THE DIFFERENTIAL EMISSION MEASURE OF DYNAMIC CORONAL LOOPS.

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by

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Grant NGL 05-020-272
Grant NAGW-92

OFFICE OF NAVAL RESEARCH
Contract N00014-75-C-0673

SUIPR Report No. 863
December 1981

INSTITUTE FOR PLASMA RESEARCH
STANFORD UNIVERSITY, STANFORD, CALIFORNIA

DISTRIBUTION STATEMENT A
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Abstract

Although most of the observations of soft x-ray and UV emission from stellar sources can be adequately explained in terms of static models of coronal loops, there are now several observations in soft x-rays of a solar-like flare from a stellar flare source, (e.g. Johnson 1981, Kahler 1981; Stern, et al.

We discuss the effects of time dependant phenomena, such as flare energization and decay, on the temperature and density structure of the transition region and, in particular, on the form of the differential emission measure. We find that unlike the case of the static models, the form of the differential emission measure can be used to determine the important physical mechanisms in the dynamic models.

I. Introduction

Since the advent of solar x-ray observations with high spatial resolution, (in particular the data from Skylab), there has been considerable interest in the so-called "quasi-static" model of coronal loops, (e.g. Landini and Fossi 1975, Rosner et. al., 1978, Craig et. al., 1978, Hood and Priest 1979, Vesecky et. al., 1979). The important assumptions in the "quasi-static" model are that (a) the plasma in static and (b) the magnetic field dominates the plasma so that a one-dimensional treatment (i.e. parallel to the field) is sufficient. Additional assumptions that are usually made are that: (c) the conductive flux is small compared to the saturated value so that the collisional thermal conductivity is valid (Spitzer 1962); (d) all radiation is optically then so that the losses calculated by e.g. Cox and Tucker (1969) or Raymond and Smith (1977), can be used; and (e) the coronal heating can be expressed as a simple function of density and temperature, (usually two power laws).

The key result of the static model is that the plasma temperature and density profiles, along the loop, are essentially independant of the assumed form for the heating function. The quantity obtained from the observations is the differential emission measure, which we define as:

$$\xi = A n^2 \left| \frac{d \ln T}{ds} \right|^{-1}$$

(1)
where \( A \) is the cross-sectional area of the loop, \( n \) is the electron density, and \( \frac{d \ln T}{ds} \) is the temperature scale height along the loop. The static model predicts that:

\[
\xi \propto T^\delta \quad \text{with} \quad \delta = \frac{3}{4} + \frac{l}{2},
\]

(2)

where \( l \) measures the slope of the radiative loss coefficient (see Cox and Tucker 1969), i.e.

\[
\lambda(T) \sim T^{-l}
\]

(3)

Since \( l \approx 1/2 \) for transition region temperature (Rosner et al., 1978), we find that \( \delta = 1 \), for a static loop.

Note that, as stated above, the differential emission measure is independent of the heating function parameters, eqtn. (2). Hence, observations of \( \xi \) are ineffective for determining the nature of the coronal heating mechanism; however, they do provide a stringent test of the assumption that the coronal plasma is static. If a value for \( \delta \) is observed that is not compatible with eqtn. (2), then this argues strongly in favour for a dynamic model. In practice, eqtn. (2) provides only an upper limit for \( \delta \) since the observed emission is unlikely to be due to a single loop, and the effect of adding the contributions from a collection of loops is to decrease the value of \( \delta \), (e.g. Antiochos 1980).

The observed value of \( \delta \) for solar quiet and active regions are generally \( \leq 1 \), and hence are compatible with the static model, (Noyes, et al., 1970, Raymond and Doyle 1981). Unfortunately, there is little data in the EUV region on stellar coronae, so that \( \delta \) has not been accurately determined for stars other than our sun. What data is available also appears to be compatible with eqtn. (2) (Zolcinski, et al., 1981). Solar flares, on the other hand, are generally not compatible with eqtn. (2). Several authors (Dere, et al., 1977, Underwood et al., 1978, Widing and Spicer 1980) have reported a large value for \( \delta \), \( \geq 3 \), in the temperature range, \( 10^5 \leq T \leq 10^7 \). This value cannot be reconciled with the static model even if a collection of loops is observed.

Of course, flares are not static so that it is not surprising that their values for \( \delta \) do not agree with static models. The important point here is that observation of the form of the
differential emission measure can be used to infer the presence of dynamic effects, even if the observations do not have sufficient temporal or spatial resolution to observe these effects directly.

In the next section we derive the value for $\delta$ that is predicted by the dynamic models. It turns out that this value is sensitive to the particular physical mechanism that dominates the dynamics.

II. Dynamic Models

The dynamics that we consider are assumed to be due basically to a time dependent coronal heating function. This time dependence could represent either an impulsive event such as a flare, or a more gradual evolution such as is expected to occur during the formation and decay of an active region loop. Note that we will not consider extremely rapid events such as chromospheric evaporation by a beam of energetic electrons, (the so-called "primary" evaporation). For such processes, the form of the differential emission measure will clearly depend on the details of the energization, (e.g. the exact time profile, energy spectrum, etc. of the electron beam), and will be different for every particular event. In addition, non-thermal effects, such as departures from ionization equilibrium, are likely to be important so that the simple differential emission measure as defined in (1) is no longer relevant to the observations. Fortunately such processes, (e.g. the hard x-ray bursts in solar flares), are typically short-lived compared to the total flare duration as measured, for example, by the soft x-ray bursts in solar flares.

We identify three important phases for the "thermal" evolution of a flare-like loop, that predict a characteristic signature for the form of the differential emission measure, i.e. for $\delta$. These phases are characterized by the ratio, $R(T)$, of the radiative flux from loop plasma at temperature $T$, to the conductive flux at that temperature:

$$R(T) = \frac{n^2 \Lambda(T) H_T}{10^{-6} T^{7/2} / H_T}$$

(4)

where $H_T$ is the temperature scale height, as given in eqtn.(1).

We define the three phases as: (a) conduction dominated, for which $R \ll 1$ throughout the loop, i.e. for $T \gtrsim 10^4$; (b) condensation dominated, for which $R \ll 1$ in the corona, i.e. for $T \gtrsim 10^6$, but $R \gtrsim 1$ in the transition region, $T < 10^6$; and (c) radiation dominated, for which $R \gtrsim 1$ throughout the loop.
IIA Conduction Dominated Models

In the early stages of a flare event, we expect conduction to dominate the dynamics. This results from the following argument. Assume a coronal loop is initially static. In this case $R = 1$ throughout the loop (Vesecky, Antiochos and Underwood 1979), so that radiation and conduction are approximately equal throughout. If the energy input to the loop begins to rise rapidly, as in a flare event, then the initial response of the plasma is to increase its temperature. The density cannot rise until material is evaporated up from the chromosphere. Since the conductive flux increases rapidly with temperature, $F = T^{7/2}$, whereas the radiative losses do not, (in fact, they decrease somewhat for $10^{5} \leq T \leq 10^{7}$), the effect of the increased energy input is to enhance conduction over radiation, i.e. $R$ decreases. If the increase in energy input is significant, as in a flare, then $R$ becomes $<< 1$ throughout the loop and conduction dominates.

This results in a large heat flux entering the chromosphere from above. Since radiation is insufficient to dissipate this flux, it must result in a large enthalpy flux, and hence mass flux, being driven up the loop. This is the process of "chromospheric evaporation" as described by Antiochos and Sturrock (1978). Under some simplifying assumptions, the temperature gradients, and therefore the differential emission measure, can be calculated quite readily for the evaporative model. We assume a one-dimensional model, neglect the effect of gravity, and assume that the heat flux is well below its saturated value so that the motions are highly subsonic. In this case the relevant equations are:

$$\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial s} \left( \frac{\partial p}{\partial \rho} \right) = 0 \quad (5)$$

$$\frac{\partial p}{\partial s} = 0 \quad (6)$$

$$\frac{3}{2} \frac{\partial p}{\partial t} + \frac{1}{4} \frac{\partial}{\partial s} \left( \frac{5}{2} \frac{\partial p}{\partial s} - 10^{-6} \frac{\partial V}{\partial t} \frac{\partial T}{\partial s} \right) = \epsilon(t) \quad (7)$$

where $\rho$ is the mass density, $A(s)$ is the cross-sectional area of the loop and $\epsilon(t)$ is the energy input function, assumed to be a function of time only. Note that we have not included the radiative losses in the heat equation since these are presumed
to be small compared to the other terms in the equation.

Imposing the boundary conditions that at the loop footpoints the velocity and heat flux become small, eqtn. (7) implies that:

\[
\frac{5pv}{2} = 10^{-6} T^{5/2} \frac{dT}{ds}
\]  

(8)

Substituting (8) into (5) yields a nonlinear equation for \( T(t,s) \) that can be solved by separation of variables. Investigating the form of the solutions of this equation, it turns out that over most of the temperature range, (i.e. except for temperatures very near the maximum temperature in the loop), the heat flux,

\[
F = 10^{-6} T^{5/2} \frac{dT}{ds} \propto T
\]  

(9)

Since

\[
\xi = p^2 \frac{T^{3/2}}{F}
\]  

(10)

the evaporative model predicts that \( \xi \propto T^{1/2} \) over most of the temperature range. Note that in deriving (10) we have used the fact that the loop is isobaric, eqtn. (6), and that from (9),

\[
F \propto T^{7/2}/H_T
\]  

(11)

In addition, we have used the result that the size scale for variations of the magnetic field, and \( A(s) \), are large compared to the size scale of the transition region.

Our main conclusion is that conduction driven mass motions tend to decrease the slope of \( \xi \), (\( \delta = 1/2 \)), from the value given by the static models (\( \delta = 1 \)). Unfortunately, the effect of observing a collection of static loops is also to decrease the value of \( \delta \). Hence, the observation that \( \delta = 1/2 \) does not necessarily imply that conduction driven evaporation is occurring, unless the observations also indicate that the loop density is increasing with time.
IIB Condensation Dominated Models

Once the flare energy input ceases, the temperature in the loop will begin to drop, whereas the density still increases as a result of conduction driven evaporation. These two effects act to increase the ratio \( R(T) \). Eventually this ratio reaches unity at some point in the loop, and chromospheric evaporation ceases since now radiation is sufficient to dissipate the downward heat flux. The temperature at which \( R \) first reaches unity is \( \approx 10^5 \) K as can be seen from Fig. 1, where we indicate the behavior of \( R(T) \) for a typical conduction dominated loop.

![Figure 1. \( R(T) \) for the conduction dominated models.](image)

When \( R(T) \) becomes of order unity at \( T \sim 10^5 \), the loop plasma at this temperature begins to cool very rapidly because the downward heat flux is no longer sufficient to sustain the large radiative losses. As a result of this rapid cooling, a pressure difference between the material at \( \sim 10^5 \) and that at \( \sim 10^7 \) is created which, in turn, generates large downward velocities. This process may be thought of as the opposite of chromospheric evaporation in that the loop plasma cools and condenses onto the chromosphere.

Assuming, again, a one-dimensional model and, in addition, assuming that in the transition region where the velocities are large (supersonic), \( 10^5 < T < 10^6.5 \), the flow is approximately in a steady-state, the relevant equations become:
\[ \rho v = \text{const.}, \quad (12) \]
\[ \rho v^2 + p = \text{const.}, \quad (13) \]

and
\[ \frac{d}{ds} \left( \frac{1}{2} \rho v^3 + \frac{5}{2} \rho v - 10^{-6} T^{5/2} \frac{dT}{ds} \right) = - n^2 \Delta(T), \quad (14) \]

where we have assumed that the loop cross-section varies insignificantly over the transition region.

The condensation model described by equations (12)-(14) has been investigated by Antiochos and Sturrock (1982). The important results of their analysis are that (a) the coronal plasma, \( T \sim 10^7 \text{K} \), cools primarily by mass motion, rather than by conduction or radiation; and (b) in the region where the flow is supersonic, \( \delta = 3.5 \). The latter result is due primarily to the fact that the flow is isochoric rather than isobaric.

The key aspect of the condensational model, for stellar purposes, is that it predicts a larger value for \( \delta \) than can be explained by the static models. In addition, we expect that it is valid during most of the decay phase of a flare, because the time scale, \( \tau \), for cooling by mass motions increases with decreasing \( T \). In particular,
\[ \tau \propto \frac{1}{C} \propto T^{-1/2} \]

where \( C \) is the sound speed of the coronal plasma. Hence, if stellar flare emission is detected, at all, it is likely to be that predicted by the condensation dominated model.

III. Radiation Dominated Models

In the later stages of flare decay, we expect that radiation cooling dominates throughout the loop. This can be seen by comparing the ratio, \( R' \), of the radiative cooling rate to that by mass motion:
\[ R' \propto \frac{n^2 \Delta(T)}{pC} \propto n/T^2 \quad (16) \]

Since, in the condensation phase, most of the coronal energy loss is due to mass motion, rather than conduction or radiation (Antiochos and Sturrock 1982), the coronal plasma cools primarily by adiabatic expansion so that
\[ p \propto n^{5/3} \text{; hence, } R' \propto T^{-1/2} \quad (17) \]

This result implies that eventually radiation will dominate even
in the corona. However, since $R'$ has a weak dependence on $T$, radiation will not dominate until the coronal plasma has cooled to quite low temperatures, probably $< 10^6$, and has lost the bulk of its energy. Hence, the total amount of emission during the radiation dominated phase is likely to be negligible compared to that radiated during the condensation phase.

In any case, we have calculated the differential emission measure to be expected for a radiative dominated loop (Antiochos 1980), and it turns out to predict a value for $\delta$ of only $\sim 1$. This value is not sufficiently different from that given by the static models so that it can be used as an observational test. Of more importance for the radiation dominated model is the rapid decay expected for the emission.

IV. Conclusions

In summary, we find that the form of the differential emission measure is a sensitive indicator of the important physical processes in the dynamic models. We tabulate the slope of $\xi(T)$ predicted by the various models below:

<table>
<thead>
<tr>
<th>Model</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>1.0</td>
</tr>
<tr>
<td>Conduction</td>
<td>0.5</td>
</tr>
<tr>
<td>Condensation</td>
<td>3.5</td>
</tr>
<tr>
<td>Radiation</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The model that has most significance for stellar observations is the condensation dominated model which predicts a large value for $\delta$ and also large supersonic velocities.

Acknowledgements

This work was supported by NASA Contracts NAGW-92 and NGL 05-020-272, and ONR Contract N0014-75-C-0673.
References