SUBTLE PERTURBATIONS AFFECTING QUARTZ RESONATORS USED FOR HIGH...
SUBTLE PERTURBATIONS AFFECTING QUARTZ RESONATORS
USED FOR HIGH-STABILITY APPLICATIONS

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The quest for high stability crystal resonators for advanced frequency control applications has led very recently to a number of far-reaching developments. As the level of frequency stability has improved over the years, various factors that were once imperceptible have become nonnegligible, and their influences have had to be eliminated, reduced, or compensated. At present the most obvious contributors to resonator performance have been enumerated. It is important to continue to catalog less obvious effects, and to evaluate critically at what level their influences make an appearance in the error budget of the overall system.
Abstract (cont)

We consider a number of influences that are more-or-less "subtle". Each is examined to see what contribution it makes to resonator performance. It is apparent that some have to be taken into account in present-day applications; others may become important as future stability specifications become even more stringent.
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INTRODUCTION

The year 1980 marks the centenary of the discovery of piezoelectricity by the brothers Curie, and the fifty-ninth year of the quartz oscillator.* The past thirty years or so have seen the development of cultured quartz from coin-sized pieces, shown in Fig. 1, to large, multi-kilogram crystals that constitute the bulk of the usage today (Fig. 2). Since the introduction of the quartz crystal-controlled oscillator by Cady, each decade of time has seen approximately one order of magnitude frequency stability improvement. Surrounding the development of high-stability oscillators incorporating crystal vibrator control has been the on-going interplay between improvements in resonator technology and causes of ever-smaller perturbations in resonator frequency and equivalent circuit parameters.1 Progress in timekeeping is shown on a broad time scale in Fig. 3, and a comparison of crystal resonator performance with atomic and molecular frequency standards is shown in Fig. 4; both Figures are adapted from Reference 2.* Further improvements will depend upon less-obvious factors than have been considered to date. This technical report treats a number of such topics.

MAGNETIC FIELDS

Quartz, and other high purity dielectric crystals, can be considered to be nonmagnetic for our purposes here. A vibrating crystal with metallic, nonmagnetic electrodes is, however, affected by external magnetic fields. Eddy currents are produced in the electrodes by their motions in the fields; these change the critical frequencies of the vibrator, and lower its Q. The situation considered is that of a resonator vibrating in a pure shear mode with applied static magnetic field parallel to plate thickness as depicted in Fig. 5. The resonator leads are attached to an active network comprising the remainder of the oscillator, and the combination adjusted in frequency so that the vibrator operates at zero reactance at all times.

For a pure shear mode, the retarding force density at the plate surfaces is

\[-j \omega \sigma B^2 u,\]

with \(\sigma\) the electrical conductivity of the electrode, and \(u\) the mechanical displacement at the surfaces. Electrodes of thickness \(h'\) will produce resistances of

\[R = A B^2 h'\]

for areas of value \(A\). Using conventional notation\(^3\)*, this leads to an expression for the imput admittance of the single mode vibrator, including the effects of electrode inertia\(^4\):

\[\text{Y}_{\text{in}} = \frac{1}{\text{R} + \frac{j}{\omega C} + \frac{1}{j\omega L}}\]

*See list of references beginning on page 72.
FIGURE 1. Early Example of Quartz Synthesis
Figure 3. Progress in Timekeeping Accuracy
Figure 4. Fractional frequency stability of several types of oscillators as function of sample time.
\[
Y_{in} = j \omega C_{n} + j \omega C_{o}.
\]

\[
((X/(k' \tan X) - 1 - X'/\mu/k')) + (jXRY_{o}/k')^{-1}.
\]

Resonance and antiresonance frequencies are determined from the zeros of the susceptance function as the roots of

\[
(X-(k' + \mu X') \tan X) \cdot (1- \mu X \tan X)
+ X \cdot (R Y_{o})^{2} \cdot \tan^{2} X = 0.
\]

When \( R = 0 \), the first expression in brackets yields the condition for resonance, and the second that for antiresonance. From the expression for \( Y_{in} \) the network in Fig. 6 is obtained, with the electrode inertia represented by the inductances

\[
L_{1} = L_{2} = L = \rho h \mu A.
\]

The network components are labeled with their physically corresponding effects. When \( R \) is not zero, the inductances are replaced by series \( R-L \) combinations, and when electrode mass-loading is neglected, the circuit appears as given in Fig. 7. Figure 8 is the bisected version of Fig. 7 with massless electrodes (\( \mu = 0 \)).

Departures from the resonance frequency as function of \( R \) are found by perturbation, with \( RY_{o} \) the small parameter. The expression for frequency shift is

\[
\Delta f/f = -(2/\pi)^{2} (RY_{o})^{2},
\]

with \( RY_{o} = \sigma h - B^{2} / (\rho v) \),

\( \sigma \) the quartz mass density, and \( v \) the acoustic velocity of the mode in question. It is seen from (6) and (7) that the frequency shift is quartic in the magnetic field strength. Figure 9 gives frequency shift values for an AT cut having 10,000 \( \AA \) copper electrodes; only for fields about ten thousand times larger than that of the earth's (the first entry) is the effect perceptible.

This analysis has been extended to the multimode case, and to the situation of a lateral magnetic field, with corresponding network representations.

**Resonator Asymmetries**

In treatments of resonator electroding, it is usually tacitly assumed that both electrodes are of equal size and thickness. The practical reality is usually just the opposite. It is very difficult to guarantee that the thicknesses are equal in the first place; to make matters worse, final frequency adjustment is usually made with a burst of material applied to a...
FIGURE 6. Analog Network of Resonator with Electrode Imbalance
FIGURE 7. Analog Network of Resonator in Magnetic Field
FIGURE 8. Bisected Analog Network
\[ \Delta F/F \approx -\left(\frac{e}{h}\right)^2 R^2 \gamma_0^2 \]

\[ R\gamma_0 = \sigma \hbar B^2 / (\rho v) \]

<table>
<thead>
<tr>
<th>( B ) ( )</th>
<th>( \Delta F/F ) ( )</th>
<th>( H ) ( )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TESLA</td>
<td>KILO-GAUSS</td>
<td>AT CUT 1( \mu )M Cu ELECTRODES</td>
</tr>
<tr>
<td>5.8 (-5)</td>
<td>0.58 (-3)</td>
<td>-2.2 (-28)</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>-1.9 (-15)</td>
</tr>
<tr>
<td>0.3</td>
<td>3</td>
<td>-1.5 (-13)</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>-1.9 (-11)</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>-1.5 (-9)</td>
</tr>
</tbody>
</table>

*FIGURE 9. Frequency Shifts Due to Magnetic Fields*
single side. Add to this situation the facts that registration of both electrodes is never perfect, that the electrode tabs are asymmetrically disposed, and that sometimes a larger electrode is used on one side to improve statistical uniformity of the motional capacitance by compensating for misregistration, then it is seen that symmetry of the plate resonator about the mid-plane is seldom, if ever, achieved.

The single-mode case is treated, by way of example, for a simple electrode mass imbalance in the one-dimensional approximation. The equivalent network is that of Fig. 6 with the mechanical transmission line ports attached to unequally sized transmission lines representing the thick electrodes. When the electrodes are thin enough for propagation of the acoustic waves within the electrodes to be neglected, the transmission lines are replaced with lumped inductances; the mass imbalance results in unequal inductance values $L_1$ and $L_2$, as shown, and prevents the network from being bisected, as in Fig. 8. When bisection can take place, the transmission line representing the crystal plate of thickness $2h$ is replaced by one of half this value and the construction for the critical frequencies is that given in Fig. 10; the formulas are those of the bracketed terms in (4) separately set to zero. Figure 11 gives the normalized frequency variable

$$X_{RO}^{(M)} = \frac{\pi}{\lambda} \left( \frac{f_{RO}^{(M)}}{f_{AO}} \right)$$

versus piezoelectric coupling $k$ for the first three harmonics $M = 1, 3, \text{ and } 5$. An AT-cut resonator with symmetrical electrodes of equal thickness produced the mode spectrograph at the top of Fig. 12. Strong responses are found at the odd harmonics, and nothing is visible at the even harmonics.

It is appropriate now to quote from Cady¹, p. 308:

"There are circumstances under which a plate may conceivably vibrate in a thickness mode at or close to an even harmonic frequency. The shape, size and location of the electrodes may be such as to produce a driving field in the plate that varies in the direction of the thickness. Or the plate may be twinned or have other defects such that the excitation is not uniform. Finally, the plate may be in contact with an electrode of considerable mass, so that in effect one has a composite resonator."

When the electrodes are of unequal mass, and circuit bisection is not possible, the construction becomes that of Fig. 13, where the critical frequencies are determined by intersection with a tangent function of argument $2X$ instead of $X$. The expression for input impedance is

$$Z_{in} = \frac{k^2 (\tan X/X)}{1^2 \cdot (1- \frac{\Delta^2 X^2}{(1+\Delta X \cot X)})},$$

(8)

$$\left((1-\Delta X \tan X) + \Delta^2 X^2 \right) \left((1+\Delta X \cot X) \right)$$

12
FIGURE 11. Normalized Resonance Frequency versus Coupling
FIGURE 12. Symmetric and Asymmetric Resonators
FIGURE 13. Construction for Antiresonant Frequencies with Imbalance

\( \mu_1 = 15\% ; \mu_2 = 5\% \)

\[ \frac{(\mu_1 + \mu_2)}{\mu_1 \mu_2} \]

BRANCHES OF TAN 2X
where \( \zeta = (\mu_1 + \mu_2)/2 \) and \( \lambda = (\mu_1 - \mu_2)/2 \). Equation (8) has been written in terms of argument \( X \) for the tangent and cotangent functions. The anti-resonant frequencies (denoted \( f_{A}^{(M)} \) for \( M \) odd, and \( f_{A}^{(M)} \) for \( M \) even) are found from the vanishing of the bracketed term in (8) as roots of

\[
\tan 2X = \frac{-\frac{1}{2} \mu_1 + \mu_2 \cdot X}{1 - \mu_1 \cdot \mu_2 \cdot X^2}.
\]

These are shown in Fig. 13 along with the high and low frequency asymptotes of the right hand side of (9). The roots for the resonant frequencies are found from the zeros of (8).

The even-harmonic resonances produced by the unequal mass-loadings on the crystal plate surfaces \( \{\mu_1 \neq \mu_2\} \) are very sharp compared to the odd harmonics as may be seen from Fig. 14, which is drawn for a case with large piezo-coupling and large mass asymmetry in order to widen the resonance regions of both the even and odd harmonics. It is thus possible to use such resonances for stabilizing the frequency of an oscillator - at least in principle. Figure 15 shows a blow-up of the second harmonic of the case of Fig. 14; Fig. 16 shows the fourth harmonic, and Fig. 17 shows the fourteenth harmonic. Compare the width of the resonance in Fig. 17 with that of Fig. 18 showing the fifteenth harmonic. In general, for large \( M \), the normalized frequency of response approaches \( (M-1) \).

When the resonator shown in the top of Fig. 12 was overcoated with electrode material on one side only, the mode spectrum shown in the bottom of the figure was produced. The second harmonic becomes present due to the imbalance, and it is sharp; however, it is also very weak. Figure 19 shows a photograph of the spectrograph given in the top of Fig. 12; Fig. 20 is a photograph of the spectrograph for the case of mass imbalance to the same scale as Fig. 20; Fig. 21 shows a magnified view of the photograph of Fig. 20; it corresponds to the lower drawing in Fig. 12.

Another unsymmetrical structure that leads to even harmonics is the composite resonator; (see Reference 5 for additional references). This structure is shown in Fig. 22, with the equivalent network given in Fig. 23. Here the quartz substrate must be represented by a transmission line because of its thickness; as it shrinks in size the situation of Fig. 6 with inductors is approached.

Apart from producing even harmonics, the presence of asymmetry in resonators (due to unequal electrodes, bevelling, contouring, etc.) is very probably a major cause of "activity dips," or "bandbreaks." These are illustrated in Fig. 24, which is taken from Reference 8. Both the frequency versus temperature and the conductance versus temperature curves suffer anomalous dips as the interfering mode" influence is manifest at a particular temperature. The second harmonic mode "divided by two" via the nonlinear elastic constants can produce dips at the fundamental.
FIGURE 14. Wideband Spectrogram of In-balanced Resonator

k = 50%
\bar{\mu} = 10%
\Delta = 5%
Figure 15. Spectrograph Near the Second Harmonic
FIGURE 17: Spectrogram Near the Fourteenth Harmonic
FIGURE 19. Photograph of Spectrum of Balanced Resonator
FIGURE 20. Photograph of Spectrum of Imbalanced Resonator
FIGURE 21. Magnification of Spectrum of Figure 20
THIN-FILM LAYER OF HIGH-COUPLING PIEZOELECTRIC CRYSTAL DRIVES THE COMPOSITE STRUCTURE.

ELECTRODES

QUARTZ SUBSTRATE

FIGURE 22. COMPOSITE RESONATOR STRUCTURE
FIGURE 23. EQUIVALENT NETWORK OF COMPOSITE RESONATOR
FIGURE 24. Frequency-Temperature Plots Showing Activity Dips

- $C_{L1} = 35 \text{ pF}$  
- $C_0 = 2.1 \text{ pF}$  
- $C_{L2} = 22 \text{ pF}$  
- $f_R = 20 \text{ MHz}$  
- $R_1 = 3 \text{ } \Omega$  
- $P = 12 \mu\text{W}$
ELECTRODE INFLUENCES

It is well-known that the frequency-temperature (f-T) curve of an AT plate resonator measured sufficiently slowly assumes a cubic shape, as depicted in Fig. 25. Small changes in orientation angle $\phi$, for rotation about the crystallographic $X_1$ axis, produce slope changes at the inflection point of $-5.08 \times 10^{-3}/K^0$. A variety of other influences lead to nonnegligible slope variations as well. Figure 25 shows the effect of placing a load capacitor in series with the vibrator to produce frequency $f_0$, as is often done for frequency trimming. The slope shift reduces the peak-to-peak frequency deviation and must be taken into account in design of temperature compensated crystal oscillators (TCXOs); it comes about because of the temperature coefficient of the piezoelectric coupling factor.$^{10}$ A similar effect occurs when a resonator is operated at different harmonics, and for the same reason.

The mass of, and intrinsic stress in electrode coatings also affects the f-T curve. Some time ago the effect of equal electrode coatings was demonstrated in regard to frequency shift and rotation of the static frequency-temperature curve of a resonator.$^{11-13}$ The calculations dealt only with the influence of inertial mass; no elastic properties were assumed for the electrodes, nor were they presumed to apply any stresses to the crystal. This was done so that the inertial influence of the electrode could be subtracted from elastic and stress effects. Results from the one-dimensional theory for the change in apparent orientation angle are given in Fig. 26 and Fig. 27 as function of harmonic number $M$ and mass loading $\mu$. A comparison of apparent orientation angle shifts as function of mass loading and harmonic number is given in Fig. 23 for AT plates of 5MHz fundamental frequency, and plate/electrode diameters of 14 and 6 mm, respectively, coated with evaporated aluminum, sputtered chromium, and sputtered indium. The electrodes are of equal thickness-s. It is seen that Al and In have slopes that agree with inertial mass theory, but are displaced; the curves for Cr are consistent among themselves, but differ in sign from the theory. The results in Fig. 28 indicate the importance of electrode material, deposition method, and conditions of deposition on the behavior of the finished resonator. Figure 29 gives the experimental data for aluminum evaporated electrodes as function of $\mu$ and $M$. Figure 30 shows the decrease in inflection temperature with increasing mass of evaporated aluminum, for resonators with the parameters described above.

It should be borne in mind that stresses in the electrodes can also produce activity dips, even when the electrodes are of equal mass. Consequently, the practice of furnishing AT quartz plates with a thin layer of Ni or Cr to insure adhesion of gold, e.g., ought to be re-examined in view of the stress levels produced by these metals, both intrinsically, and as a function of temperature.
FIGURE 25. Frequency-Temperature Plots of AT-Cut Resonators
FIGURE 26. Apparent Angle Shift versus Mass Loading

AT-CUT QUARTZ
k = 8.796 %
FIGURE 27. Angle Shift versus Loading for Harmonic Transitions
FIGURE 28. Experimental Angle Shifts versus Mass Loading
### Figure 29. Apparent Angle Shift in Minutes of Arc

<table>
<thead>
<tr>
<th>Harmonic Number</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>M = 0%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+9.4</td>
<td>+3.13</td>
<td>+2.6</td>
<td>+2.1</td>
</tr>
<tr>
<td>M = 0.3%</td>
<td>-6.31</td>
<td>-0.53</td>
<td>-0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+1.73</td>
<td>+1.37</td>
<td>+1.2</td>
<td>+1.68</td>
</tr>
<tr>
<td>M = 0.9%</td>
<td>-6.63</td>
<td>-0.7</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+1.29</td>
<td>+1.25</td>
<td>+1.76</td>
<td></td>
</tr>
<tr>
<td>M = 1.5%</td>
<td>-6.5</td>
<td>-0.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+2.16</td>
<td>+2.25</td>
<td>+3.19</td>
<td></td>
</tr>
<tr>
<td>M = 2.3%</td>
<td>-6.58</td>
<td>+0.75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
- Values are in minutes of arc.
ANGULAR MISORIENTATION

Subtle influences on the static frequency-temperature curve were briefly discussed in the previous section in terms of apparent orientation angle changes in $\phi$, for AT resonators. The most general class of thickness mode plate resonators is the family of doubly rotated cuts. These are compared in Fig. 31 with singly rotated cuts, of which the AT cut is the most often used. Also shown in the figure are the loci of zero temperature coefficient, and the angular coordinates of popular quartz cuts. The line $\phi = 0$ is a line of symmetry, so that singly rotated cuts are only quadratically sensitive to misorientations in $\phi$. For the AT cut, misorientations $-\Delta \phi$ may be compensated for by changes $\Delta \phi$, obtained from

$$\Delta \phi = -1.77 \times 10^{-3} (\Delta \phi)^2,$$

with $\Delta \phi$ in minutes of arc, and $\Delta \phi$ in seconds of arc; this relationship is portrayed in Fig. 32.

X-RAY ANGLE CHANGES

The angles locating the X-ray planes in a crystal are functions of temperature and external stress. For example, the plane at $38^\circ 13'$ used for orienting AT cuts is only found at this angle at a certain temperature. For high precision applications the angle shift with temperature is not negligible. We give an approximate calculation to establish the order of magnitude of the angle change.

Consider a rotated-Y-cut plate of crystal in the rhombic, tetragonal, trigonal, hexagonal, or cubic systems. (These have no off-diagonal thermoelastic coefficients.) At temperature $T$ the plate angle will be determined from

$$\tan \epsilon = (X_2/X_3). \tag{10}$$

With a change in temperature to $T+\Delta T$, the axes become, in the linear approximation

$$X_2' = X_2 (1 + a_{22} \Delta T) \tag{11}$$
$$X_3' = X_3 (1 + a_{33} \Delta T)$$

We are thus led to a relation between $\Delta \phi$ and $\Delta T$:

$$\tan \Delta \phi = \{(A-1) \tan \phi / (1 + A \tan^2 \phi )\} \tag{12}$$

where

$$A = (1 + a_{22} \Delta T)/(1 + a_{33} \Delta T). \tag{13}$$
FIGURE 31. Doubly Rotated Cuts and Zero Temperature Coefficient Loci
FIGURE 32. Delta Theta versus Delta Phi for AT Cuts
As a consequence of (12) and (13) we have

$$\Delta \theta = \frac{1}{2} (a_{11} - a_{33}) \cdot \Delta T \cdot \sin 2\theta. \quad (14)$$

Figure 33 gives the X-ray angle change with temperature; beyond about 10 kelvins the linear approximation is not accurate except as to order of magnitude. For the tetragonal, trigonal and hexagonal systems, since \( a_{12} = a_{11} \), the result is independent of angle \( \phi \). In particular, for the upper zero temperature locus for thickness mode plates \( \theta \) is approximately constant and the result holds therefore for the AT, FC, IT, and SC cuts (cf. Fig. 31). Relation (14) shows that \( \Delta \theta \) depends on the difference between \( a_{11} \) and \( a_{33} \) for rotated-Y-cuts (or doubly rotated cuts when \( \alpha_{12} = a_{11} \)). For the cubic system this difference is zero, so \( \Delta \theta \) is always zero in this system. For quartz, the effective value of \( \alpha \) in the thickness direction of rotated-Y-cut plates is shown in Fig. 34; it is given by

$$\alpha = a_{11} \cos^2 \theta + a_{33} \sin^2 \theta; \quad (15)$$

for the Y cut, \( \alpha = a_{12} = 13.72 \times 10^{-6}/K \), while for the Z cut (\( \phi = 90^\circ \)), \( \alpha = a_{33} = 7.48 \times 10^{-6}/K \).

TWINNING

The types of twinning that may occur in quartz are shown in Fig. 35 along with the changes in axes and handedness.\(^{16,11}\) We briefly explore the effects of twinning on the piezoelectric constants of crystals in Class 18, which includes quartz. For the three types of twinning considered, the operations involved are

(a) Dauphiné: \( X_1 \rightarrow X_1; X_2 \rightarrow X_2; X \rightarrow X_3 \),

\[ e_{11} \rightarrow e_{11}; e_{14} \rightarrow e_{14}. \]

(b) Brazil: \( X_1 \rightarrow X_1; X_2 \rightarrow X_2; X \rightarrow X_3 \),

\[ e_{11} \rightarrow -e_{11}; e_{14} \rightarrow e_{14}. \]

(c) Combined: \( X_1 \rightarrow X_1; X_2 \rightarrow X_2; X \rightarrow X_3 \),

\[ e_{11} \rightarrow -e_{11}; e_{14} \rightarrow e_{14}. \]

Consider \( e_{-26} \), the piezoelectric constant driving the pure shear mode in rotated-Y-cuts of quartz:

Untwinned: \( e_{-26} = (e_{11} \cos \theta + e_{14} \sin \theta) \cdot (-\cos \theta) \).

(a) Dauphine: \( e_{-26} = (-e_{11} \cos \theta + e_{14} \sin \theta) \cdot (-\cos \theta) \).

(b) Brazil: \( e_{-26} = (e_{11} \cos \theta + e_{14} \sin \theta) \cdot (+\cos \theta) \).

(c) Combined: \( e_{-26} = (e_{11} \cos \theta - e_{14} \sin \theta) \cdot (-\cos \theta) \).
ANGLE $\theta$ AT TEMPERATURE $T$

$\Delta \theta = \frac{1}{2} (\alpha_{e} - \alpha_{c}) \cdot \Delta T \cdot \sin 2\theta$

<table>
<thead>
<tr>
<th>AT-FC-IT-SC CUTS</th>
<th>$\Delta T (K)$</th>
<th>$\Delta \theta (\text{sec.})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6</td>
<td>-0.6</td>
</tr>
<tr>
<td>3</td>
<td>1.8</td>
<td>-1.9</td>
</tr>
<tr>
<td>10</td>
<td>6.1</td>
<td>-6.4</td>
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<tr>
<td>30</td>
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<td>-19.1</td>
</tr>
<tr>
<td>100</td>
<td>60.6</td>
<td>-63.6</td>
</tr>
</tbody>
</table>

FIGURE 33. Change of Orientation with Angle
FIGURE 34. Thermoelastic Coefficient versus Theta
FIGURE 35. Electrical and Optical Twinning
Consider further a plate of unity area, with a fraction "a" of this area twinned. For simplicity, assume the twinning is complete throughout the thickness, and that the distribution of motion is uniform even though twinned (although we keep in mind Cady's remark, above). Then the composite $e^{-26}$ should be:

(a) Dauphiné:
$$e^{-26} = ((1-2a) e_{11} \cos \theta + e_{14} \sin \theta) \cdot (-\cos \theta).$$

(b) Brazil:
$$e^{-26} = (1-2a) e_{11} \cos \theta + (1-2a) \sin \theta \cdot (-\cos \theta).$$

(c) Combined:
$$e^{-26} = (e_{11} \cos \theta + (1-2a) e_{14} \sin \theta) \cdot (-\cos \theta).$$

In the case of Brazil twinning:
$$e^{-26} \text{ (twinned)} = (1-2a) \cdot e^{-26} \text{ (untwinned)}.$$  \hspace{1cm} (16)

Equation (16), when written in terms of the resonator capacitance ratio becomes:
$$r \text{ (twinned)} = r \text{ (untwinned)} / (1-2a)^2.$$  \hspace{1cm} (17)

**DYNAMIC THERMAL EFFECTS**

It has been known for many years that abrupt temperature changes produce frequency changes in quartz vibrators that are unpredicted by static f-T characteristics such as those of Fig. 25. The dynamic thermal effect has been given a phenomenological explanation by a modification of the cubic curve to include time-dependent terms that take into account thermal gradients in the resonator and support structure.\(^1\) The classical f-T equation
$$\frac{\Delta f}{f} = a_o \Delta T + b_o \Delta T^2 + c_o \Delta T^3$$  \hspace{1cm} (18)

is augmented by the addition of the term
$$\left( a \Delta T + \Delta T \right) \cdot \frac{dT}{dt}$$  \hspace{1cm} (19)

A temperature ramp ($dT/dt = \text{constant}$) applied to the resonator both rotates the f-T curve, through $\Delta T$, and translates it, by means of $\Delta T$. Figure 36 defines the desired reference temperature (which may be at either zero-slope point) at which the oven ought to be set, along with the actual oven setting point achieved, and the oven cycling range. The cycling range is
a measure of the quality of the crystal oven. The result of sinusoidal temperature variations of magnitude \( \Delta T = 60K \) about the inflection temperature of the static AT cut f-T curve is shown in Fig. 37. Temperature varies as \( \sin \omega t \); various \( \omega \) values are shown. In Fig. 38 is given the result of variations for \( \Delta T = 10K \) about the upper turning point. Figure 39 gives the result when the temperature magnitude is reduced to \( 5mK \). In Fig. 40 the effects of \( \Delta T = 10mK \) about the turning point of the static f-T curve of Fig. 25. With a period of 8 hours, the frequency change is about \( 3 \times 10^{-16} \); when the period is lengthened to 1 week, the change is still about \( 1 \times 10^{-14} \), irrespective of cycling rate. Since thermal transients, temperature cyclings, and fluctuations cannot be entirely avoided, the dynamic temperature effect is a very important consideration for high precision resonators. The magnitude of the effect can fortunately be reduced by recourse to doubly rotated quartz cuts, specifically the SC-cut.

AMBIENT VIBRATIONS

Insensitivity to acceleration is one of the most prominent criteria by which to judge high stability resonators. At present, the rule-of-thumb number relating frequency shift to applied acceleration is \( 2 \times 10^{-11} \) per g, although efforts are underway to reduce this value. (See references given in Reference 20.) It appears that a sensitivity of \( 1 \times 10^{-11} / g \) is a reasonable goal for the next few years. These two sensitivity figures have been used in conjunction with published values for acceleration levels in various environments to produce the frequency shift entries shown in Fig. 42 and Fig. 43.

From Fig. 42 and Fig. 43 it is seen that the decrease in acceleration sensitivity to parts in \( 10^{11} \) per g and beyond is highly desirable. The smallest entry in the figures is due to microseisms. Figure 44, taken from Reference 24, shows the very interesting spectrum of microseisms observed at the earth's surface. It appears that the spectrum of displacement versus period plotted on equal log-log scales consists of portions of +2 and -2 slope peaking rather sharply at a period of about 7 seconds. The maximum amplitude of acceleration is approximately 4 micro-g's, and is constant along the +2 slope line. For oscillators with high acceleration sensitivity this source of disturbance would have to be eliminated by mounting supports even for isolated, underground installations. Building vibrations are far greater than microseisms, and their noise spectra must be considered in the installation of precision oscillators. In certain locations, the accelerations due to earthquakes must be allowed for, although the vibrations found in urban environments produce accelerations associated with earthquakes of magnitude 3-4 on the Richter scale. This scale is defined, for shallow shocks, as the log_{10} of the maximum trace amplitude in micrometers with which the standard short-period torsion seismometer would register that earthquake at an epicentral distance of 100km.

Two crystal resonator structures having decreased acceleration sensi-
FIGURE 38. Dynamic Thermal Effect: $\Delta T = 10K$
AT CUT QUARTZ

\[ \omega = 2 \times 10^{-3} \]

\[ \Delta f/f \]

\[ (T - T_{\mu^+}) \text{ KELVINS} \]

Figure 20: Dynamic Thermal Effect; \( \Delta T_{\mu} = 5\text{mK} \)
Figure 40. Dynamic Thermal Effect: $\Delta T_0 = 0.5$ and 1 mK

$\Delta f = -2.27 \times 10^{-7} S/K^2$

$\omega = 10^{-4}$ and $10^{-5}$

(T - $T_\mu$) KELVINS

CCW ORBITS

AT CUT QUARTZ

$\sigma = 2 - 10^{-2}$
FIGURE 41. Dynamic Thermal Effect; $\Delta T_0 = 0.5$ K

$\frac{\Delta f}{f} = 2.2 \times 10^{-7}$ S/K²

$\omega = 2 \times 10^{-4}$

$\Delta T = 10^{-4}$

$\Delta T = 10^{-5}$
<table>
<thead>
<tr>
<th>ENVIRONMENT</th>
<th>ACCELERATION G UNITS</th>
<th>OSCILLATOR SENSITIVITY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$2 \times 10^{-9}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FREQUENCY SHIFT</td>
</tr>
<tr>
<td>ROAD VEHICLES</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TRACTOR-TRAILER (3-80 Hz)</td>
<td>0.2 PEAK</td>
<td>4 (-10)</td>
</tr>
<tr>
<td>PANEL TRUCK</td>
<td>0.02 RMS</td>
<td>4 (-11)</td>
</tr>
<tr>
<td>ARMORED PERSONNEL CARRIER</td>
<td>0.5 TO 3 RMS</td>
<td>6 (-9)</td>
</tr>
<tr>
<td>RAILROADS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>STANDARD</td>
<td>1 PEAK</td>
<td>2 (-9)</td>
</tr>
<tr>
<td>SOFT-RIDE</td>
<td>0.1 PEAK</td>
<td>2 (-10)</td>
</tr>
<tr>
<td>CROSSING TRACK/SWITCHES</td>
<td>6 PEAK</td>
<td>1.2 (-8)</td>
</tr>
<tr>
<td>NORMAL COUPLING &amp; SWITCHES</td>
<td>30 PEAK</td>
<td>6 (-8)</td>
</tr>
<tr>
<td>SHIPS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NORMAL CALM</td>
<td>0.02 TO 0.8</td>
<td>1.6 (-9)</td>
</tr>
<tr>
<td>ROUGH SEAS</td>
<td>0.8</td>
<td>1.6 (-9)</td>
</tr>
<tr>
<td>SLAM &amp; EMERGENCY MANEUVERS</td>
<td>TO 3</td>
<td>6 (-9)</td>
</tr>
</tbody>
</table>

**FIGURE 42.** Ambient Accelerations: Road/Rail/Ship
<table>
<thead>
<tr>
<th>ENVIRONMENT</th>
<th>ACCELERATION G UNITS</th>
<th>OSCILLATOR SENSITIVITY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2 x 10⁻⁹ G</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FREQUENCY</td>
</tr>
<tr>
<td>AIRCRAFT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PROPELLER</td>
<td>0.3 to 5 RMS</td>
<td>1 (-8)</td>
</tr>
<tr>
<td>HELICOPTER</td>
<td>0.1 to 7 RMS</td>
<td>1.4 (-8)</td>
</tr>
<tr>
<td>JET</td>
<td>0.02 to 2 RMS</td>
<td>4 (-9)</td>
</tr>
<tr>
<td>ROCKETS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MISSILES (BOOST PHASE)</td>
<td>15</td>
<td>3 (-8)</td>
</tr>
<tr>
<td>ABLE-I (MULTI-STAGE)</td>
<td>30</td>
<td>6 (-9)</td>
</tr>
<tr>
<td>VENUS ENTRY</td>
<td>310 MAX</td>
<td>6.2 (-7)</td>
</tr>
<tr>
<td>SEISMIC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EARTHQUAKE (EL CENTRO, CA, 1940)</td>
<td>0.33 MAX.</td>
<td>6.6 (-10)</td>
</tr>
<tr>
<td>MICROSEISMS</td>
<td>4 x 10⁻⁶ MAX.</td>
<td>8 (-15)</td>
</tr>
<tr>
<td>BUILDINGS (QUIESCENT)</td>
<td>0.02 RMS</td>
<td>4 (-9)</td>
</tr>
</tbody>
</table>

FIGURE 43. Ambient Accelerations: Aircraft/Rocket/Seismic
PERIOD T (SECONDS)

FIGURE 44. SEISMIC NOISE OF THE EARTH’S SURFACE

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tivity for accelerations in the thickness direction are the ring-supported and grooved configurations.\textsuperscript{20} Several varieties of these are displayed in Fig. 45 and Fig. 46; Fig. 45 gives the underlying rationale for using ring-supported resonators when the acceleration is applied in the thickness direction. The frequency shift is proportional to displacement of the central membrane, and because of the double cantilever edge conditions this displacement is smaller than in conventional simply supported resonators. Figure 47 defines the parameters pertinent to the geometry of the plano-convex ring-supported resonator. These parameters are listed below for resonator B-4: (in mm)

\[
\begin{align*}
\phi_o &= 7.950; \quad \phi_i = 6.020; \quad \phi_e = 4.064; \\
t_o &= 1.000; \quad t_i = 0.662; \quad t_c = 0.700; \\
R &= 120.
\end{align*}
\]

The measured frequency constant was \( N = 1.770 \text{ MHz} \cdot \text{mm} \). In Fig. 48 is shown the experimental frequency shift versus acceleration for resonator T-4, an AT-cut, plano-convex, ring-supported resonator with acceleration applied along the X axis. The frequency shifts are seen to saturate at high g levels, leading to the conclusion that the acceleration-frequency sensitivity coefficient \( i \) is nonlinear. Figure 49 gives the corresponding curves for resonator B-4. The concavo-convex ring-supported geometry in Fig. 46 is a preferred structure for unidirectional acceleration loads directed toward the radii of curvature. The resonator center is thicker than the junction to the ring to permit energy trapping.

**ACOUSTIC VISCOSITY**

One of the foremost reasons for using quartz as a vibrator material is its extremely low acoustic attenuation, as manifested in the large values of quality factor \( Q \) measured, typically as much as:

\[
Q = 10^7 / f_o, \quad (20)
\]

at room temperature, where \( f_o \) is the operating frequency in MHz. This is shown in Fig. 50. The values in (20) are met, and exceeded, only by very careful design of the mounting structure at frequencies below a few tens of MHz\textsuperscript{20}; above these frequencies the intrinsic internal friction of the material predominates. Calculation of the intrinsic loss follows from the measurement of the elements \( \eta_{ij} \) of the acoustic viscosity tensor.\textsuperscript{29}

For the rotated-Y-cuts of quartz, the piezoelectrically driven shear mode has an effective viscosity of

\[
\eta^e = \eta_{66} \cos^2 \theta + \eta_{44} \sin^2 \theta + 2 \eta_{14} \sin \theta \cos \theta. \quad (21)
\]

Substituting the elastic stiffness \( c_{ij} \) for the viscosities in (21) yields
CRYSTAL RESONATORS

CONVENTIONAL

RING-SUPPORTED

OUT-OF-PLANE ACCELERATION

BOUNDARY CONDITIONS AT EDGES

SIMPLY SUPPORTED:
(a) EDGE FORCES; ZERO DISPLACEMENTS
(b) NO EDGE TORQUES; NONZERO SLOPES

DOUBLE CANTILEVER:
(a) EDGE FORCES; ZERO DISPLACEMENTS
(b) EDGE TORQUES; ZERO SLOPES

FIGURE 45. Conventional and Ring-Supported Resonators
FIGURE 46. Ring-Supported and Grooved Crystal Resonators
FIGURE 47. Definitions of Ring-Supported Resonator Parameters
FUNDAMENTAL MODE \( f_0 = 3.967207 \text{ MHz} \)

ACCELERATION FREQUENCY EFFECTS OF \( X \)-AXIS DIRECTION FOR SAMPLE NO. T-4

\[(g-01) \frac{0}{\sqrt{V}}\]
Figure 49. Acceleration-frequency effects of X-axis direction for sample No. B-4

\[ f_0 = 7.338163 \text{ MHz} \]

3rd harmonic; B-4
the corresponding effective stiffness $c'$ for this pure mode. Then the
motional time constant $\tau_c$ representing the intrinsic losses is

$$\tau_c = \frac{1}{\omega_c^2 c'}. \quad (22)$$

Since $\tau_c$ is also equal to

$$\tau_c = \frac{R}{C} = \frac{1}{\omega_c Q}, \quad (23)$$

both the intrinsic $Q$ at an operating frequency $f = \omega_c / 2\pi$ and the motional
resistance $R$ of the equivalent electrical network may be computed, as the
motional capacitance is separately calculated.

When mounting losses cannot be neglected, it is found that these loss-
es depend upon the plate diameter-to-thickness ratio $\zeta$ in the following
manner:

$$\tau_c (\zeta) = \frac{1 + \left(\zeta / \zeta_0\right)^2}{\left(1 + \left(\zeta / \zeta_0\right)^2\right)^2}, \quad (24)$$

where $\zeta_0$ is the intrinsic value of $\zeta$, and depends on the material used,
being 10 to 15 femtoseconds for room temperature quartz, and $\zeta_0$ is a func-
tion of the resonator design, with $\zeta_0 = 60$ a good average. This is demon-
strated by Fig. 51 and Fig. 52, where the data of Fig. 50 and additional
data are plotted against $\zeta$. It is seen that for $\zeta < \zeta_0$ the loss, as
reflected in $\tau_c$, is governed by a curve with -2 slope; beyond $\zeta_0$, the curve
is constant, with zero slope. The calculation of $\tau_c$ has been extended to
the general doubly rotated thickness mode case.\textsuperscript{14}

COSMIC RAYS

Cosmic rays, and other unshieldable radiation sources, produce cumu-
lative contributions to long-term aging. Knowing the radiation sensitivity
of natural and cultured quartz under a variety of treatments permits an
evaluation of resonator performance due to this source.

The raw data are contained in Fig. 53, Fig. 54 and Fig. 55. Using the
values for the photon flux given and a number of simplifying assumptions,
the frequency shift due to this source is given in Fig. 56 for two values
of quartz sensitivity. The unit "rad" is defined as "The standard unit of
absorbed dose, equal to energy absorption of 100 ergs per gram (0.01 joules
per kilogram)."\textsuperscript{12} The assumptions used are (a) unprotected crystal, (b)
total incident energy is absorbed, and (c) sea level flux. $F_p$ is the photon
flux in photons / cm$^2$ - sec. The fractional frequency shift $\Delta f/f$ is spec-
ified as $10^{-5}$; the time $i(s)$ for this shift to occur is

$$i(s) = 5 \times 10^{12-5} \text{ years.} \quad (25)$$

Similarly for neutrons, assumptions (a) and (b) listed above, plus
the sensitivity value $\Delta f/f \sim 10^{-21}$ cm$^2$/n. lead to the order-of magnitude
figures given in Fig. 57 for three values of altitude. $\bar{J}$ $n$ is the neutron

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FIGURE 51. TIME CONSTANT, $\tau_s$, FOR AT-CUT QUARTZ PLATES AS A FUNCTION OF DIAMETER-THICKNESS RATIO, $d/L$, AVERAGE VALUES
FIGURE 52. TIME CONSTANT, $\tau_t$, FOR AT-CUT QUARTZ PLATES IN VACUUM AS A FUNCTION OF DIAMETER-THICKNESS RATIO, $\phi_o/t$. 

INFLUENCE OF ENERGY TRAPPING ON UNITS (n=5) WITH COMPARABLE MOUNTING LOSSES.
FIGURE 53. Cosmic Ray Flux versus Particle Energy at Sea Level
FIGURE 54. Flux versus Energy in MeV at Sea Level
FIGURE 55. Flux versus Energy at 3 and 10 km Height
SPECIFICATION: $\Delta F/F = 10^{-8}$

TIME FOR SPECIFIED SHIFT: $\tau (s) \approx 5 \cdot 10^{12}$-S YEARS

<table>
<thead>
<tr>
<th></th>
<th>NATURAL QUARTZ</th>
<th>SWEPT CULTURED</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta F/F$</td>
<td>$\sim 10^{-11}/$RAD</td>
<td>$\sim 10^{-12}$/RAD</td>
</tr>
<tr>
<td>$F_p$</td>
<td>$\sim 1$</td>
<td>$\sim 1$</td>
</tr>
<tr>
<td>$\tau (12)$</td>
<td>$\sim 5$ YRS.</td>
<td>$\sim 50$ YRS.</td>
</tr>
<tr>
<td>$\tau (14)$</td>
<td>$\sim 5$ HOURS!</td>
<td>$\sim 0.5$ YRS.</td>
</tr>
</tbody>
</table>

**NOTES:**
1/ UNPROTECTED CRYSTAL
2/ TOTAL ENERGY ABSORBED
3/ SEA LEVEL
4/ $F_p$ IN PHOTONS/CM$^2$-SEC

FIGURE 56. Frequency Shift for Photons
SPECIFICATION: $\Delta F/F = 10^{-6}$

TIME FOR SPECIFIED SHIFT: $\tau(s) \approx 10^{14-S/4T_N}$ YEARS

<table>
<thead>
<tr>
<th></th>
<th>SEA LEVEL</th>
<th>3 KM</th>
<th>10 KM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_N$</td>
<td>$\sim 10^{-3}$</td>
<td>$\sim 10^{-2}$</td>
<td>$\sim 0.6$</td>
</tr>
<tr>
<td>$\tau$ (12)</td>
<td>$\sim 25,000$ YRS.</td>
<td>$\sim 2,500$ YRS.</td>
<td>$\sim 4$ YRS.</td>
</tr>
<tr>
<td>$\tau$ (14)</td>
<td>$\sim 250$ YRS.</td>
<td>$\sim 25$ YRS.</td>
<td>$\sim 16$ DAYS!</td>
</tr>
</tbody>
</table>

NOTES: 1/ UNPROTECTED CRYSTAL
2/ TOTAL ENERGY ABSORBED
3/ $\Delta F/F \sim 10^{-21}$ CM$^2$/NEUTRON
4/ $F_N$ IN NEUTRON-SEC/CM$^2$

FIGURE 57. Frequency Shift for Neutrons
fluct in neutron-sec/cm\(^2\). The fractional frequency shift is specified as \(10^{-s}\), and the time \(\tau(s)\) to reach this shift is

\[ \tau(s) = 10^{14-s}/4\pi^2 \text{ years.} \]  

(26)

CONCLUSION

Over the years, the stabilities of precision quartz oscillators have improved according to a simple law. With frequency stability written as \(\Delta f/f = 10^{-s}\), \(s\) is given by \(s = 6 + 0.1(Y-X); Y\) is the year date and \(x = 1920\) for large-scale production models; this is seen in Fig. 58. Laboratory oscillators, therefore, can be projected to have stabilities in the \(10^{-14}\) range in the time frame 2000 if the current rate of improvement is maintained. There do not appear to be intrinsic impediments to achieving \(10^{-15}\) stabilities with quartz elements by 2000, but in order to do so the various "subtle" effects affecting stability will have to be subjected to increasingly fine scrutiny. Figure 59 gives a broad view of technology progression for crystal resonators from 1945 onward. The most interesting developments are yet to come.
FIGURE 5B. Frequency Stability versus Year

PRECISION QUARTZ OSCILLATORS

MBS * SC

LABORATORY VERSIONS

LIMITED COMMERCIAL

LARGE SCALE PRODUCTION
FIGURE 59. Progression in Crystal Technology
REFERENCES


