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Volume IV

DURABILITY METHODS DEVELOPMENT

VOLUME IV
INITIAL QUALITY REPRESENTATION

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MODERN ANALYSIS INC.
RIDGEWOOD, NEW JERSEY

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This technical report has been reviewed and is approved for publication.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The initial fatigue quality of the durability critical parts of an aircraft structure is one of the key input parameters for the durability analysis to be performed in the present investigation. In the present investigation, we consider that either the EIFS (Equivalent Initial Flaw Size) or TTCI (Time-to-Crack-Initiation) represents such an initial quality that controls the durability of (cont.)

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) aircraft. A statistical characterization of the EIFS data is attempted by fitting various probability distribution functions thereto. The EIFS data considered here consist of three samples of size 37, 37 and 38. These data sets are examined to see if they fit (a) the distributions of the Johnson family including the log-normal distribution, (b) the Weibull distribution, (c) the distributions of the Pearson family, (d) the asymptotic distributions of largest values and (3) the TTCI compatible distributions. It is found that, with respect to the data examined, the Weibull compatible, log-normal compatible, and second asymptotic distributions provide the best fit.

FOREWORD

This program is conducted by General Dynamics, Fort Worth Division with George Washington University (Dr. J.N. Yang) and Modern Analysis Inc. (Dr. M. Shinozuka) as associate investigators. This program is being conducted in three phases with a total duration of fifty (50) months.

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This report (Volume IV) presents the accomplishments of the task entitled "Initial Quality Representation" of this program. Four other volumes are written to describe the summary of and the progress made in Phase I. They are:

Volume I - Phase I Summary

Volume II - Durability Analysis: State-of-the-Art Assessment

Volume III - Structural Durability Survey: State-of-the-Art Assessment

Volume V - Durability Analysis Methodology Development

This report is published only for the exchange and stimulation of ideas. As such, the views expressed herein are not necessarily those of the United States Air Force or Air Force Flight Dynamics Laboratory.

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SECTION I

INTRODUCTION

The initial fatigue quality of the durability critical parts of an aircraft structure is one of the key input parameters for the durability analysis to be performed in the present investigation. Indeed, in the present investigation, we consider that either the EIFS (Equivalent Initial Flaw Size) or TTCI (Time To Crack Initiaion) represents such an initial quality that controls the durability of aircraft. A statistical characterization of the EIFS data is attempted by fitting various probability distribution functions thereto. The EIFS data considered here are from the "Fastener Hole Quality" program [1] and are listed in Table 1. These data consist of data sets XQPF, XWPF and WPF of size 37, 37 and 38, respectively.

To be more specific, these data sets are examined to see if they fit (a) the distributions of the Johnson family including the long-normal distribution, (b) the Weibull distribution, (c) the distributions of the Pearson family, (d) the asymptotic distributions of largest values and (e) the TTCI compatible distributions. It is found that, with respect to the data examined, the Weibull compatible, log-normal compatible, and second asymptotic distributions provide the best fit.

Table 1 Ordered Observations, Their Plotting Positions, and Some Sample Statistics (EIFS in mils)

TEST SERIES = XQPF

NO.	EIFS	CUM.PROB.
1	.026	.0263
2	.031	.0526
3	.047	.0789
4	.050	.1053
5	.058	.1316
6	.059	.1579
7	.060	.1842
8	.063	.2105
9	.063	.2368
10	.068	.2632
11	.068	.2895
12	.090	.3158
13	.096	.3421
14	.105	.3684
15	.110	.3947
16	.113	.4211
17	.120	.4474
18	.130	.4737
19	.150	.5000
20	.170	.5263
21	.200	.5526
22	.220	.5789
23	.270	.6053
24	.300	.6316
25	.536	.6579
26	.536	.6842
27	.611	.7105
28	.612	.7368
29	.650	.7632
30	1.090	.7895
31	1.090	.8158
32	1.090	.8421
33	1.100	.8684
34	1.140	.8947
35	1.240	.9211
36	3.000	.9474
37	7.700	.9737

MEAN= .6233
AM2= 1.7214
AM3= 9.8704
AM4=68.7061
STDV= 1.3120
SQRT(B1)= 4.3701
B2=23.1853
B1=19.0982

TEST SERIES = XWPF

NO.	EIFS	CUM.PROB.
1	.093	.0263
2	.145	.0526
3	.145	.0789
4	.160	.1053
5	.175	.1316
6	.180	.1579
7	.180	.1842
8	.190	.2105
9	.190	.2368
10	.210	.2632
11	.210	.2895
12	.240	.3158
13	.240	.3421
14	.240	.3684
15	.240	.3947
16	.250	.4211
17	.290	.4474
18	.295	.4737
19	.295	.5000
20	.295	.5263
21	.295	.5526
22	.330	.5789
23	.330	.6053
24	.330	.6316
25	.420	.6579
26	.420	.6842
27	.420	.7105
28	.470	.7368
29	.470	.7632
30	.540	.7895
31	.612	.8158
32	.700	.8421
33	.810	.8684
34	.810	.8947
35	.870	.9211
36	.940	.9474
37	1.280	.9737

MEAN= .3868
AM2= .0694
AM3= .0285
AM4= .0241
STDV= .2634
SQRT(B1)= 1.5598
B2= 5.0060
B1= 2.4331

TEST SERIES = WPF

NO.	EIFS	CUM.PROB.
1	.140	.0256
2	.236	.0513
3	.280	.0769
4	.280	.1026
5	.320	.1282
6	.367	.1538
7	.367	.1795
8	.420	.2051
9	.450	.2308
10	.450	.2564
11	.450	.2821
12	.482	.3077
13	.518	.3333
14	.520	.3590
15	.520	.3846
16	.560	.4103
17	.560	.4359
18	.600	.4615
19	.647	.4872
20	.647	.5128
21	.650	.5385
22	.698	.5641
23	.698	.5897
24	.698	.6154
25	.698	.6410
26	.698	.6667
27	.698	.6923
28	.754	.7179
29	.754	.7436
30	.817	.7692
31	1.040	.7949
32	1.140	.8205
33	1.250	.8462
34	1.250	.8718
35	1.490	.8974
36	1.640	.9231
37	2.730	.9487
38	3.830	.9744

MEAN= .7986
AM2= .4660
AM3= .9179
AM4= 2.6276
STDV= .6826
SQRT(B1)= 2.8860
B2=12.1022
B1= 8.3292

SECTION II

STATISTICAL ANALYSIS PROCEDURES

2.1 Standard Measure of Skewness $\sqrt{\beta_1}$ and Standardized Measure of Peakedness (Kurtosis) β_2

With respect to a random variable X, write μ_i' for the i-th moment about the origin and μ_i for the i-th moment about the mean;

$$\mu_i' = E\{X^i\}, \qquad \mu_i = E\{(X - E[X])^i\} \tag{1}$$

The standardized measures of skewness $\sqrt{\beta_1}$ and of peakedness β_2 are then given by

$$\sqrt{\beta_1} = \mu_3/(\sqrt{\mu_2})^3 \qquad \beta_2 = \mu_4/(\mu_2)^2 \tag{2}$$

The estimations of the moments can be made on the basis of the observed data consisting of a sample size n; $x_1, x_2, \ldots, x_n$. In fact, introducing the sample moments m_i' and m_i as

$$m_i' = (\sum_{k=1}^{n} x_k^i)/n \qquad m_i = \sum_{k=1}^{n} (x_k - m_i')^i/n \tag{3}$$

the population moments μ_i' and μ_i may be estimated by their corresponding sample moments m_i' and m_i and also skewness $\sqrt{\beta_1}$ and peakedness β_2 respectively by $\sqrt{b_1}$ and b_2:

$$\sqrt{b_1} = m_3/(\sqrt{m_2})^3 \qquad b_2 = m_4/(m_2)^2 \tag{4}$$

2.2 Use of the β_1-β_2 Plane for the Selection of Distribution Functions

A large number of distribution functions may be considered for the current investigation to characterize the EIFS data. An engineering approach has been used here to select from these distribution functions only those distributions that have the prospect of passing further tests of goodness-of-fit, while eliminating those that are obviously incompatible with the given EIFS data. The approach is to use the β_1-β_2 plane as shown in Fig. 1 [2] where each of these distributions may be identified either as a point, curve, or region. Although Fig. 1 indicates the relationships between β_1 and β_2 of some of the better known distribution functions, they have been established analytically and do not necessarily indicate the relationships between the estimates b_1 and b_2 of β_1 and β_2. Nevertheless, Fig. 1 and similar figures have been used, whenever appropriate, to single out those distribution functions which probably will fit well to the observed data. This is done by examining whether the point (b_1, b_2) plotted in the β_1-β_2 plane is inside the region (or close to the point or the curve) associated with the distribution function for which the goodness-of-fit is to be considered.

2.3 Test of Goodness-of-Fit

Although there are a number of possible ways in which a test can be performed on goodness-of-fit (for example, the χ^2 test and the Kolmogorov-Smirnov test), the ω^2 method [3] is chosen for the present investigation. Let $F(x)$ = distribution to be tested and $F_n(x)$ = empirical distribution based on a sample of size n, and then form a statistic

$$n\omega_n^2 = n \int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 dF(x) \tag{5}$$

This statistic is distribution-free and some of the percentiles of its asymptotic distribution are listed below [3] and are also plotted in Fig. 2.

Table 2 The ω^2 Test

$P\{n\omega_n^2 \leqq a\}$	0.80	0.85	0.90	0.95	0.98	0.99
a	0.241	0.285	0.347	0.461	0.620	0.743

For practical computations, the following simpler form of $n\omega_n^2$ has been used.

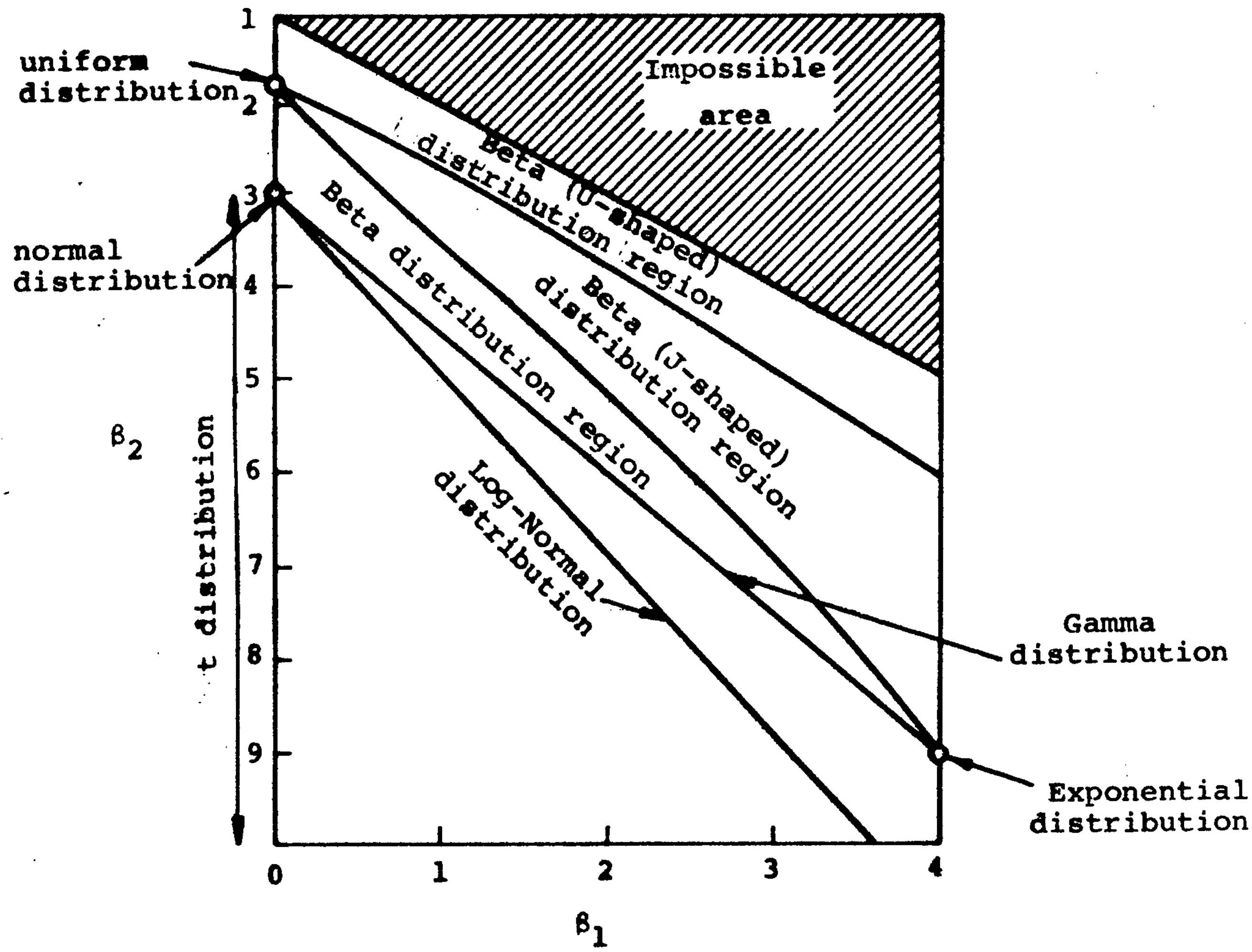


Fig. 1 Regions of various distributions in the (β_1, β_2) plane

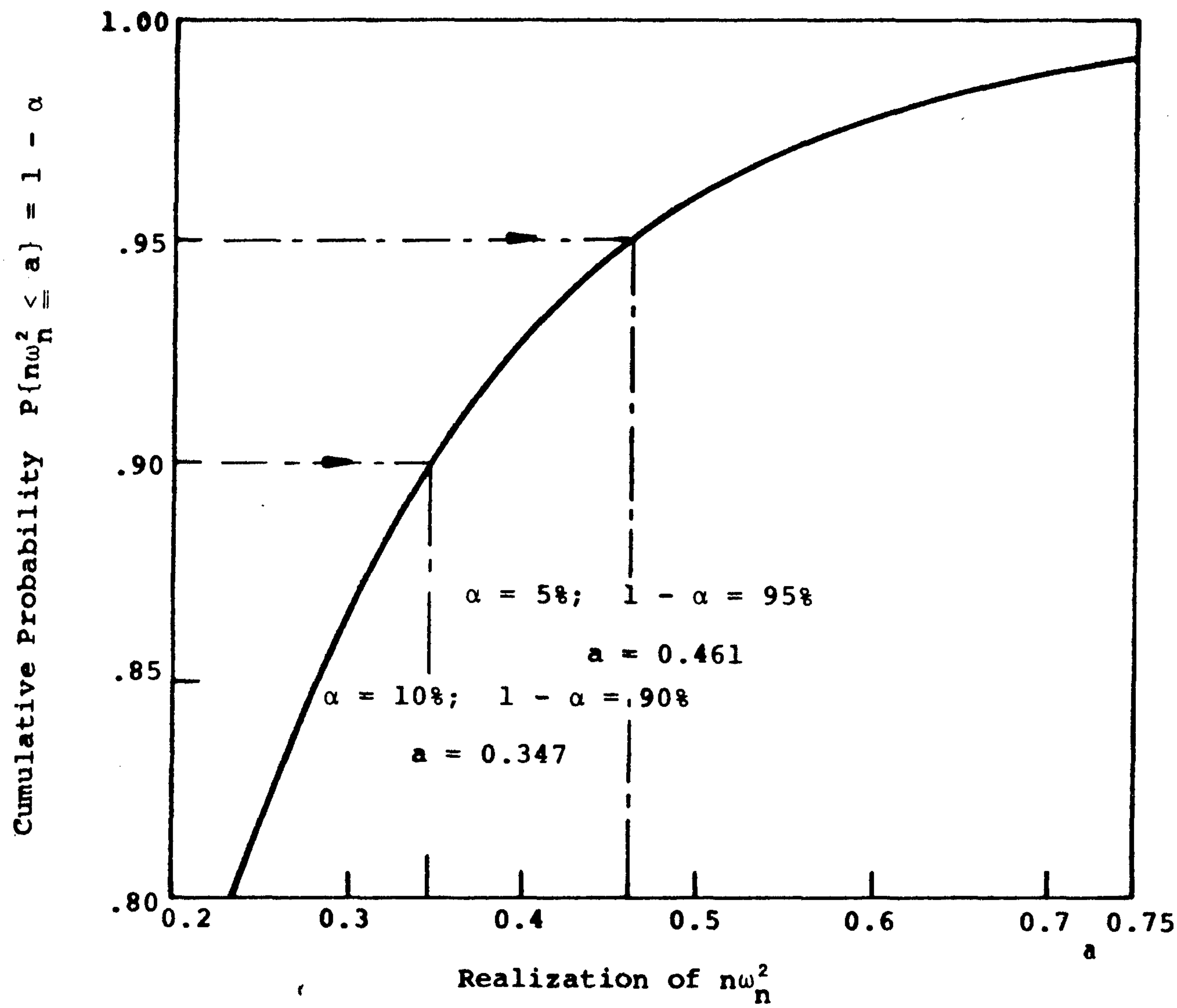


Fig. 2 Probability Distribution of Statistic $n\omega_n^2$

$$n\omega_n^2 = 1/(12n) + \sum_{i=1}^{n} [(2i - 1)/(2n) - F(x_i)]^2$$

$$= 1/(12n) + \sum_{i=1}^{n} [(2i - 1)/(2n) - F(x_{(i)})]^2 \qquad (6)$$

where $x_{(i)}$ $(i=1,2,\ldots,n)$ are the observations arranged in ascending order; $x_{(1)}$ is the smallest, $x_{(2)}$ the second smallest, ..., $x_{(n)}$ the largest.

The preceding table indicates that the proposed distribution should be rejected if the corresponding $n\omega_n^2$ value is larger than 0.461 (0.347) for a significance level of 5% (10%).

2.4 Statistics of Observed Data

There are three sets of observed data: (i) XQPF, (ii) XWPF and (iii) WPF, where X specifies "Load Transfer", W specifies "Winslow Drilled", P specifies "Proper Technique," F specifies "Fighter Spectrum" and Q specifies "Quackenbush Drill and Ream." All data values are given in mils (10^{-3} inch).

The observed data, XQPF, XWPF and WPF are arranged in ascending order as shown in Table 1 with the cumulative probability or the plotting position defined by

$$F(x_{(i)}) = i/(n + 1) \qquad (7)$$

Estimates of some fundamental statistics are also listed in Table 1. Symbols used therein signify the following: MEAN = m_1', AM2 = m_2, AM3 = m_3, AM4 = m_4, STDV = s (unbiased standard deviation), SQRT(B1) = $\sqrt{b_1}$, B2 = b_2 and B1 = b_1.

SECTION III

FITTING TO THE JOHNSON DISTRIBUTION FAMILY

3.1 Method of Translation

The Johnson distribution family was derived by Johnson [2] with the aid of a method of translation which takes advantage of possible transformations of non-Gaussian random variables into Gaussian (normal) variables. The method is outlined below.

We say that a random variable X has been transformed to the normality if a function $G(\cdot)$ transforms X into the standardized normal variable Z.

$$Z = G(X) \tag{8}$$

with the density function of Z being

$$f_Z(z) = \phi(z) = 1/\sqrt{2\pi}\, \exp(-z^2/2) \tag{9}$$

Such a transformation can be performed in two steps. First, we perform a linear transformation of X into Y such that

$$Y = (X - \varepsilon)/\lambda \tag{10}$$

with λ being positive and then transform Y into the standardized normal variable Z by

$$Z = G(X) = \gamma + \eta g(Y) \tag{11}$$

where η is assumed to be positive. The density function of Y can then be shown to be

$$\begin{aligned} f_Y(y) &= \eta g'(y) f_Z(\gamma + \eta g(y)) \\ &= \eta g'(y)/\sqrt{2\pi}\, \exp[-\{\gamma + \eta g(y)\}^2/2] \end{aligned} \tag{12}$$

where $g(y)$ has been assumed to be a non-decreasing function of y and $g'(y)$

$= dg(y)/dy$. We note that there are four parameters ε, λ, γ and η involved in the transformation $Z = G(X)$.

Since X and Y are linearly related through Eq. 10, it is easy to show that the density function of X is given by

$$f_X(x) = \eta/(\sqrt{2\pi}\lambda)g'(\frac{x-\varepsilon}{\lambda}) \exp[-\{\gamma + \eta g(\frac{x-\varepsilon}{\lambda})\}^2/2] \qquad (13)$$

The transformation $Z = G(X) = \gamma + \eta g(Y)$ and the relationship between $f_Z(z)$ and $f_X(x)$ are schematically illustrated in Fig. 3. Beyond the obvious fact that the analytical form of $g(\cdot)$ precisely determines the distribution function of Y when $\eta = 1$ and $\gamma = 0$, we observe from Fig. 3 that the skewness and the kurtosis of the distribution of Y are greatly influenced by γ and η respectively.

3.2 The Johnson Distribution Family

Different distribution functions can be generated by using different functions for $g(y)$. In fact, Johnson [2] proposed three different types of distribution functions referred to as the Johnson S_L, S_B and S_U distributions by respectively employing the following three functions for $g(y)$:

(a) $g(y) = \ell n(y) \qquad$ for $y > 0$ (14)

(b) $g(y) = \ell n(\frac{y}{1-y}) = 2\tanh^{-1}(2y - 1) \qquad$ for $0 < y < 1$ (15)

(c) $g(y) = \sinh^{-1}(y) = \ell n\{y + \sqrt{y^2 + 1}\} \qquad$ for $-\infty < y < \infty$ (16)

3.2.1 The Johnson S_L Family

The density function of the Johnson S_L family can be defined with the aid of Eqs. 13 and 14 as

$$f_{SL}(x) = \eta/[\sqrt{2\pi}(x - \varepsilon)] \exp\{-[\gamma + \eta\ell n(\frac{x-\varepsilon}{\lambda})]^2/2\} \qquad \text{for } x > \varepsilon \qquad (17)$$

where

$$\eta > 0,\ |\gamma| < \infty,\ \lambda > 0,\ |\varepsilon| < \infty$$

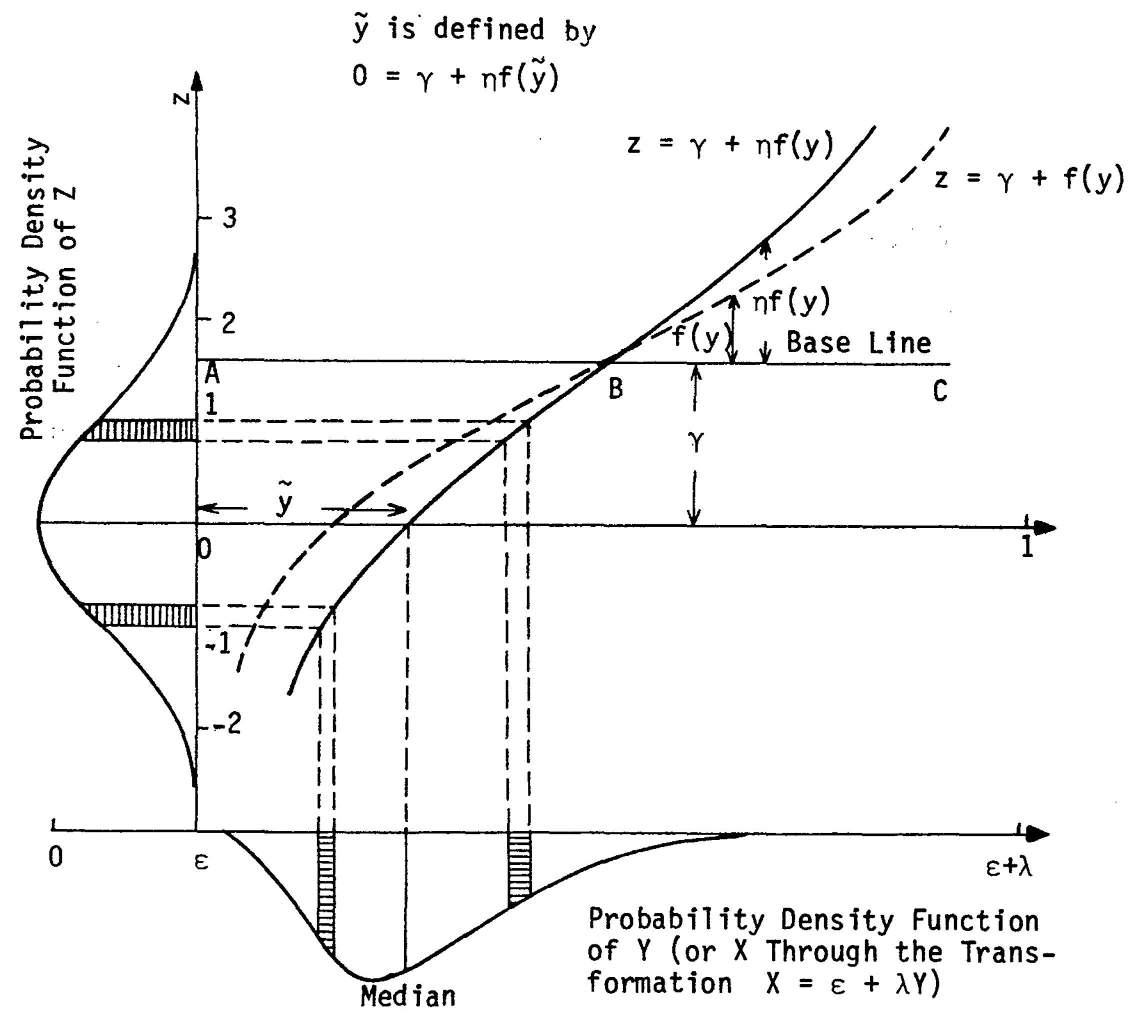


Fig. 3 Schematic Representation of the Method of Translation

Setting

$$\gamma^* = \gamma - \eta \ell n \lambda \tag{18}$$

we can rewrite Eq. 17 in the following form:

$$f_{SL}(x) = \eta/[\sqrt{2\pi}\,(x - \varepsilon)]\ \exp\{-\tfrac{1}{2}\eta^2[\gamma^*/\eta + \ell n(x - \varepsilon)]^2\},$$
$$x > \varepsilon \tag{19}$$

where

$$\eta > 0,\ |\gamma^*| < \infty,\ |\varepsilon| < \infty$$

We can recognize Eq. 19 as the three-parameter log-normal distribution. Indeed, introducing σ and μ so that

$$\eta = 1/\sigma \qquad \text{and} \qquad \gamma^* = -\mu/\sigma \tag{20}$$

we can derive the familiar form of the three-parameter log-normal density function:

$$f_{SL}(x) = 1/[\sqrt{2\pi}\sigma\,(x - \varepsilon)]\ \exp\{-[\ell n(x - \varepsilon) - \mu]^2/(2\sigma^2)\} \tag{21}$$

With the form of the transformation function given in Eq. 14, we can show that the i-th moment about the origin μ_i' of variable Y is given as

$$\mu_i'(y) = 1/(\sqrt{2\pi})\ \int_{-\infty}^{\infty} \exp\{i(z - \gamma)/\eta\}\ \exp(-z^2/2)dz$$
$$= \exp\{(i/\eta)^2/2 - \gamma(i/\eta)\} \tag{22}$$

It then follows that β_1 (square of skewness) and β_2 (peakedness) are given by

$$\left.\begin{aligned} \beta_1 &= (w - 1)(w + 2)^2 \\ \beta_2 &= 3 + (w - 1)(w^3 + 3w^2 + 6w + 6) \end{aligned}\right\} \tag{23}$$

where $w = e^{-\sigma^2}$ with σ indicating the standard deviation of the log-normal distribution. When this relationship is plotted on the β_1-β_2 plane, we obtain a curve indicating those values of β_1 and β_2 that represent the log-normal distribution as shown in Fig. 4.

3.2.2 The Johnson S_B Family

On the basis of the function $g(\cdot)$ defined by Eq. 15, we can construct

$$f_{SB}(x) = 1/\sqrt{2\pi} \cdot \lambda/\{(x - \varepsilon)(\lambda - x + \varepsilon)\} \exp\{-[\gamma + \eta \ln(\frac{x-\varepsilon}{\lambda-x+\varepsilon})]^2/2\}$$

$$\text{for } \varepsilon < x < \varepsilon + \lambda \qquad (24)$$

where

$$\eta > 0, \ |\gamma| < \infty, \ \lambda > 0, \ |\varepsilon| < \infty$$

The probability distributions with the density function given by Eq. 24 are said to belong to the Johnson S_B family. These distributions involve four independent parameters and consequently β_1 and β_2 that represent this family of distributions can take on those values within the domain designated by "Johnson S_B distribution" in Fig. 4. Indeed, this domain is bound by the curve representing the log-normal distribution and a straight line $\beta_2 - \beta_1 - 1 = 0$. Above this straight line is the domain representing those values of β_1 and β_2 that are not realizable.

It follows from Eq. 12 that the density function of $Y = (X - \varepsilon)/\lambda$ is given by

$$f_{YSB}(y) = \eta/[\sqrt{2\pi}y(1 - y)] \exp\{-[\gamma + \eta \ln(\frac{y}{1-y})]^2/2\}$$

$$\text{for } 0 < y < 1 \qquad (25)$$

It also follows from Eq. 11 that Y is expressed in terms of Z as

$$Y = \{1 + \exp[-(Z - \gamma)/\eta]\}^{-1} \qquad (26)$$

Hence, the median value $\tilde{y}$ of Y is

$$\tilde{y} = [1 + \exp(+\gamma/\eta)]^{-1} \qquad (27)$$

The necessary and sufficient conditions for bimodality irrespective of the sign of γ are

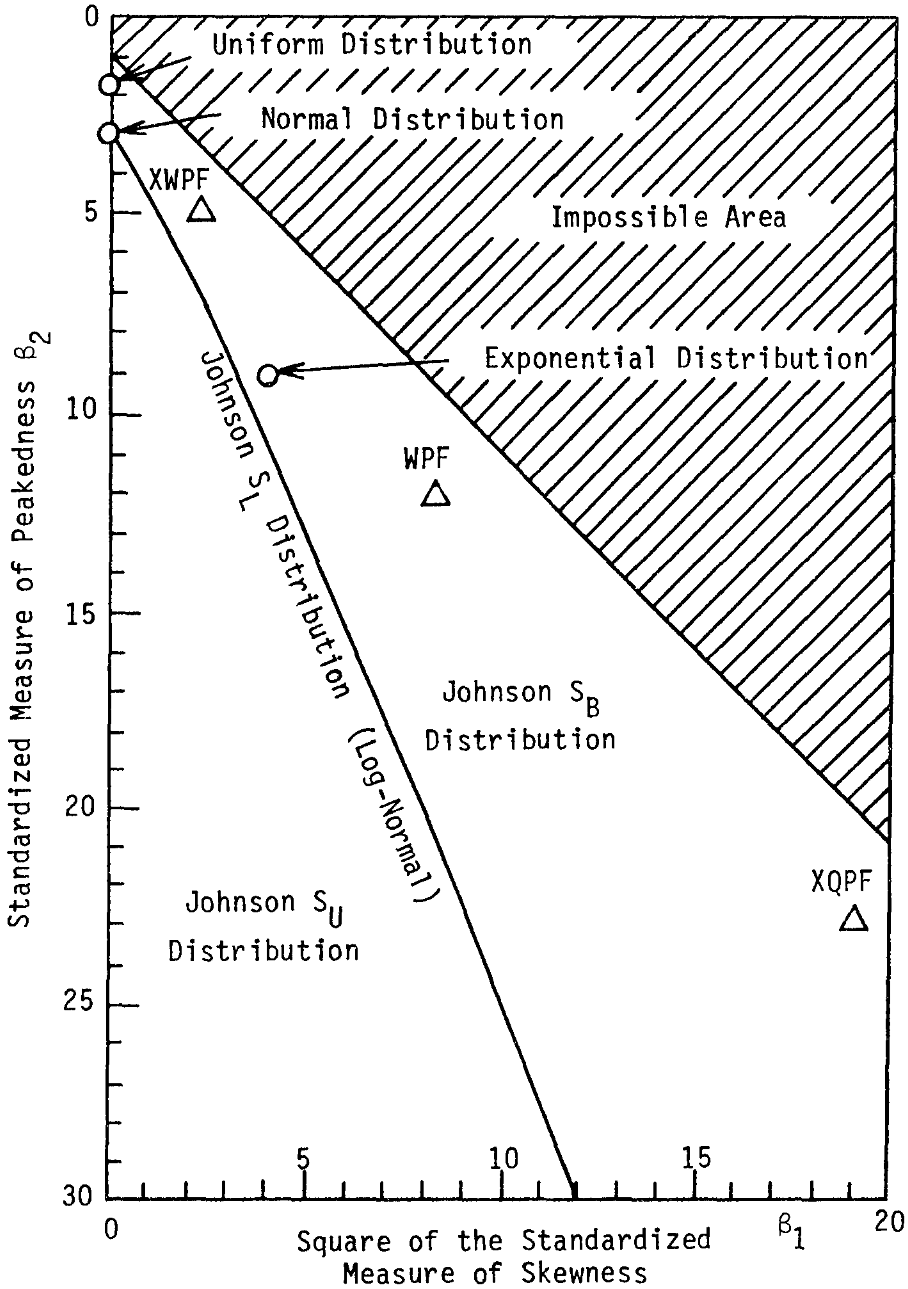


Fig. 4 Estimated Result of (β_1, β_2) for Data XQPF, XWPF and WPF (Johnson Distribution Family)

$$\eta < 1/\sqrt{2},\ |\gamma| < (1/\eta)\sqrt{1 - 2\eta^2} - 2\eta\tanh^{-1}\sqrt{1 - 2\eta^2} \quad (28)$$

3.2.3 The Johnson S_U Family

Use of Eq. 16 in Eq. 12 immediately results in the density function of Y:

$$f_{YSU}(y) = \eta/\sqrt{2\pi(y^2 + 1)}\ \exp\{-[\gamma + \eta\ln(y + \sqrt{y^2 + 1})]^2/2\}$$

$$\text{for } -\infty < y < \infty \quad (29)$$

Similarly, use of Eq. 16 in Eq. 13 produces the density function of X:

$$f_{SU}(x) = \eta/\sqrt{2\pi\{(x - \varepsilon)^2 + \lambda^2\}}$$

$$\times \exp\{-[\gamma + \eta\ln\{(x - \varepsilon)/\lambda + \sqrt{(x - \varepsilon)^2/\lambda^2 + 1}\}]^2\}$$

$$\text{for } -\infty < x < \infty \quad (30)$$

where

$$\eta > 0,\ -\infty < \gamma < \infty,\ \lambda > 0,\ -\infty < \varepsilon < \infty$$

The probability distributions having the density function given by Eq. 30 are said to belong to the Johnson S_U family. As in the case of the Johnson S_B family, the values which β_1 and β_2 of the distributions of this family can take are confined in a domain below the log-normal curve as designated by "Johnson S_U distribution" in the β_1-β_2 plane in Fig. 4.

3.3 Fitting Data to Distributions of the Johnson Family

Observed experimental data can be fitted to the distribution functions of the Johnson family by proceeding with the following steps:

(1) Determine which of the three distribution families is to be used.

(2) Estimate the parameters of the selected distribution.

(3) Compute the expected cumulative frequencies of the fitted distribution.

(4) Perform a goodness-of-fit test using the ω^2 method.

The objective of Step (1) can be accomplished by plotting on the β_1-β_2 plane the estimates b_1 and b_2 of β_1 and β_2, respectively, evaluated with the aid of Eq. 4. Observe whether or not the point is in the S_B domain, S_U domain, or close to or on the S_L curve. It appears prudent to assume that the data fit the S_B (S_U) distributions if the point (b_1,b_2) falls in the S_B (S_U) domain, and that the log-normal distribution is a likely candidate if the point falls on or close to the S_L curve. Indeed, for the observed data listed in Table 1, all three points representing (b_1,b_2) for XQPF, XWPF and WPF fall in the S_B domain. Therefore, these data are expected to fit well to the S_B distribution. Also, while the three points are not particularly close to the S_L curve, their general proximity to the curve suggests that reasonable fits may be expected between the data and the three-parameter log-normal distribution. In fact, the data plotted on the log-normal probability paper (assuming that the minimum flaw size = 0) in Fig. 5 suggests that a more than satisfactory fit may be observed particularly for XWPF and WPF if the three-parameter log-normal distribution is used.

For the reasons described above and for a greater familiarity with the log-normal distribution on the part of engineers, the Johnson S_L (three-parameter log-normal) distribution is considered first for the purpose of fitting the observed data in Table 1. Then, a fitting procedure will be described for the Johnson S_B distribution.

3.3.1 The Johnson S_L (Three-Parameter Log-Normal) Distribution

Expressing the log-normal probability density function in the form of Eq. 21 with parameters μ, σ and ε, we can establish the parameter estimation procedure in one of the following manners, depending on whether or not the location parameter ε is known. The number of unknown parameters is equal to two when ε is known; otherwise it is equal to three.

(a) When ε is assumed to be known

Since $\ell n(X - \varepsilon)$ is a normally distributed random variable with mean μ and standard deviation σ, the estimates $\hat{\mu}$ and $\hat{\sigma}$ of μ and σ, respectively, can be obtained in a manner analogous to that for estimating the parameters of the normal distribution:

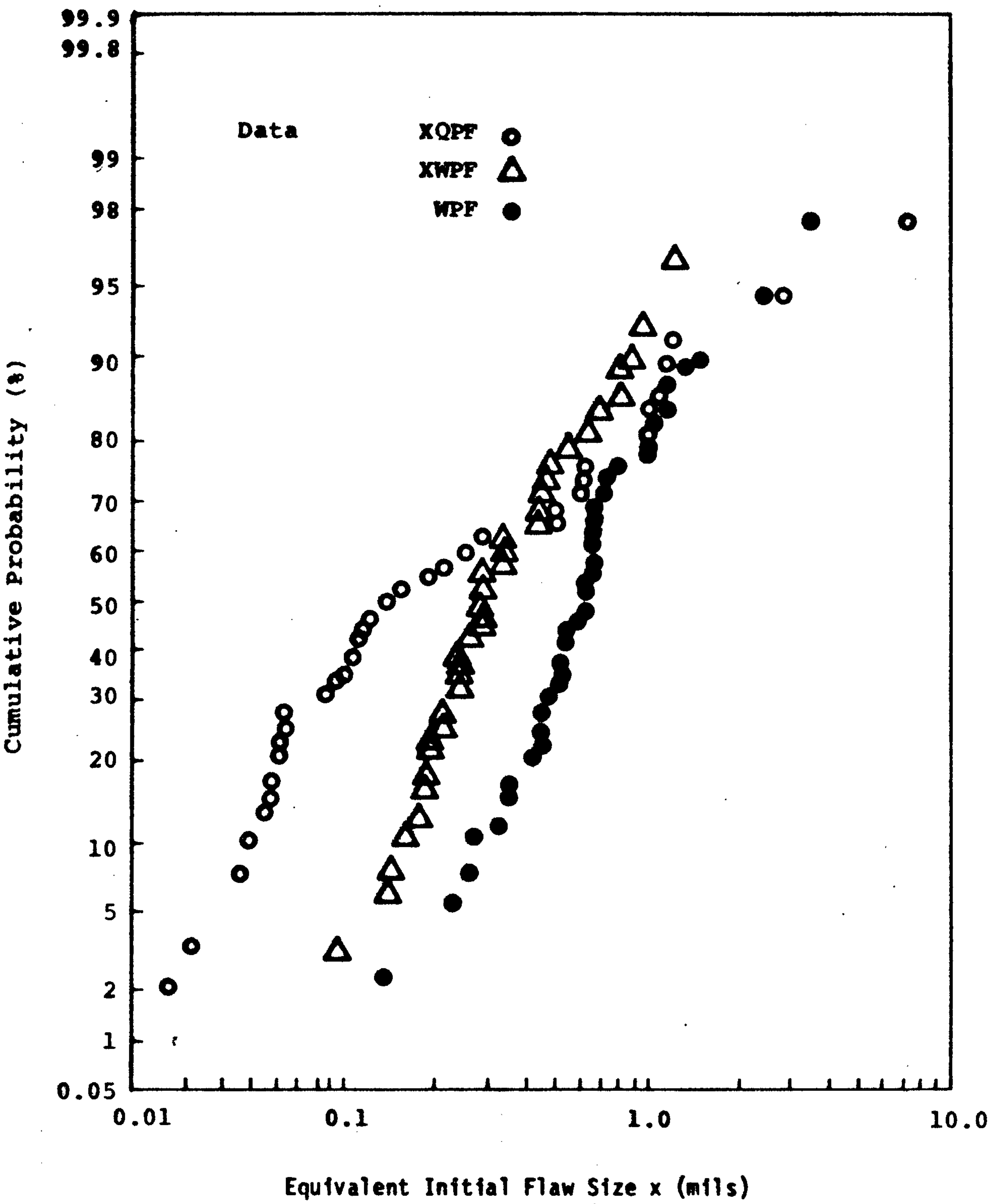


Fig. 5 Log-Normal Probability Plots of Data Sets XQPF, XWPF and WPF

$$\hat{\mu} = (1/n) \sum_{i=1}^{n} x_i^* \tag{31}$$

$$\hat{\sigma} = [\{\sum_{i=1}^{n} (x_i^* - \hat{\mu})^2\}/n]^{\frac{1}{2}} = [\{\sum_{i=1}^{n} x_i^{*2}\}/n - (\hat{\mu})^2]^{\frac{1}{2}} \tag{32}$$

or

$$s = [\{\sum_{i=1}^{n} (x_i^* - \hat{\mu})^2\}/(n - 1)]^{\frac{1}{2}} = \sqrt{n/(n - 1)}\, \hat{\sigma} \tag{33}$$

where $x_i^* = \ell n(x_i - \varepsilon)$ and s is the unbiased estimate of standard deviation. The estimates of η and γ^* in Eq. 20 are then obtained as

$$\hat{\eta}_1 = 1/\hat{\sigma} \qquad \text{or} \qquad \hat{\eta}_2 = 1/s \tag{34}$$

and

$$\hat{\gamma}_1^* = -\hat{\mu}/\hat{\sigma} \qquad \text{or} \qquad \hat{\gamma}_2^* = -\hat{\mu}/s \tag{35}$$

Because of the general theoretical advantage, s in Eq. 33 is used more frequently than $\hat{\sigma}$ in Eq. 32 for an estimate of the standard deviation. For a large value of n, however, there is little difference between these two as Eq. 33 indicates. In the present section, however, both $\hat{\sigma}$ and s are used for comparison.

As is well known, the estimate $\hat{\mu}$ in Eq. 31, when realizations x_i's are replaced by corresponding random variables X_i's, becomes an estimator and will vary from sample to sample. Therefore, it is standard practice that the adequacy of $\hat{\mu}$ as an estimate for the unknown parameter μ is indicated in terms of the confidence interval given by

$$\begin{matrix}\mu_U \\ \mu_L\end{matrix} = \hat{\mu} \pm (t_{1-\alpha,n-1}) s/\sqrt{n} \tag{36}$$

where $\hat{\mu}$ and s are obtained from Eqs. 31 and 33, respectively, and $t_{1-\alpha,n-1}$ is the two-sided $100(1-\alpha)$ percentile of the Student's t distribution of (n-1) degrees-of-freedom. The quantity $(1-\alpha)$ is referred to as the confidence level and has the following significance.

$$P\{\hat{\mu} - (t_{1-\alpha,n-1})s/\sqrt{n} < \mu < \hat{\mu} + (t_{1-\alpha,n-1})s/\sqrt{n}\} = 1 - \alpha \tag{37}$$

From the (two-sided) 100(1-α)% confidence interval of $\mu = E[\ell n(X - \varepsilon)]$ given in Eq. 36, we can obtain the lower bound X_L and upper bound X_U of the corresponding interval for X from

$$\mu_L = \ell n(X_L - \varepsilon), \qquad \mu_U = \ell n(X_U - \varepsilon) \tag{38}$$

as

$$X_L = \varepsilon + \exp\{\hat{\mu} - (t_{1-\alpha,n-1})s/\sqrt{n}\} \tag{39}$$

$$X_U = \varepsilon + \exp\{\hat{\mu} + (t_{1-\alpha,n-1})s/\sqrt{n}\} \tag{40}$$

(b) <u>When ε is assumed to be unknown</u>

It follows from Eqs. 11, 14, and 18 that

$$Z = \gamma^* + \eta \ell n(X - \varepsilon) \tag{41}$$

where Z is the standardized normal variable. The three parameters γ^*, η and ε in Eq. 41 can be estimated in the following manner as suggested by Hahn and Shapiro [4]: choose three probability values p, q and r, find $A = \Phi(z_A)$ where $\Phi(\cdot)$ = standardized Gaussian distribution function, with A = p, q and r, estimate x_A such that $P\{X \leq x_A\} = A$, construct three equations of the form

$$z_A = \hat{\gamma}^* + \hat{\eta} \ell n(x_A - \hat{\varepsilon}) \qquad (A = p, q \text{ and } r) \tag{42}$$

and finally solve Eq. 42 for the estimates $\hat{\gamma}^*$, $\hat{\eta}$ and $\hat{\varepsilon}$. In practice, the quantity x_A defined above is estimated with the aid of the ordered sample $x_{(1)} < x_{(2)} < \ldots . < x_{(n)}$ where $x_{(i)}$ is the i-th smallest in the sample of size n with the plotting position i/(1+n). Indeed, if it so happens that

$$A = i/(1+n) \tag{43}$$

then

$$x_A = x_{(i)} \tag{44}$$

while if

$$i/(1 + n) < A < (1 + i)/(1 + n) \tag{45}$$

then by interpolation

$$x_A = x_{(i)} + \{x_{(i+1)} - x_{(i)}\}\{(1 + n)A - i\} \tag{46}$$

In the present study, we choose $p = 1-\alpha$, $q = 0.5$ and $r = \alpha$ with α being $i/(1+n)$ $(i=1,2,\ldots,8)$. Then, it is a relatively simple matter to derive the following expressions for $\hat{\gamma}^*$, $\hat{\eta}$ and $\hat{\varepsilon}$:

$$\hat{\gamma}^* = \hat{\eta}\ln\{(1 - e^{-z'/\hat{\eta}})/(x_{0.5} - x_{\alpha})\} \tag{47}$$

$$\hat{\eta} = z'/\ln\{(x_{1-\alpha} - x_{0.5})/(x_{0.5} - x_{\alpha})\} \tag{48}$$

$$\hat{\varepsilon} = x_{0.5} - \exp(-\hat{\gamma}^*/\hat{\eta}) \tag{49}$$

Obviously, these values are different for different values of α. In the analysis that follows later, we use as our estimates those values of $\hat{\gamma}^*$, $\hat{\eta}$ and $\hat{\varepsilon}$ that produce the smallest value of $n\omega_n^2$. In Eqs. 47 and 48, $z' = z_{1-\alpha} = -z_{\alpha}$ represents the 100 x $(1-\alpha)$-th percentile of the standardized Gaussian distribution.

(c) Results of estimation

First, we consider the case where the location parameter ε is assumed to be known. In this case, we perform the estimation presuming that ε is a fraction of the smallest observation $x_{(1)}$; $\varepsilon = x_{(1)} \cdot i/10$ $(i=0,1,2,\ldots,9)$. Using Eqs. 31-35, we then estimate η and γ^* based on both $\hat{\sigma}$ and s for each of these ten different values of ε; $\varepsilon = 0, 0.1x_{(1)}, \ldots, 0.9x_{(1)}$. For each set of $\hat{\eta}$ and $\hat{\gamma}^*$ thus estimated, Eq. 6 is used to evaluate $n\omega_n^2$, and we choose as our best estimate the set of ε, $\hat{\eta}$ and $\hat{\gamma}^*$ that produce the smallest value of $n\omega_n^2$. Table 3 lists estimated parameters $\hat{\eta}_1$ (written as ETA 1), $\hat{\gamma}_1^*$ (GAMMA 1), $\hat{\eta}_2$ (ETA 2), $\hat{\gamma}_2^*$ (GAMMA 2) and $n\omega_n^2$ (NWN2) for the data XQPF, XWPF and WPF under ten different cases of ε; Case 1 for $\varepsilon = 0$, Case 2 for $\varepsilon = 0.1x_{(1)}$, ..., Case 10 for $\varepsilon = 0.9x_{(1)}$. The $n\omega_n^2$ values are considerably larger for XQPF than for XWPF and WPF. The $n\omega_n^2$

Table 3 Values of η, γ and $n\omega_n^2$ as Functions of ε (The Johnson S_L Distribution)

FOR DATA SERIES XQPF

CASE	E	ETA1	GAMMA1	NWN2	ETA2	GAMMA2	NWN2
1	.0000	.736	1.119	1.630	.726	1.104	1.646
2	.0026	.725	1.120	1.673	.715	1.105	1.690
3	.0052	.714	1.120	1.720	.704	1.105	1.738
4	.0078	.702	1.120	1.771	.692	1.105	1.790
5	.0104	.689	1.120	1.828	.680	1.104	1.849
6	.0130	.676	1.118	1.892	.666	1.103	1.914
7	.0156	.661	1.115	1.965	.652	1.100	1.988
8	.0182	.644	1.110	2.050	.635	1.095	2.075
9	.0208	.623	1.101	2.155	.615	1.086	2.182
10	.0234	.595	1.083	2.300	.587	1.069	2.331

FOR DATA SERIES XWPF

CASE	E	ETA1	GAMMA1	NWN2	ETA2	GAMMA2	NWN2
1	.0000	1.671	1.905	.451	1.648	1.879	.455
2	.0093	1.621	1.904	.471	1.599	1.879	.475
3	.0186	1.569	1.901	.493	1.548	1.876	.497
4	.0279	1.516	1.895	.519	1.495	1.870	.523
5	.0372	1.460	1.886	.548	1.440	1.860	.553
6	.0465	1.402	1.872	.582	1.383	1.846	.587
7	.0558	1.340	1.851	.622	1.321	1.826	.628
8	.0651	1.271	1.822	.673	1.254	1.797	.680
9	.0744	1.191	1.776	.741	1.175	1.751	.748
10	.0837	1.085	1.691	.846	1.070	1.668	.854

FOR DATA SERIES WPF

CASE	E	ETA1	GAMMA1	NWN2	ETA2	GAMMA2	NWN2
1	.0000	1.586	.714	.330	1.565	.704	.333
2	.0140	1.548	.738	.343	1.527	.728	.347
3	.0280	1.507	.761	.358	1.487	.751	.362
4	.0420	1.465	.783	.374	1.445	.772	.378
5	.0560	1.420	.802	.393	1.401	.792	.398
6	.0700	1.372	.820	.415	1.354	.809	.421
7	.0840	1.320	.834	.443	1.302	.823	.448
8	.0980	1.260	.844	.477	1.243	.833	.483
9	.1120	1.188	.846	.525	1.172	.835	.532
10	.1260	1.085	.828	.606	1.071	.817	.614

values for the last two sets are not significantly different. Hence, the three-parameter log-normal distribution can be used for the XWPF data with approximately the same level of goodness-of-fit as for the WPF data, while at a considerably less satisfactory level for the XQPF data, a trend that can easily be observed from Fig. 5. Table 3 also shows that within each set of data, the $n\omega_n^2$ values decrease as ε decreases thus indicating the choice of $\varepsilon = 0$ together with the corresponding values of $\hat{\eta}$ and $\hat{\gamma}^*$ as statistically the best. The value of $n\omega_n^2$ associated with the significance level of 5% is 0.461 from Table 2. Therefore, observing from Table 3 that the $n\omega_n^2$ values for $\varepsilon = 0$ are smaller than 0.461 for XWPF and WPF, we may accept the hypothesis (with a significance level of 5%) that XWPF and WPF data have been taken from the three-parameter log-normal populations with $\varepsilon = 0$ and corresponding values of $\hat{\eta}_1$ and $\hat{\gamma}_1^*$ (or $\hat{\eta}_2$ and $\hat{\gamma}_2^*$). However, we must reject (with the same level of significance) the hypothesis that XQPF data have been taken from the three-parameter log-normal populations since the smallest $n\omega_n^2$ value associated with $\varepsilon = 0$ is in this case larger than 0.461. It is of interest to note that if the significance level is raised to 10%, only WPF data will survive the test. These results are summarized in Table 4.

Figures 6-8 show the values of $\hat{\gamma}_1^*$, $\hat{\gamma}_2^*$, $\hat{\eta}_1$ and $\hat{\eta}_2$ as functions of the location parameter ε for XQPF, XWPF and WPF data, respectively. We observe from these figures that the difference between the estimates based on $\hat{\sigma}$ and s are neglibibly small. Fig. 9 illustrates how the values of $\hat{\eta}_1$ and $\hat{\gamma}_1^*$ compare "data set by data set" as functions of ε when $\hat{\sigma}$ is used, while Fig. 10 illustrates the same when s is used. These two figures also show that the values of both $\hat{\eta}$ and $\hat{\gamma}^*$ are generally largest for XWPF, larger for WPF and smallest for XQPF over practically the entire range of ε considered. Table 4 also lists the upper and lower bounds, X_U and X_L, corresponding to the confidence bounds μ_U and μ_L with a confidence level of 0.9, while Table 5 lists the bounds corresponding to confidence levels of 0.9, 0.95 and 0.99. Fig. 11 plots the $n\omega_n^2$ values as functions of ε respectively for XQPF, XWPF and WPF and in essence reiterates the result of the test of hypothesis mentioned earlier. Note that the $n\omega_n^2$ values based on $\hat{\sigma}$ and s are indistinguishable in Fig. 11 for the same

Table 4 Best Estimates (The Johnson S_L Distribution with $\varepsilon = 0$)

Data	ε	$\hat{\eta}$	$\hat{\gamma}$*	$n\omega_n^2$*	X_L**	X_U**
XQPF	0	.726	1.10	1.65	.163	.294
XWPF	0	1.65	1.88	.455	.281	.364
WPF	0	¯1.57	.704	.333	.557	.730

* 5% (10%) significance level = .461 (.347)

** 90% confidence bounds in mils

Table 5 Lower and Upper Bounds X_L and X_U in mils Corresponding to μ_L and μ_U for Several Confidence Levels $1-\alpha$ (When $\varepsilon = 0$)

FOR DATA XQPF (N=37)

1-ALPHA	XL	XU
0.90	.1627	.2938
0.95	.1492	.3204
0.99	.1260	.3793

FOR DATA XWPF (N=37)

1-ALPHA	XL	XU
0.90	.2808	.3643
0.95	.2703	.3785
0.99	.2509	.4077

FOR DATA WPF (N=38)

1-ALPHA	XL	XU
0.90	.5571	.7299
0.95	.5354	.7595
0.99	.4957	.8203

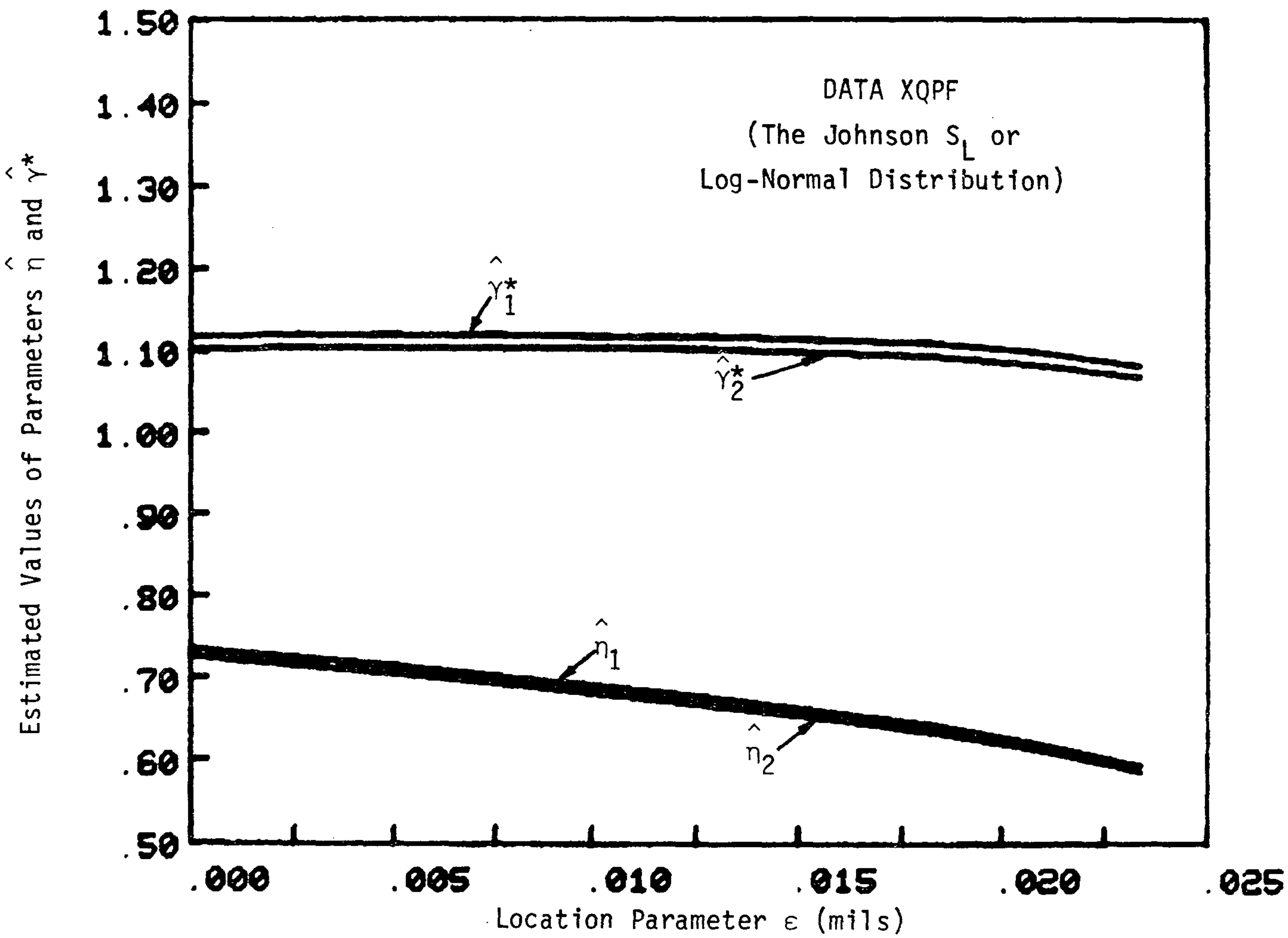


Fig. 6 Variation of Estimated Parameters $\hat{\eta}$ and $\hat{\gamma}^*$ for Data XQPF (When ε is Known)

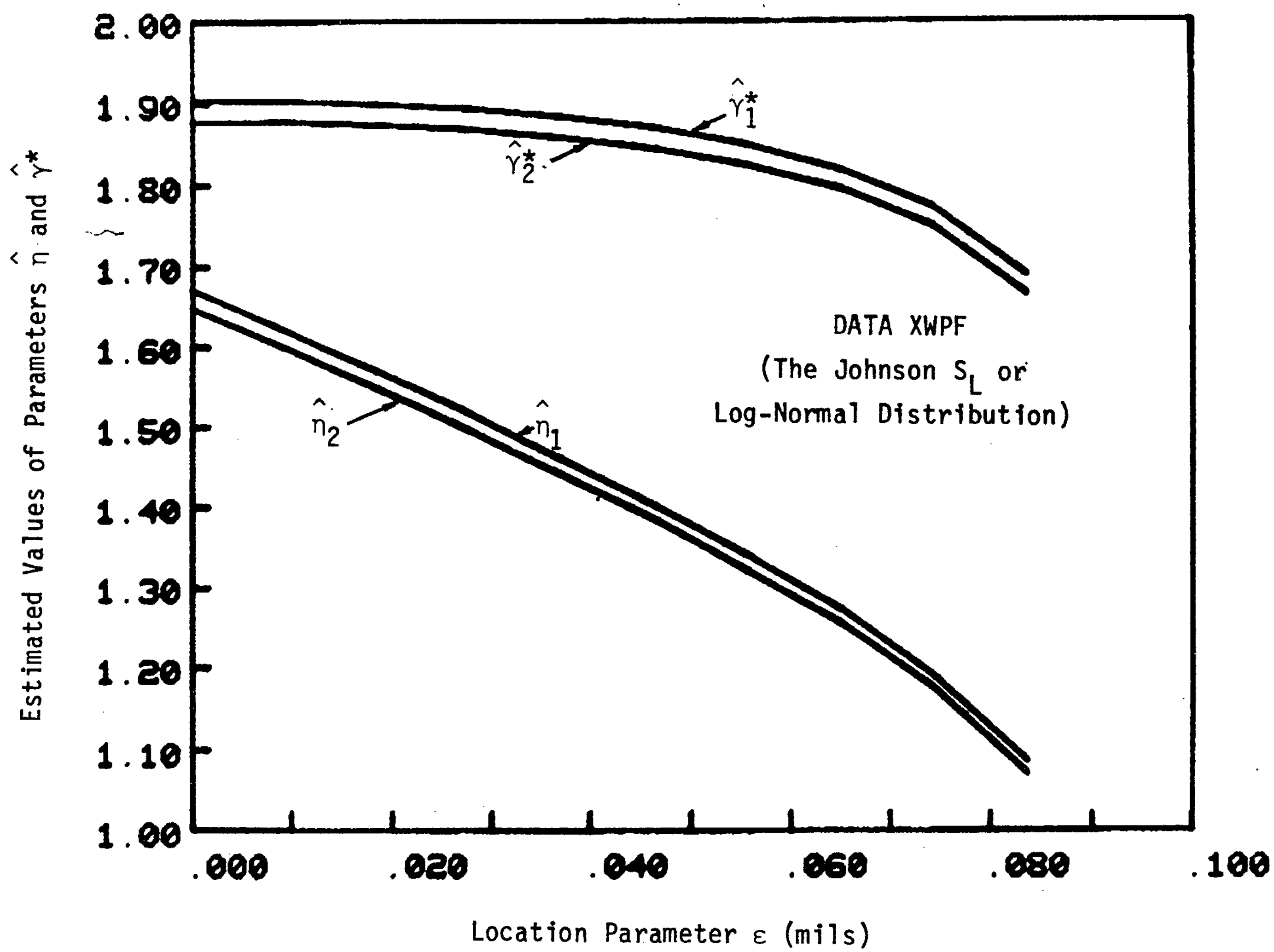


Fig. 7 Variation of Estimated Parameters $\hat{\eta}$ and $\hat{\gamma}^*$ for Data XWPF (When ε is Known)

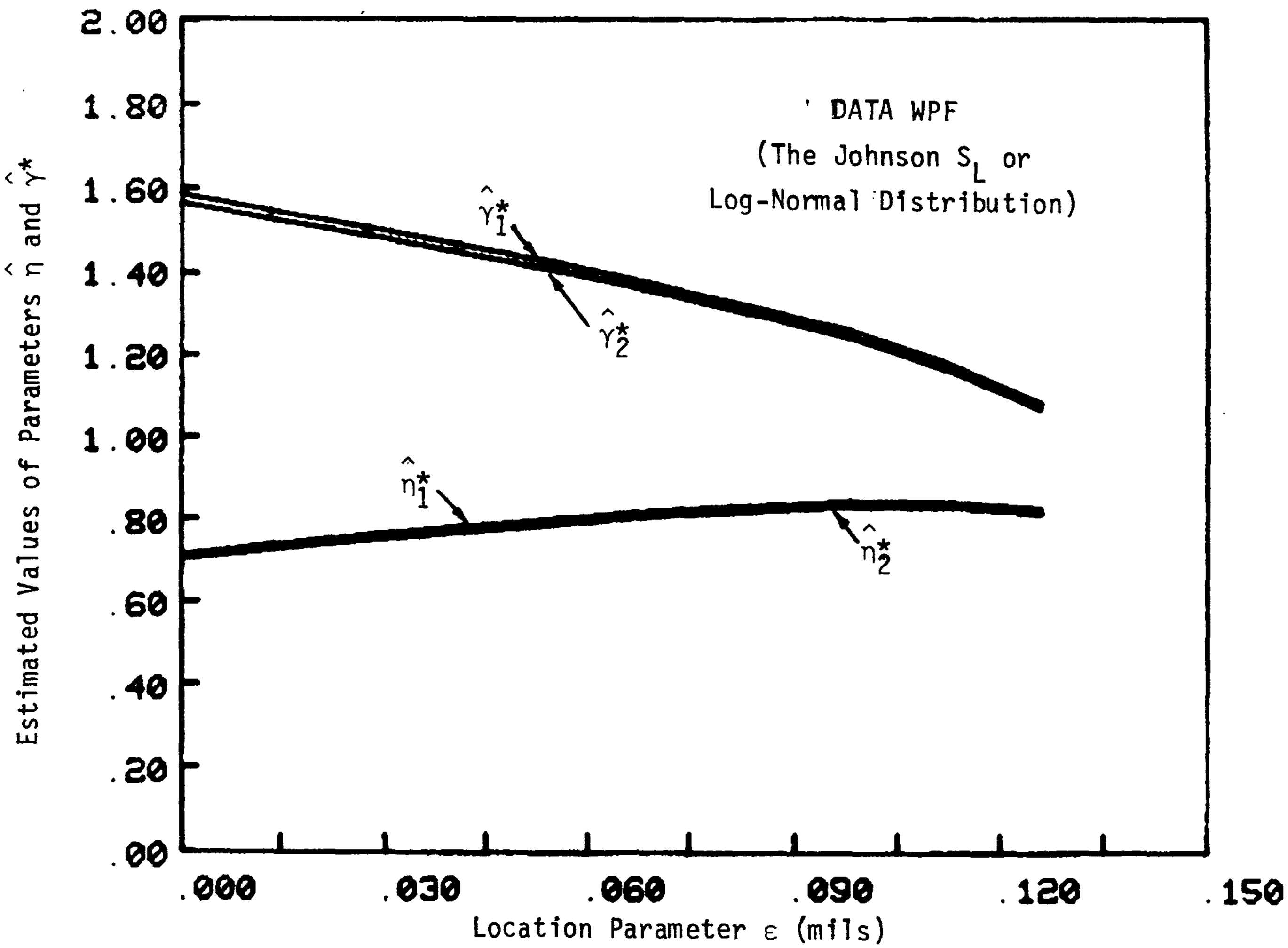


Fig. 8 Variation of Estimated Parameters $\hat{\eta}$ and $\hat{\gamma}^*$ for Data WPF (When ε is Known)

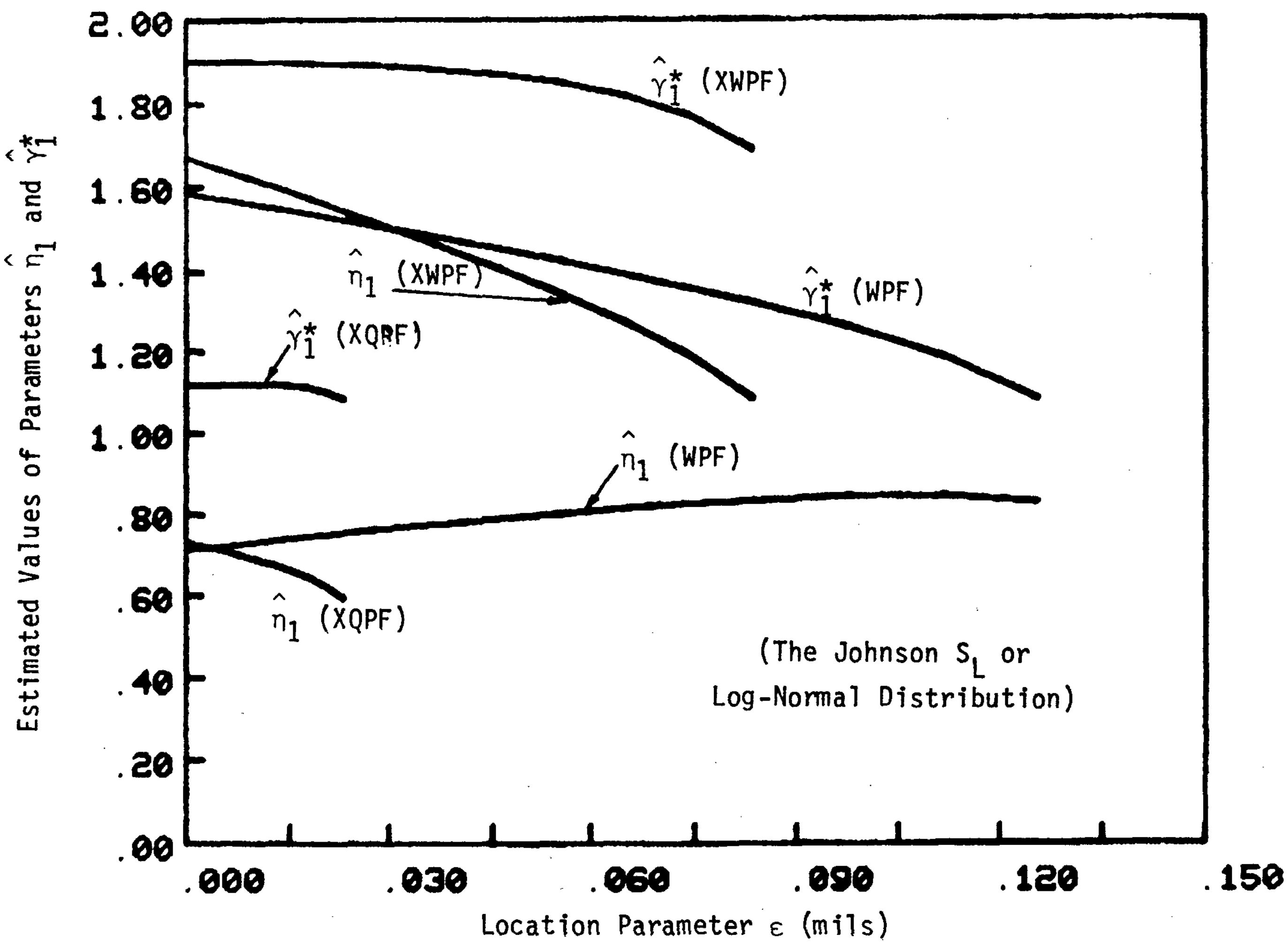


Fig. 9 Variation of Estimated Parameters $\hat{\eta}_1$ and $\hat{\gamma}_1^*$ Based on $\hat{\sigma}$ (When ε is Known)

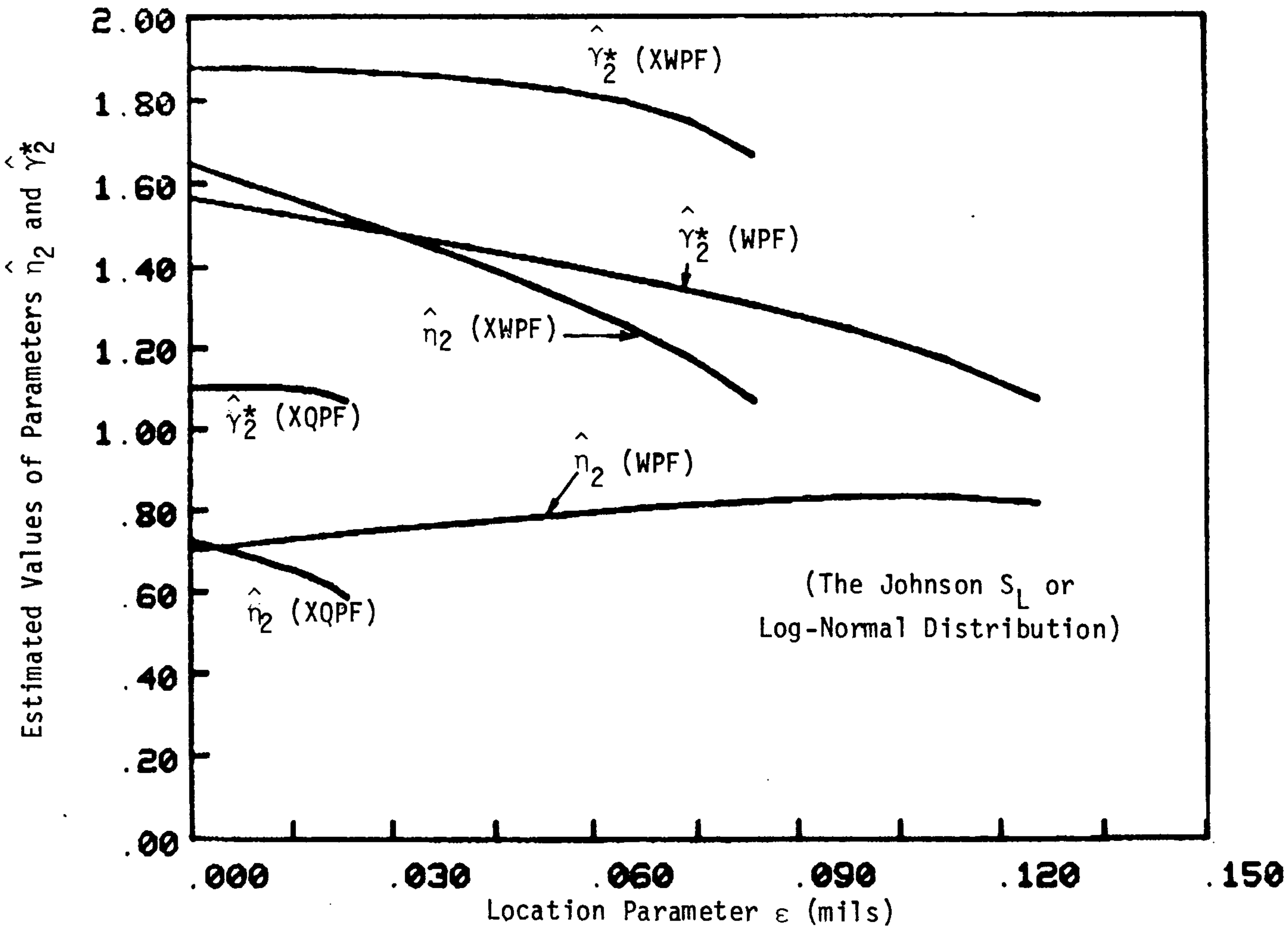


Fig. 10 Variation of Estimated Parameters $\hat{\eta}_2$ and $\hat{\gamma}_2^*$ Based on Unbiased Statistic s (When ε is Known)

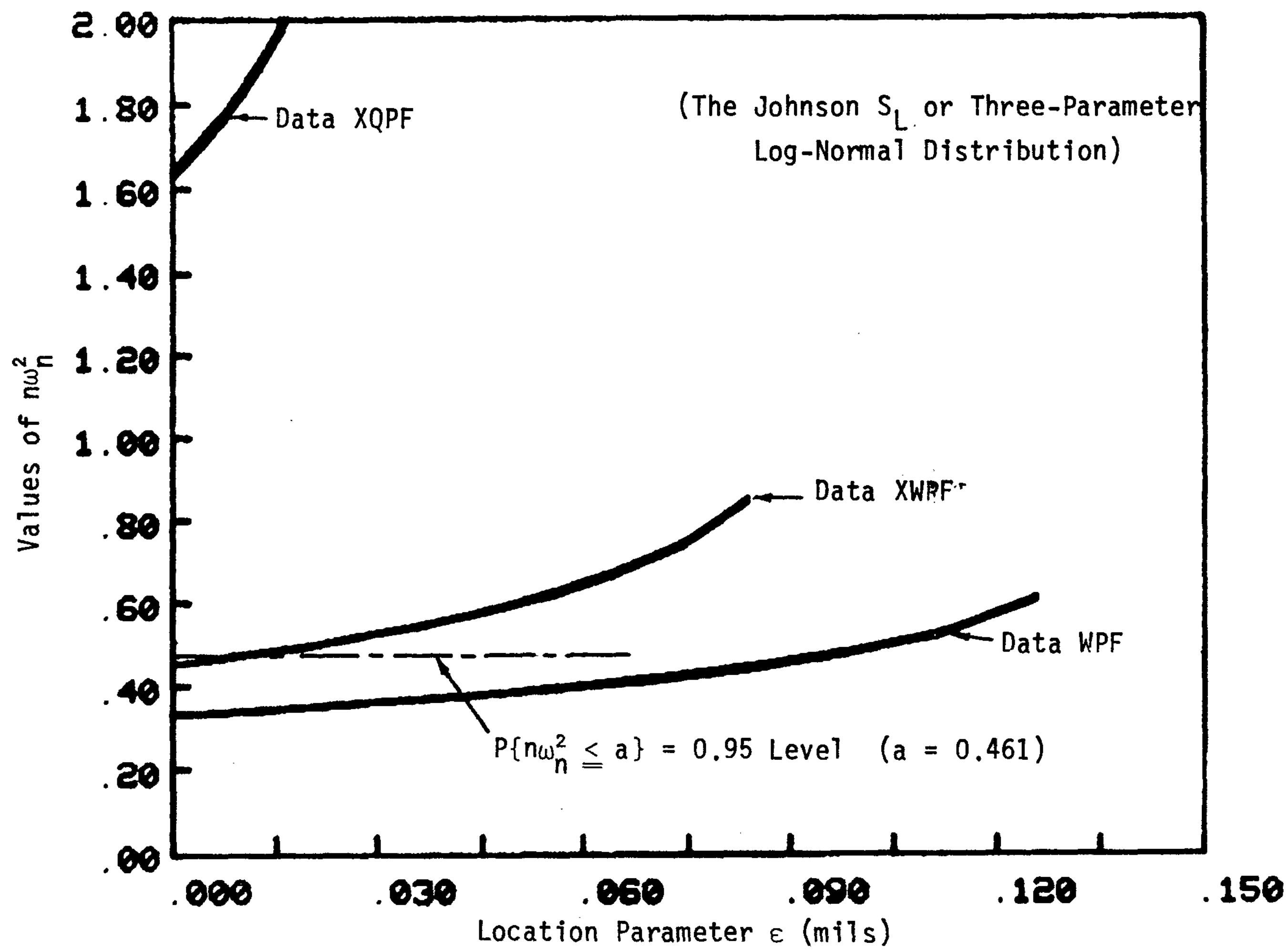


Fig. 11 Effect of the Assumed Location Parameter ε on the Goodness-of-Fit Test Statistic $n\omega_n^2$

data sets.

We now turn to the case where ε is assumed to be unknown. In this case, Eqs. 47-49 are used to estimate $\hat{\varepsilon}$, $\hat{\eta}$ and $\hat{\gamma}^*$. As mentioned earlier, different sets of $\hat{\varepsilon}$, $\hat{\eta}$ and $\hat{\gamma}^*$ will be obtained depending on the values of α to be used in Eqs. 47 and 48. In the present study, we choose the following eight different values for α; $\alpha = i/(1+n)$ $(i=1,2,\ldots,8)$. Making use of either Eqs. 43 and 44 or Eqs. 45 and 46, the values of x_α, $x_{1-\alpha}$ and $x_{0.5}$ are evaluated and listed in Table 6 for each case of these α values; Case i for $\alpha = i/(1+n)$. Table 7 lists, for each data set, the estimated parameters $\hat{\eta}$ as ETA, $\hat{\gamma}^*$ as GAM, $\hat{\varepsilon}$ as E and it further lists the value of $n\omega_n^2$ computed with the aid of Eq. 6. The values of $\hat{\eta}$, $\hat{\gamma}^*$ and $\hat{\varepsilon}$ for XQPF, XWPF and WPF are plotted respectively in Figs. 12-14 as functions of the probability level α. Fig. 15 plots the value of $n\omega_n^2$ for each data set as a function of α and shows that, as in the case of Fig. 11, the values are largest for XQPF, larger for XWPF and smallest for WPF, again reflecting the degrees of "goodness-of-fit" observed in Fig. 5. Fig. 15 further shows that the $n\omega_n^2$ values assume minimum at $\alpha = 3/38 = 0.0789$ for XQPF, at $\alpha = 1/38 = 0.0263$ for XWPF and at $\alpha = 5/39 = 0.1316$. The set of $\hat{\varepsilon}$, $\hat{\eta}$ and $\hat{\gamma}^*$ corresponding to each of these α values is chosen as the best estimate for the respective data set. If we use the best estimate of ε thus obtained in place of ε in Eqs. 39 and 40, the lower and upper bounds, X_L and X_U, will result as listed in Table 8. In evaluating X_L and X_U from Eqs. 39 and 40, $\hat{\mu}$ and s are needed and they are computed with the aid of Eqs. 31 and 33.

It is concluded from Table 7 and Fig. 15 that with the significance level of 5% we may accept the hypothesis that the WPF data have been taken from the three-parameter log-normal population but we must reject the other two data sets as taken from the three-parameter log-normal populations. For the WPF data, the best estimates are $\hat{\eta} = 1.855$, $\hat{\gamma}^* = 0.624$ and $\hat{\varepsilon} = -0.067$. Incidentally, the WPF data will also pass the hypothesis testing under the significance level of 10%. These results are summarized in Table 9.

Table 6 Values of $x_{.50}$, x_α and $x_{1-\alpha}$
(The Johnson S_L Distribution)

		XQPF	XWPF	WPF
	X(.50)	.150	.295	.647
CASE 1	X(.03)	.026	.093	.140
	X(.97)	7.700	1.280	3.830
CASE 2	X(.05)	.031	.145	.236
	X(.95)	3.000	.940	2.730
CASE 3	X(.08)	.047	.145	.280
	X(.92)	1.240	.870	1.640
CASE 4	X(.11)	.050	.160	.280
	X(.90)	1.140	.810	1.490
CASE 5	X(.13)	.058	.175	.320
	X(.87)	1.100	.810	1.250
CASE 6	X(.16)	.059	.180	.367
	X(.85)	1.090	.700	1.250
CASE 7	X(.18)	.060	.180	.367
	X(.82)	1.090	.612	1.140
CASE 8	X(.21)	.063	.190	.420
	X(.79)	1.090	.540	1.040

Table 7 Values of η, γ, ε and $n\omega_n^2$
(The Johnson S_L Distribution)

	DATA	XQPF	XWPF	WPF
	ETA	.472	1.223	1.061
CASE 1	GAM	.977	1.676	.537
	E	.024	.041	.044
	NWN2	3.098	.719	.465
	ETA	.510	1.111	1.006
CASE 2	GAM	1.064	1.813	.673
	E	.026	.100	.135
	NWN2	2.898	.838	.517
	ETA	.599	1.051	1.433
CASE 3	GAM	1.301	1.676	.775
	E	.036	.092	.065
	NWN2	2.408	.872	.340
	ETA	.546	.935	1.524
CASE 4	GAM	1.200	1.588	.656
	E	.039	.112	-.003
	NWN2	2.600	.999	.322
	ETA	.479	.768	1.855
CASE 5	GAM	1.095	1.425	.624
	E	.048	.139	-.067
	NWN2	2.717	1.190	.278
	ETA	.430	.797	1.330
CASE 6	GAM	.986	1.457	.863
	E	.049	.134	.124
	NWN2	2.940	1.178	.359
	ETA	.383	.887	1.622
CASE 7	GAM	.885	1.519	.703
	E	.050	.115	-.001
	NWN2	3.299	1.059	.306
	ETA	.338	.950	1.500
CASE 8	GAM	.793	1.609	.932
	E	.054	.111	.110
	NWN2	4.077	.983	.322

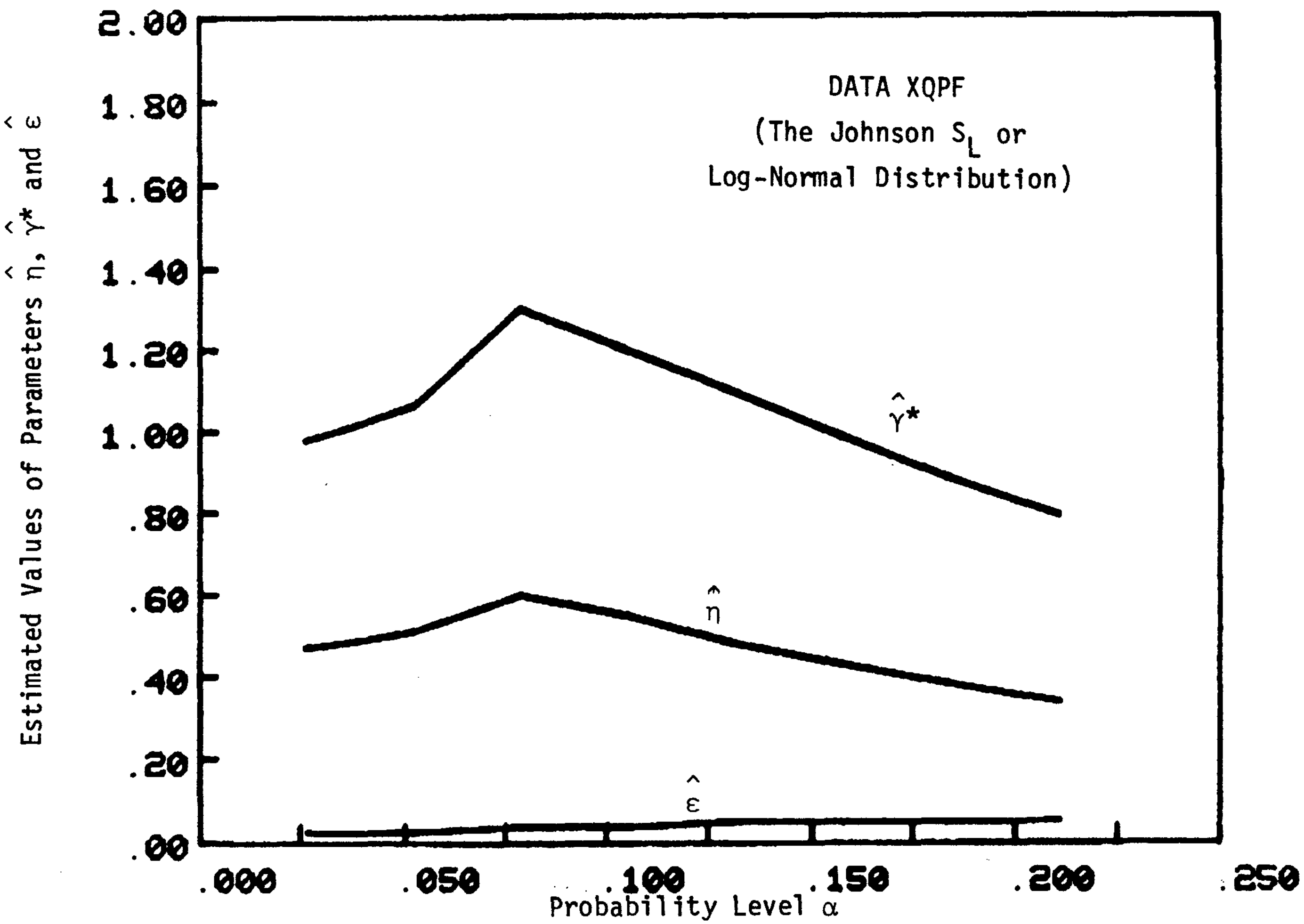


Fig. 12 Variation of Estimated Parameters $\hat{\eta}$, $\hat{\gamma}^*$ and $\hat{\varepsilon}$ for Data XQPF (When ε is Unknown)

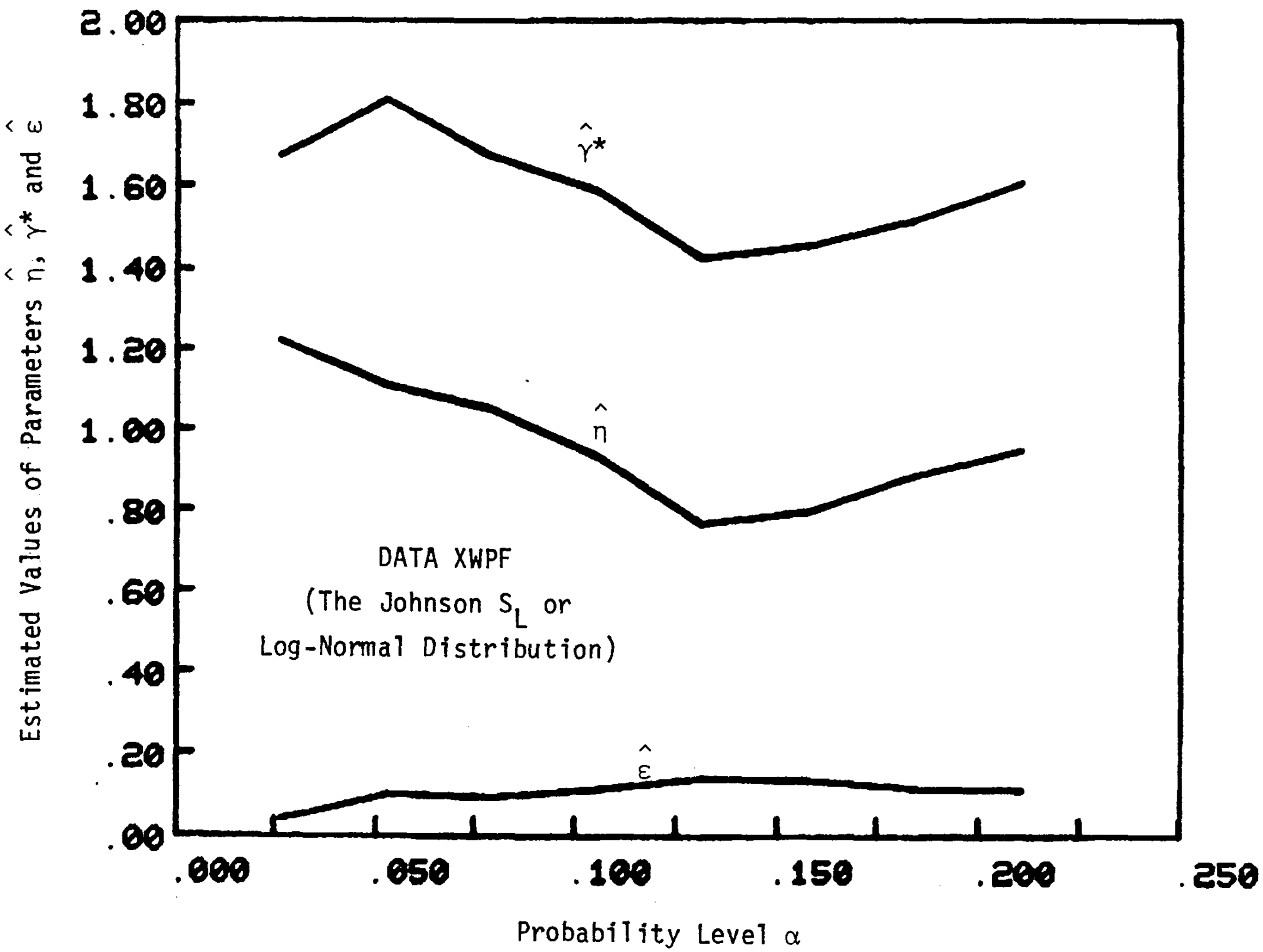


Fig. 13 Variation of Estimated Parameters $\hat{\eta}$, $\hat{\gamma}^*$ and $\hat{\varepsilon}$ for Data XWPF (When ε is Unknown)

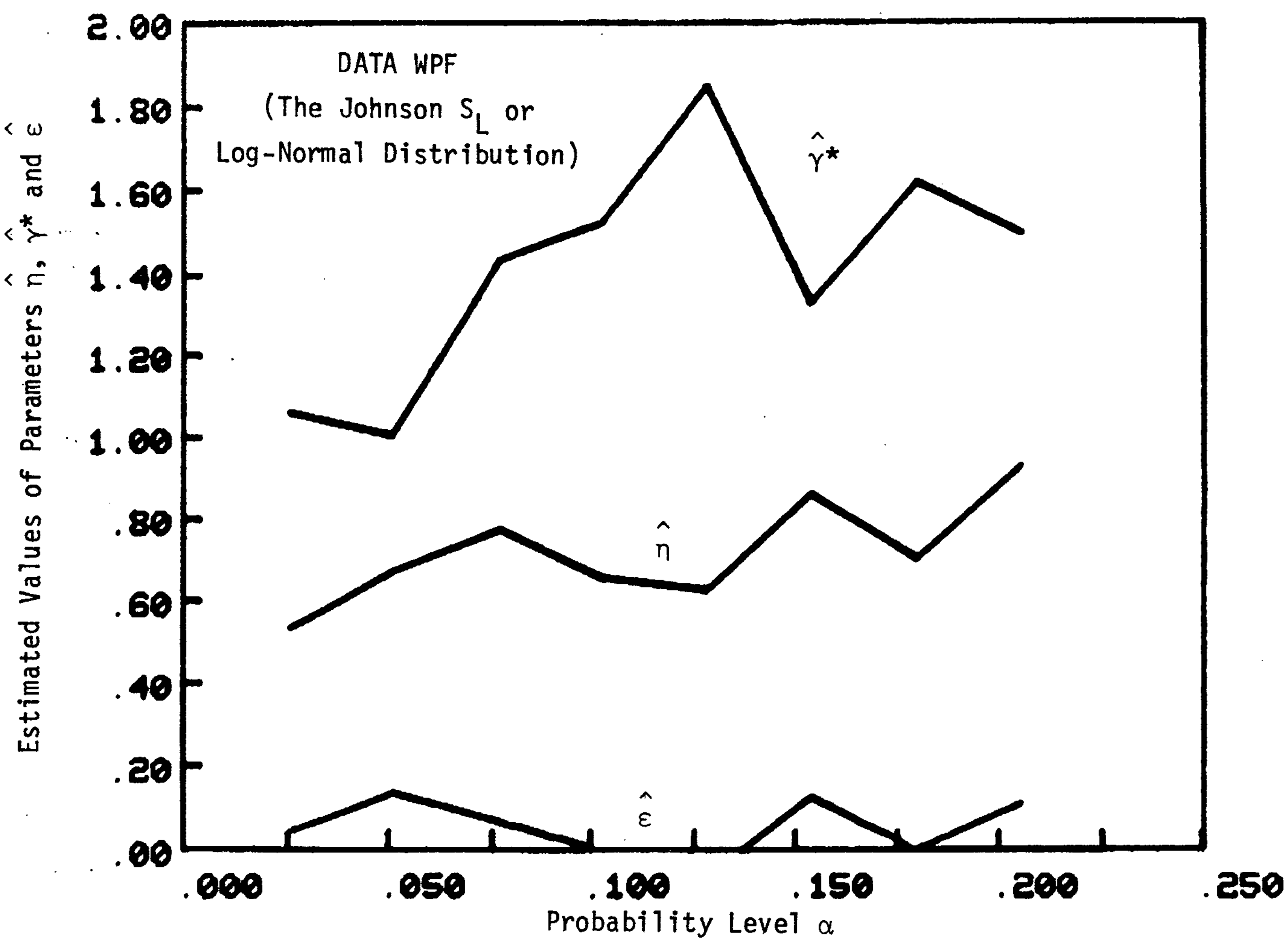


Fig. 14 Variation of Estimated Parameters $\hat{\eta}$, $\hat{\gamma}^*$ and $\hat{\varepsilon}$ for Data WPF
(When ε is Unknown)

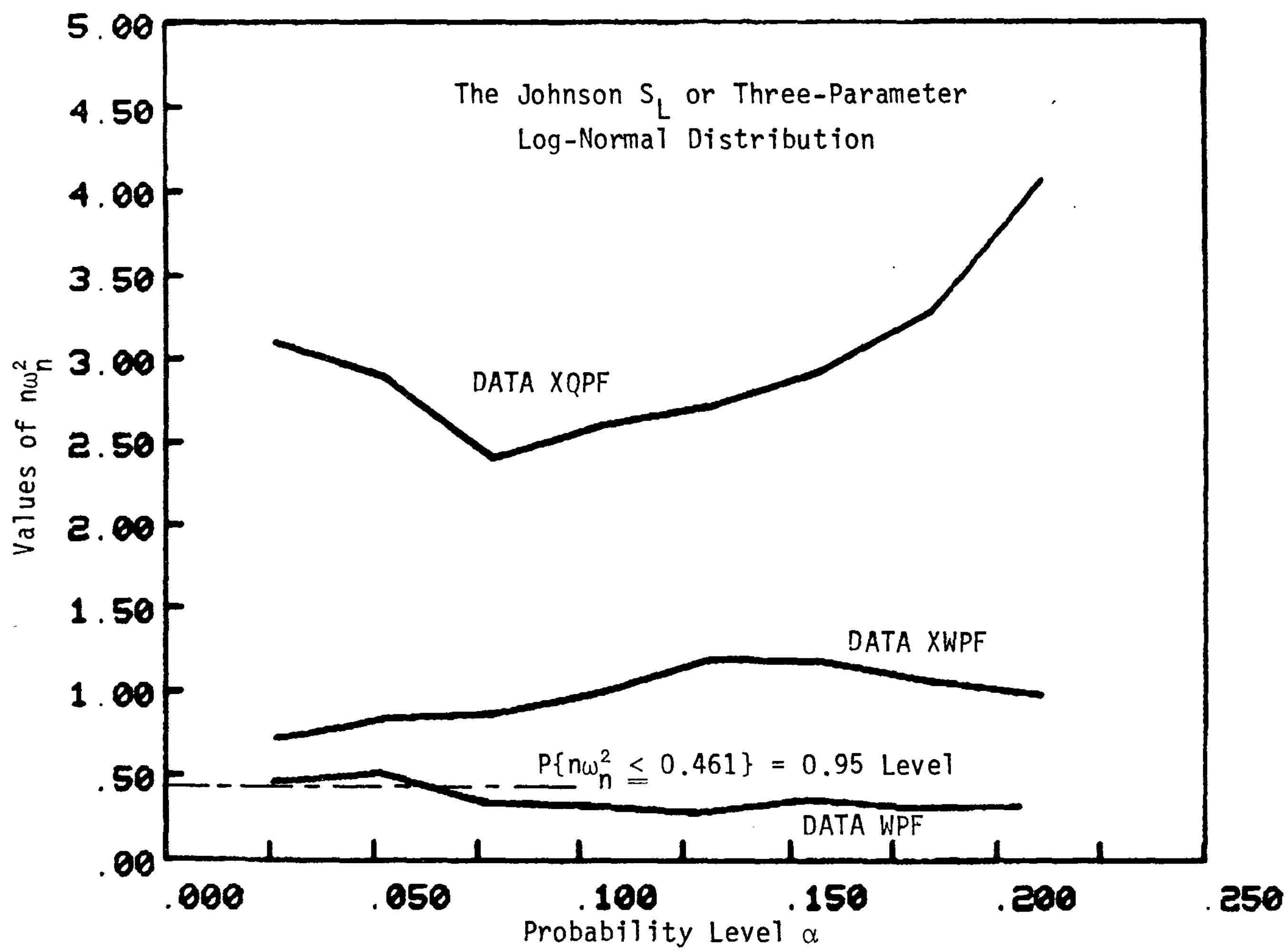


Fig. 15 Effect of the Probability Level α on the Goodness-of-Fit Test Statistic $n\omega_n^2$

Table 8 Lower and Upper Bounds X_L and X_U in mils Corresponding to μ_L and μ_U for Several Confidence Levels 1-α (The Johnson S_L Distribution; When ε is Assumed to be Unknown)

FOR DATA XQPF (N=37)

1-ALPHA	XL	XU
0.90	.1655	.2988
0.95	.1528	.3276
0.99	.1314	.3929

FOR DATA XWPF (N=37)

1-ALPHA	XL	XU
0.90	.2737	.3546
0.95	.2637	.3687
0.99	.2455	.3978

FOR DATA WPF (N=38)

1-ALPHA	XL	XU
0.90	.5673	.7425
0.95	.5450	.7721
0.99	.5039	.8325

Table 9 Best Estimates (The Johnson S_L Distribution With Unknown ε)

Data	$\hat{\eta}$	$\hat{\gamma}$*	$\hat{\varepsilon}$	$n\omega_n^2$ *	X_L**	X_U**
XQPF	.599	1.30	.036	2.41	.166	.299
XWPF	1.22	1.68	.041	.719	.274	.355
WPF	1.86	.624	-.067	.278	.567	.743

* 5% (10%) significance level = .461 (.347)

** 90% confidence bounds

3.3.2 The Johnson S_B Distribution

The Johnson S_B variable is limited between the lower bound ε and the upper bound $\varepsilon + \lambda$ as indicated in Eq. 24. Therefore, the following four possibilities arise with respect to the upper and lower bounds: (i) Both bounds are known, (ii) only the upper bound is known, (iii) only the lower bound is known and (iv) neither bound is known. In the present study, we formulate the procedures of parameter estimation assuming that either case (iii) or case (iv) will prevail, although actual estimations are performed only for case (iii). It is noted in this connection that the flaw size can never be negative and therefore the lower bound may be assumed to be zero, an assumption that generally produces a conservative result. While such an assumption offers a considerable mathematical convenience, there is no definite reason, physically or otherwise, to believe that the lower bound of the initial flaw size distribution must be equal to zero.

(a) <u>When ε is assumed to be known</u>

In this case, we estimate the parameters λ, η and γ assuming that $\varepsilon = x_{(1)} \cdot i/10$ $(i=0,1,\ldots,9)$ as was done when dealing with the S_L distribution. Following then the same method that produced Eqs. 47-49 from Eq. 42, we obtain the three equations below for the estimates $\hat{\lambda}$, $\hat{\eta}$ and $\hat{\gamma}$.

$$\left[\frac{(x_{0.5} - \varepsilon)(\varepsilon + \hat{\lambda} - x_{1-\alpha'})}{(\varepsilon + \hat{\lambda} - x_{0.5})(x_{1-\alpha'} - \varepsilon)}\right]^{-z_\alpha/z_{1-\alpha'}} = \frac{(x_\alpha - \varepsilon)(\varepsilon + \hat{\lambda} - x_{0.5})}{(\varepsilon + \hat{\lambda} - x\ )(x_{0.5} - \varepsilon)} \tag{50}$$

$$\hat{\eta} = (z_{1-\alpha'} - z_\alpha)/\ln \frac{(x_{1-\alpha'} - \varepsilon)(\varepsilon + \hat{\lambda} - x_\alpha)}{(x_\alpha - \varepsilon)(\varepsilon + \hat{\lambda} - x_{1-\alpha'})} \tag{51}$$

$$\hat{\gamma} = z_{1-\alpha'} - \hat{\eta}\ \ln \frac{x_{1-\alpha'} - \varepsilon}{\varepsilon + \hat{\lambda} - x_{1-\alpha'}} \tag{52}$$

where, as before, z_A and x_A are such that $z_A = \Phi(A)$ and $P\{X \leq x_A\} = A$ with $\Phi(\cdot)$ denoting the standardized normal distribution function. If $\alpha \neq \alpha'$, these three equations must be solved by trial and error. If, however, we take an identical value for α and α' ($\alpha = \alpha'$), then Eq. 50 can immediately be solved for $\hat{\lambda}$ producing

$$\hat{\lambda} = (x_{0.5} - \varepsilon)\{(x_{0.5} - \varepsilon)(x_\alpha - \varepsilon) + (x_{0.5} - \varepsilon)(x_{1-\alpha} - \varepsilon) - 2(x_\alpha - \varepsilon)(x_{1-\alpha} - \varepsilon)\}/\{(x_{0.5} - \varepsilon)^2 - (x_\alpha - \varepsilon)(x_{1-\alpha} - \varepsilon) \tag{53}$$

The estimates $\hat{\eta}$ and $\hat{\gamma}$ are then given by

$$\hat{\eta} = 2z_{1-\alpha}/[\ell n\{(x_{1-\alpha} - \varepsilon)(\varepsilon + \hat{\lambda} - x_\alpha)\} - \ell n\{(x_\alpha + \varepsilon)(\varepsilon + \hat{\lambda} - x_{1-\alpha})\}] \tag{54}$$

$$\hat{\gamma} = z_{1-\alpha} - \hat{\eta}\{\ell n(x_{1-\alpha} - \varepsilon) - \ell n(\varepsilon + \hat{\lambda} - x_{1-\alpha}) \tag{55}$$

We point out again that the estimated values of these parameters depend on the value of α and that the set of estimated values producing the smallest $n\omega_n^2$ value will be considered as the best estimate.

(b) <u>When neither upper nor lower bound is known</u>

It follows from Eqs. 11, 15 and 18 that the Johnson S_B variable X and the standardized normal variable Z are related by

$$Z = \gamma + \eta\ \ell n\{(X - \varepsilon)/(\varepsilon + \lambda - X)\} \tag{56}$$

With the aid of z_A and x_A, we then derive the following four equations from which the estimates $\hat{\varepsilon}$, $\hat{\lambda}$, $\hat{\gamma}$ and $\hat{\eta}$ can be solved.

$$z_A = \hat{\gamma} + \hat{\eta}\ \ell n\{(x_A - \hat{\varepsilon})/(\hat{\varepsilon} + \hat{\lambda} - x_A) \qquad (A=p,q,r \text{ and } u) \tag{57}$$

The similarity between Eqs. 56 and 57 and Eqs. 41 and 42 is obvious. As before, depending on the probability levels p, q, r and u to be used, different sets of estimates will be obtained.

However, we have not pursued this avenue of investigation since the preliminary result indicated that the fit of the observed data to the S_B distribution with four unknown parameters would probably not be particularly better, if not worse, than the fit to the same distribution with three unknown parameters. In this connection, we recall the S_L distribution result where considering ε as an unknown does not really improve the goodness-of-fit.

(c) Confidence Interval

Using the estimated parameters $\hat{\lambda}$, $\hat{\gamma}$, $\hat{\eta}$ and $\hat{\varepsilon}$ (ε in case it is assumed to be known), write the following equation

$$Z = \hat{\gamma} + \hat{\eta}\ \ell n\{(X - \hat{\varepsilon})/(\hat{\varepsilon} + \hat{\lambda} - X)\} \tag{58}$$

Since Z is the standardized normal variable, the unbiased estimates of its expected value μ and standard deviation σ are respectively given by

$$\mu = \sum_{i=1}^{n} z_i/n \tag{59}$$

$$s = [\{\sum_{i=1}^{n} (z_i - \hat{\mu})^2\}/(n - 1)]^{\frac{1}{2}} \tag{60}$$

where

$$z_i = \hat{\gamma} + \hat{\eta} \times \sum_{i=1}^{n} \ell n\{(x_i - \hat{\varepsilon})/(\hat{\varepsilon} + \hat{\lambda} - x_i)\} \tag{61}$$

The upper bound μ_U and the lower bound μ_L of the confidence interval of $\hat{\mu}$ can then be established on the basis of the Student's t distribution as in the case of the S_L distribution. Indeed, they are also given by Eq. 36. The corresponding bounds X_L and X_U are obtained from

$$X_L = \hat{\varepsilon} + \hat{\lambda}\ \ \exp\{(\mu_L - \hat{\gamma})/\hat{\eta}\}/[1 + \exp\{(\mu_L - \hat{\gamma})/\hat{\eta}\}] \tag{62}$$

$$X_U = \hat{\varepsilon} + \hat{\lambda}\ \ \exp\{(\mu_U - \hat{\gamma})/\hat{\eta}\}/[1 + \exp\{(\mu_U - \hat{\gamma})/\hat{\eta}\}] \tag{63}$$

(d) Results of estimation

Dealing with the case where ε is assumed to be known, we list in Table 10 the estimated values of λ, η and γ of the S_B distribution. In this table,

Table 10 Results of Estimation for the Johnson S_B Distribution When ε is Assumed to be Known

FOR DATA SERIES XWPF

CASE	E	ETA1	GAMMA1	RAM	NWN2
1	.0000	—	—	—	—
2	.0093	—	—	—	—
3	.0186	—	—	—	—
4	.0279	—	—	—	—
5	.0372	1.354	4.774	9.009	.664
6	.0465	1.260	3.454	4.103	.669
7	.0558	1.171	2.703	2.644	.675
8	.0651	1.088	2.182	1.939	.683
9	.0744	1.009	1.791	1.522	.692
10	.0837	.935	1.484	1.244	.703

FOR DATA SERIES WPF

CASE	E	ETA1	GAMMA1	RAM	NWN2
1	.0000	1.512	3.568	7.503	.283
2	.0140	1.447	3.143	6.187	.285
3	.0280	1.385	2.787	5.253	.287
4	.0420	1.324	2.484	4.555	.289
5	.0560	1.265	2.221	4.012	.291
6	.0700	1.281	2.832	5.838	.316
7	.0840	1.221	2.480	4.852	.319
8	.0980	1.163	2.185	4.140	.322
9	.1120	1.486	8.033	119.569	.322
10	.1260	1.407	4.907	17.577	.323

Case i indicates the use of $\varepsilon = x_{(1)} \cdot (i-1)/10$, and E is written for ε, ETA 1 for $\hat{\eta}$, GAMMA 1 for $\hat{\gamma}$, RAM for $\hat{\lambda}$ and NWN2 for $n\omega_n^2$. As mentioned earlier, the S_B variable is limited between ε and $\varepsilon + \lambda$ ($\lambda > 0$). We have found, however, that the estimation procedure used here produces negative values for the estimate $\hat{\lambda}$ depending on the assumed values of ε. For example, the values of the estimate $\hat{\lambda}$ have turned out to be negative for all ten cases of ε with respect to XQPF data and for the first four cases with respect to XWPF data. If an assumed value of ε, say ε_0, produces a negative $\hat{\lambda}$, we interpret that to be an indication of the unacceptability of the S_B distribution with $\varepsilon = \varepsilon_0$ for the data considered. Therefore, the results of the estimation for XQPF data have not been listed in Table 10. Also, the results for the first four cases of ε values with respect to XWPF data have been indicated by bars (-).

Based on the $n\omega_n^2$ values listed in Table 10, we conclude that the best estimates are obtained from Case 5 for XWPF:

$$\varepsilon = 0.0372, \ \hat{\lambda} = 9.009, \ \hat{\eta} = 1.354, \ \hat{\gamma} = 4.774$$

and from Case 1 for WPF:

$$\varepsilon = 0.000, \ \hat{\lambda} = 7.503, \ \hat{\eta} = 1.512, \ \hat{\gamma} = 3.568$$

Figures 16 and 17 plot the values of estimates $\hat{\lambda}$, $\hat{\eta}$ and $\hat{\gamma}$ as functions of ε, respectively for XWPF and WPF. Fig. 18 plots $n\omega_n^2$ values as functions of ε for both XWPF and WPF. This figure shows that the S_B distribution can be accepted for WPF data but not for XWPF data if the significance level of 5% is assumed. Table 11 lists the values of X_L and X_U for XWPF and WPF with the aid of Eqs. 62 and 63. These results are summarized in Table 12.

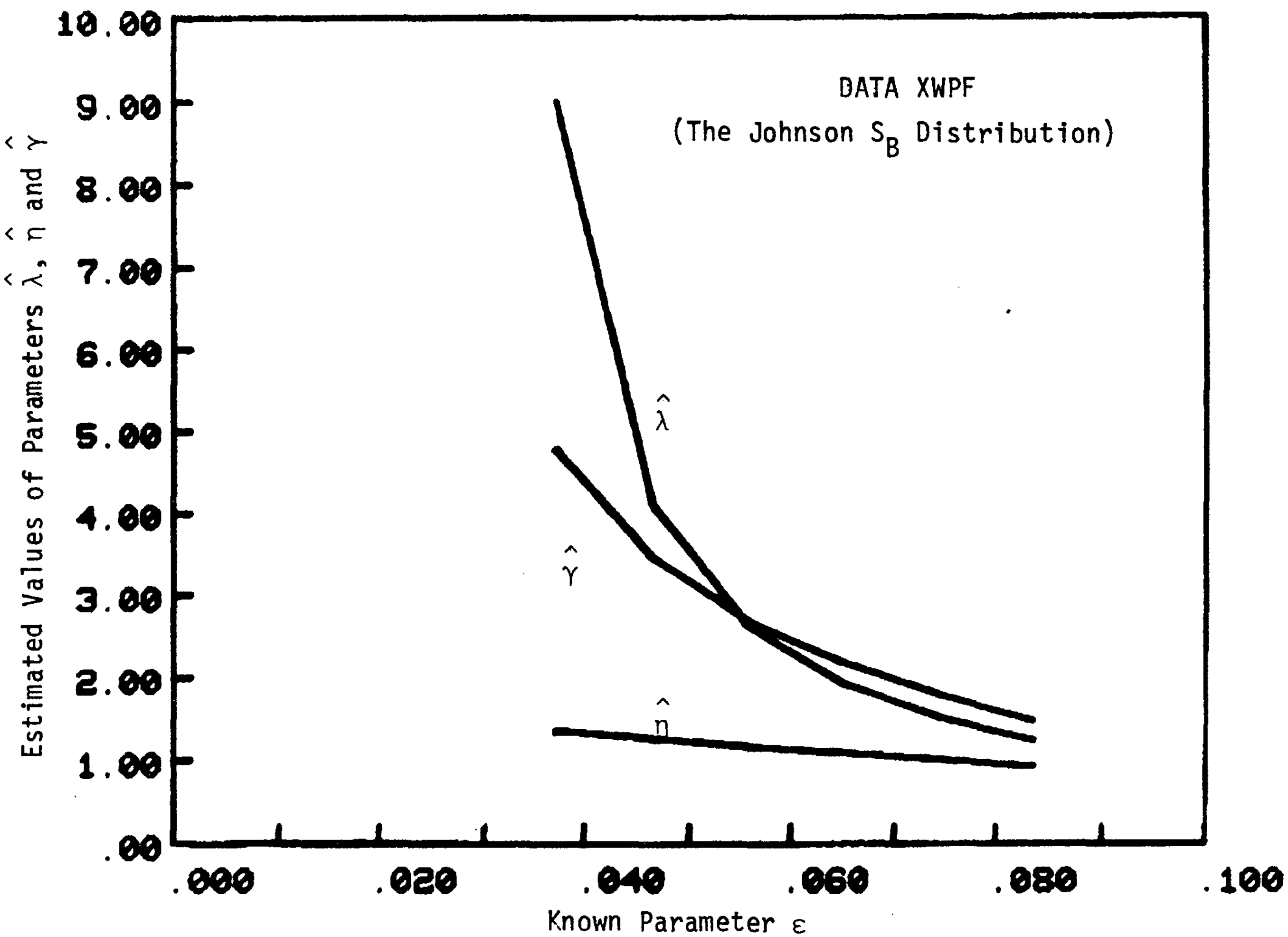


Fig. 16 Variation of Estimated Parameters $\hat{\lambda}$, $\hat{\eta}$ and $\hat{\gamma}$ in the Johnson S_B Distribution for Data XWPF (When ε is Known)

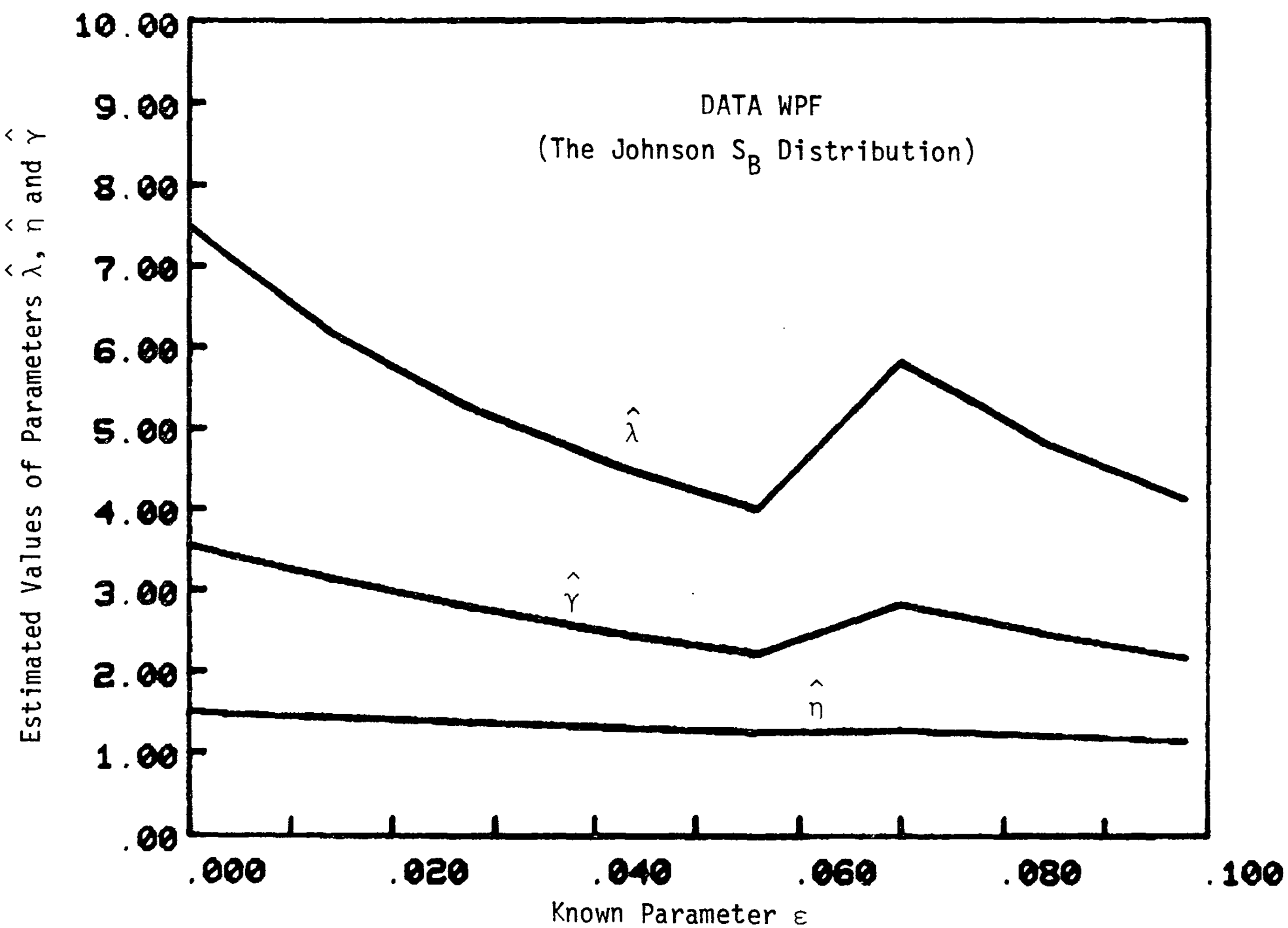


Fig. 17 Variation of Estimated Parameters $\hat{\lambda}$, $\hat{\eta}$ and $\hat{\gamma}$ in the Johnson S_B Distribution for Data WPF (When ε is Known)

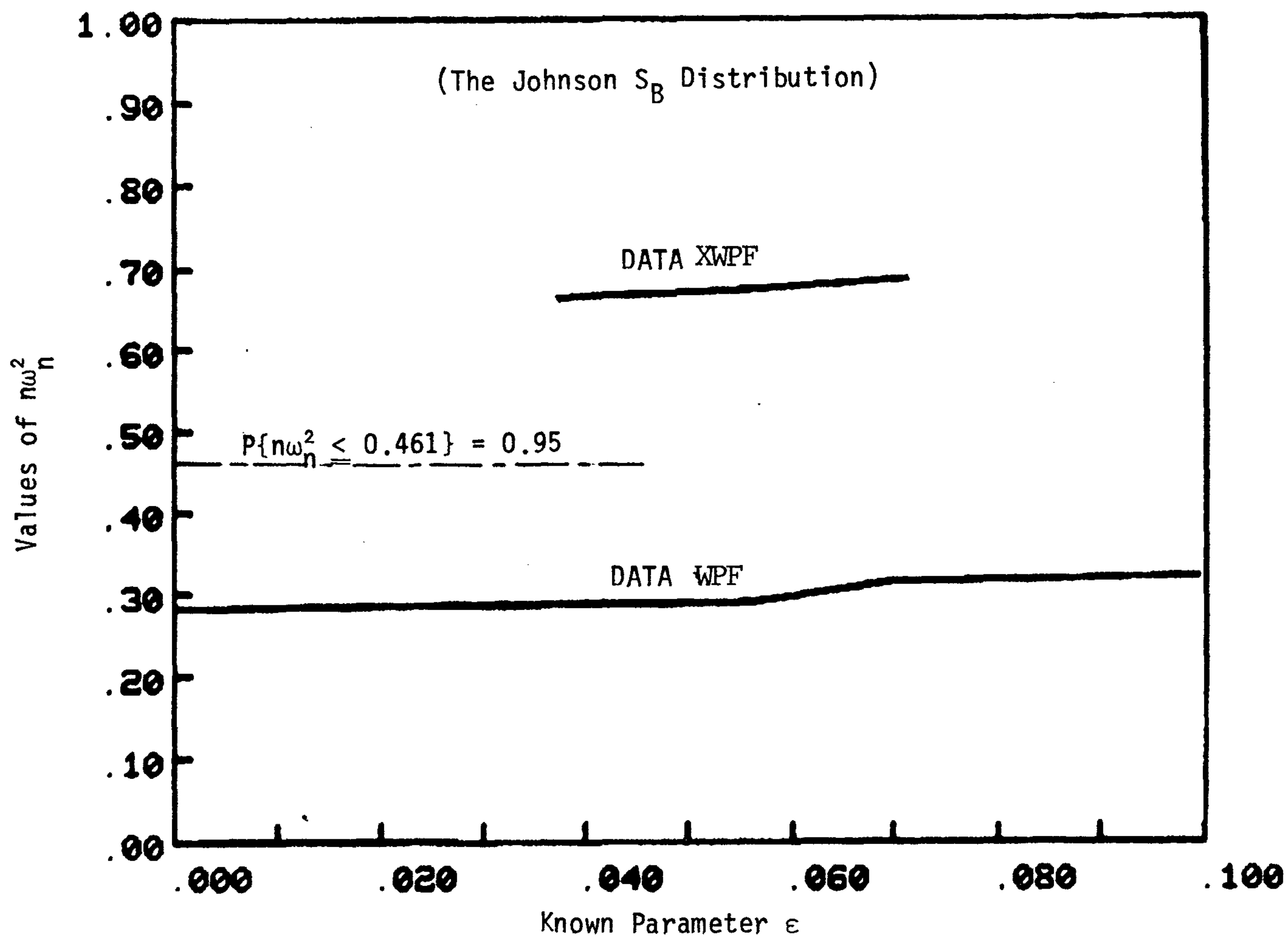


Fig. 18 Effect of the Assumed Parameter ε on the Goodness-of-Fit Test Statistic $n\omega_n^2$ for the Johnson S_B Distribution

Table 11 Lower and Upper Bounds X_L and X_U in mils Corresponding to μ_L and μ_U for Several Confidence Levels 1-α (The Johnson S_B Distribution; When ε is Assumed to be Known)

FOR DATA XQPF (N=37)

1-ALPHA	XL	XU
0.90	.9587	1.0314
0.95	.9485	1.0425
0.99	.9289	1.0645

FOR DATA XWPF (N=37)

1-ALPHA	XL	XU
0.90	.5270	.8752
0.95	.4892	.9429
0.99	.4232	1.0898

FOR DATA WPF (N=38)

1-ALPHA	XL	XU
0.90	.5613	.9586
0.95	.5189	1.0370
0.99	.4455	1.2079

Table 12 Best Estimates (The Johnson S_B Distribution)

Data	$\hat{\lambda}$	$\hat{\eta}$	$\hat{\gamma}$	ε	$n\omega_n^2$*	X_L**	X_U**
XQPF	$\hat{\lambda}$ is negative; unacceptable						
XWPF	9.01	1.35	4.77	.0372	.664	.527	.875
WPF	7.50	1.51	3.57	.00	.283	.561	.959

* 5% (10%) significance level = .461 (.347)

** 90% confidence bounds

SECTION IV

FITTING TO THE WEIBULL DISTRIBUTION FUNCTION

An attempt is made to fit the data shown in Table 1 to the Weibull distribution. For the present investigation, we consider the following three-parameter Weibull distribution.

$$F_X(x) = 1 - \exp\{-[(x - \varepsilon)/\lambda]^{\eta}\} \qquad \text{for } x \geqq \varepsilon \quad . \tag{64}$$

where η = shape parameter, λ = scale parameter and ε = location parameter.

It is often a practice to use the maximum likelihood method to estimate these parameters. Its use for the three-parameter Weibull distribution, however, not only creates numerical problems requiring possibly highly expensive iterative procedures but also may produce an awkward result in which the estimated location parameter ε is larger than the smallest observation $x_{(1)}$.

In the present study, therefore, a convenient curve fitting procedure described below is used in conjunction with the least square method under the assumption that the location parameter is known.

Transforming Eq. 64 into the form

$$\ell n \ell n\{1/(1 - F_X(x))\} = \eta\{\ell n(x - \varepsilon) - \ell n\lambda\} \tag{65}$$

and setting

$$\left.\begin{aligned} y &= \ell n \ell n\{1/(1 - F_X(x))\} \\ u &= \ell n(x - \varepsilon) \\ \lambda^* &= -\eta \ell n \lambda \end{aligned}\right\} \tag{66}$$

we can reduce Eq. 64 into

$$y = \eta u + \lambda^* \tag{67}$$

which is a linear relationship between y and u. Using the observations $x_{(i)}$ and corresponding cumulative probabilities $F(x_{(i)}) = i/(1 + n)$ listed in Table 1, we can compute u_i and y_i which are the values of u and y in Eq. 66 with x replaced by $x_{(i)}$. These u_i and y_i are used to construct the square error E^2 of the form

$$E^2 = \sum_{i=1}^{n} \{y_i - (\eta u_i + \lambda^*)\}^2 \tag{68}$$

The parameter values $\hat{\eta}$ and $\hat{\lambda}^*$ which produce the least value of E^2 are then chosen as our estimates. Explicit expressions for these estimates can be obtained by solving the equations $\partial E^2/\partial\eta = \partial E^2/\partial\lambda^* = 0$ as

$$\hat{\eta} = \{n \sum_{i=1}^{n} u_i y_i - (\sum_{i=1}^{n} u_i)(\sum_{i=1}^{n} y_i)\}/\{n \sum_{i=1}^{n} u_i^2 - (\sum_{i=1}^{n} u_i)^2\} \tag{69}$$

$$\hat{\lambda}^* = \{(\sum_{i=1}^{n} u_i^2)(\sum_{i=1}^{n} y_i) - (\sum_{i=1}^{n} u_i)(\sum_{i=1}^{n} u_i y_i)\}/\{n \sum_{i=1}^{n} u_i^2 - (\sum_{i=1}^{n} u_i)^2\} \tag{70}$$

The scale parameter λ is then estimated from Eq. 66 as

$$\hat{\lambda} = \exp\,(-\hat{\lambda}^*/\hat{\eta}) \tag{71}$$

The results of such estimations are presented in Tables 13, 14 and 15 respectively for the samples XQPF, XWPF and WPF. In deriving these results, we have assumed that the location parameter $\varepsilon = 0,\ 0.2x_{(1)},\ 0.4x_{(1)},\ 0.6x_{(1)}$ and $0.8x_{(1)}$. Figs. 19, 20 and 21 plot y_i against u_i (although the probability and the logarithmic scales are used) respectively for XQPF, XWPF and WPF. Each data point in these figures represents $x_i - \varepsilon$ along the abscissa and $i/(1 + n)$ along the ordinate. Therefore, at each probability level $i/(1 + n)$, we see five points corresponding to five different values of ε. For each assumed value of ε, we evaluate the value of $n\omega_n^2$ from Eq. 6 using the corresponding estimates $\hat{\eta}$ and $\hat{\lambda}$. In Tables 13, 14, and 15, we also list the values of least square error as well as the values of $n\omega_n^2$. Both of these values decrease as ε increases. This suggests that the goodness-of-fit is more satisfactory if we assume larger ε values. However, it is apparent from Fig. 2 as well as Table 2 that all the Weibull distributions

Table 13 Statistical Analysis of Data XQPF for Weibull Distribution

```
ANALYSIS OF EQUIVALENT INITIAL FLAW SIZE DISTRIBUTION
IN THE CASE OF WEIBULL DISTRIBUTION
--------------------
FOR DATA SERIES XQPF
--------------------


WEIBULL SHAPE    =  .76818E+00
WEIBULL SCALE    =  .44256E+00
WEIBULL LOCATION =  .00000E+00
LEAST SQUARE ERROR =  .72664E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .24760E+00


WEIBULL SHAPE    =  .75091E+00
WEIBULL SCALE    =  .42824E+00
WEIBULL LOCATION =  .52000E-02
LEAST SQUARE ERROR =  .66314E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .23008E+00


WEIBULL SHAPE    =  .73159E+00
WEIBULL SCALE    =  .41314E+00
WEIBULL LOCATION =  .10400E-01
LEAST SQUARE ERROR =  .58844E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .21037E+00


WEIBULL SHAPE    =  .70878E+00
WEIBULL SCALE    =  .39705E+00
WEIBULL LOCATION =  .15600E-01
LEAST SQUARE ERROR =  .49732E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .18742E+00


WEIBULL SHAPE    =  .67785E+00
WEIBULL SCALE    =  .37988E+00
WEIBULL LOCATION =  .20800E-01
LEAST SQUARE ERROR =  .37987E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .15869E+00
```

Table 14 Statistical Analysis of Data XWPF for Weibull Distribution

```
ANALYSIS OF EQUIVALENT INITIAL FLAW SIZE DISTRIBUTION
IN THE CASE OF WEIBULL DISTRIBUTION
--------------------
FOR DATA SERIES XWPF
--------------------

WEIBULL SHAPE    =  .17920E+01
WEIBULL SCALE    =  .43272E+00
WEIBULL LOCATION =  .00000E+00
LEAST SQUARE ERROR =  .50286E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .23084E+00

WEIBULL SHAPE    =  .16945E+01
WEIBULL SCALE    =  .40981E+00
WEIBULL LOCATION =  .18600E-01
LEAST SQUARE ERROR =  .44298E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .20895E+00

WEIBULL SHAPE    =  .15891E+01
WEIBULL SCALE    =  .38653E+00
WEIBULL LOCATION =  .37200E-01
LEAST SQUARE ERROR =  .37610E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .18437E+00

WEIBULL SHAPE    =  .14695E+01
WEIBULL SCALE    =  .36297E+00
WEIBULL LOCATION =  .55800E-01
LEAST SQUARE ERROR =  .30437E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .15642E+00

WEIBULL SHAPE    =  .13150E+01
WEIBULL SCALE    =  .34009E+00
WEIBULL LOCATION =  .74400E-01
LEAST SQUARE ERROR =  .24880E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .12513E+00
```

Table 15 Statistical Analysis of Data WPF for Weibull Distribution

```
ANALYSIS OF EQUIVALENT INITIAL FLAW SIZE DISTRIBUTION
IN THE CASE OF WEIBULL DISTRIBUTION
-------------------
FOR DATA SERIES WPF
-------------------


WEIBULL SHAPE    =  .16945E+01
WEIBULL SCALE    =  .87823E+00
WEIBULL LOCATION =  .00000E+00
LEAST SQUARE ERROR =  .57231E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .30361E+00


WEIBULL SHAPE    =  .16208E+01
WEIBULL SCALE    =  .84345E+00
WEIBULL LOCATION =  .28000E-01
LEAST SQUARE ERROR =  .51375E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .28299E+00


WEIBULL SHAPE    =  .15373E+01
WEIBULL SCALE    =  .80884E+00
WEIBULL LOCATION =  .56000E-01
LEAST SQUARE ERROR =  .45602E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .26271E+00


WEIBULL SHAPE    =  .14362E+01
WEIBULL SCALE    =  .77523E+00
WEIBULL LOCATION =  .84000E-01
LEAST SQUARE ERROR =  .40810E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .24514E+00


WEIBULL SHAPE    =  .12923E+01
WEIBULL SCALE    =  .74648E+00
WEIBULL LOCATION =  .11200E+00
LEAST SQUARE ERROR =  .40969E+01
GOODNESS-OF-FIT TEST STATISTICS NWN2=  .24091E+00
```

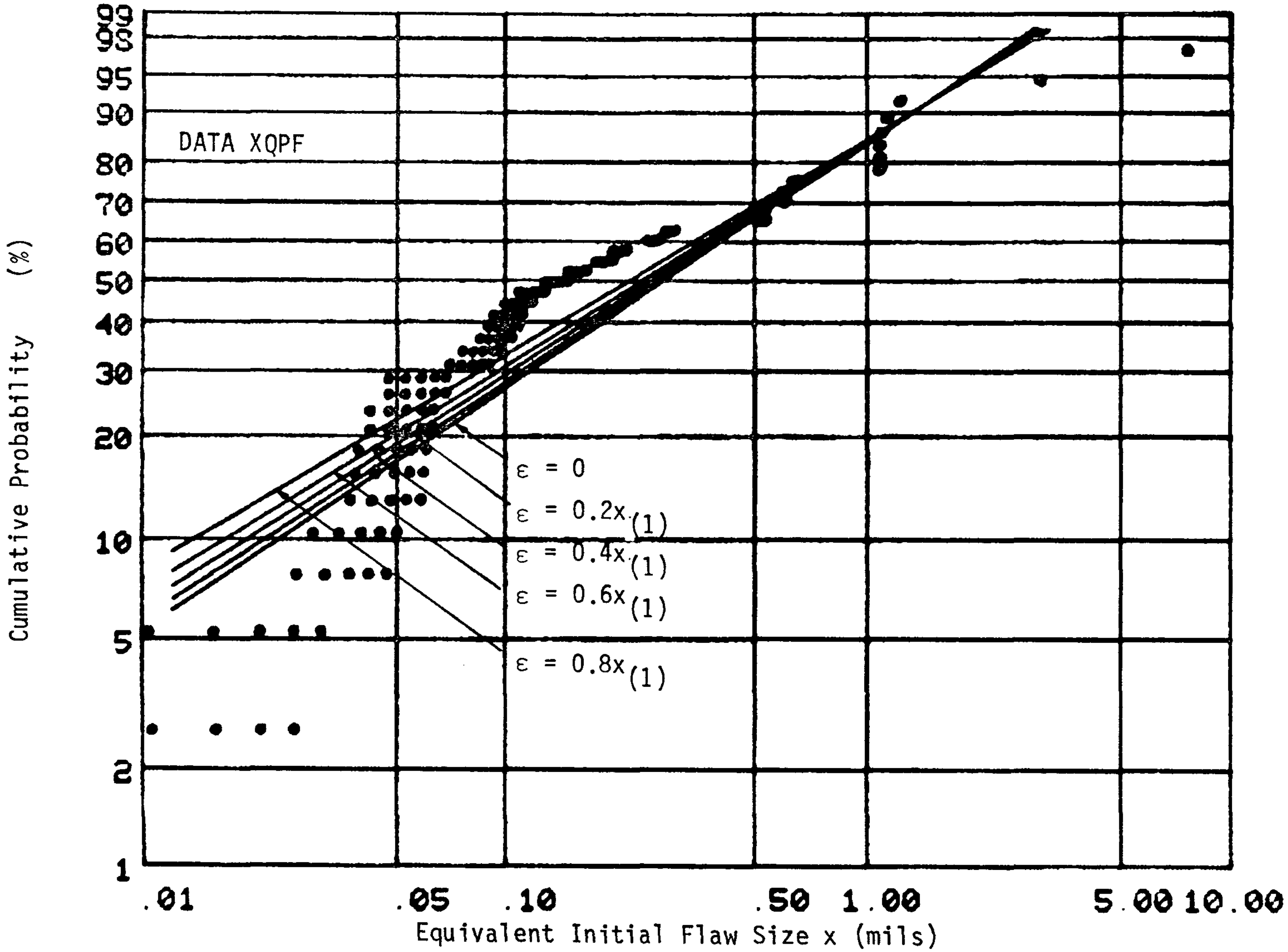


Fig. 19 Weibull Probability Plots for Data XQPF

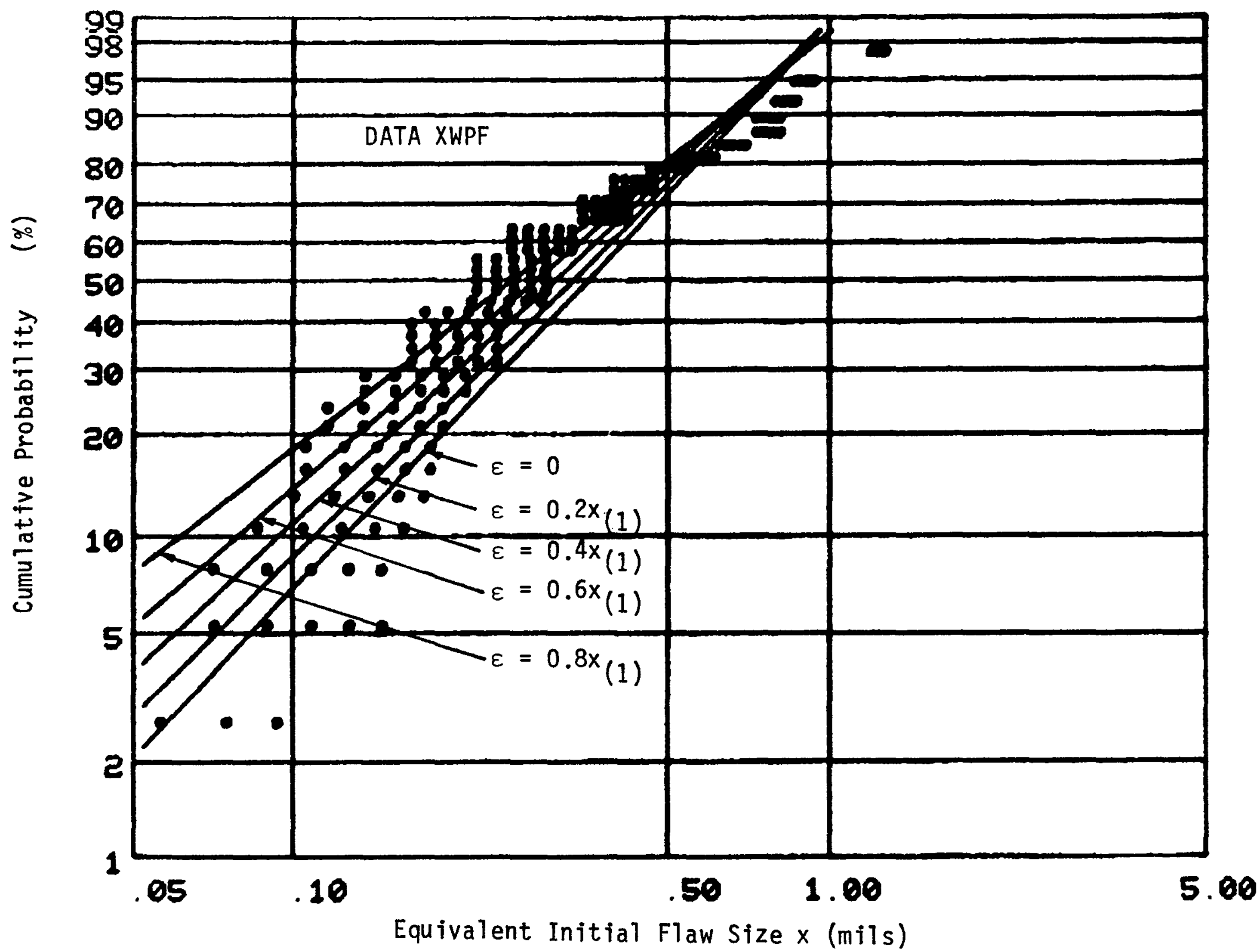


Fig. 20 Weibull Probability Plots for Data XWPF

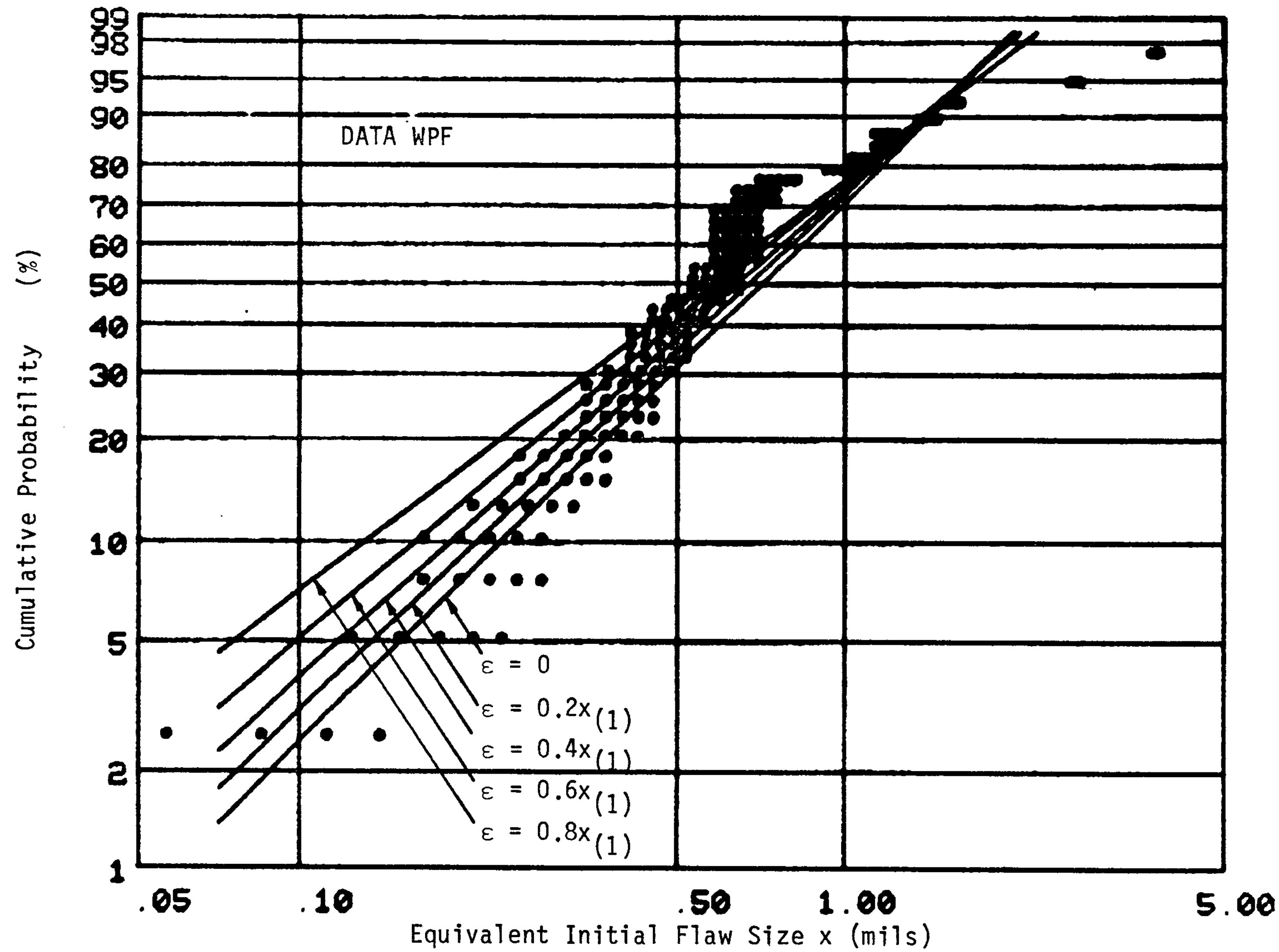


Fig. 21 Weibull Probability Plots for Data WPF

with estimated shape and scale parameter values and with assumed location parameter values (including those with $\varepsilon = 0$) can be accepted as the population distribution if the significance level α (for rejection) is 0.10. Indeed, all but one (WPF with $\varepsilon = 0$) can be accepted even under $\alpha = 0.15$. These results are summarized in Table 16.

Table 16 Best Estimates (The Weibull Distribution)

Data	$\hat{\eta}$	$\hat{\lambda}$	$n\omega_n^{2}$*	E++	ε
XQPF	.768	.443	.248	7.27	0**
	.678	.380	.159	3.80	.0208+
XWPF	1.79	.433	.231	5.03	0**
	1.32	.340	.125	2.49	.0744+
WPF	1.69	.878	.304	5.72	0**
	1.29	.746	.241	4.10	.112+

* 5% significance level = .461
10% significance level = .347

** Assumed minimum value of ε
\+ Assumed maximum value of ε
++ Least square error

SECTION V

FITTING TO THE PEARSON DISTRIBUTION FAMILY

5.1 The Pearson Distribution Family

The probability distribution functions with the following twelve types of density belong to the Pearson family [5, 6, 7].

Type	Density	Origin	Range	
I	$f(x) = c\,x^p(1 - x/a)^q$	$x = 0$	$0 \leq x \leq a$	(72)
II	$f(x) = c\{1 - (x/a)^2\}^p$	Mean(=Mode)	$-a \leq x \leq a$	(73)
III	$f(x) = c\,x^p \exp(-x/a)$	$x = 0$	$0 \leq x < \infty$	(74)
IV	$f(x) = c\{1 + (x/a)^2\}^{-p} \times \exp\{-b \tan^{-1}(x/a)\}$	Mean $+ \frac{1}{2}ab/(p-1)$	$-\infty < x < \infty$	(75)
V	$f(x) = c\,x^{-p} \exp(-a/x)$	$x = 0$	$0 \leq x < \infty$	(76)
VI	$f(x) = c\,x^q(1 + x/a)^{-p}$	$x = 0$	$0 \leq x < \infty$	(77)
VII	$f(x) = c\{1 + (x/a)^2\}^{-p}$	Mean(=Mode)	$-\infty < x < \infty$	(78)
VIII	$f(x) = c(1 + x/a)^{-p}$	$x = 0$	$-a \leq x \leq 0$	(79)
IX	$f(x) = c(1 + x/a)^p$	$x = 0$	$-a \leq x \leq 0$	(80)
X	$f(x) = c \exp(-x/\sigma)$	$x = 0$	$0 \leq x < \infty$	(81)
XI	$f(x) = c\,x^{-p}$	$x = b$	$b \leq x < \infty$	(82)
XII	$f(x) = c\left[\dfrac{\sigma(\sqrt{3+\beta_1} + \sqrt{\beta_1}) + x}{\sigma(\sqrt{3+\beta_1} - \sqrt{\beta_1} - x}\right]^{\sqrt{\beta_1}/(3+\beta_1)}$	Mean	$-\sigma(\sqrt{3+\beta_1} + \sqrt{\beta_1}) \leq x \leq \sigma(\sqrt{3+\beta_1} - \sqrt{\beta_1})$	(83)

Each of these distribution functions is identified either as a point, a curve (possibly a straight line) or a region in the β_1 and β_2 plane as shown in Fig. 22 which is constructed by extending the work presented in [5, 6, 7]. Fig. 22 plots the (b_1, b_2) points for the data XQPF, XWPF and WPF and suggests that the Pearson distribution function of Type I is supposed to represent all these data well.

Following the procedures suggested in [5, 6, 7], four parameters c, a, p and q of Type I written in the form

$$f(x) = c\, x^p (a - x)^q \tag{84}$$

are estimated as

$$a = \tfrac{1}{2}(\mu_2 r_2)^{\frac{1}{2}} \tag{85}$$

$$\begin{matrix} p \\ q \end{matrix} = \tfrac{1}{2}\{r_1 - 2 \mp r_1(r_1 + 2)(\beta_1/r_2)^{\frac{1}{2}}\} \tag{86}$$

$$c = a^{-(p+q+1)}\Gamma(p + q + 2)/\{\Gamma(p + 1)\Gamma(q + 1)\} \tag{87}$$

where

$$r_1 = 6(\beta_2 - \beta_1 - 1)/(6 + 3\beta_1 - 2\beta_2) \tag{88}$$

$$r_2 = \beta_1(r_1 + 2)^2 + 16(r_1 + 1) \tag{89}$$

and μ_2, β_1 and β_2 are given by Eqs. 1 and 2 while $\Gamma(\cdot)$ indicates the gamma function.

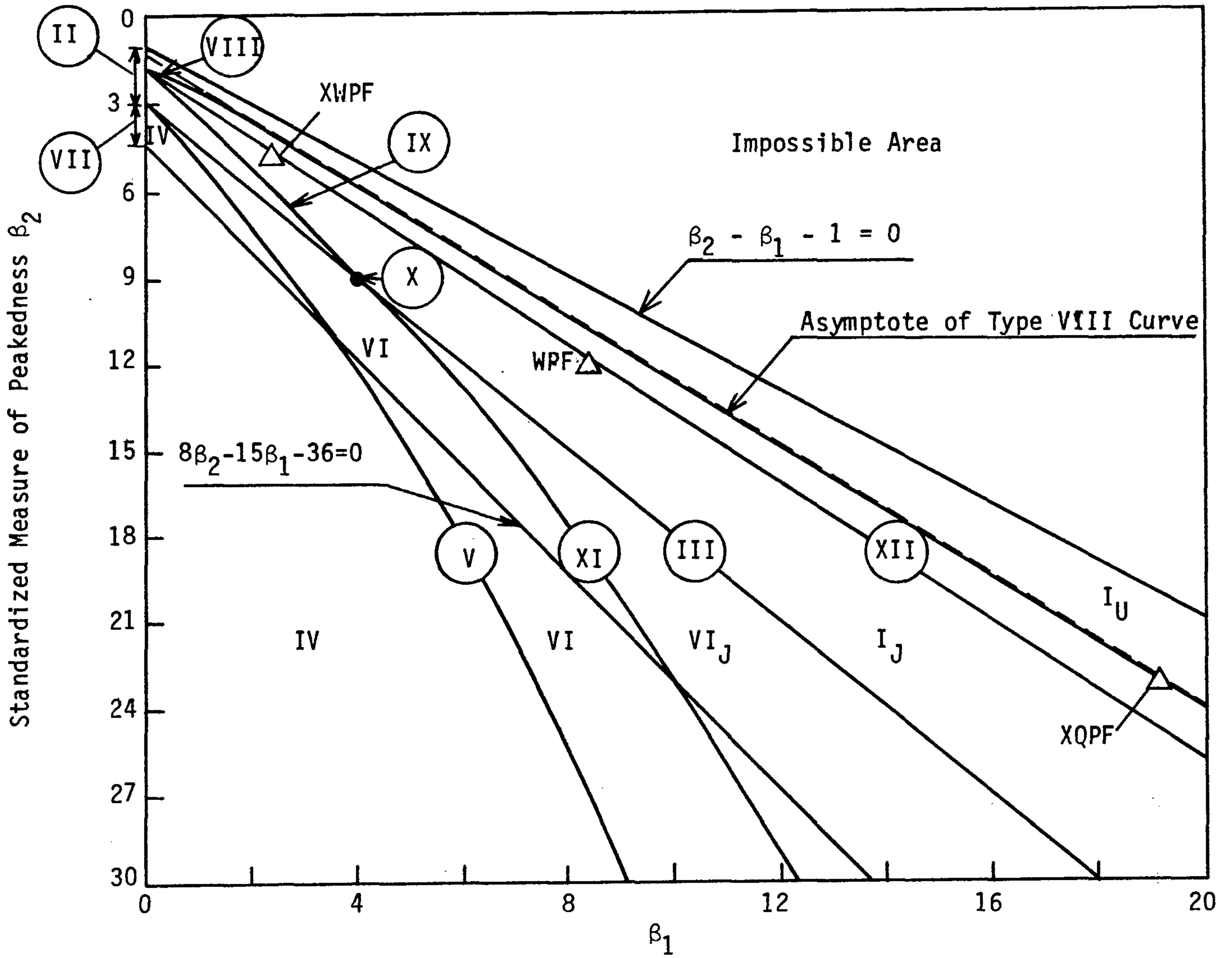
5.2 Results of Estimation

The parameters a, p, q and c of the Type I distribution function are estimated and used in Eq. 84 resulting in the following density functions.

For XQPF, $$f(x) = 0.0358x^{-0.956}(9.65 - x)^{0.0503} \tag{90}$$

For XWPF, $$f(x) = 0.351x^{-0.562}(1.44 - x)^{1.43} \tag{91}$$

For WPF , $$f(x) = 0.0187x^{-0.835}(5.07 - x)^{1.29} \tag{92}$$



Square of the Standardized Measure of Skewness

Fig. 22 Estimated Result of (β_1, β_2) for Data XQPF, XWPF and WPF (Pearson Distribution Family)

Unfortunately, the resulting values of $n\omega_n^2$ are large: 6.64 for XQPF, 3.02 for XWPF and 6.03 for WPF. These large values of $n\omega_n^2$ clearly indicate that the Pearson distribution of Type I cannot apply to either of those data. These density functions are plotted in Figs. 23-25 together with the corresponding normalized histograms for comparison.

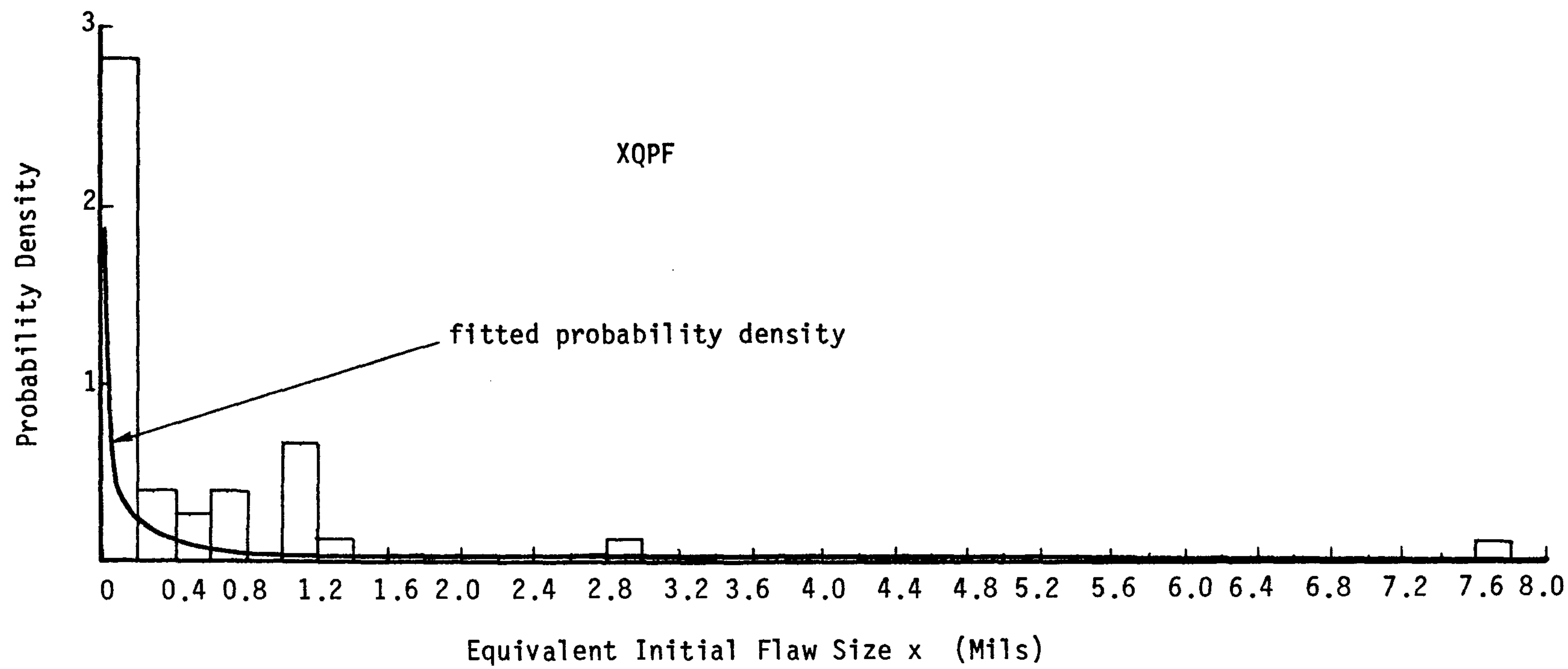


Fig. 23 Pearson Type I Fit for XQPF Data

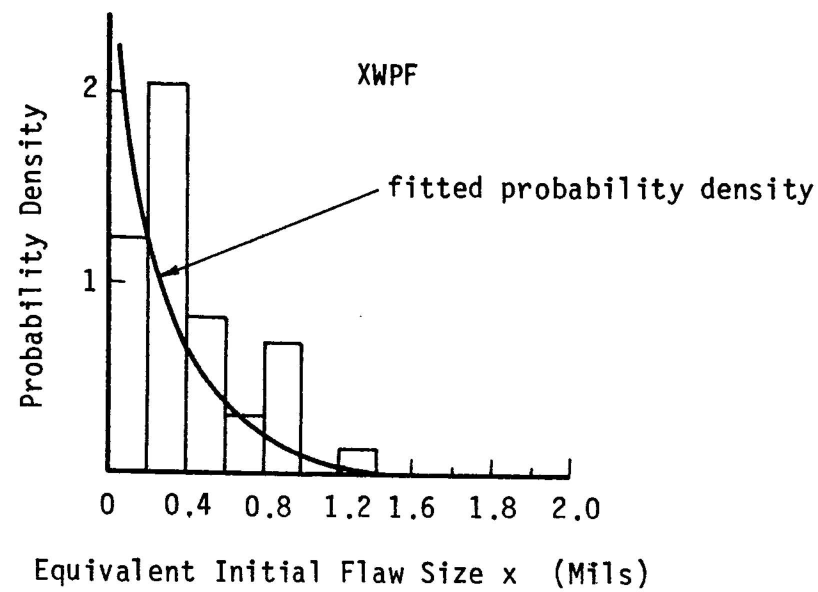


Fig. 24 Pearson Type I Fit for XWPF Data

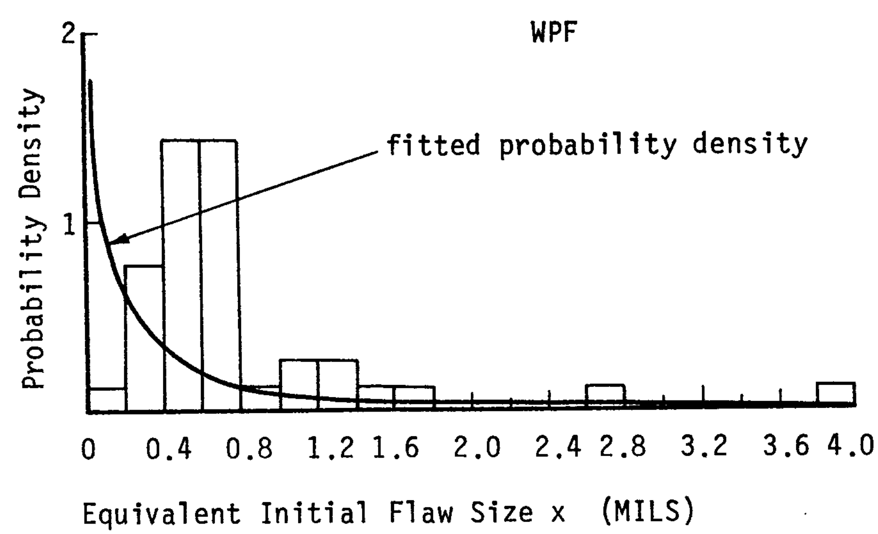


Fig. 25 Pearson Type I Fit for WPF Data

SECTION VI

FITTING TO THE ASYMPTOTIC DISTRIBUTIONS OF LARGEST VALUES

6.1 Asymptotic Distributions of Largest Values

The following three asymptotic distributions of largest values are tested for their goodness-of-fit with respect to the observed EIFS data.

(a) The First Asymptote

$$F_X(x) = e^{-e^{-\alpha_1 x - \beta_1}} \qquad -\infty < x < \infty \tag{93}$$

where α_1(> 0) and β_1 are the parameters to be estimated.

(b) The Second Asymptote

$$F_X(x) = \exp[-(x/x_2)^{-\alpha_2}] \qquad 0 < x \tag{94}$$

in which α_2 (> 0) and x_2 (> 0) are the parameters.

(c) The Third Asymptote

$$F_X(x) = \exp\left[-\left(\frac{w - x}{x_3}\right)^{\alpha_3}\right] \tag{95}$$

with the parameters α_3 (> 0), x_3 (> 0) and w(> 0).

For each of these asymptotic distribution functions, we can find the appropriate transformations of the probability scale and of the random variate so that the relationship between the transformed is linear, although the upper bound w in the case of the third asymptotic distribution must be known.

6.2 The First Asymptote

Using a procedure similar to that for the Weibull distribution fit, transform Eq. 93 into the form

$$y = -\alpha_1 x - \beta_1 \tag{96}$$

where

$$y = \ell n \ell n\{1/F_X(x)\} \tag{97}$$

Using the observations $x_{(i)}$ and the corresponding cumulative probabilities $F(x_{(i)}) = i/(1 + n)$ as listed in Table 1, we can construct $y_{(i)}$ from Eq. 97 as

$$y_{(i)} = \ell n \ell n\{(1 + n)/i\} \tag{98}$$

and construct the square error E^2 as

$$E^2 = \sum_{i=1}^{n} \{y_{(i)} - (-\alpha_1 x_{(i)} - \beta_1)\}^2 \tag{99}$$

The parameter values $\hat{\alpha}_1$ and $\hat{\beta}_1$ which produce the least value of E^2 are chosen as our best estimates. Explicit expressions for these estimates can be obtained by solving the equations $\partial E^2/\partial\alpha_1 = \partial E^2/\partial\beta_1 = 0$ and are given by

$$\hat{\alpha}_1 = -\{n \sum_{i=1}^{n} x_i y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)\}/\{n \sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2\} \tag{100}$$

$$\hat{\beta}_1 = -\{(\sum_{i=1}^{n} x_i^2)(\sum_{i=1}^{n} y_i) - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} x_i y_i)\}/\{n \sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2\} \tag{101}$$

in which x_i and y_i are written in place of $x_{(i)}$ and $y_{(i)}$ for simplicity of notation.

The results of the parameter estimation and test of goodness-of-fit are summarized in Table 17 which lists the values of the least square E^2 and $n\omega_n^2$ as well as the best estimates of the two parameters for the data sets XQPF, XWPF and WPF. The results clearly indicate that neither of the data sets fits to the first asymptotic distribution at the significance level of 5% or 10%. Fig. 26 plots these data sets on the Gumbel probability paper.

Table 17 Best Estimates (The Asymptotic Distributions)

(a) The First Asymptote

Best Estimates

Data	$\hat{\alpha}_1$	$\hat{\beta}_1$	$n\omega_n^{2*}$	E^{2+}
XQPF	.642	.142	1.48	21.3
XWPF	.415	-.106	2.63	3.41
WPF	.147	-.629	.697	10.9

(b) The Second Asymptote

Best Estimates

Data	$\hat{\alpha}_2$	$\hat{x}_2$	$n\omega_n^{2*}$	E^{2+}
XQPF	.822	.113	.0672	1.44
XWPF	1.87	.240	.0306	1.03
WPF	1.76	.469	.140	2.13

(c) The Third Asymptote

Best Estimates

Data	$\hat{\alpha}_3$	$\hat{x}_3$	ω (MIL)	$n\omega_n^{2*}$	E^{2+}
XQPF	16.3	30.3	30	1.62	23.0
XWPF	105.4	29.8	30	.370	4.33
WPF	40.0	29.6	30	.779	11.7

* 5% significance level = .461
10% significance level = .347

\+ Least square error

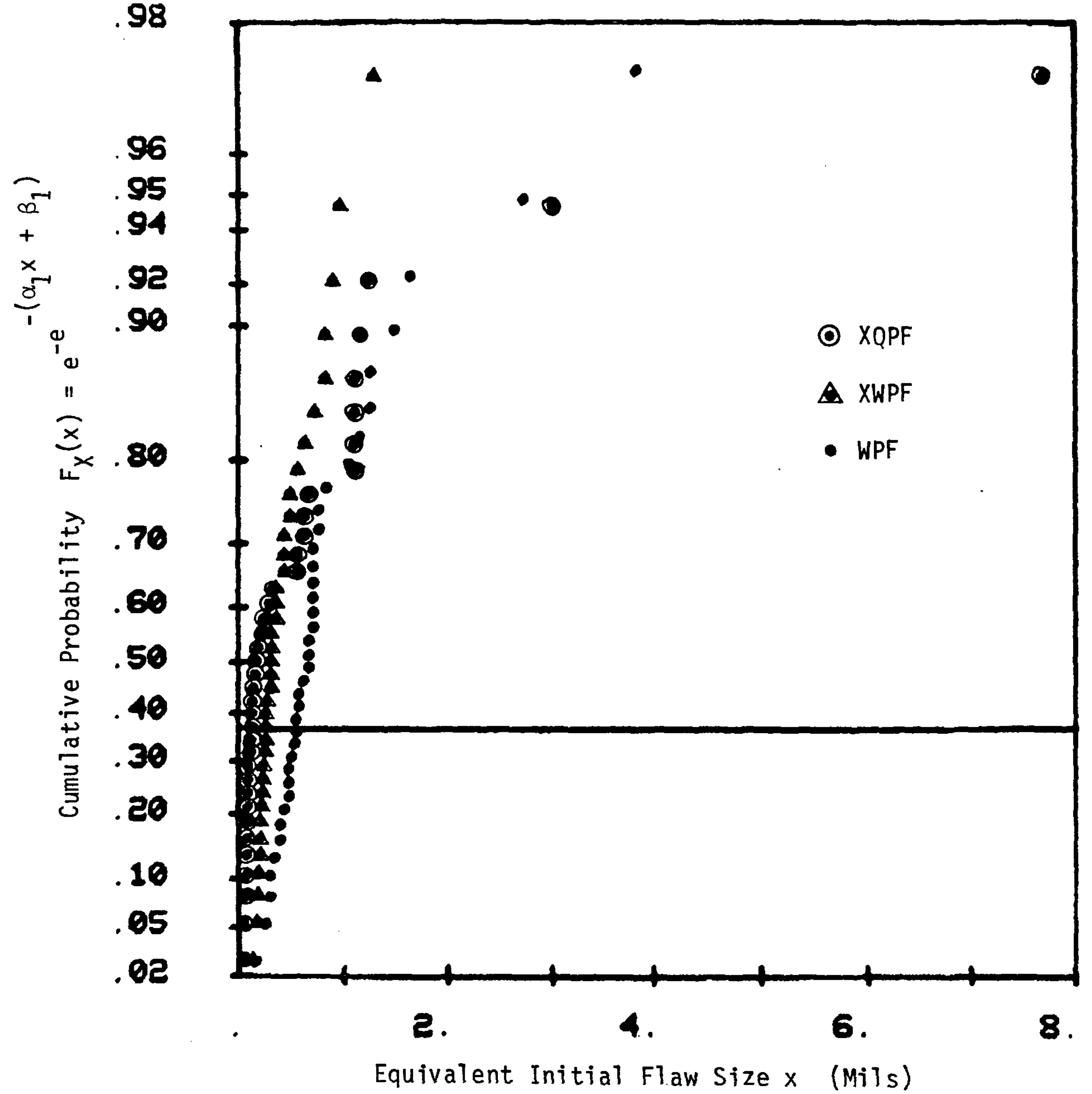


Fig. 26 Fitting to First Asymptotic Distribution of Largest Values

6.3 The Second Asymptote

For the second asymptotic distribution, consider the following transformation of the random variable X into Y

$$Y = \ln X \tag{102}$$

Then, the distribution function of Y can be written as

$$F_Y(y) = e^{-e^{-\alpha_2 y - \beta_2}} \tag{103}$$

where

$$\beta_2 = -\alpha_2 \ln x_2 \tag{104}$$

Analytically, Eq. 103 is identical with Eq. 93 and therefore the same method of estimation as used for the parameters of the first asymptotic distribution can be employed in this case as well: Eqs. 100 and 101 can be solved for the best estimates $\hat{\alpha}_2$ and $\hat{\beta}_2$ for α_2 and β_2 if in these equations $\hat{\alpha}_1$ and $\hat{\beta}_1$ are replaced by $\hat{\alpha}_2$ and $\hat{\beta}_2$ and also $x_i = x_{(i)}$ by $u_i = \ln x_i = \ln x_{(i)}$.

Once $\hat{\alpha}_2$ and $\hat{\beta}_2$ are found, the best estimate $\hat{x}_2$ for x_2 can be evaluated as

$$\hat{x}_2 = e^{-\hat{\beta}_2/\hat{\alpha}_2} \tag{105}$$

The estimated parameters and the values of the least square E^2 and $n\omega_n^2$ are listed in Table 17 which indicates that all the data sets fit extremely well to the second asymptotic distribution: Observe the very low values of $n\omega_n^2$. Fig. 27 plots these data sets on the Gumbel probability paper.

6.4 The Third Asymptote

Using a transformation similar to Eq. 102;

$$Y = \ln(w - X) \tag{106}$$

the distribution function of Y can be obtained from Eq. 95 as

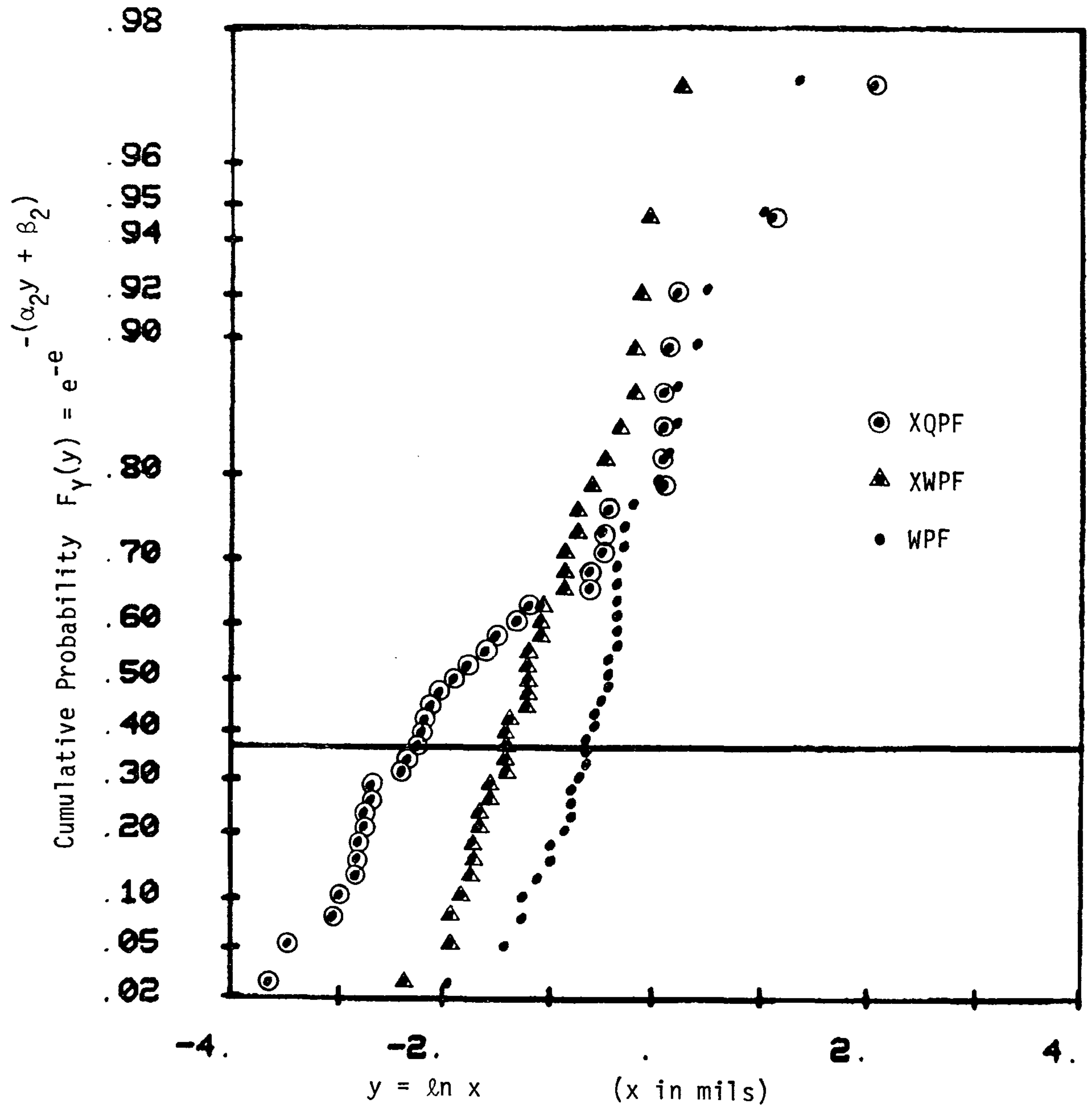


Fig. 27 Fitting to Second Asymptotic Distribution of Largest Values

$$F_Y(y) = 1 - e^{-e^{\alpha_3 y+\beta_3}} \tag{107}$$

where

$$\beta_3 = -\alpha_3 \ \ell n\ x_3 \tag{108}$$

Transforming Eq. 107 further into the following form

$$z = \alpha_3 y + \beta_3 \tag{109}$$

with

$$z = \ell n\ \ell n[1/\{1 - F_Y(y)\}] \tag{110}$$

we can construct the square error E^2 as

$$E^2 = \sum_{i=1}^{n} \{z_{(i)} - (\alpha_3 y_{(i)} + \beta_3)\}^2 \tag{111}$$

in which

$$z_{(i)} = \ell n\ \ell n[(1+n)/(1+n-i)] \tag{112}$$

and

$$y_{(i)} = \ell n(w - x_{(i)}) \tag{113}$$

The best estimates $\hat{\alpha}_3$ and $\hat{\beta}_3$ respectively for the parameters α_3 and β_3 can then be obtained as those values of α_3 and β_3 that minimize the square error E^2 in Eq. 111. Hence, these best estimates can again be obtained from Eqs. 100 and 101 by replacing $\hat{\alpha}_1$, $\hat{\beta}_1$, x_i and y_i respectively by $-\alpha_3$, $-\beta_3$, $y_i = y_{(i)}$ and $z_i = z_{(i)}$. The best estimate $\hat{x}_3$ of x_3 can then be evaluated as

$$\hat{x}_3 = e^{-\hat{\beta}_3/\hat{\alpha}_3} \tag{114}$$

This estimation procedure requires, however, the knowledge of the upper bound w. In the present study, we assume that the upper bound is $w = 1.1x_{(n)}, 1.2x_{(n)}, \ldots, 3.0x_{(n)}$. The $n\omega_n^2$ values associated with these assumed upper bounds are computed after the corresponding parameter values are estimated and listed in Table 18. The table indicates that the $n\omega_n^2$ values are smaller as the upper bound w gets larger. Therefore, for the upper bound w, we assume a crack size of 30 mils, a value to be used in Section VII as the (smallest visible) crack size at the time of crack initiation. The values of the estimated parameters, $n\omega_n^2$ and E^2 under the assumption of w = 30 mils are then listed in Table 17. The table indicates that, for the data sets XQPF and WPF, the $n\omega_n^2$ values are smaller under the assumption of w = 30 mils than those listed in Table 18; this is consistent with the trend observed in Table 18 that the $n\omega_n^2$ values are smaller for larger values of w. However, the $n\omega_n^2$ value for the data set XWPF under the assumption of w = 30 mils is larger than those associated with $w = 2.3x_{(n)}, \ldots, 3.0x_{(n)}$ in Table 18 against the trend.

The following conclusions can be drawn from these observations: The third asymptotic distribution fits to neither of the WQPF and WPF data sets, whether at the 5% or the 10% significance level. However, the distribution fits to the XWPF data at the 5% significance level under the assumption of $w = 1.6x_{(n)} \sim 3.0x_{(n)}$ and w = 30 mils while at the 10% significance level under $w = 2.7x_{(n)} \sim 3.0x_{(n)}$. How well the third asymptotic distribution fits to the XWPF data under the assumption of $w > 3.0x_{(n)}$ still remains to be investigated. No further study has been pursued, in this respect however, in view of the generally poor degree of goodness-of-fit exhibited by the distribution for the current data sets.

Figs. 28-30 plot respectively the data sets XQPF, XWPF and WPF on the probability paper for $F_Y(y) = 1 - \exp[-\exp(\alpha_3 y + \beta_3)]$ when $w = 1.1x_{(n)}$, $2.0x_{(n)}$ and $3.0x_{(n)}$. Note that the abscissa of the probability paper plots $\ell n(w - x)$.

Table 18 Values of $n\omega_n^2$ as a Function of w (The Third Asymptotic Distribution)

w	XQPF	XWPF	WPF
$1.1 \times x_{(n)}$	2.28	0.813	1.64
1.2	2.14	0.648	1.43
1.3	2.05	0.566	1.31
1.4	1.98	0.516	1.22
1.5	1.93	0.480	1.16
1.6	1.89	0.454	1.11
1.7	1.86	0.434	1.07
1.8	1.83	0.418	1.04
1.9	1.80	0.404	1.01
2.0	1.78	0.393	0.985
2.1	1.77	0.384	0.966
2.2	1.75	0.375	0.948
2.3	1.73	0.368	0.933
2.4	1.72	0.362	0.920
2.5	1.71	0.356	0.908
2.6	1.70	0.352	0.899
2.7	1.69	0.347	0.887
2.8	1.68	0.343	0.879
2.9	1.67	0.339	0.870
3.0	1.67	0.336	0.864

$x_{(n)}$ = 7.70 mils for XQPF, 1.28 mils for XWPF and 3.83 mils for WPF

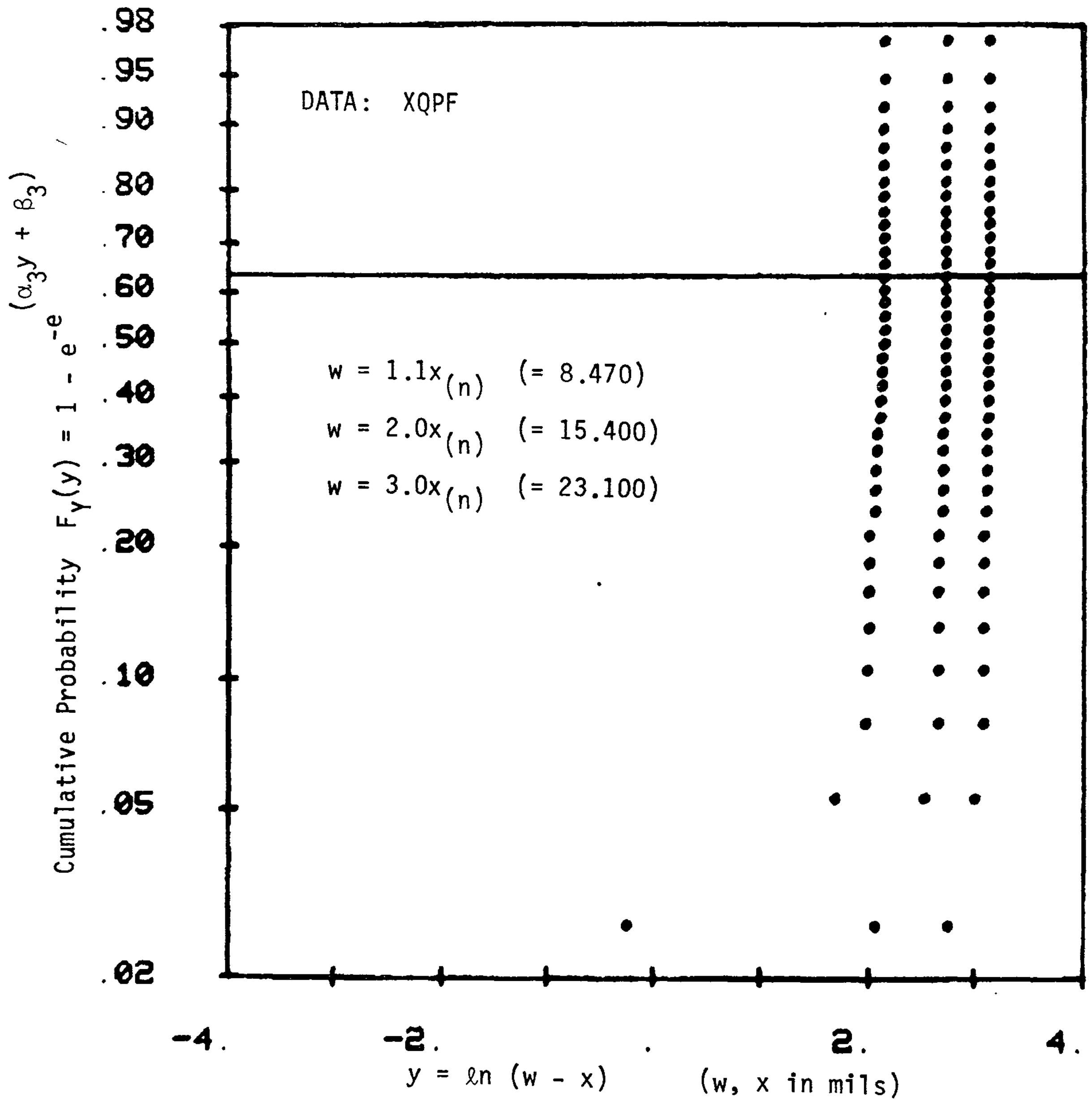


Fig. 28 Fitting XQPF Data to Third Asymptotic Distribution of Largest Values

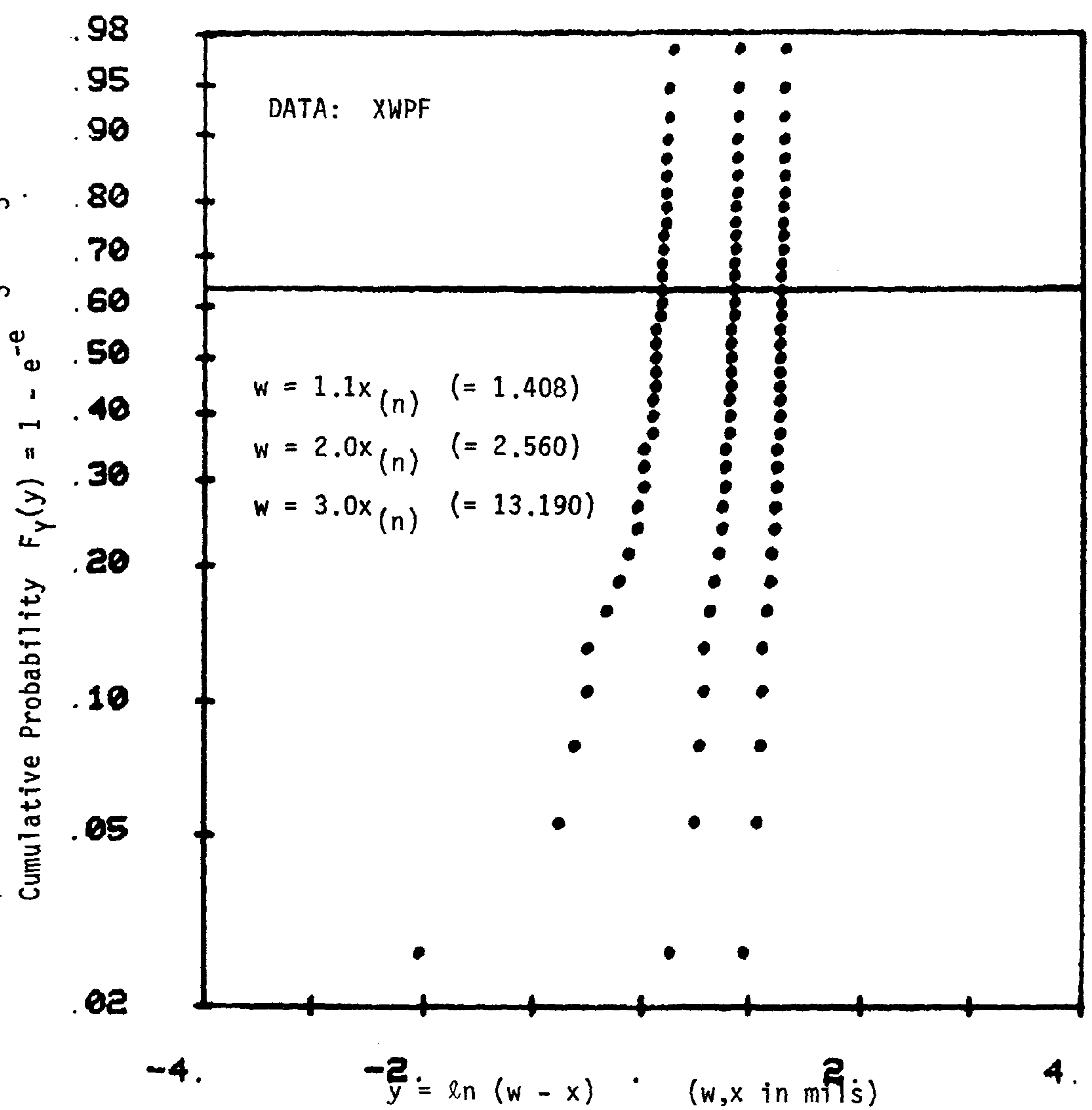


Fig. 29 Fitting XWPF Data to Third Asymptotic Distribution of Largest Values

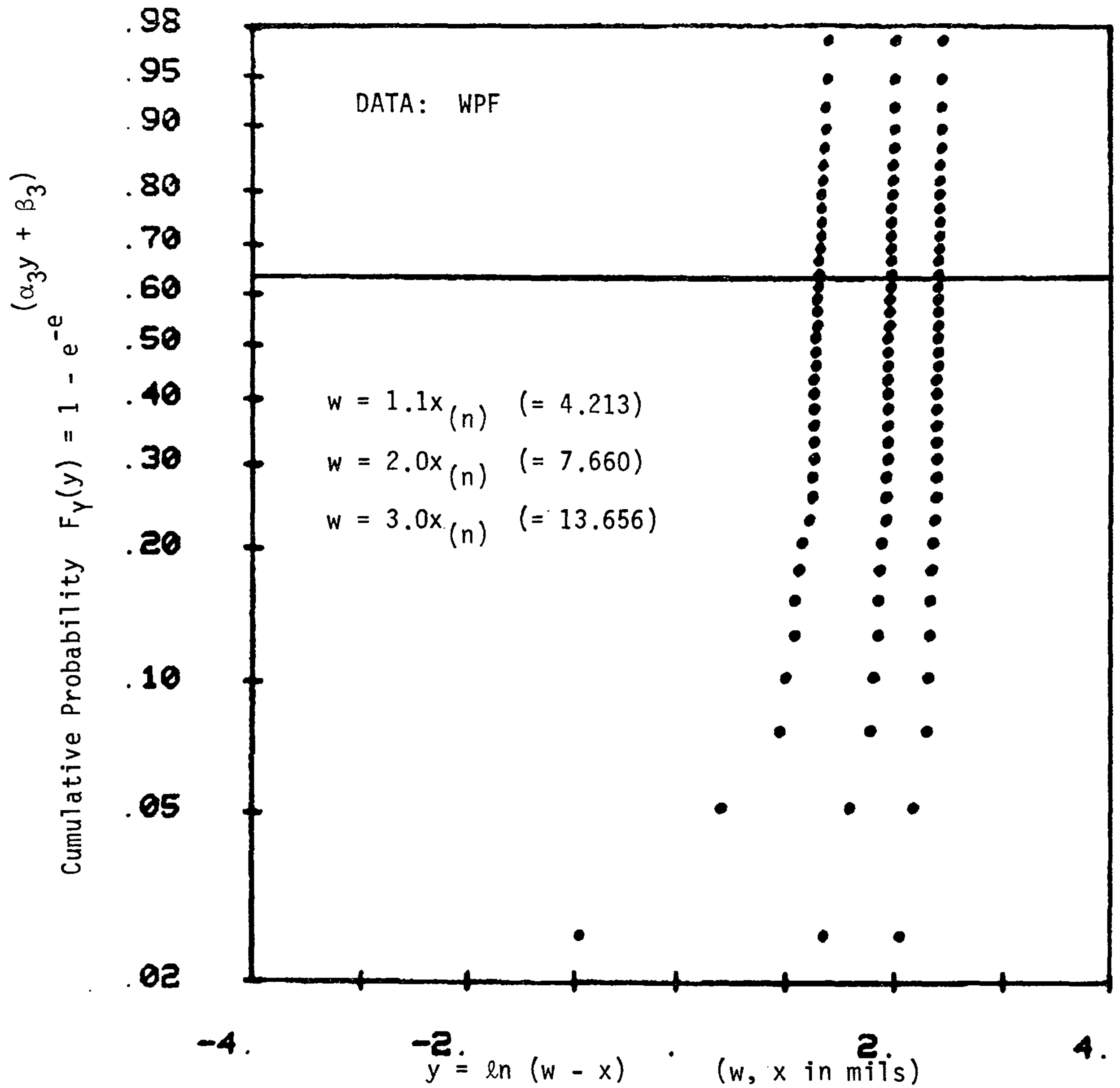


Fig. 30 Fitting WPF Data to Third Asymptotic Distribution of Largest Values

SECTION VII

FITTING TO TTCI COMPATIBLE EIFS DISTRIBUTIONS

7.1 Compatibility Between EIFS and TTCI Distributions

We now follow Shinozuka [8] to demonstrate the existence of compatibility between the EIFS distribution function $F_X(x)$ and the TTCI distribution function $F_T(t)$. The key to the establishment of such compatibility is the interpretation that the crack size $a_0 = a(T)$ at time T of crack initiation is specifiable (say, $a_0 = 0.03"$) and that the crack size a_0 is reached as a result of crack growth from the initial crack size $X = a(0)$. Assuming that the crack growth is governed by

$$da(t)/dt = Q[a(t)]^b \tag{115}$$

and integrating from t = 0 to t = T, we obtain the following relationship among $X = a(0)$, $a_0 = a(T)$ and T.

$$X = a(0) = a_0/(1 + a_0^c\ cQT)^{1/c} \tag{116}$$

where $c = b-1 > 0$. With a_0 specified, Eq. 116 provides the transformation necessary to establish the compatibility. Indeed, we can derive on the basis of Eq. 116

$$F_X(x) = 1 - F_T\{(x^{-c} - a_0^{-c})/(cQ)\} \tag{117}$$

A number of distribution functions have been used for $F_T(t)$. Particularly notable are the log-normal and Weibull distribution functions. It is well documented that both of these can usually describe the observed fatigue data reasonably well, especially in the central range of scattered fatigue data. They represent, however, widely differing underlying mechanical-statistical models: The log-normal distribution implies a failure rate that increases at first and decreases after reaching a maximum, while the failure rate associated with the Weibull distribution is a monotonically increasing function with time provided that the shape parameter is larger than unity. Since monotonically increasing failure rates are more amenable to physical interpretation, the Weibull distribution is often preferred to the log-normal distribution. This preference is sometimes reinforced further by the fact that the Weibull distribution has an analytical form that is mathematically easy to manipulate.

7.2 Weibull Compatible Distribution

Shinozuka [8] applied Eq. 117 to the two-parameter Weibull (TTCI) distribution and computed the probability density and distribution functions under the assumption that $Q = 2.0 \times 10^{-8}$, $a_0 = 0.04"$ and $c = 4.0$. Yang suggests that the same equation be applied to the three-parameter Weibull distribution

$$F_T(t) = 1 - \exp\{-[(t - \varepsilon)/\beta]^{\alpha}\} \qquad t \geqq \varepsilon \tag{118}$$

in which case, the Weibull compatible distribution is obtained as

$$F_X(x) = \exp\{-\left(\frac{x^{-c} - a_0^{-c} - cQ\varepsilon}{cQ\beta}\right)^{\alpha}\} \qquad x \leqq x_U \tag{119}$$

with

$$x_U = (a_0^{-c} + cQ\varepsilon)^{-1/c} \tag{120}$$

In the present investigation, the following values of b and Q are assumed, based on some experimental evidence.

Table 19 Values of Crack Propagation Parameters

DATA	XQPF	XWPF	WPF
b	1.26	1.26	1.22
Q	2.33×10^{-3}	2.33×10^{-3}	9.25×10^{-3}

Note: $c = b - 1$

As in the case of the three-parameter Weibull (EIFS) distribution, the parameter ε is assumed to be known and the shape and scale parameter, α and β, of the parent Weibull distribution are estimated using the following estimation procedure. First, transfer the random variable $X = a(0)$ into

$$Y = X^{-c} = \{a(0)\}^{-c} \tag{121}$$

then the distribution function of Y can be written in the form of a three-parameter Weibull distribution:

$$F_Y(y) = 1 - \exp\left\{-\left(\frac{y - \gamma}{\delta}\right)^{\alpha}\right\} \quad ; \qquad y \geq \gamma \tag{122}$$

where

$$\gamma = x_\mu^{-c} = a_0^{-c} + cQ\varepsilon \tag{123}$$

and

$$\delta = cQ\beta \tag{124}$$

Realizations of X, $x_{(1)} < x_{(2)} < \ldots < x_{(n)}$, are transformed through Eq. 121 into realizations of Y, $y_{(n)} = x_{(1)}^{-c} > y_{(n-1)} = x_{(2)}^{-c} > \ldots > y_{(1)} = x_{(n)}^{-c}$. Note that the largest, the second largest, ... of the X sample become respectively the smallest, the second smallest, ... of the Y sample as shown below:

Observations of EIFS		Corresponding Y	
Largest	$a(0)_{(n)}$	Smallest	$y_{(1)} = \{a(0)_n\}^{-c}$
Second largest	$a(0)_{(n-1)}$	Second smallest	$y_{(2)} = \{a(0)_{n-1}\}^{-c}$
	⋮		⋮
Smallest	$a(0)_{(1)}$	Largest	$y_{(n)} = \{a(0)_1\}^{-c}$

Defining $\Delta\gamma$ as

$$\Delta\gamma = (y_{(1)} - a_0^{-c})/5 \tag{125}$$

we assume that the location parameters ε of the TTCI (Eq. 118) and γ of the distribution of Y (Eq. 122) take respectively the following values

$$\varepsilon_j = (j - 1)\Delta\gamma/(cQ) \qquad (j=1,2,\ldots,5) \tag{126}$$

and

$$\gamma_j = a_0^{-c} + (j - 1)\Delta\gamma \qquad (j=1,2,\ldots,5) \tag{127}$$

Taking advantage of the same procedure as used in the case of the Weibull distribution fit, we introduce

$$z = \alpha v + \delta^* \tag{128}$$

where

$$z = \ell n\, \ell n\{1/(1 - F_Y(y))\} \tag{129}$$

$$v = \ell n\, (y - \gamma) \tag{130}$$

and

$$\delta^* = -\alpha\, \ell n\, \delta \tag{131}$$

By means of the least square method, we can then evaluate the best estimates $\hat{\alpha}$ and $\hat{\delta}^*$ of α and δ from Eqs. 69 and 70 by replacing $\hat{\eta}$, $\hat{\lambda}^*$, u_i and y_i there respectively with $\hat{\alpha}$, $\hat{\delta}^*$, v_i and z_i where

$$v_i = \ell n(y_{(i)} - \gamma) \tag{132}$$

and

$$z_i = \ell n\, \ell n\{(1 + n)/(1 + n - i)\} \tag{133}$$

The best estimates $\hat{\beta}$ and $\hat{\delta}$ of β and δ can be found respectively from Eqs. 124 and 131:

$$\hat{\beta} = \hat{\delta}/(cQ) \tag{134}$$

$$\hat{\delta} = e^{-\hat{\delta}^*/\hat{\alpha}} \tag{135}$$

Table 20 indicates the values of $n\omega_n^2$ corresponding to γ_j $(j=1,2,\ldots,5)$ for each of the three data sets. In all cases, they are smaller for smaller

Table 20 Values of $n\omega_n^2$ as a Function of γ_j (Weibull Compatible Distribution)

Data XQPF

j	γ_j	$n\omega_n^2$	$\hat{\alpha}$	$\hat{\beta}$	ε
1	2.49	0.869×10^{-1}	2.04	0.133×10^5	0
2	2.70	0.941×10^{-1}			
3	2.91	1.06×10^{-1}			
4	3.12	1.29×10^{-1}			
5	3.33	1.89×10^{-1}			

Data XWPF

j	γ_j	$n\omega_n^2$	$\hat{\alpha}$	$\hat{\beta}$	ε
1	2.49	0.345×10^{-1}	4.89	0.103×10^5	0
2	3.12	0.381×10^{-1}			
3	3.75	0.449×10^{-1}			
4	4.39	0.588×10^{-1}			
5	5.02	0.950×10^{-1}			

Data WPF

j	γ_j	$n\omega_n^2$	$\hat{\alpha}$	$\hat{\beta}$	ε
1	2.14	1.51×10^{-1}	4.33	0.155×10^5	0
2	2.38	1.61×10^{-1}			
3	2.62	1.80×10^{-1}			
4	2.86	2.18×10^{-1}			
5	3.10	3.14×10^{-1}			

values of γ and hence for smaller values of ε indicating that the best fit is obtained when $\varepsilon = 0$. (Table 20 shows only the values of $\hat{\alpha}$ and $\hat{\beta}$ for $\varepsilon = 0$).

This implies, by virtue of Eq. 120, that the best fit is observed when a_0 (in this case 0.03") is taken as the upper bound of the distribution of the initial crack size a(0).

The results of estimation are given in Table 21 where the best estimates of α and β and the corresponding values of $n\omega_n^2$ and ε are listed.

Table 21 Best Estimates (Weibull Compatible EIFS Distribution)

Data	ε	$\hat{\alpha}$	$\hat{\beta}$	$n\omega_n^2$
XQPF	0	2.04	1.33×10^4	.0869
XWPF	0	4.89	1.03×10^4	.0345
WPF	0	4.33	1.55×10^4	.0151

Extremely small values of $n\omega_n^2$ above indicate that the Weibull compatible EIFS distribution fits superbly well to all the data sets.

Figures 31-33 plot z_i against v_i using the Weibull probability paper respectively for the data sets XQPF, XWPF and WPF. As in the case of Figs. 19-21, each data point in these figures represents $y_i - \gamma$ along the abscissa and i/(1+n) along the ordinate and therefore at each probability level i/(1+n) we see five points corresponding to five different values of γ.

7.3 Log-Normal Compatible Distribution

If the TTCI distribution $F_T(t)$ is a log-normal distribution of the form

$$F_T(t) = \Phi\left\{\frac{\ln(t - \varepsilon) - \mu}{\sigma}\right\} \qquad t \geq \varepsilon \qquad (136)$$

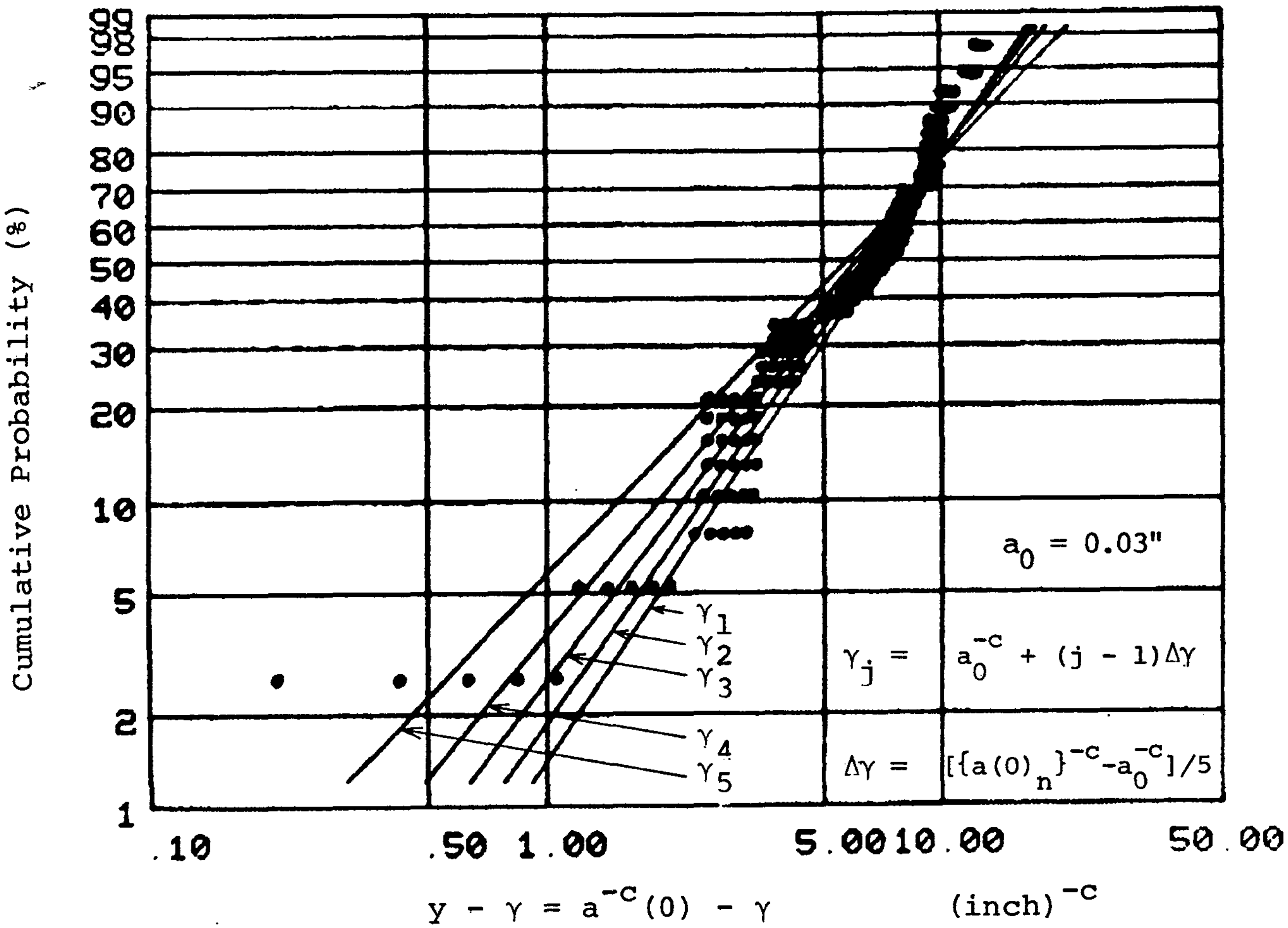


Fig. 31 Fitting XQPF Data to Weibull Compatible Distribution

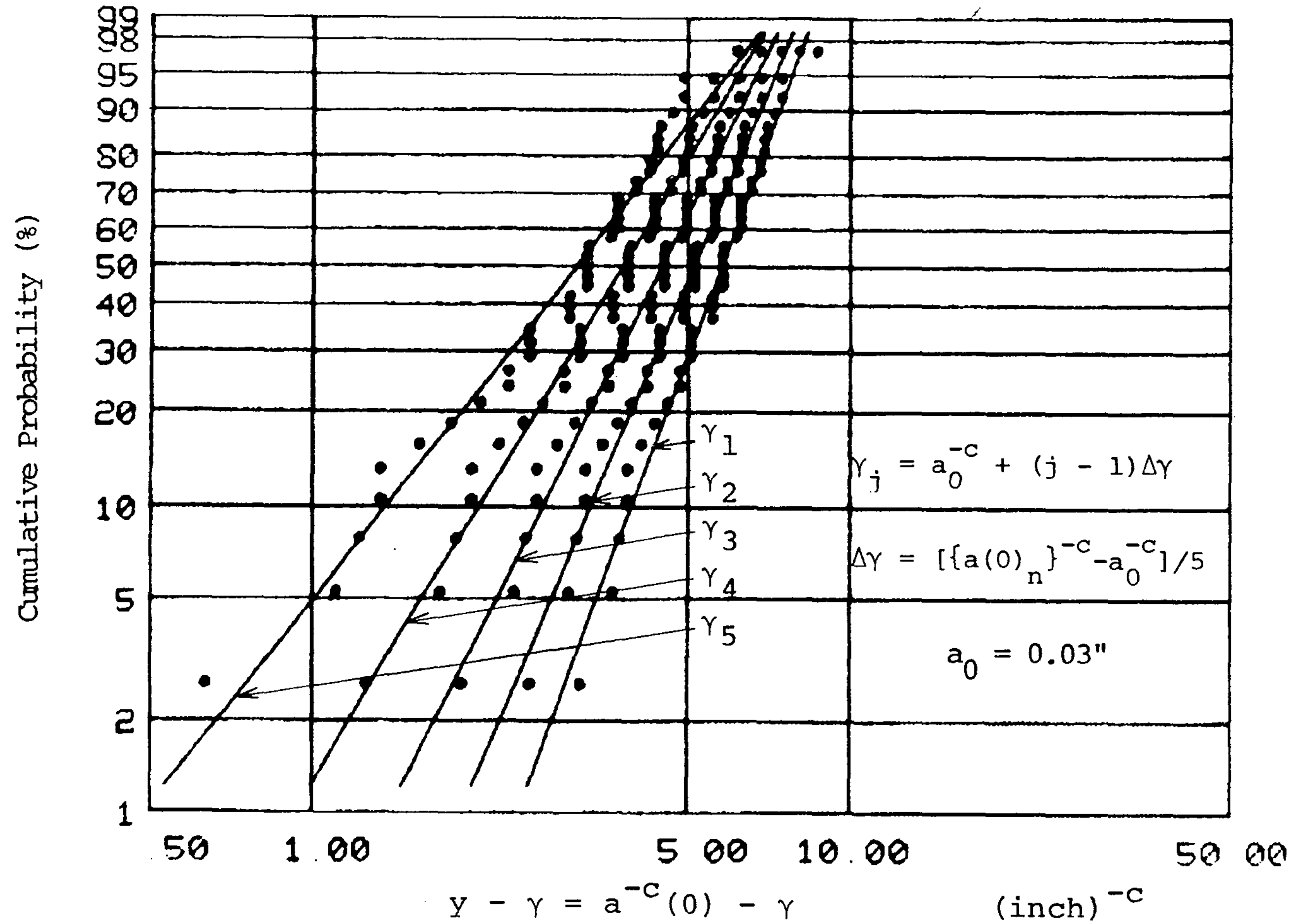


Fig. 32 Fitting XWPF Data to Weibull Compatible Distribution

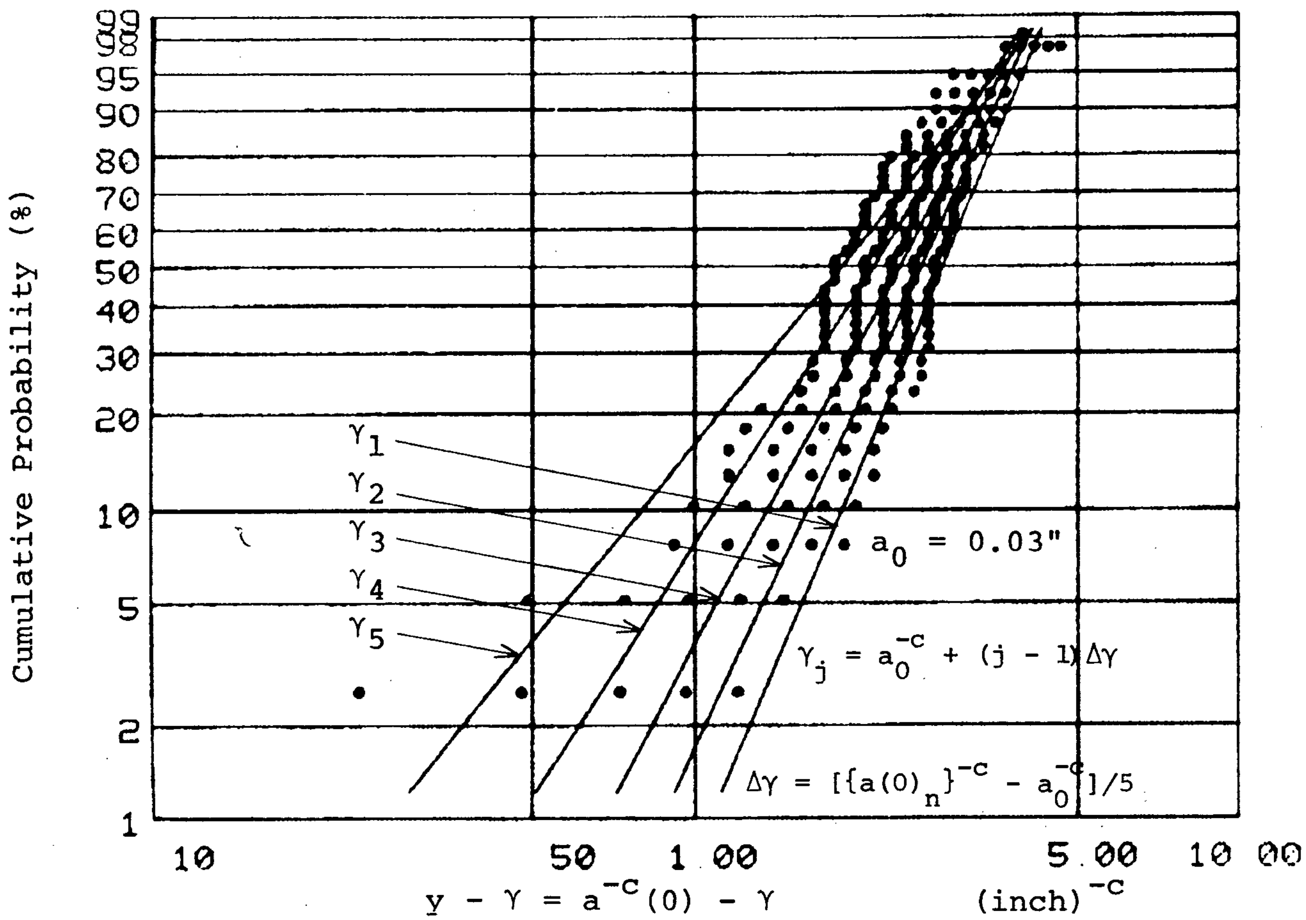


Fig. 33 Fitting WPF Data to Weibull Compatible Distribution

then, the corresponding EIFS distribution $F_X(x)$ can, by virtue of Eq. 117, be written as

$$F_X(x) = \Phi\left\{- \frac{\ell n[(x^{-c} - \gamma)/(cQ)] - \mu}{\sigma}\right\} \qquad x \leqq x_U \tag{137}$$

where γ and x_U are the quantities defined in Eqs. 123 and 120, respectively. Performing then the same transformation as shown in Eq. 121, we can derive the distribution function of Y as

$$F_Y(y) = \Phi[\{\ell n(y - \gamma) - \xi\}/\sigma] \tag{138}$$

where

$$\xi = \ell n(cQ) + \mu \tag{139}$$

Using the same b and Q values given in Table 19 and following the same procedure as used in the case of the Weibull compatible distribution, the parameters ξ and σ are estimated under the assumption that the parameter ε is known. Once, these estimates $\hat{\xi}$ and $\hat{\sigma}$ are found, the estimate $\hat{\mu}$ of μ can be found from Eq. 139 as

$$\hat{\mu} = \hat{\xi} - \ell n(cQ) \tag{140}$$

The location parameters ε and γ are assumed to take exactly the same values as given in Eqs. 126 and 127. Then, as in the case of the three-parameter log-normal distribution fit in Section III (see Eqs. 31 and 32), we evaluate the estimates $\hat{\xi}$ and $\hat{\sigma}$ respectively of ξ and σ using the following expressions:

$$\hat{\xi} = (1/n) \sum_{i=1}^{n} y_i^* \tag{141}$$

$$\hat{\sigma} = [\sum_{i=1}^{n} (y_i^* - \hat{\xi})^2/n]^{\frac{1}{2}} = [\{\sum_{i=1}^{n} y_i^{*2}\}/n - (\hat{\xi})^2]^{\frac{1}{2}} \tag{142}$$

with

$$y_i^* = \ell n(y_{(i)} - \gamma) \tag{143}$$

Table 22 indicates the values of $n\omega_n^2$ corresponding to γ_j (j=1,2,...5) for each of the three data sets. As in the case of the Weibull compatible distribution, the table indicates that (a) the $n\omega_n^2$ values are smaller for smaller values of γ and hence for smaller values of ε, suggesting that the best fit is obtained when $\varepsilon = 0$ and (b) the best fit at $\varepsilon = 0$ implies that the upper bound of the EIFS distribution is a_0 (in this case 0.03").

The results of estimation are summarized in Table 23 where the best estimates of μ and σ and the corresponding values of $n\omega_n^2$ and ε are listed. This table indicates that the goodness-of-fit is highly acceptable; the $n\omega_n^2$ values are much smaller than 0.347, the value associated with the 10% significance level, although the Weibull compatible distribution has resulted in even smaller $n\omega_n^2$ values.

Figure 34 plots the three data sets on the Gaussian probability paper after they are transformed into $\ell n(y_{(i)} - \gamma) = \ell n(x_{(i)}^{-c} - a_0^{-c})$.

The upper and lower bounds ξ_U and ξ_L of the confidence interval for the expected value ξ of $\ell n(Y - \gamma)$ in Eq. 138 can be written as

$$\begin{matrix}\xi_U\\ \\ \xi_L\end{matrix} = \hat{\xi} \pm (t_{1-\alpha,n-1})s/\sqrt{n} \tag{144}$$

where $\hat{\xi} = \hat{\mu} + \ell n(cQ)$, $t_{1-\alpha,n-1}$ = two-sided $100(1-\alpha)$ percentile of the Student's t distribution of (n-1) degrees-of-freedom and s = unbiased standard deviation. The lower bound Y_L and upper bound Y_U of the corresponding interval for Y are then obtained from

$$\xi_L = \ell n(Y_L - \gamma), \qquad \xi_U = \ell n(Y_U - \gamma) \tag{145}$$

as

$$Y_L = \gamma + \exp\{\hat{\xi} - (t_{1-\alpha,n-1})s/\sqrt{n}\} \tag{146}$$

$$Y_U = \gamma + \exp\{\hat{\xi} + (t_{1-\alpha,n-1})s/\sqrt{n}\} \tag{147}$$

Table 22 Values of $n\omega_n^2$ as a Function of γ_j
(Log-Normal Compatible Distribution)

Data XQPF

j	γ_j	$n\omega_n^2$	$\hat{\mu}$	$\hat{\sigma}$	ε
1	2.49	0.184	9.23	0.55	0
2	2.70	0.191			
3	2.91	0.201			
4	3.12	0.218			
5	3.33	0.256			

Data XWPF

j	γ_j	$n\omega_n^2$	$\hat{\mu}$	$\hat{\sigma}$	ε
1	2.49	0.109	9.12	0.23	0
2	3.12	0.121			
3	3.75	0.139			
4	4.39	0.168			
5	5.02	0.224			

Data WPF

j	γ_j	$n\omega_n^2$	$\hat{\mu}$	$\hat{\sigma}$	ε
1	2.14	0.181	9.52	0.26	0
2	2.38	0.200			
3	2.62	0.229			
4	2.86	0.275			
5	3.10	0.369			

Table 23 Best Estimates (Log-Normal Compatible Distribution)

Data	ε	$\hat{\mu}$	$\hat{\sigma}$	$n\omega_n^2$
XQPF	0	9.23	0.55	0.184
XWPF	0	9.12	0.23	0.109
WPF	0	9.52	0.26	0.181

Table 24 The Lower and Upper Bounds X_L and X_U (in mils) Corresponding to Those μ_L and μ_U for Several Confidence Levels $1-\alpha$ (Log-Normal Compatible Distribution)

For Data XQPF (N = 37)

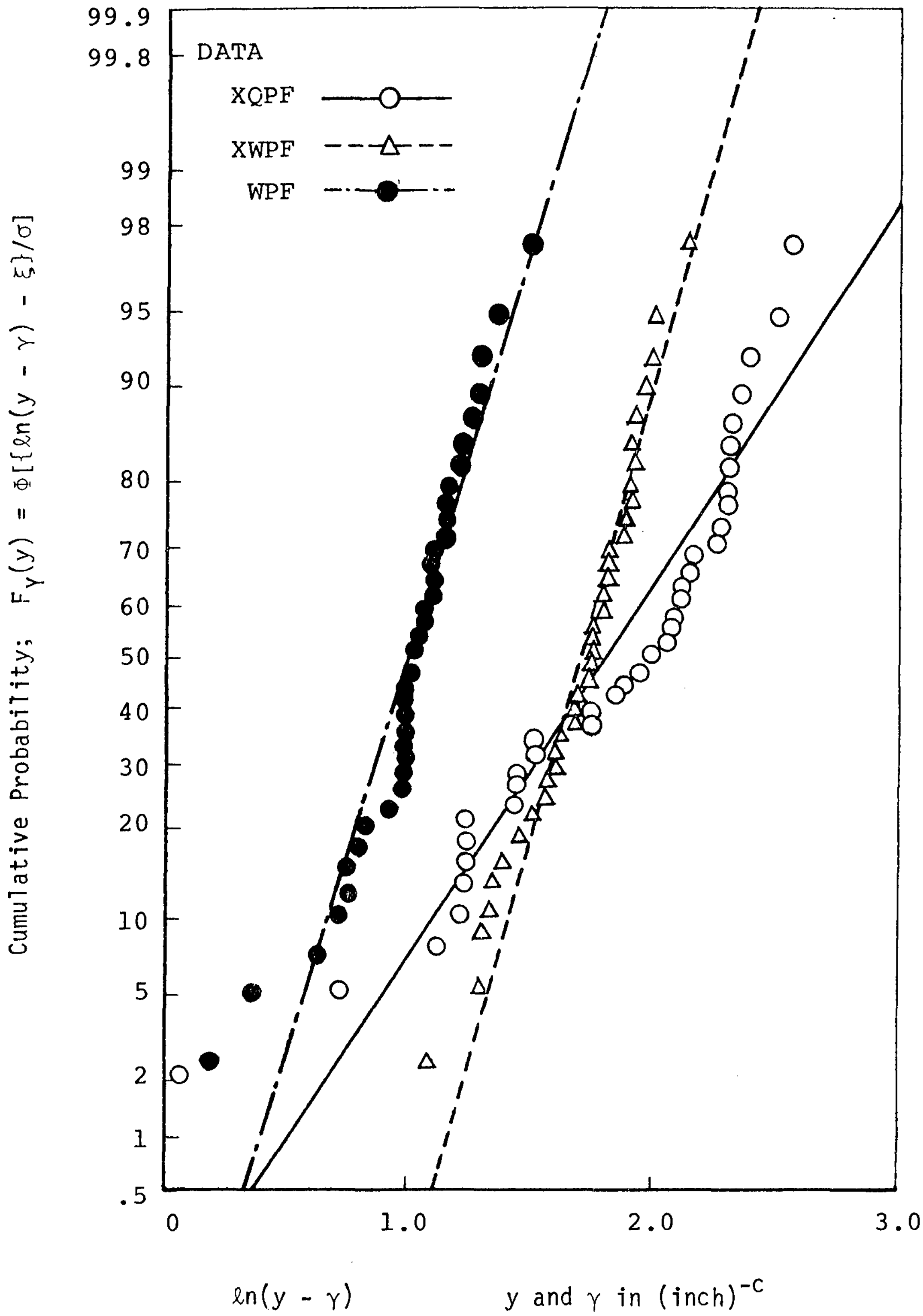
1-ALPHA	X_L	X_U
0.90	.177	.340
0.95	.161	.373
0.99	.132	.444

For Data XWPF (N = 37)

1-ALPHA	X_L	X_U
0.90	.286	.373
0.95	.275	.388
0.99	.254	.417

For Data WPF (N = 38)

1-ALPHA	X_L	X_U
0.90	.573	.762
0.95	.549	.794
0.99	.504	.858



Log-Normal Compatible Distribution

Fig. 34 Fitting the Data to Log-Normal Compatible Distribution

Then, finally, the upper and lower bounds X_U and X_L of the corresponding interval for $X = a(0)$ are evaluated as

$$X_U = Y_L^{-1/c}, \qquad X_L = Y_U^{-1/c} \tag{148}$$

Table 24 lists these upper and lower bounds for all the data sets at the 90, 95 and 99% confidence levels.

SECTION VIII

CONCLUSIONS

Distribution functions of the Johnson family,(including log-normal distribution functions) Pearson family, Weibull, Asymptotic (the First, Second and Third), and TTCI compatible EIFS (Weibull and log-normal compatible EIFS) have been examined by means of the ω^2 method for their acceptability in describing the statistical characteristics of EIFS. The results are summarized in Table 25 below where "unacceptable", "marginal", "acceptable" and "highly acceptable" signify the following unless other significances are indicated in the notes.

Unacceptable: $n\omega_n^2 > 0.461$; unacceptable even at the 5% significance level

Marginal: $0.461 \geqq n\omega_n^2 > 0.347$; acceptable at the 5% significance level but unacceptable at the 10% significance level

Acceptable: $n\omega_n^2 \leqq 0.347$; acceptable at the 10% significance level.

Highly acceptable: acceptable at the significance level higher than 20%.

Notes (see Table 25):

1. This is the result when a number of values are assumed for the location parameter ε and corresponding values of two other parameters are estimated. When we treat all three parameters as unknown and perform parameter estimations, resulting $n\omega_n^2$ values indicate that the Johnson S_L distribution is "unacceptable" for XQPF and XWPF. The distribution is "unacceptable" for WPF, however, in the sense that the estimated location parameter ε is negative (thus physically unacceptable), although the corresponding $n\omega_n^2$ value is in the statistically "acceptable" range.

2. This is the result when a number of values are assumed for the lower bound ε of the distribution and corresponding values of the three other parameters are estimated. This distribution is "unacceptable" for XQPF in the sense that the estimated values of the upper bound λ are negative for all assumed values of ε.

3. This is the result when the location parameter ε is assumed to be zero. When we assume that $\varepsilon = 0.8x_{(1)}$, this distribution is "highly acceptable" for all the data sets.

4. Only Type I has been tested. Other types are eliminated on the basis of poor comparison between (b_1, b_2) and (β_1, β_2)

5. Use of these distributions requires the knowledge of parameters b and Q in the crack growth model $da/dt = Qa^b$.

Table 25 Summary of Goodness-of-Fit Tests

Data	XQPF	XWPF	WPF	Notes
Johnson S_L	unacceptable	marginal	acceptable	1
Johnson S_B	unacceptable	unacceptable	acceptable	2
Weibull	acceptable	acceptable	acceptable	3
Pearson	unacceptable	unacceptable	unacceptable	4
Asymptote I	unacceptable	unacceptable	unacceptable	
Asymptote II	highly acceptable	highly acceptable	highly acceptable	
Asymptote III	unacceptable	acceptable	unacceptable	
Weibull Compatible	highly acceptable	highly acceptable	highly acceptable	5
Log-Normal Compatible	highly acceptable	highly acceptable	highly acceptable	5

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