

Hence, for the case of defocus Maréchal's approximation is in agreement with Rayleigh's criterion.

2.2.2 Relationship between Maréchal's approximation and focal shift due to a lens temperature change

An expression for \( I_{\text{approx}} \) to include longitudinal focal shift is obtained by substitution of (2.1) and (2.10) into (2.7) which yields

\[ I_{\text{approx}} = \left[ 1 - \frac{n'^2 \pi^2}{24} \frac{\sin^4 \alpha'}{\lambda^2} \Delta^2 \right]^2 \]

(2.11)

This equation enables an easy evaluation of the state of correctness of an optical system as a function of focal shift \( \Delta \). In the situation studied here \( \Delta \) is considered as a function of temperature. For the case of a thin lens in air, substitution of \( A(iii) \) into (2.11) yields

\[ I_{\text{approx}} = \left[ 1 - \frac{1}{2\pi} \left\{ \pi \frac{\sin^2 (\alpha')}{\lambda(n - 1)} + \frac{\partial n}{\partial T} \Delta T \right\} \right]^2 \]

(2.12)

2.3 OPTICAL TRANSFER FUNCTION

A more complete way to describe the performance of an optical system is by the optical transfer function (OTF). This complex function expresses both the contrast reduction \( C < 1.0 \) and phase shift \( \Theta \) as a function of frequency components \( (s,t) \), that comprise an object. The OTF may be written
\[ D(s,t) = C(s,t) e^{i0(s,t)} \]
\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(u',v') \exp \left[ -i2\pi(u's + v't) \right] du'dv' =
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(u',v') du'dv'
\]

(2.12)

after Hopkins\(^3\) (1955), where \(G(u',v')\) is the PSF of (2.4). The modulus of \(D(s,t)\) is known as the MTF and is frequently used to describe the quality of an optical system. This paper shows how the MTF is altered with \(O'W_{20}\), where \(O'W_{20}\) is related to a lens temperature change.

2.4 RELATIONSHIP BETWEEN WAVEFRONT ABERRATION AND LENS TEMPERATURE CHANGE

A thin spherical lens in air is considered here which will give an overestimation of \(O'W_{20}\) by

\[
\left( \frac{n-1}{n} \right)^2 \frac{\tan}{(r_2 - r_1)} \times 100\%
\]

(2.14)

In this study a lens is considered as thin if (2.14) yields a value of \(\leq 5\%\). The accuracy is then comparable with that of (2.11) in the lower limit of I. An expression for \(O'W_{20}\) in fractions of a wavelength is obtained from equation A(viii) which yields

\[
O'W_{20} = \frac{\sin^2 (\alpha')}{2\lambda} \frac{f}{(n - 1)} \frac{3n}{\partial T} \Delta T
\]

(2.15)

2.4.1 The effect of lens material upon focal shift

A comparison of the thermal sensitivity to defocus of materials suitable for use in optics for thermal imagers (RSRE Applied Physics Newsletter August 1976) is made using (2.15). For a unit lens temperature change, a given wavelength and numerical aperture (2.15) yields

\[
O'W_{20} = \frac{3n}{\partial T} \frac{1}{(n - 1)}
\]

(2.16)
From (2.16) the quantity $\gamma$ is defined where

$$\gamma = \frac{(n - 1)}{(\delta n/\delta T)} K^0$$

(2.17)

an increase in $\gamma$ between lens materials yields a reduced focal shift. Table A contains a list of 8-14 $\mu$m transmitting materials with their corresponding $\gamma$-number, where the sign change indicates an opposing focal shift. The $\gamma$ value of a material will vary with wavelength due to the change in dielectric susceptibility with frequency (Shaskanka S Mitra and Yet-ful Tsay 1972).5 With the exception of the chalcogenide glass (TI 1173) the materials listed in Table A may be divided into two types. First semiconductors which exhibit a small change in $\gamma$ with wavelength, and secondly ionic crystals which show a larger variation. The change in $\gamma$ with wavelength between 8.5 $\mu$m and 11.5 $\mu$m is also shown in Table A where data was available.

Table A

<table>
<thead>
<tr>
<th>Material</th>
<th>$\gamma$ at 10.6 $\mu$m $\times 10^{-3}$</th>
<th>$(\gamma$ 8.5 $\mu$m $-$ $\gamma$ 11.5 $\mu$m) $\times 10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge</td>
<td>7.50</td>
<td>0.006</td>
</tr>
<tr>
<td>Si</td>
<td>15.73</td>
<td>0.004</td>
</tr>
<tr>
<td>CdTe</td>
<td>17.99</td>
<td>0.161</td>
</tr>
<tr>
<td>ZnS</td>
<td>27.74</td>
<td>0.919</td>
</tr>
<tr>
<td>ZnSe</td>
<td>29.23</td>
<td>0.379</td>
</tr>
<tr>
<td>TI 1173</td>
<td>57.09</td>
<td>0.411</td>
</tr>
<tr>
<td>KRS5</td>
<td>-5.83</td>
<td>NA ionic crystal</td>
</tr>
<tr>
<td>KBr</td>
<td>-13.13</td>
<td>NA ionic crystal</td>
</tr>
<tr>
<td>KCl</td>
<td>-15.25</td>
<td>2.11</td>
</tr>
<tr>
<td>NaCl</td>
<td>-19.64</td>
<td>10.73</td>
</tr>
</tbody>
</table>

Changes in $\omega_{20}$ with temperature may be minimised by using a combination of materials, optics so corrected are referred to as athermalised. The values of $\gamma$ for germanium determined by experiment and reported in this paper are mean values in the stated waveband.
EXPERIMENT

3.1 MEASUREMENT OF FOCAL SHIFT AS A FUNCTION OF LENS TEMPERATURE

The experimental system is shown schematically in figure 3.

The optic used was a very well corrected antireflection coated single element polycrystalline germanium meniscus lens with a clear aperture of 180 mm and a numerical aperture of 0.243. Temperature was measured using platinum resistance probes. The incident energy from a modulated thermal source at infinity was focused onto a cooled photoconductive CdHgTe detector element of linear dimensions 115 x 125 μm. The signal from the detector was amplified and displayed on an oscilloscope. The detector position was obtained by using a stable voltage reference, a precision linear potentiometer and a digital voltmeter, where the minimum detectable movement of 50 μm corresponded to 2 mV. The detector position for a maximum signal was determined as a function of lens temperature in the 8-12 μm and in the 3-5 μm waveband.

RESULTS

4.1 FOCAL SHIFT AS A FUNCTION OF TEMPERATURE IN THE 8-12 μm AND IN THE 3-5 μm WAVEBAND

The detector position for a maximum signal as a function of lens temperature is shown in figure 4 and figure 5 for the 8-12 μm and for the 3-5 μm wavebands respectively. Calculation of the gradient using a least squares best fit straight line yields

\[
\frac{\partial f}{\partial T} (8-12 \text{ μm}) = -50.95 \text{ μm/K}
\]

and

\[
\frac{\partial f}{\partial T} (3-5 \text{ μm}) = -60.45 \text{ μm/K}
\]
4.1.1 Determination of the temperature coefficient of refractive index and $\gamma$ for germanium

By substituting for $\Delta f/\Delta T$ and the lens data (Appendix A) into $A(vi)$ and (2.17) the following results are obtained

(i) For the 8-12 $\mu$m waveband

$$\frac{\partial n}{\partial T} = (4.3 \pm 0.3) \times 10^{-4} \text{ K}^{-1}$$

and

$$\gamma = (7.0 \pm 0.5) \times 10^{3} \text{ K}^{0}$$

(ii) For the 3-5 $\mu$m waveband

$$\frac{\partial n}{\partial T} = (5.2 \pm 0.1) \times 10^{-4} \text{ K}^{-1}$$

and

$$\gamma = (5.8 \pm 0.1) \times 10^{3} \text{ K}^{0}$$

4.1.2 Strehl ratio vs. lens temperature for various optics

$I_{\text{approx}}$ as a function of lens temperature change is evaluated using the results for $\Delta$ and equation (2.11). Figure 6 shows a plot of $I_{\text{approx}}$ against lens temperature for positive $\Delta T$ only since there is symmetry about $\Delta T = 0$. The results are extended to various optical arrangements using (2.12).

Figure 7 shows the effect of a change in numerical aperture for a given lens diameter (100 mm), and figure 8 the effect of a change in lens diameter for a given numerical aperture (0.2425). The results here show that any increase in relative aperture and/or in lens diameter gives a reduced lens temperature change for a $\lambda/4$ defocus.

4.1.3 Modulation transfer function as a function of lens temperature changes for various optics

Using the definition (2.13) the MTF following an introduction of wavefront aberration is determined. Figure 9 shows MTF against normalised spatial frequency ($\sigma$) for various amounts of defocus expressed as fractions
of a wavelength. A conversion to real spatial frequencies is made using

$$N = \frac{\sigma(NA)}{\lambda} \text{ line pairs/mm} \quad (4.1)$$

In this paper a lens temperature change $\Delta T$ is related to MTF by evaluating the temperature change that will introduce a wavefront aberration $0 W_{20} = \lambda/4$.

Figure 10(a) shows values of $\Delta T$ against the diameter of a germanium lens for various numerical apertures. The lens temperature change required for different values of $0 W_{20}$ is given by the fractional change in $0 W_{20}$, since from (2.15) $0 W_{20} \propto \Delta T$. The effect of different lens materials upon $\Delta T$ is evaluated using the $\gamma$ values of Table A; since $\Delta T \propto \gamma$, $\Delta T$ following a change of material scales with the fractional change in $\gamma$.

Figure 10(b) shows $\Delta T$ for different lens materials against lens diameter for a numerical aperture $= 0.2425$ and $0 W_{20} = \lambda/4$.

The sensitivity of MTF to a change in lens temperature is seen by using figure 9 in conjunction with, figure 10(a) from the aspect of optical configuration, and with figure 10(b) for the effect of lens material.

4.1.4 Effect upon a thermal imager following a change in the optical MTF as a result of a longitudinal focal shift

Figure 9 shows the effect upon MTF of an increase in wavefront aberration $0 W_{20}$; this has been related graphically to a change in lens temperature for various lenses and materials. To show how such changes will affect a thermal imager the relative increase in the minimum resolvable temperature difference MRTD as a function of $\sigma$ is considered. Following Rosell

$$\text{MRTD} \propto (\text{MTF})^{-1} \quad (4.2)$$

at any given spatial frequency. If (4.2) is applied to the change in the optical MTF following an introduction of $0 W_{20}$ figure 11 is obtained, where the change in MRTD relative to the case of diffraction limited optics is shown for $0 W_{20} = \lambda/4, \lambda/2, 3\lambda/4$ and $\lambda$ as a function of $\sigma$. In the case of $0 W_{20} = 3\lambda/4$ and $\lambda$ the asymptotes indicate frequencies where a contrast reversal occurs.
To relate figure 11 to a thermal imager it is necessary to determine the range of spatial frequencies that an imager can resolve. A consideration of frequencies up to the first minimum of the detector frequency response is made, this gives

$$\sigma_{\text{Detector cut off}} = \frac{\lambda}{(NA)d}$$  \hspace{1cm} (4.3)

The largest $\sigma$ resolved by a thermal imager for a fixed waveband is given by the smallest $(NA)d$ product. Here considering the smallest numerical aperture and detector width to be 0.164 and 50 µm respectively, (4.3) yields for $\lambda = 10$ µm

$$\sigma_{\text{Detector cut off}} = 1.22$$

A study of figure 11 over the range of spatial frequencies $0 < \sigma < 1.22$ shows that it is important that $O_{20} \leq \lambda/4$ to maintain optimum thermal resolution. The severe degradation that occurs following an increase in $O_{20}$ from $\lambda/4$ to $\lambda/2$ is considered here at a frequency equal to half that given by (4.2), this frequency may be referred to as half detector cut-off. From figure 11 for $O_{20} = \lambda/4$ a 27% increase in MRTD is found compared to a 190% increase for $O_{20} = \lambda/2$. Generally for thermal imaging systems $\sigma_{\text{Detector cut off}} < 1.22$ and therefore there is a significant reduction in the 27% increase in MRTD at half detector cut-off.

5 CONCLUSIONS

The temperature coefficient of refractive index is evaluated for a germanium lens from measurements of focal shift with lens temperature, the results agree well with other published data (H W Icenogle et al 1976)\(^2\). Using $\Delta n/\Delta T$ the $\gamma$-value for germanium has been evaluated and compared with values for other materials.

The change in MTF following a defocus that occurs due to a lens temperature change is related to the relative change in the MRTD of a thermal imaging system. The results here show that the optical criterion for a system to perform near to the diffraction limit may be applied to a thermal imager with respect to the relative change in MRTD, that is an optimum performance will occur for $O_{20} \leq \lambda/4$. Thus the Strehl ratio and the Maréchal approximation may be applied to a thermal imaging system to infer the MRTD performance.
There is no general tolerance upon a change in lens temperature, however the following can be considered as a guide to the lower limit. For a 300 mm diameter 0.2425 numerical aperture germanium lens a 1 K lens temperature change will introduce $0.20 \times \lambda/4$ for 10 $\mu$m radiation.

Finally to conclude, it is necessary to consider each optical system separately, the formulae contained herein enable this to be done simply for different optical configurations and materials. In a general optical system where there are many elements the total wavefront aberration is given by the sum of the contributions from each individual element. Further any analysis should first be applied to the largest diameter elements, having the smallest $\gamma$, since any defocus effects will be greatest there.

In this paper I have considered defocus resulting from a lens temperature change in the 3-5 $\mu$m and in the 8-12 $\mu$m waveband. Dispersion may also give a significant reduction in the optical performance of a thermal imaging system, particularly in the 3-5 $\mu$m waveband. However, for a stated waveband this will represent a fixed quantity, with any thermal effects adding to the wavefront aberration.

In the case of a thermal imaging system where a focus adjustment is included any temperature effect to the optics can generally be removed. However it is desirable to keep operator adjustments to a minimum, where focusing is limited to a near/far range control.

The $\gamma$ values of Table A may be used to design optical systems where the magnitude of a lens temperature change necessary to produce significant defocus is increased substantially; the temperature change required to introduce $0.20 \times \lambda/4$ is obtained from (2.16) and (2.14) which yield $|\Delta T| = \lambda \gamma/2f(NA)^2$.

For the case of a thermal radiometer where no operator feedback can achieve a best focus it is essential to correct the effect of a lens temperature change if an optimum operational performance is to be maintained.

ACKNOWLEDGEMENTS

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REFERENCES

1 RSRE Applied Physics Newsletter, August 1976.


APPENDIX A

The case of a spherical germanium lens in air and mounted in an aluminium holder, figure 2 is considered here. Analysis is for a thick lens, but where appropriate the thin lens case is considered. The focal length of a thick lens is given by

\[ f = \left[ (n - 1) \left( \frac{1}{\frac{1}{r_1} - \frac{1}{r_2} + t_o \left( \frac{n - 1}{nr_1 r_2} \right) \right} \right]^{-1} \] \hspace{1cm} A(i)

A.1 FOCAL SHIFT WITH TEMPERATURE VIA REFRACTIVE INDEX

Differentiating A(i) and rearranging gives

\[ df_n = - \frac{f^2}{r_1 r_2} \left( \frac{r_2 - r_1 + t_o \left( 1 - \frac{1}{n^2} \right) \right)} dn \] \hspace{1cm} A(ii)

where

\[ dn = \frac{\alpha n \Delta T}{\beta T} \]

For the case of a thin lens A(ii) reduces to

\[ df_n = - \frac{f}{(n - 1)} \frac{3n \Delta T}{\beta T} \] \hspace{1cm} A(iii)

A.1.2 FOCAL SHIFT WITH TEMPERATURE VIA EXPANSION OF THE LENS

Differentiating A(i) and rearranging gives

\[ df_L = - \frac{f^2}{r_1 r_2} \left( \frac{n - 1}{n} \right)^2 dt_o \] \hspace{1cm} A(iv)

where

\[ dt_o = t_o a_L \Delta T \]

A.1
A.1.3 FOCAL SHIFT WITH TEMPERATURE VIA EXPANSION OF THE LENS HOUSING

Here
\[ df_H = L_o \alpha_H \Delta T \] \hspace{1cm} A(v)

A.1.4 TOTAL FOCAL SHIFT WITH TEMPERATURE

Using the relationship (2.2), and \( df_e = df_L + df_H \), substituting for A(ii), A(iv) and A(v) yields

\[ df = -\frac{f^2}{r_1 r_2} \left\{ \left[ r_2 - r_1 + t_o \left( 1 - \frac{1}{n^2} \right) \right] \frac{3n}{3T} + \frac{t_o \alpha_L (n-1)^2}{n} \right\} \Delta T + \]
\[ + \frac{L_o \alpha_H}{\Delta T} \] \hspace{1cm} A(vi)

A.1.5 TOTAL CONTRIBUTION TO THE WAVEFRONT ABERRATION COEFFICIENT \( W_{20} \) VIA TEMPERATURE EFFECTS

Here A(vi) is combined with (2.1), which for a lens in air yields

\[ W_{20} = -\frac{\sin^2(a')}{2} \frac{f^2}{r_1 r_2} \left\{ \left[ r_2 - r_1 + t_o \left( 1 - \frac{1}{n^2} \right) \right] \frac{3n}{3T} + \right\}
\[ + \frac{t_o \alpha_L (n-1)^2}{n} \Delta T + \frac{\sin^2(a')}{2} L_o \alpha_H \Delta T \] \hspace{1cm} A(vii)

For the case of a thin lens A(vii) reduces to

\[ W_{20} = -\frac{\sin^2(a')}{2} \frac{f}{(n-1)} \frac{3n}{3T} \Delta T \] \hspace{1cm} A(viii)

A.1.6 EVALUATION OF THE RELATIVE CONTRIBUTION TO FOCAL SHIFT

The following lens and holder is considered

\[ n = 4.0174 \quad \alpha_H = 23 \times 10^{-6} \text{K}^{-1} \]
\( \lambda = 4.7 \text{\mu m} \)

\[ n = 4.0022 \quad \alpha_L = 6.1 \times 10^{-6} \text{K}^{-1} \]
\( \lambda = 10.6 \text{\mu m} \)

\[ r_1 = 334.2 \text{mm} \quad L_o = 40 \text{mm} \]
Substituting into A(ii), A(iv) and A(v) gives for a unit temperature change

\[ |df_n| = 47.53 \, \mu m \]

\[ |df_L| = 0.14 \, \mu m \]

\[ |df_H| = 0.92 \, \mu m \]

The result is obtained that for a germanium lens in an aluminium holder

\[ df \sim df_n \]

provided that \( f \gg l_o \).
FIG. 1. LONGITUDINAL FOCAL SHIFT

FIG. 2. SPHERICAL LENS IN AIR
FIG. 3. FOCAL SHIFT EXPERIMENT
FIG. 4. DETECTOR POSITION AGAINST LENS TEMPERATURE FOR THE 8.12 µm WAVEBAND
FIG. 5. DETECTOR POSITION AGAINST LENS TEMPERATURE FOR THE 3.5 μm WAVEBAND
FIG. 6. STREHL RATIO (I APPROX) AGAINST LENS TEMPERATURE CHANGE FOR THE 
8-12 μm AND FOR THE 3-5 μm WAVEBANDS
FIG. 7: STREHL RATIO (1 APPROX) AGAINST LENs TEMPERATURE CHANGE FOR A 100 mm DIAMETER OPTIC AND VARIOUS NUMERICAL APERTURES.
FIG. 8. STREHL RATIO (\(I_{\text{APPROX}}\)) AGAINST LENS TEMPERATURE CHANGE FOR A 0.2425 NUMERICAL APERTURE OPTIC AND VARIOUS LENS DIAMETERS

- **1** LENS DIAMETER = 250 mm
- **2** LENS DIAMETER = 200 mm
- **3** LENS DIAMETER = 150 mm
- **4** LENS DIAMETER = 100 mm
- **5** LENS DIAMETER = 50 mm
FIG. 9. THE FREQUENCY RESPONSE OF A DEFOCUSED OPTICAL SYSTEM
FIG. 9. THE FREQUENCY RESPONSE OF A DEFOCUSED OPTICAL SYSTEM

1. $\Delta \omega = 0$
2. $\Delta \omega = \lambda / 4$
3. $\Delta \omega = \lambda / 2$
4. $\Delta \omega = 3 \lambda / 4$
5. $\Delta \omega = \lambda$

NORMALISED SPATIAL FREQUENCY

CONTRAST

0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0

0 0.2 0.4 0.6 0.8 1.0

0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0

0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
FIG. 10(a). TEMPERATURE CHANGE REQUIRED TO INTRODUCE $\lambda/4$ ($\lambda = 10.6 \mu m$) OF $oW20$ FOR A GERMANIUM LENS
THE RESULTS ARE FOR A LENS OF NA 0.2425. THE DASHED LINES INDICATE A MATERIAL WHERE THE FOCAL LENGTH INCREASES WITH INCREASING TEMPERATURE.

FIG. 10(b). TEMPERATURE CHANGE REQUIRED TO INTRODUCE $\lambda/4$ ($\lambda = 10.6 \mu m$) OF $0\pi$ FOR LENSES MADE OF DIFFERENT MATERIALS.
FIG. 11. RELATIVE CHANGE IN MRTD OF A THERMAL IMAGER FOR A
DEFOCUSSED OPTICAL SYSTEM