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AIRCRAFT AND CREW SCHEDULING DURING AIRLIFT OPERATIONS

by

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ABSTRACT

The Air Force is frequently placed in a situation where a large quantity of goods must be transported between various locations in a specified time period by Air Force personnel. The goods are of different and interrelated types; thus, the sequence of missions is specified to maintain a proper balance of goods at each location. In addition to aircraft maintenance, health and alertness maintenance for pilots and personnel is a vital consideration in scheduling. Nevertheless, economy in operations needs to be demonstrated. Thus, the problem addressed in this paper is one of minimizing the number of crews used in the airlift, subject to crew rest requirements and the completion of all missions within the specified time frame.

KEY WORDS

Scheduling
Networks
Heuristics
Aircraft scheduling
INTRODUCTION

An Air Force airlift operation consists of transporting large quantities of people and equipment between various locations in a specified time period by Air Force personnel. A certain balance must be maintained between the different types of equipment and people to provide a working mixture. For example, a cargo of tanks is useless without the supporting operators and ammunitions. Health and alertness maintenance for pilots and personnel is a vital consideration in scheduling since overall military readiness as well as safety in flight operations must be considered.

The scheduling problem encountered in an airlift operation and one encountered by commercial airlines differ in the lack of restrictions on the possible routes for a crew. Normally air crews have a home base and spend the majority of time resting at this base. During an airlift, an assignment of a crew to a flight is limited mainly by rest requirements between flights to satisfy health and safety desiderata. Another difference between crew scheduling for an airlift and standard crew scheduling is the lack of a fixed schedule for the aircraft. Missions must be flown in a specified sequence and completed within a specified time frame; however, an aircraft is not otherwise restricted in landing or takeoff. The goods are of different and interrelated types; thus, the sequence of missions is specified to maintain a proper balance of goods at each location. In addition to aircraft maintenance, health and alertness maintenance for pilots and personnel is a vital consideration in scheduling. Nevertheless economy in operations needs to be demonstrated. Thus, the problem addressed in this paper is one of minimizing the number of crews used in the airlift, subject to crew rest requirements and the completion of all missions within the specified time frame.
Our model assumes that the mission sequence is given for each aircraft. The assignment of missions to aircraft is a separate problem that will not be considered here. This aspect of the problem could be incorporated by an extension of our model. It is not, however, since an effective assignment can be obtained manually, because the balance of missions is specified. Although our model does not require assumptions of cyclic scheduling, the pioneering work of Bartlett [1] and Bartlett and Charnes [2] in 1957 have suggested ideas to us.

Model

The objective of the model is to minimize the number of crews required to complete all missions within the specified time period subject to health, aircraft maintenance and connection considerations[8]. Once a crew assignment has been determined, then the minimum amount of time to complete all the missions is obtained. This minimal time cannot be more than the specified time. In other words, minimizing completion time is a secondary goal and minimizing the number of crews is a primary goal. As will be seen, this secondary goal is not part of the model, but, rather, is a specification achieved from the departure and arrival times derived from the solution algorithms.

Notation

The following are given parameters for the problem:

\[ m \]  
\[ n_i \] The number of aircraft in the system.

\[ n_i \]  
\[ t_{ij} \] The number of legs to be flown by aircraft \( i \) during the time period, \( i=1,2,\ldots,m \).

\[ t_{ij} \]  
\[ t_{ij} \] The flying time of aircraft \( i \) during leg \( j \); \( i=1,2,\ldots,m \);
\[ j=1,\ldots,n_i \].
Forced delays to prepare aircraft $i$ for leg $j$. These forced delays may result from such considerations as maintenance, refueling, loading aircraft and reconfiguration; $i=1,2,\ldots,m$; $j=1,\ldots,n_i$.

The minimal amount of time that will elapse between the arrival of aircraft $i$ on leg $j-1$ and the arrival of aircraft $i$ on leg $j$; $i=1,2,\ldots,m$; $j=1,2,\ldots,n_i$.

The minimum ground time required for the crew that arrived on leg $j$ of aircraft $i$. This may include rest time, post-flight time, travel time, pre-flight time, etc.

An upper bound on the amount of time to complete all missions.

The index set with each entry in the set having the form $(i,j,k,\ell)$. An appearance of $(i,j,k,\ell)$ in $Q$ means that the possibility of the crew on aircraft $i$ leg $j$ going out on aircraft $k$ leg $\ell$ should be considered. The possibility should not be considered unless leg $j$ of aircraft $i$ arrives at the same base from which leg $\ell$ of aircraft $k$ leaves.

Also, the estimated arrival and departure times should be within some reasonable range of each other.

Thus, $Q_{ij}$ is the index set defining all flights with which the crew of aircraft $i$ leg $j$ might connect.

Thus, $Q^{k\ell}$ is the index set defining all flights from which the crew for aircraft $k$ leg $\ell$ might be obtained.
The following are variables which are to be determined by the model.

\( X_{ij} \)

The arrival time of aircraft \( i \) on leg \( j \).

\[ Y_{ijk} = \begin{cases} 
1 & \text{if the crew on aircraft } i \text{ leg } j \text{ departs on aircraft } k \text{ leg } 2 \\
0 & \text{otherwise} 
\end{cases} \]

\( d_{ij} \)

The ground delay time before the departure of aircraft \( i \) on leg \( j \) that is in excess of \( f_{ij} \). This is a logical variable in the model and is not entered explicitly. The derivation of \( d_{ij} \) will be discussed after the model is presented.
The model is the following.

\[
\begin{align*}
\text{(P1)} \\
\text{Maximize} & \quad \sum_{(i,j,k,\ell) \in Q} Y_{ijkl} \\
\text{subject to} & \quad X_{ij} - X_{i,j-1} \geq t_{ij} + f_{ij} \\
& \quad i = 1, 2, \ldots, m \quad j = 1, 2, \ldots, n_i \quad X_{i,n_i} \leq T, \quad i = 1, 2, \ldots, m \quad (X_{k\ell} - t_{k\ell}) - X_{ij} \geq R_{ij} Y_{ijkl} \\
& \quad (k,\ell) \in Q_{ij} \quad (i,j) \in Q_{k\ell} \\
& \quad \sum_{(k,\ell) \in Q_{ij}} Y_{ijkl} \leq 1, \quad i = 1, 2, \ldots, m \quad j = 1, 2, \ldots, n_i \\
& \quad \sum_{(i,j) \in Q_{k\ell}} Y_{ijkl} \leq 1, \quad k = 1, 2, \ldots, m \quad \ell = 1, 2, \ldots, n_k \\
& \quad Y_{ijkl} = 0 \text{ or } 1 \quad (i,j,k,\ell) \in Q \\
& \quad X_{ij} \geq 0, \quad i = 1, 2, \ldots, m \quad j = 1, 2, \ldots, n_i 
\end{align*}
\]

The objective function (1) maximizes the number of crews that connect from other flights. Since the failure of connecting a crew from another flight means that a new crew must be added to the system, this objective is equivalent to minimizing the number of crews in the system. The set $Q_{k\ell}$ may be empty for some $(k,\ell)$ pairs and a crew must be added to the system for aircraft $k$ leg $\ell$. 
Constraint (2) imposes a restriction that the arrival time of aircraft $i$ leg $j$ be greater-than-or-equal-to the arrival time of aircraft $i$ leg $j-1$ plus flying time and forced delays. The discretionary delay time $d_{ij}$ can be obtained from the equation.

$$d_{ij} = X_{ij} - X_{i,j-1} - t_{ij} - f_{ij}$$

Constraint (3) prevents the arrival time of the last leg for each aircraft from exceeding the allotted completion time $T$.

Constraint (4) requires the departure time of aircraft $k$ leg $\ell$ to be greater-than-or-equal-to the arrival time of aircraft $i$ leg $j$ plus the ground time for the crew. This constraint is enforced only when $Y_{ijk\ell}$ equals 1.

Constraint (5) says that the crew arriving on aircraft $i$ leg $j$ can leave on at most one aircraft.

Constraint (6) says that at most one crew can leave on aircraft $k$ leg $\ell$. Slack in (6) means that a new crew is added to the system.

Constraint (7) requires the crew of aircraft $i$ leg $j$ to leave either on aircraft $k$ leg $\ell$ or not leave on aircraft $k$ leg $\ell$.

Constraint (8) forces all arrival times to be non-negative. In particular, $X_{i0}$ will be zero unless there is slack in the system.

**Practical Dimensions**

Practical airlift problems encountered by the U.S. Air Force involve no less than 2,000 total legs with a leg allowing for an average of 5 possible connections for the crew. This gives rise to a mixed-integer programming problem with about 32000 constraints, 35000 linear variables (including logicals) and 10000 zero-one variables. The problem size precludes the use of standard branch-and-bound or cutting plane procedures [5]. Benders decomposition [3] can be utilized, but the Master Problem is still of formidable size, with 10000 zero-one variables. Also, modifications to
schedules may be required during the airlift and a rapid response time is essential. Thus, the algorithm presented here utilizes a decomposition procedure which may take a solution to a linear programming relaxation as a starting point. The decomposition considers the following two models.

Model with $X$ fixed

When $X_{ij} = \bar{X}_{ij}$, the arrival time for leg $j$ or aircraft $i$ is fixed. If all $X_{ij}$ variables are fixed, the following problem arises.

(P2) Maximize $\sum_{(i,j,k,l) \in Q} Y_{ijkl}$

subject to $Y_{ijkl} \leq S_{ijkl}$

(5), (6) and (7)

where $S_{ijkl} = (\bar{X}_{kl} - t_{kj} - \bar{X}_{ij})/R_{ij}$ for $(k,l) \in Q_{ij}$

This problem can be solved without the benefit of a linear programming algorithm. First note that an $S_{ijkl}$ less than zero forces $Y_{ijkl}$ to be equal to zero. Values can be assigned to the remaining $Y_{ijkl}$ variables in a first-in/first-out manner. Each crew has the proper training to take any flight; thus, the assignment of a crew to a flight will not limit later assignments. Let $N$ be the number of flights (legs) in the airlift operation. Assume that all flights have been sorted, based on departure time. The four subscripts used to identify a flight are dropped and the flights are denoted by $j = 1, 2, \ldots, N$.

Define the following for each flight in the system.

$DT_j$ The departure time for flight $j$

$FRBASE_j$ The base where the $j$-th flight begins

$TOBASE_j$ The base where the $j$-th flight ends

$FT_j$ The flying time for the $j$-th flight
CR<sub>j</sub>  
The crew assigned to flight j-th (this must be determined)

RT<sub>j</sub>  
The ground time required for the crew after the j-th flight.

Define the following for each crew in the system.

PL<sub>i</sub>  
The base where the i-th crew is currently located

RD<sub>i</sub>  
The time at which the i-th crew will be available to depart

M  
The number of crews currently in the system

An algorithm for solving P2 is now stated.

**STEP 1**  
Set \( j = 1 \) and \( M = 1 \).  
\( \text{PL}_1 = \text{TOBASE}_1 \)
\( \text{RD}_1 = \text{DT}_1 + \text{FT}_1 + \text{RT}_1, \text{CR}_1 = 1 \)

**STEP 2**  
\( j = j + 1 \)  
If \( j > N \), then stop; otherwise go to **STEP 3**.

**STEP 3**  
If \( \text{PL}_i = \text{FRBASE}_j \) and \( \text{RD}_i < \text{DT}_j \) for some \( 1 \leq i \leq M \), go to **STEP 4**; otherwise, go to **STEP 5**.

**STEP 4**  
Assign the i-th crew to flight j; i.e., \( \text{CR}_j = i \).  
\( \text{PL}_i = \text{TOBASE}_{j_i}, \text{RD}_i = \text{DT}_j + \text{FT}_j + \text{RT}_j \).  
Go to **STEP 2**.

**STEP 5**  
Add a new crew to the system and assign the crew to flight j.  
\( M = M + 1 \).  
\( \text{CR}_j = M, \text{PL}_M = \text{TOBASE}_j, \text{RD}_M = \text{DT}_j + \text{FT}_j + \text{RT}_j \).  
Go to **STEP 2**.

Several variations can be used to choose the crew to assign to a flight when more than one crew is available. The two rules that we have worked with are to: 1) choose the crew that has been at the base the longest, and 2) choose the crew with the smallest total flying time. It is also possible to modify the algorithm to accommodate an initial assignment of crews to specified bases. This modification has been used to "polish" assignments and to work from existing crew stagings.

**Model with Y fixed**

When \( Y_{ijkl} = \bar{Y}_{ijkl} \), the crew to fly on leg \( \ell \) of aircraft \( k \) is fixed. If all \( Y_{ijkl} \) variables are fixed, objective function value is determined and only
feasibility is a concern. An alternate objective of minimizing the total completion time can be instituted; thus, constraint set (3) will be dropped and replaced with another set of constraints. The problem is the following:

(P3) Minimize $Z$

subject to

$X_{i,n} \leq Z, \ i = 1, 2, \ldots, m \quad (9)$

$X_{k\ell} - X_{ij} \geq R_{ij} + t_{k\ell} \quad (i,j,k,\ell) \in \tilde{Q} \quad (10)$

where $\tilde{Q} = \{(i,j,k,\ell): \gamma_{ijk\ell} = 1\}$

It can be shown that the dual of P3 is a transhipment problem and, thus can be solved as a network. But, as was the case with P2, P3 can be solved in a direct manner without the benefit of linear programming. As the airlift operation moves through time, an aircraft may be dispatched as soon as it and the assigned crew are available. There is no motivation for delaying aircraft because crew assignments have already been made. The following may be obtained from the crew assignments.

$W_{k\ell} = \begin{cases} (i,j) & \text{if the crew for aircraft } k \text{ leg } \ell \text{ is connecting from aircraft } i \text{ leg } j \\ 0 & \text{otherwise} \end{cases}$

Relating to the model (P1), $W_{k\ell} = (i,j)$ means that $Y_{ijk\ell} = 1$.

The following additional definitions are needed.

LEG$_i$ The sequence number of the leg that will be flown next by aircraft $i$.

DONE The number of aircraft that have completed their assigned missions.

An algorithm for solving P3 is now stated.
STEP 1. Set \(X_{ij} = 0, i = 1, \ldots, m, j = 0, 1, 2, \ldots, n_i\)

\[\text{LEG}_i = 1, i = 1, 2, \ldots, m \quad \text{DONE} = k = 0\]

STEP 2

\(k = k + 1\) \quad \text{If } k > m, \text{ then } k = 1.\)

STEP 3

\(\text{If } \text{LEG}_k = -1 \text{ then go to STEP 2.}\)

STEP 4

Set \(\ell = \text{LEG}_k.\) \quad \text{If } W_{k\ell} = 0 \text{ then go to STEP 5; otherwise, go to STEP 6.}\)

STEP 5

\[X_{k\ell} = X_{k,\ell-1} + t_{k\ell} + f_{k\ell} \quad \text{Go to STEP 7}\]

STEP 6

Assign \((i, j) = W_{k\ell}.\) \quad \text{If } X_{ij} = 0 \text{ then go to STEP 2; otherwise}\n
\[X_{k\ell} = \max \{X_{ij} + R_{ij} + t_{k\ell}, X_{k,\ell-1} + f_{k\ell} + t_{k\ell}\} \]

STEP 7

\[\text{LEG}_k = \text{LEG}_k + 1. \quad \text{If } \text{LEG}_k \leq n_k \text{ go to STEP 4; otherwise go to STEP 8.}\]

STEP 8

\[\text{LEG}_k = -1 \text{ and DONE} = \text{DONE} + 1. \quad \text{If } \text{DONE} = m \text{ then STOP; otherwise}\]

\text{go to STEP 2}.\)

A Decomposition Algorithm and Computational Results

Problem (P1) is a large-scale mixed-integer problem and the effort required to implement an algorithm that will generate an optimal solution does not seem appropriate. Thus, two decomposition approaches which utilize the algorithms for (P2) and (P3) are proposed. In the first algorithm, the integer restrictions on the variables are relaxed (i.e., constraint set (7) is dropped) and the linear programming approximation of (P1) is solved. The optimal solution of the linear program does provide a lower bound on (P1). This permits an efficiency measure for the integer solutions which the decomposition method obtains. After the linear program is solved, \(X\) is fixed at the optimal value for the relaxed problem and (P2) is solved. This will provide a feasible crew assignment. If the number of crews used is equal to the lower bound provided by the linear program, the procedure stops; otherwise, \(Y\) is fixed at the determined crew assignments and (P3) solved.
After (P3) is solved, slack in the aircraft schedule is evenly distributed over all flights to permit the possibility of an improved schedule. The method continues to alternate back and forth between (P2) and (P3) until an iteration limit is reached, a solution is repeated, or the lower bound is achieved. The second algorithm is the same as first except that the linear programming relaxation of (P1) is not solved. The initial plane schedule is obtained by evenly spacing all unforced delays over the time frame. The second algorithm was instituted because of the size of the linear program. We did not have a computer or computer code capable of solving the larger problems in an acceptable amount of time. The effectiveness of the decomposition methods is assessed in the remainder of this section.

Comparison of Algorithms 1 and 2

The larger problems that could be solved with algorithm 1 required approximately twenty crews to complete all missions in the time period allocated. Algorithm 2 required from zero to ten percent more crews than algorithm 1. Thus, when available, the linear programming solution did improve the performance of the decomposition approach. Also, algorithm 1 was able to provide a lower bound on the number of crews required and an upper bound on how far a generated solution could be from optimal. The lower bound was achieved on only a few of the smaller problems and the linear programming solution generally had several fractional assignments for the zero-one variables. A ten percent different between the lower bound provided by the linear program and the best integer solution was common.

An alternative linear programming model was considered. This model had an objective function which maximized the slack in constraint set (4) and included an additional constraint fixing the number of crews. The purpose of this alternative model was to produce a solution with fewer fractional values.
for variables by encouraging the zero-one variables to be nonbasic (the slacks to be basic). This approach was effective in reducing fractional solutions but had the disadvantage that the problem must be solved several times with the number of crews set at different levels. Also, little or no improvement was seen when using the solution obtained from this second linear programming model as a starting point for the iterative decomposition procedures. In any case, the effectiveness of alternative linear programming models was often a mute point because the solution of the linear program was not practical for large problems.

The solution methods for problem (P2) and (P3) were coded in standard FORTRAN and each required less than forty lines of code. The process of solving (P2) and then solving (P3) took under a second of CPU time on a DEC 20. This was true even for problems with 10000 integer variables and 35000 continuous variables. No improvement in objective value was ever obtained after five iterations between (P2) and (P3). Thus, excluding input/output time, no more than five seconds of CPU time was needed to obtain crew assignments and aircraft schedules. The speed of the algorithm is important because unanticipated delays may make an earlier schedule inoperable. The rescheduling must take place during the airlift. As mentioned earlier, the algorithm for (P2) is easily modified to accommodate initial staging of crews. However, in certain instances the value of T must be expanded to complete all missions with the number of crews allocated to the system from an earlier solution.

Comparison with Simulation

In the past, the Air Force has used a simulation to assess the performance of airlift operations. The initial staging of the crews and the scheduling were performed more or less manually. To assume the reliability of
the simulation estimates many simulations using different initial staging and scheduling policies have to be performed to insure that assessed poor airlift capability is actually due to resource constraints rather than suboptimal staging and scheduling policies. These simulations usually consumed many hours of computer time. The simulation measured effectiveness in terms of aircraft utilization which is the percentage of time an aircraft is in the air from the beginning of the airlift to the end of the airlift. When the initial crew staging from the solution obtained by algorithm 2 was used by the simulation, the aircraft utilization was within one percent of the best utilization obtained by the simulation. Table 1 shows sample results with a large airlift operation. The main disadvantage of the simulation exercise with manual staging was that computer time was measured in hours, whereas with algorithm 2 it is measured in seconds.

Conclusions

Scheduling is one of the most frequently considered class of problems in the management science literature. However, the mathematical programming problems arising from scheduling applications are often difficult to solve and workable schedules must be obtained from heuristics [4,7]. The algorithm presented in this paper is also heuristic. The problem is partitioned into the problem of scheduling aircraft and then, based on the schedule, crew assignments to flight legs are made. The aircraft scheduling problem is again considered subject to the most recently obtained crew assignments. This "flip-flop" approach between crew assignments and aircraft schedules continues until no progress is made in improving the schedule. The algorithm can be started from an available feasible schedule or an initial schedule will be generated automatically. By using a linear programming solution to start the algorithm reduced the number of crews required by up to ten percent. However, the size of the linear program and the need for a rapid response often precluded the solution of the linear program.
The decomposition algorithm is definitely effective from the standpoint of solution time and empirical results indicate the worth of the generated schedules. The initial stagings obtained from the decomposition algorithm are evaluated as being essentially equivalent to the best stagings generated by simulation. The simulation exercise using manual staging consumed several hours of computer time while using the decomposition approach only a few seconds were consumed.
Table 1. Achieved aircraft utilization rates (hours/day-plane) using different staging policies. A 1/n rule implies that approximately one crew is staged at a base for every n legs departing from the base.

<table>
<thead>
<tr>
<th>Days</th>
<th>Proposed Algorithm</th>
<th>&quot;1/18&quot; Rule</th>
<th>&quot;1/36&quot; Rule</th>
<th>&quot;1/55&quot; Rule</th>
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<tr>
<td>1 - 15</td>
<td>9.85</td>
<td>9.11</td>
<td>10.28</td>
<td>9.51</td>
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<tr>
<td>16 - 30</td>
<td>9.94</td>
<td>9.43</td>
<td>9.97</td>
<td>9.28</td>
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<td>31 - 45</td>
<td>10.27</td>
<td>8.89</td>
<td>10.21</td>
<td>9.37</td>
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<td>9.77</td>
<td>9.42</td>
<td>9.54</td>
<td>9.52</td>
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<tr>
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<td>10.24</td>
<td>9.36</td>
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References


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