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SURFACE WAVES FROM UNDERGROUND EXPLOSIONS WITH SPALL: ANALYSIS OF ELASTIC AND NONLINEAR SOURCE MODELS

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Anomalous surface wave observations from underground explosions have been widely attributed to the phenomenon of spall, i.e., the detachment and subsequent slapdown of near-surface layers in response to explosion-induced tensile stresses. We argue, on the basis of a simple equivalent-force model, that, while spall can enhance explosion surface waves at very short periods, it cannot contribute significantly to the teleseismic surface waves radiated by underground explosions at periods exceeding about 10 seconds. Previous theoretical results to the
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contrary are in error, having been based on source models which do not conserve momentum.

A nonlinear, two-dimensional (axisymmetric) finite difference simulation of a buried explosion in granite further supports the conclusion that spall cannot contribute to long-period surface waves. The simulation exhibits extensive spall; nonetheless, predicted fundamental mode Rayleigh wave spectra are nearly identical to those obtained from a spherically symmetric simulation (which does not include spall), at periods exceeding about 10 seconds. At shorter periods, the two-dimensional simulation predicts some Rayleigh wave enhancement, compared to the one-dimensional simulation. The maximum enhancement, about a factor of 2, occurs at a period of approximately 2.5 seconds. Synthetic long-period Rayleigh wave seismograms, at 3000 km range, show no perceptible phase or amplitude anomalies due to spall. These results are in excellent agreement with the predictions of the equivalent-force model.

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TABLE OF CONTENTS

Section		Page
I.	INTRODUCTION	1
II.	REVIEW OF THE PROBLEM	3
III.	ANALYSIS OF THE ELASTIC MODELS	7
IV.	AN ELASTIC SOURCE MODEL FOR SPALL	13
V.	NONLINEAR NUMERICAL SIMULATION OF SPALL	19
VI.	DISCUSSION	29
VII.	REFERENCES	32

LIST OF ILLUSTRATIONS

Figure		Page
1.	Geometry for the surface-integral representation of seismic radiation from an axisymmetric explosion source	8
2.	Equivalent elastic surface load for an idealized spall model	15
3.	(A) Theoretical Rayleigh wave enhancement factor for the equivalent elastic load model of spall. (B) Reduced velocity potential for a 61 KT one-dimensional explosion simulation	17
4.	Vertical velocity time-histories obtained from the two-dimensional simulation, compared with recorded velocities for PILEDRIVER.	21
5.	Total momentum in the source volume, V_S, compared with the total impulse on its boundary for the two-dimensional explosion simulation	24
6.	(A) Fundamental-mode Rayleigh wave displacement spectra for one- and two-dimensional explosion simulations. (B) Rayleigh wave enhancement factor obtained analytically for the equivalent-point-force model	26
7.	Synthetic fundamental-mode Rayleigh waves for the one-and two-dimensional explosion simulations at a range of 3000 km.	28

LIST OF TABLES

Table		Page
1	PROPAGATION PATH MODEL FOR SYNTHETIC SEISMOGRAMS IN FIGURE 7	27

I. INTRODUCTION

A number of investigations have provided evidence that simple, spherically symmetric source representations are inadequate to explain surface wave generation by underground explosions. For example, Murphy (1977) demonstrates that a spherically symmetric source model subject to the depth/yield scaling proposed by Mueller and Murphy (1971) is consistent with a wide variety of near-regional and teleseismic body wave data; he shows, however, that the same model is inconsistent with long-period surface wave observations. In particular, the source scaling relationships predict that surface wave magnitude will increase with yield (W) less rapidly than log W, whereas studies by Basham and Horner (1973), Springer and Hannon (1973) and others consistently give yield exponents exceeding 1.0 (about 1.1 to 1.3). A second example pertains to anomalies in the Rayleigh waveform, rather than amplitude. Rygg (1979) demonstrates that the Rayleigh waves from an eastern Kazakh explosion exhibit a polarity reversal and time shift relative to similar, nearby explosions.

In both these cases, the authors proposed that spall closure was responsible for the anomalous surface wave observations. To quote Rygg, "The only source that appears capable of introducing both phase reversal and time delay is the phenomenon of spall closure." Such anomalous surface wave observations have been widely attributed to the effect of spall closure, and two principal reasons can be cited. Firstly, spall of near-surface material is a well-documented and nearly ubiquitous component of contained underground explosions (for example, Eisler and Chilton, 1964; Viecelli, 1973; Springer, 1974). Secondly, Viecelli has purportedly shown theoretically that spall closure can be a major source of Rayleigh wave radiation from buried explosions.

If spall were the primary source of explosion-induced surface waves, or an important secondary source, this fact would have important consequences for the seismic estimation of explosion yield. Shots with anomalously large spall, say due to underburial, would in turn have anomalously large surface wave excitation. In order to reliably exploit surface wave magnitudes in the seismic determination of explosion yield, it is essential to understand the potential effects of spall on the surface wave signal.

The purpose of this paper is to point out and correct a fundamental flaw in previous theoretical work on the subject of surface wave excitation by surface spall. By means of a simple physical argument, it is demonstrated that spall is incapable of generating a significant contribution to long-period (greater than about ten seconds) surface waves. It is shown that previous conclusions to the contrary have been based on source descriptions which do not conserve momentum. An equivalent elastic point-source representation is derived for spall which corrects this momentum imbalance. This simple source model predicts that spall can enhance surface wave excitation only for relatively short-period waves -- on the order of a few seconds period -- with negligible effect on 20-second radiation.

We believe that the argument against long-period surface wave enhancement by spall is conclusive, since it rests only on the very fundamental principle of conservation of momentum. In particular, the conclusion would seem to be independent of assumptions about the nonlinear rheology of the source region. There remains the possibility, however, that the occurrence of spall might indirectly modify the surface wave signal by altering the effective reduced displacement potential (RDP), or monopole component, of the explosion. To test the generality of our conclusion, therefore, we compare the simple model with predictions from a nonlinear numerical simulation of a buried explosion.

II. REVIEW OF THE PROBLEM

Spalling is defined (Eisler and Chilton, 1964) as the parting of near-surface layers, which were originally in contact, in response to the stress waves produced by a contained underground explosion. The explosion-generated stresses become tensile upon reflection at the free surface, and spall results when the tensile strength of the near-surface geologic material is exceeded. The detacned layer then decelerates under the force of gravity, and eventually slaps down.

It is the slapdown phase that has been the focus of attention as a potential generator of surface waves. Near-field acceleration pulses, radiated upon slapdown ("spall closure") can be identified on free-surface accelerometer recordings (Eisler and Chilton, 1964; Rimer, et al., 1979), from which the shape and duration of the spall gap can be estimated. Eisler and Chilton have analyzed travel time data from such recordings, from which they deduced that the shape of the spall gap is roughly conical, with sides concave upward. For large explosions, ballistic periods as long as two seconds are inferred (Viecelli, 1973).

Viecelli (1973) studied surface accelerometer records from nuclear explosions in order to estimate the mass and momentum of the spall region. He determined the following approximate empirical relationships between the explosion yield (W), and the spall mass (m_s) and momentum (I_s):

$$m_s \approx 1.6 \times 10^9 \text{ W kg} \tag{1}$$

$$I_s \approx 4.6 \times 10^9 \text{ W Nt-sec} \quad , \tag{2}$$

where W is in kilotons. These estimates formed the basis for his analysis of the surface wave contribution from spall closure.

To estimate the surface wave excitation, Viecelli approximated the slapdown phenomenon as a vertical load F applied at the free surface. The time history was assumed to be

$$F(t) = I_s \tau_0 / \pi (t^2 + \tau_0^2) \quad ,$$

where τ_0 is a characteristic time representing the duration of the spall impact (about twice the spall thickness divided by the P-wave velocity). Actually, since τ_0 is short, on the order of a few tenths of a second at most, and the surface wave periods of interest are many times longer, this time history can be viewed as essentially a delta function with area I_s. Using the analytic solution for the surface waves from a vertical load on a uniform elastic halfspace, and the empirical estimate for I_s, Equation (2), it was found that this impulsive-load model predicts surface wave amplitudes comparable to those observed for underground explosions. Even without adding the direct contribution from the explosive monopole, it was argued, the spall momentum is sufficient to produce surface waves of the observed amplitude. Thus, spall would appear, on this basis, to be a potential contributor to anomalous surface wave amplitude and phase observations.

A different theoretical approach to estimating the spall effect on surface waves was taken by Harkrider and Bache (1979). We can refer to this approach as the wave-suppression method. Analytic methods for computing synthetic surface wave seismograms for buried sources in layered elastic media represent the source by means of a stress-displacement discontinuity at the source depth (for example, Harkrider, 1964). This representation is equivalent to specifying the up- and downgoing waves emitted at the source. The surface waves associated with each of these components can be calculated separately, as described by Harkrider and Bache.

In the wave-suppression approach, the effect of spall is modeled by partial or complete suppression of the source contribution from upgoing waves generated by the explosion. This reduction of the upgoing source wavefield is intended to simulate, in an approximate way, the loss of energy from the upgoing wave due to spallation or other near-surface nonlinear phenomena. Bache, Goupillaud and Mason (1977) used this technique to model spall for explosions in NTS source materials. They found that complete suppression of the upgoing waves reduced the predicted M_s (measured at about 20 seconds period) by about 0.4 units for explosions in granite. For explosions in tuff, the effect was to enhance M_s by about 0.1 to 0.2 units. Harkrider (1981) points out that, for an explosion in a soft material overlying a hard material, suppression of the upgoing wave can reverse the polarity of the Rayleigh wave.

Both these theoretical approaches, the impulsive-load model and the wave-suppression model, predict that spall can substantially modify the surface waves generated by a contained explosion. The actual physical processes involved in spall are complex, and the predictions of the simple elastic theories are fairly sensitive to assumptions which are difficult to test. For example, there is no physical basis for determining what fraction of the upgoing waves to use in the wave-suppression model.

For this reason, numerical explosion simulations should be an important theoretical tool for evaluating the spall effect and testing the elastic theories. Viecelli (1973) compared two finite difference simulations, one in which tension failure, and therefore spall, was permitted, and a second in which only elastic behavior was permitted. Gravitation was included, so the spall model included closure. The computed Rayleigh wave amplitude, observed in the finite difference grid at a range of 15 km, was 2.7 times larger for the spall model than for the elastic model, apparently corroborating the results of the elastic theory. An important feature of the result, however, is evident in Viecelli's Figure 10: the Rayleigh wave amplitude has been measured at a predominant period of about three seconds. We will remark on this further in a later section.

Both the simple elastic theories and Viecelli's numerical simulation agree in predicting a potentially important spall effect on teleseismic surface waves. Yet, we find in the next section that the elastic theories proposed so far are flawed, and their results consequently are misleading. The results of Viecelli's numerical simulation, on the other hand, should simply be reinterpreted. In a later section, we analyze a new simulation in terms of its teleseismic surface wave radiation, using the procedures of Bache, Day and Swanger (1982). These results will be found to compare favorably with a corrected elastic theory.

III. ANALYSIS OF THE ELASTIC MODELS

In the last section we reviewed two elastic source models which have been used to approximate the effect of spall on surface waves. Although the physical process of spall is highly nonlinear, it is a valid analytical approach to approximate it by elastic loading. It is well known, of course, that in general, elastic sources exist whose radiation is exactly equivalent to a given nonlinear seismic source. Our critique of the proposed elastic models centers on their failure to conserve momentum. It will be shown that when this deficiency is corrected, a simple elastic model of spall predicts negligible surface wave excitation at periods of teleseismic interest.

For now, we avoid tying the analysis to any assumptions about the source-region rheology by imagining the source region, V_s, to be enclosed by a surface, Σ, lying wholly outside the zone of nonlinear response (Figure 1). The surface-wave generation can be expressed in terms of the elastic properties outside Σ and the displacements (u_α) and surface tractions (τ_α) on Σ. The displacements and tractions on Σ do depend on the rheology of the source region, but we will be able to sidestep this problem temporarily.

Bache, Day and Swanger (1982) give an expression for Rayleigh wave spectral displacement $\bar{u}_\alpha^R$, in the low-frequency limit, for a general axisymmetric explosion source. It is assumed that the only body force acting on the source volume is gravity, and that the source-generated displacements and tractions are taken relative to their initial, static-equilibrium values. Then we have

$$\lim \bar{u}_\alpha^R(r,z) = \mathscr{G}_\alpha(r,z)\,[I_1 + I_2] \tag{3}$$

where

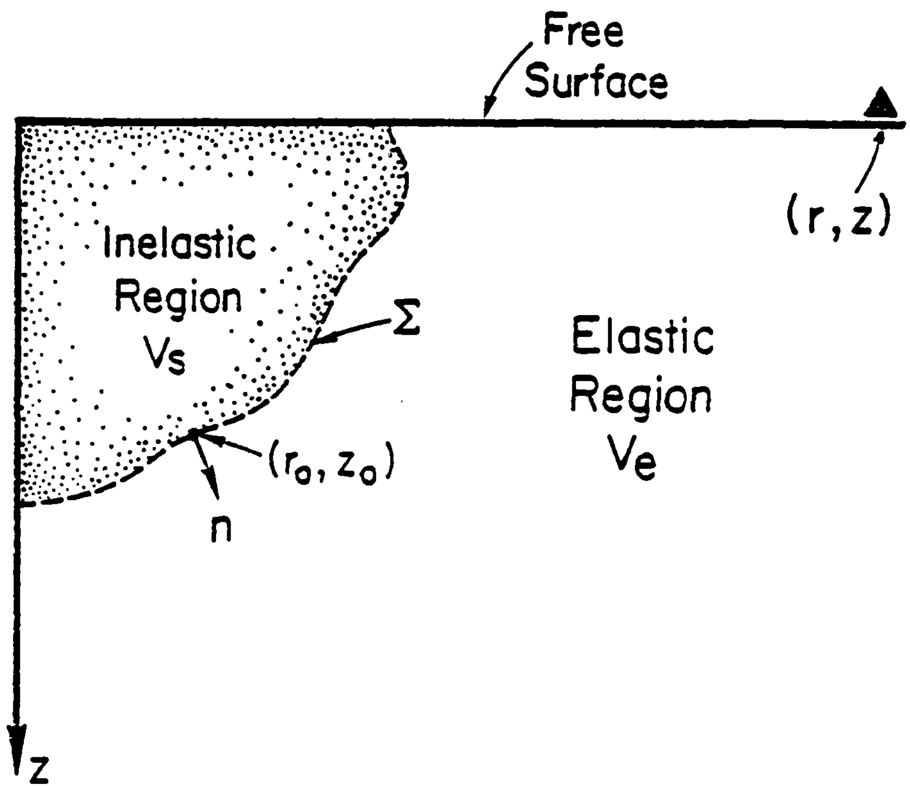

Figure 1. Geometry for the surface-integral representation of the seismic radiation from an axisymmetric explosion source. The receiver point is (r,z), and a typical point lying on the imaginary surface Σ is (r_0,z_0). The surface lies wholly outside the nonlinear region surrounding the source.

$$I_1 = \int_\Sigma dS_0 \, \bar{\tau}_z(r_0, z_0)$$

$$I_2 = \omega \frac{\epsilon_0}{c_R} \int_\Sigma dS_0 \left[\frac{\lambda}{(\lambda+2\mu)} z_0 \, \bar{\tau}_z \, (r_0, z_0) - \frac{1}{2} r_0 \, \bar{\tau}_r \, (r_0, z_0) \right.$$

$$\left. + \mu \frac{(3\lambda+2\mu)}{(\lambda+2\mu)} n_r \, \bar{u}_r \, (r_0, z_0) \right] \quad .$$

The notation is as follows:

α	= cylindrical component (r or z)
c_R	= phase velocity of the Rayleigh wave mode
ϵ_0	= surface ellipticity of the mode
λ, μ	= Lamé constants on Σ
r_0, z_0	= cylindrical coordinates on Σ
r, z	= cylindrical coordinates of the receiver point
n_α	= component of the unit normal to Σ

$$\mathscr{C}_\alpha \, (r,z) = \begin{cases} -\frac{i}{2} \, \underset{\sim}{A}_R \, \epsilon_0 \left[\frac{\dot{u}^*(z)}{\dot{w}_0} \right] H_1^{(2)} \left(\frac{\omega}{c_R} r \right), & \text{for } \alpha = r \\[2ex] -\frac{i}{2} \, \underset{\sim}{A}_R \left[\frac{\dot{w}(z)}{\dot{w}_0} \right] H_0^{(2)} \left(\frac{\omega}{c_R} r \right), & \text{for } \alpha = z \quad . \end{cases}$$

In $\mathscr{C}_\alpha$ (r,z), the factors $\dot{w}/\dot{w}_0$, $\dot{u}^*/\dot{w}_0$, and A_R are as defined by Harkrider (1964): the normalized vertical and horizontal eigenfunctions and the source-independent amplitude response of the structure, respectively; $H_0^{(2)}$ and $H_1^{(2)}$ are Hankel functions. The factor $\mathscr{C}_\alpha$ depends only on the layered elastic structure outside Σ, and the integrals I_1 and I_2 depend on the source-generated displacements and tractions on Σ. At times long compared to the source duration, the displacements and tractions reach some static values on Σ, so in general their transforms $\bar{u}_\alpha$ and $\bar{\tau}_\alpha$ are proportional to ω^{-1} at low frequency. Therefore, $I_2(\omega)$ is of order 1:

$$I_2 \sim O(1) \quad .$$

The I_1 term, on the other hand, is simply the spectrum of the total vertical force on Σ(minus its initial value in equilibrium with gravity). Conservation of momentum demands that the time integral of the total vertical force on Σ go to zero at times long compared to the source duration; this is true irrespective of the nonlinearities inside Σ. Therefore, I_1 goes to zero at least as fast as ω:

$$I_1 \sim O(\omega^\gamma) \quad , \quad \gamma \geq 1 \quad .$$

The source duration will be dominated by the ballistic period of the spall, which is at most a few seconds. So, for periods of interest for teleseismic surface waves, we expect I_2 to dominate, being of order 1, and I_1 should be negligible.

Now the unacceptability of the impulsive-load elastic model is apparent. The explosive monopole imparts zero net momentum to the continuum, and, correctly, makes a contribution of order 1 only to I_2. The impulsive surface load, however, imparts a net downward momentum equal to I_s, the spall momentum. Thus, for the impulsive-load model, we have

$$I_1(0) = I_s \quad ;$$

so the I_1 term is of the same order as the I_2 term. The large spall contribution predicted by the impulsive load model is therefore directly attributable to its inherent momentum imbalance, which cannot occur for any physical model.

The wave-suppression model is similarly flawed by failure to conserve momentum. Like the impulsive-load model, the wave-suppression model imparts a net downward momentum, again resulting in a spurious long-period contribution to I_1. To see the momentum imbalance, we examine the source wavefield radiated by a monopole explosion source. The source strength is defined by a reduced displacement potential $\bar{\Psi}(\omega)$, such that the radiated spectral displacement $\bar{\underline{u}}$ is

$$\bar{\underline{u}}(r,z,\omega) = -\underline{\nabla}\left[R^{-1} e^{i \omega R/\alpha} \bar{\Psi}\right] \quad ,$$

where α is the P-wave speed, $R = (r^2 + z^2)^{1/2}$, and we have taken the coordinate origin to coincide with the source point. Expanding the term in brackets via the Sommerfeld integral gives

$$\bar{\underline{u}} = \bar{\Psi} \int_0^\infty dk\, k \left[\mathrm{sgn}(z)\, J_0(kr)\, \hat{\underline{z}} - \left(\frac{k}{i\, \nu_\alpha}\right) J_1(kr)\, \hat{\underline{r}}\right] e^{i \nu_\alpha |z|} , \tag{4}$$

where $\hat{\underline{r}}$ and $\hat{\underline{z}}$ are unit vectors in the radial and vertical directions, respectively, and

$$\nu_\alpha = (\omega^2/\alpha^2 - k^2)^{1/2} \quad .$$

From Equation (4) we can obtain the traction vector $\bar{\underline{\tau}}$ in the z plane:

$$\bar{\underline{\tau}} = \bar{\Psi} \int_0^\infty dk\, k \left[\mu \frac{(2k^2 - \omega^2/\beta^2)}{i\, \nu_\alpha} J_0(kr)\, \hat{\underline{z}} \right.$$

$$\left. + \mathrm{sgn}(z)\, 2\mu k\, J_1(kr) \hat{\underline{r}}\right] e^{i \nu_\alpha |z|} \quad . \tag{5}$$

The vertical traction is continuous, so no net vertical force acts, and momentum is conserved. However, suppressing a fraction η of the upgoing wave requires multiplying the exponential factors in Equations (4) and (5) by $[1 - 0.5\eta\ (1 - \mathrm{sgn}(z))]$ (taking z positive downward). The result is to introduce a discontinuity in the vertical traction at $z = 0$, with Fourier-Bessel transform $\delta\tilde{\tau}_z$ given by

$$\delta\tilde{\tau}_z(k,\omega) = \eta\mu\ (i\ \nu_\alpha)^{-1}\ (2k^2 - \omega^2/\beta^2)\Psi(\omega) \quad . \tag{6}$$

The total momentum I due to this discontinuity in traction is just proportional to $\delta\tilde{\tau}_z$ at zero frequency and zero wavenumber:

$$\begin{aligned} I &= -\ 2\pi\ \delta\tilde{\tau}_z(0,0) \\ &= \eta\ 2\pi\ \rho\alpha\ \ \Psi_\infty, \end{aligned} \tag{7}$$

where Ψ_∞ is $-i\omega\bar{\Psi}(0)$, the late-time limit of $\Psi(t)$.

Thus, the wave-suppression model does not conserve momentum, and the momentum imbalance makes a spurious low-frequency contribution of order 1 to I_1 of Equation (3). The net momentum imparted is downward. Using Murphy's (1977) value of $\Psi_\infty = 4.2 \times 10\ \ m^3$ for a 100 KT event in tuff, the magnitude of the momentum imbalance for the wave-suppression model, with fractional suppression $\eta \approx 1/4$, is approximately equal to Viecelli's estimate of the slapdown momentum.

IV. AN ELASTIC SOURCE MODEL FOR SPALL

The impulsive-load and wave-suppression models of spall are both unphysical, in that each imputes a net momentum to the source. Furthermore, long-period surface-wave predictions are seriously influenced by this momentum imbalance, as shown by Equation (3). Here we consider a very simple model of spall and construct its equivalent elastic loads. It will be seen that this model predicts a spall contribution, proportional to $\omega^2 I_s$, which is certain to be negligible at 20 second periods.

The impulsive-load model reviewed earlier accounts for the inertial forces delivered to the intact continuum by the spall mass upon closure. Its failure to conserve momentum is due to neglect of (1) the inertial forces imparted during spall opening and (2) the relaxation of gravitational forces during the ballistic period (we take the initial equilibrium configuration as the reference stress state). If the spalled material is severed at an average upward velocity V_0, and if the period over which the mass accelerates to V_0 is negligible compared to the periods of interest, we can approximate the inertial force of spall opening, f_1, by a downward delta function,

$$f_1 = I_s \delta(t)$$

where I_s is the spall momentum $m_s V_0$. Subsequent to the spall opening, the continuum experiences a relaxation of the gravitational forces exerted by the spall mass, f_2, given by

$$f_2 = - m_s g \left[H(t) - H(t-T_s)\right] \quad ,$$

where m_s is the spall mass, g is the gravitational acceleration, and T_s is the average ballistic period,

$$T_s = \frac{2 I_s}{m_s g} \quad .$$

Finally, the slapdown rapidly applies inertial forces f_3 with time integral I_s, which we can again approximate by the delta function:

$$f_3 = I_s \quad \delta(t) \quad .$$

These equivalent forces are sketched in Figure 2. Obviously, when all three terms $f_1 + f_2 + f_3 = f_s$ are considered, the equivalent forces correctly conserve momentum. Our elastic source model for spall consists of applying these equivalent forces as point loads at ground zero, superposed on the explosion monopole. We neglect the finite spatial distribution of the equivalent forces, and, as already stated, we are ignoring the finite duration of the inertial forces of spall opening and closing. This approximation will be adeouate for periods substantially greater than the time for a wave to cross the spalled region, say one or two seconds.

The Rayleigh wave vertical displacement due to a monopcle at depth z_0 with reduced displacement potential $\bar{\Psi}(\omega)$ is (Harkrider, 1964)

$$\bar{u}_z^x(r,z) = -2\pi i \rho \alpha^2 \, \bar{\Psi} \, A_R \, H_0^{(2)}\left(\frac{\omega r}{c_R}\right) \left[\frac{\omega}{c_R} \; \frac{\dot{u}^*(z_0)}{\dot{w}_0} + \frac{\partial(\dot{w}/\dot{w}_0)}{\partial z_0}\right] ,$$

and the Rayleigh wave due to the vertical surface load $\bar{f}_s(\omega)$ is

$$\bar{u}_z^s(r,z) = -\frac{i}{2} \; \bar{f}_s \, A_R \, H_0^{(2)}\left(\frac{\omega}{c_R} \; r\right) \quad .$$

Therefore, the presence of spall modifies the monopole Rayleigh wave spectrum by the factor

$$A = \frac{\bar{u}_z^x + \bar{u}_z^s}{\bar{u}_z^x} = 1 + \frac{\bar{f}_s}{\bar{\Psi}} \, (4\pi\rho \, \alpha^2)^{-1} \left[\frac{\omega}{c_R} \; \frac{\dot{u}^*(z_0)}{\dot{w}_0} + \frac{\dot{w}'(z_0)}{\dot{w}_0}\right]^{-1} \tag{8}$$

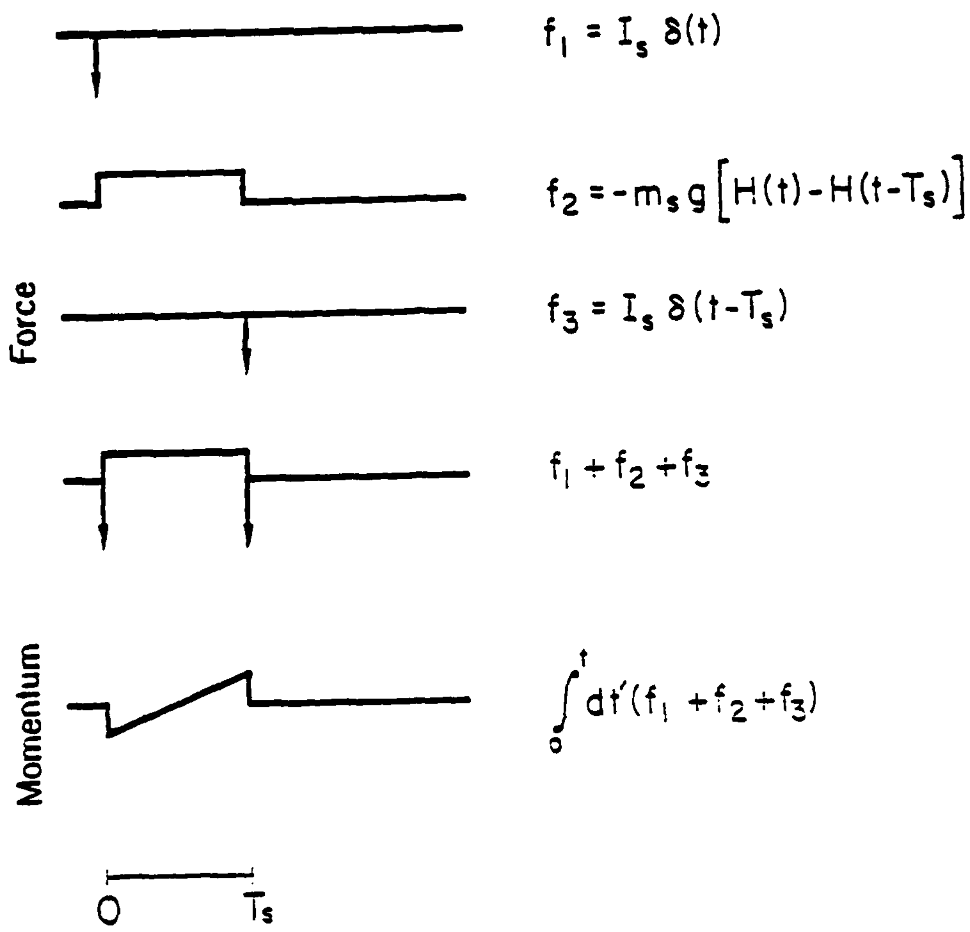

Figure 2. Equivalent elastic surface load for an idealized spall model. Correctly accounting for both inertial and gravitational forces associated with the spall mass insures zero net source momentum.

where

$$\bar{F}_s = 2I_s\, e^{-i\omega(\varepsilon+T_s/2)}\left[\cos\frac{\omega T_s}{2} - \left(\frac{2}{\omega T_s}\right)\ \sin\left(\frac{\omega T_s}{2}\right)\right], \qquad (9)$$

and ε is the delay between detonation and spall opening, which we will approximate as z_0/α. The fractional contribution of the spall, A-1, is therefore proportional to ω^2 at low frequency:

$$A\text{-}1 \sim O(\omega^2) \quad ,$$

(since $\bar{\Psi}$ is normally of order ω^{-1}). This result is in contrast to the impulsive-load and wave-suppression models, for which we have seen tnat A-1 ~ O(1).

Figure 3a shows the surface wave enhancement factor, A, as a function of frequency, for several different values of the spall momentum and spall mass. The $\bar{\Psi}(\omega)$ used in Equation (8) was obtained from a numerical simulation of a spherically symmetric, 61 KT explosion in granite ($\Psi_\infty = 1.6 \times 10^4\ m^3$). The spectrum of the reduced velocity potential is shown in Figure 3b. The values of I_s and m_s as given in Figure 3a have been normalized to Viecelli's mean empirical estimates for 61 KT (Equations 1 and 2). In no event does spall make a significant contribution to the 20 second Rayleigh wave. The maximum enhancement occurs at a period of about 1.4 T_s. This short-period enhancement might be important for interpreting regional and near-regional surface-wave data, but will have no effect on teleseismic determinations of M_s.

Now we can reexamine Viecelli's (1973) numerical simulation of spall in light of the foregoing analytical results. Recall that the numerical simulation showed Rayleigh wave amplitude enhancement of a factor of 2.7, which appeared to support the impulsive-load elastic model. However, the Rayleigh wave amplitude was measured very near the source (15 km), where the dominant period of the computed motion was about 3 seconds. From Figure 3, we see that this is in

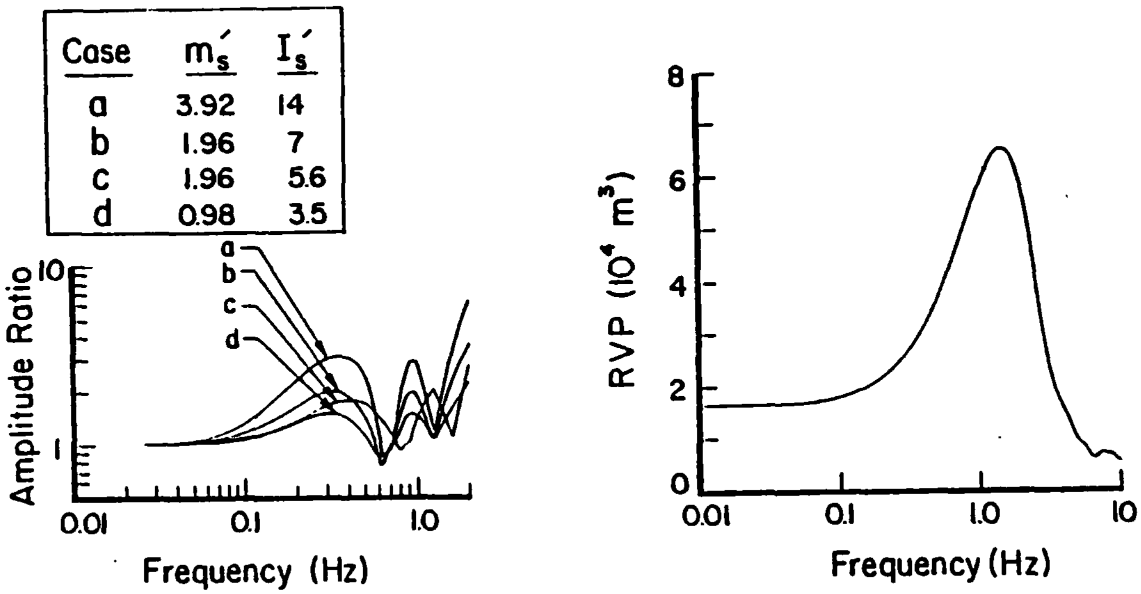

Figure 3. (A) Theoretical Rayleigh wave enhancement factor for the equivalent elastic load model of spall described in the text and in Figure 2. The m_s and I_s values represent spall mass and momentum, divided by Viecelli's (1973) mean empirical estimates for 61 KT. (B) Reduced velocity potential for a 61 KT one-dimensional explosion simulation, which was used in Equation 8 to construct the Rayleigh wave enhancement factors shown in (A).

precisely the period range where enhancement is predicted by the momentum-conserving equivalent-force model. The results of Viecelli's numerical simulation are therefore consistent with the equivalent-force model, and need not be interpreted as supporting the impulsive-load model.

V. NONLINEAR NUMERICAL SIMULATION OF SPALL

The preceding theoretical argument against long-period surface wave enhancement by spall appears to be quite conclusive, since it relies only on the very fundamental principle of conservation of momentum. In particular, the conclusion would seem to be independent of assumptions about the nonlinear rheology of the source region. There remains the possibility, however, that the occurrence of spall might indirectly modify the long-period surface wave signal by altering the effective reduced displacement potential of the explosion. Such a phenomenon was envisioned by Aki, Bouchon and Reasenberg (1974), for example, who suggested that spall slapdown might compact the nonlinear source volume, leading to a reduction of the long-period level of the RDP.

To test the generality of our conclusion, we have to rely on numerical simulations of contained explosions, which can incorporate the nonlinear material response of the source region. Since we are interested in nonlinear phenomena, it is important that the simulations preserve relatively short wavelength features of the near-source disturbance, even though our ultimate interest is in the excitation of surface waves with wavelengths of tens or hundreds of kilometers. To model the spall process, the source simulation must be two-dimensional (at least), so as to include the free surface, gravity, and the depth dependence of overburden pressure; these phenomena are absent from one-dimension (spherically symmetric) explosion simulations. Comparison of two-dimensional simulations with one-dimensional simulations which use the same constitutive models then provides a means of quantifying the effects of spall on the seismic radiation.

The finite difference method is suitable for treating the two-dimensional, nonlinear problem, but it is not feasible to compute the wavefield at distances beyond a kilometer or so from the source by this method. Provided the finite difference simulation extends out to the range of linear material response, however, the teleseismic signal can be synthesized from the close-range finite difference solution using Green's function methods. This approach was taken by Bache, Day and Swanger (1982), who synthesized teleseismic Rayleigh waves from two-dimensional explosion simulations which included spall. Comparing synthetics from one- and two-dimensional explosion simulations, they noted that two-dimensional source phenomena had very little effect on either the amplitude or waveform of the long-period teleseismic Rayleigh wave, but they observed some enhancement of very short period (2 to 4 seconds) components. This result is in excellent agreement with the momentum-conserving elastic model of spall introduced in the last section. However, the explosions considered by Bache, et al. were overburied (depths of 188 $W^{1/3}$ meters, compared to 118 $W^{1/3}$ meters for PILEDRIVER, for example); so we still cannot exclude the possibility that spall from shots at normal containment depth (~ 120 $W^{1/3}$ meters) mignt modify the long-period surface-wave signal.

To address this issue, a two-dimensional (axisymmetric) simulation of a buried explosion in granite was performed using an explicit finite difference method. The source-region geologic structure was a three-layered halfspace representing the geology of the 1966 PILEDRIVER explosion at the Nevada Test Site (NTS). The nonlinear material model used in the calculation is described by Rimer, et al. (1979), and includes tensile failure, shear failure, irreversible pore crushup, and an effective stress law. The explosion depth and yield were the same as for PILEDRIVER, 463 meters and 61 KT, respectively (i.e., depth = 118 $W^{1/3}$ meters). The initial stress field was taken to be lithostatic.

Figure 4 shows the computed vertical velocity time histories obtained from the simulation at various depths directly below ground zero and at various ranges along the free surface. These are compared with recorded velocities for the PILEDRIVER shot. The spall occurs at all the sites, immediately after the first velocity peak, as evidenced by the -1 g slopes of the velocity waveforms. Spall closure can be identified with the termination of the -1 g

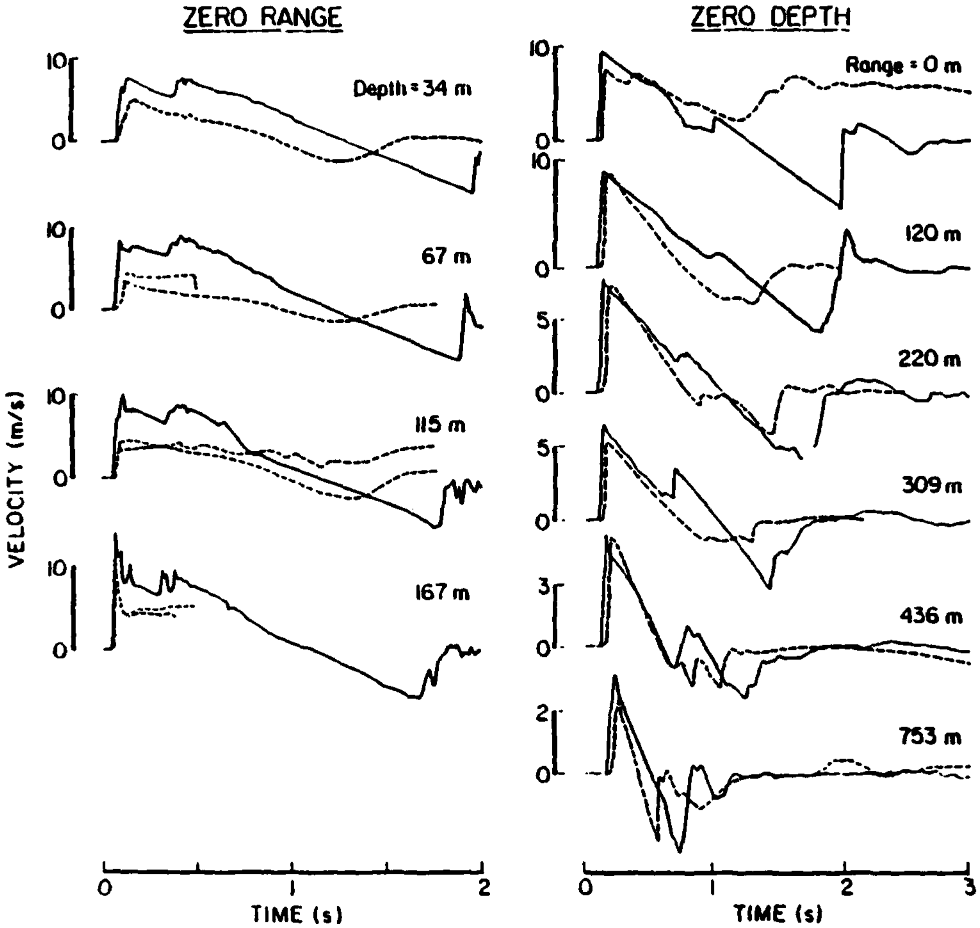

Figure 4. Vertical velocity time-histories (positive up) obtained from the two-dimensional simulation, compared with recorded velocities for PILEDRIVER. The data are from Hoffman and Sauer (1969). Occurrence of spall is apparent from the -1 g slopes in the velocity waveforms.

slope. From the figure, we can see that the simulation somewhat over-predicts the initial spall velocity, and therefore the ballistic period. This is especially noticeable near ground zero, less so at large range. The representative ballistic period near ground zero, for the simulation, appears to be about 1.7 seconds, compared to the observed period for PILEDRIVER which appears to be about 1.2 seconds. The simulation replicates very well the decrease of ballistic period with increasing range, as well as the decay of peak particle velocity. Overall, the simulation represents the main features of the PILEDRIVER recordings fairly well, and we proceed to analyze the teleseismic Rayleigh wave from the simulation.

A one-dimensional (spherically symmetric) explosion simulation was also performed, using the same nonlinear constitutive model as was used for the two-dimensional simulation. The amplitude spectrum of the reduced velocity potential obtained for this source was shown in Figure 3b.

We compute the teleseismic Rayleigh wave from the near-source results of the numerical simulation, using the procedure of Bache, Day and Swanger (1982). This procedure is analogous to the surface-integral formulation of Equation (3), but is exact rather than asymptotic. The surface of integration, Σ, is a cylinder intersecting the free surface and enclosing the nonlinear source volume, with radius and depth of 1209 meters. As shown in our discussion of Equation (3), any failure of the near-source solution to conserve momentum will lead to a long-period error in the surface wave prediction by this method. In principle, our finite difference method conserves momentum, but there are several potential sources of error. First, if the velocities in the source region (the interior of Σ) are still significantly nonzero at the time the numerical solution is terminated, the vertical traction integral on Σ will be nonzero at the final time step, and this will lead to a nonzero spectral estimate of $I_1(\omega)$ at $\omega = 0$. A second possibility is that the calculation may be run to completion, but accumulated roundoff errors might render $I_1(0)$ significantly nonzero. A third possibility is that the initial stress field assumed at the start of the calculation may be slightly out of equilibrium. If such disequilibrium is small and random, it may have insignificant effect on $I_1(0)$, but even quite small systematic errors in the initial stress field can lead to substantial errors in $I_1(0)$. This is because the effect of any net forces on Σ will be integrated over the duration of the simulation.

To verify that momentum is conserved, to sufficient precision for purposes of computing teleseismic surface waves, we plot in Figure 5 the total vertical momentum enclosed by Σ, as a function of time. Also plotted is the time integral of the total vertical force (with force taken relative to its initial equilibrium value) exerted on Σ by the exterior continuum. In the absence of numerical errors, these curves should coincide. The slight tendency for the momentum curve to lead the impulse curve is an artifact of the time series processing to which the stress histories on Σ were subjected before being integrated, i.e., causal low-pass filtering followed by decimation. The agreement is entirely satisfactory, and the terminal value of momentum is negligible.

Fundamental-mode Rayleigh wave spectral amplitudes were computed at a range of 1000 km for the two-dimensional explosion simulation using a halfspace with P wave speed α, S wave speed β, and density ρ equal to 6 km/sec, 3.5 km/sec and 2.7 gm/cm^3, respectively. In the uppermost 0.2 km, the shear quality factor Q_β was 10, with Q_β equal to 300 elsewhere. The result is shown in Figure 6a, together with the analogous result for the one-dimensional explosion simulation. The spectral differences reflect the influence of two-dimensional source effects, particularly spall, on the Rayleigh wave. As predicted by the equivalent-point-force elastic model of spall, the influence of spall is most important between about 2 and 5 seconds period, and is negligible at periods in excess of 10 seconds. The maximum enhancement is about a factor of 2, and occurs at a period of approximately 2.5 seconds.

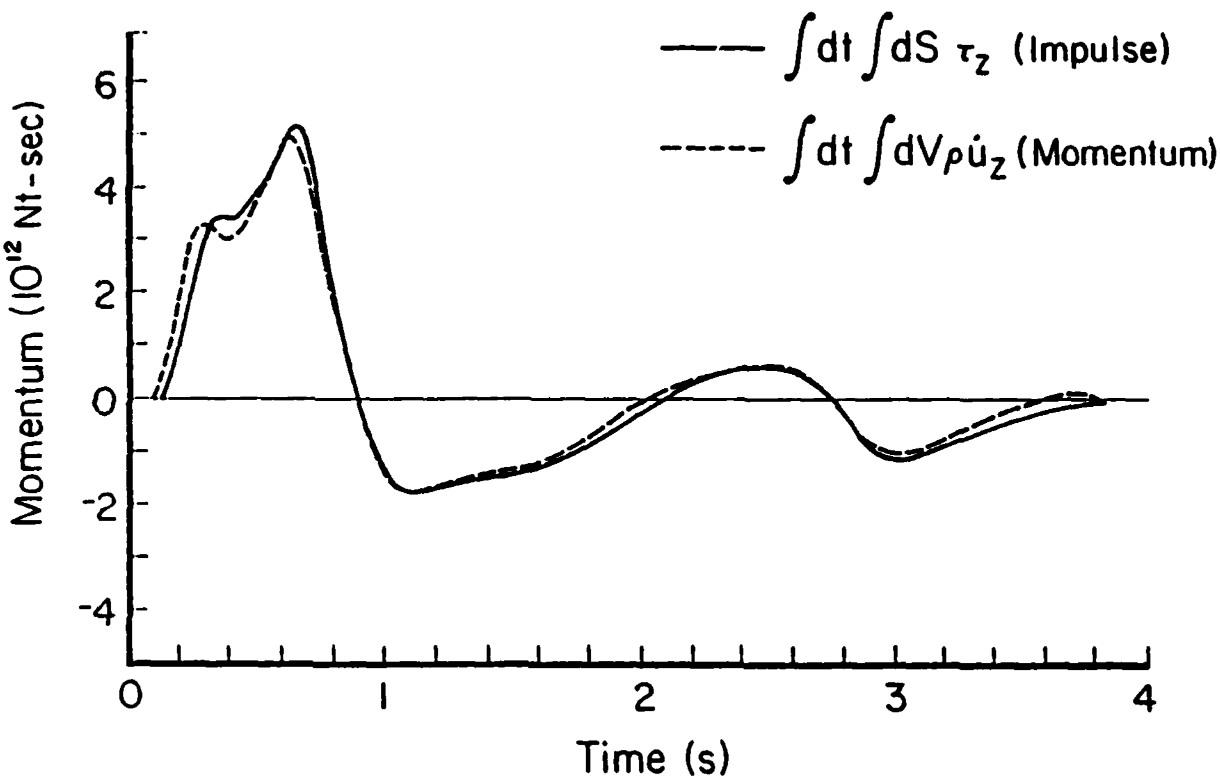

Figure 5. Total momentum (positive up) in the source volume, V_S, compared with the total impulse on its boundary, Σ, for the two-dimensional explosion simulation. The fact that the curves are nearly coincident and approach zero at late time demonstrates that the computation properly conserves momentum.

A comparison of the Rayleigh wave prediction for the simple equivalent-point-force model of spall with Rayleigh waves from the nonlinear model is made in Figure 6. The solid curve in this figure shows the Rayleigh wave enhancement due to spall as computed from the nonlinear simulation; that is, it plots the ratio of the two curves in Figure 6a. This spectral ratio is compared to the enhancement factor $|A|$ from the equivalent-point-force model (given by Equation 8), which is shown as a dashed curve. Equation (8) was evaluated using the $\bar{\Psi}(\omega)$ obtained from the one-dimensional numerical simulation. Both spall mass m_s and ballistic period T_s were adjusted so that the enhancement factor agrees as well as possible with the spectral ratio obtained from the numerical simulation. The resulting value of $T_s = 1.6$ seconds is in very good agreement with the ballistic periods inferred from the near-source waveforms shown in Figure 4. The resulting value of $m_s = 2.5 \times 10^{11}$ kg is about a factor of 2.5 greater than Viecelli's mean empirical estimate (Equation 1). The two curves are in remarkably good agreement, especially considering the simplicity of the equivalent-point-force model. We conclude that even when source-region rheology is accounted for in detail, as in the explosion simulations analyzed here, spall makes no contribution to the teleseismic surface wave spectral amplitude at periods above 10 seconds.

To verify that spall has not altered the long-period Rayleigh waveform, we compare fundamental mode synthetic seismograms for the one- and two-dimensional explosion simulations. The three-layered earth structure used for the synthesis is shown in Table 1. The synthetics, at 3000 km range, and including the long-period LRSM seismometer response, are shown in Figure 7. The waveforms for the two simulations are indistinguishable, and peak-to-peak amplitudes agree within less than half of a percent. Thus, for teleseismic surface waves, at periods greater than 10 seconds, we can exclude the possibility of phase anomalies, as well as amplitude anomalies, due to spall.

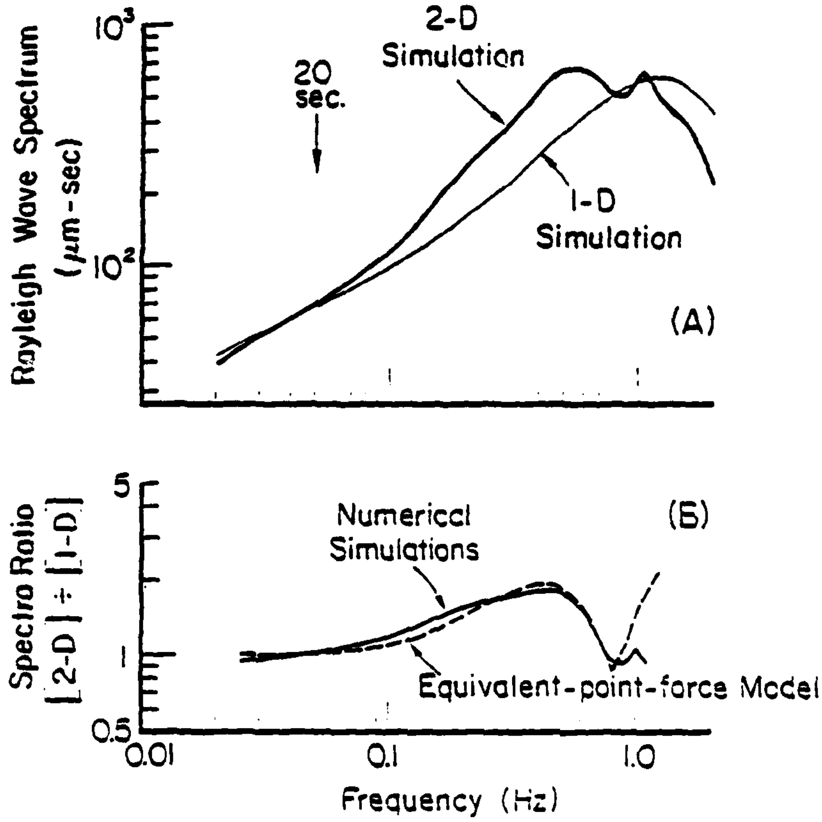

Figure 6. (A) Fundamental-mode Rayleigh wave displacement spectra for the one- and two-dimensional explosion simulations. (B) Rayleigh wave enhancement factor obtained analytically for the equivalent-point-force model, compared to the ratio of the one- and two-dimensional numerical results in (A).

TABLE 1

PROPAGATION PATH MODEL FOR THE SYNTHETIC SEISMOGRAMS IN FIGURE 7

Layer Thickness (km)	α (km/sec)	β (km/sec)	ρ (gm/cm^3)	Q_β
25	5.5	3.18	2.65	200
20	6.4	3.70	2.90	500
∞	8.1	4.68	3.50	1000

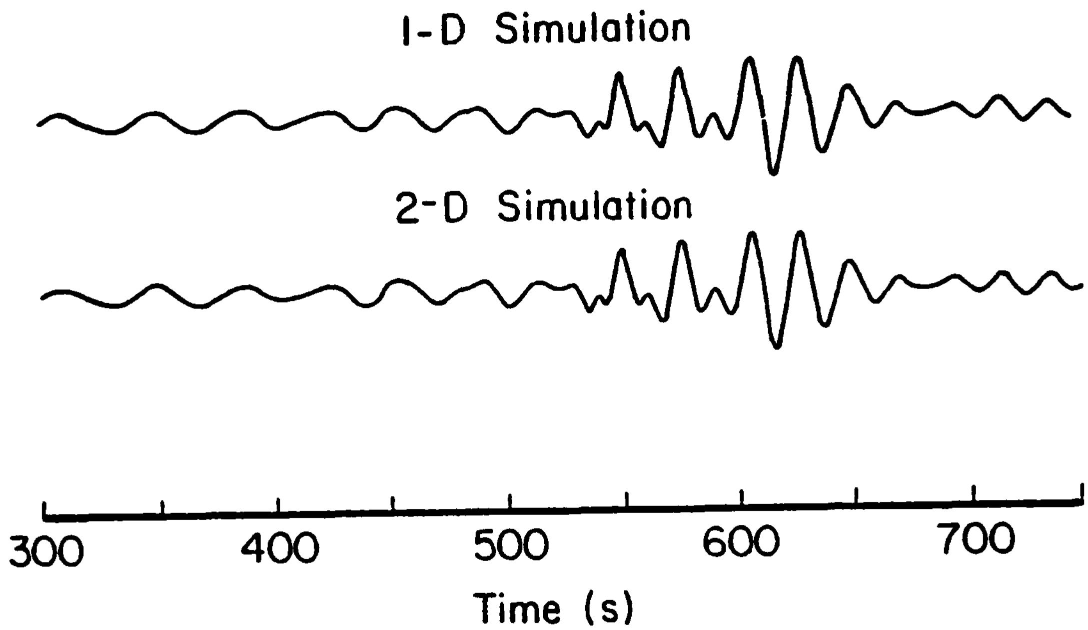

Figure 7. Synthetic fundamental-mode Rayleigh waves for the one- and two-dimensional explosion simulations, at a range of 3000 km. Two-dimensional effects, including spall, have had no perceptible effect on the amplitude or waveform. Peak amplitudes differ by less than half a percent.

VI. DISCUSSION

The main conclusion of this study is that spall cannot contribute significantly to the teleseismic surface waves radiated by buried explosions at periods in excess of about 10 seconds. Therefore, the spall phenomenon cannot explain observations of anomalous surface wave amplitude and phase.

It has been shown that previous analytical studies which have found a potentially significant surface wave signal from spall are based on source descriptions which fail to conserve momentum. This momentum imbalance exists for both Viecelli's (1973) impulsive-load model and Harkrider and Bache's (1979) wave-suppression source model, and it is directly responsible for the long-period surface wave anomalies predicted by those models.

The impulsive-load model has been modified to produce an equivalent-point-force model for spall which includes both the gravitational and inertial forces associated with detachment and closure. This model, which conserves momentum, predicts surface wave enhancement at short periods, but predicts no significant effect at periods above about 10 seconds. This simple analytical model thereby reconciles the apparently conflicting results deduced from numerical simulations by Viecelli (1973) and Bache, Day and Swanger (1982), respectively. Viecelli observed enhancement of peak Rayleigh wave amplitude, because he measured peak amplitude very near the source (within the finite difference grid) at a dominant period of about 3 seconds; Bache, et al. found that spall had no effect on peak Rayleigh wave amplitude, because they measured peak amplitude from synthetic teleseismic waveforms which have much longer dominant periods.

The wave-suppression model could be similarly modified to conserve momentum. This could be accomplished by assigning a frequency spectrum to the suppression coefficient. We have seen that suppressing a fraction η of the upgoing wave introduces a discontinuity in vertical traction proportional to η. The requirement imposed by conservation of momentum is that the resulting vertical traction discontinuity $\delta\tilde{\tau}_z$ (Equation 6) go to zero at long period at least as fast as ω. Furthermore, the numerical simulation indicates that virtually no momentum remains trapped in the nonlinear region longer than about 4 seconds following detonation; therefore, the long-period decay of $\delta\tilde{\tau}_z$ must begin with periods on the order of 4 seconds.

Under these constraints, only short-period upgoing waves can be significantly suppressed relative to downgoing waves. With a suppression coefficient which goes to zero at least as fast as ω for periods longer than several seconds, the wave-suppression model might provide a useful technique for simulating energy loss in the region between an underground explosion and the surface; this region is probably highly attenuative, as a result of the tensile cracking associated with spall. The foregoing analysis demonstrates, however, that such increased attenuation cannot be modeled simply as a frequency-independent suppression of the upgoing wave. The same reasoning also precludes approximating near-surface losses as a frequency-independent suppression of the free-surface reflection coefficient. The near-surface environment may be highly absorptive, but any seismic source must conserve momentum irrespective of such energy losses.

We note in passing that Peppin's (1977) model of explosion cavity collapse as a downward force impulse is physically unrealizable for the same reason. He argued that the balancing upward impulse due to detachment of the collapse mass would be preferentially dissipated, since the cavity ceiling would consist of highly shattered material, compared to the cavity bottom. This violation of momentum conservation might be an acceptable approximation for modeling high-frequency waveforms, but it must be ruled out as a source model for teleseismic surface waves. Similarly, Peppin's proposed explosion source function, which consists of an upward impulse added to a monopole, is physically unacceptable, and will lead to misleading results if applied to teleseismic surface waves.

The numerical simulation performed for this study represents the most complete multi-dimensional theoretical model of a contained explosion of which we are aware. It includes the depth dependence of overburden pressure, the free surface, gravity, depth-varying geologic structure, and several modes of material failure. The calculation was time-stepped well beyond spall closure, as verified by the momentum curves in Figure 5. Nonetheless, its long-period Rayleigh waveform and amplitude are indistinguishable from those of a spherically symmetric explosion, according to Figure 7. Furthermore, the short-period Rayleigh wave spectral enhancement predicted by the numerical simulation is nearly identical in shape to the enhancement predicted by the equivalent-point-force model (Figure 6b). The latter result is particularly surprising in view of the complexity of the numerical model and the highly idealized nature of the equivalent-force model. It appears that the surface wave prediction depends in a fairly simple way on the total mass and momentum of spalled material. Refinements of the nonlinear material models may affect the amount of short-period enhancement, but will not alter our conclusion that periods above 10 seconds are uninfluenced by spall.

Clearly, we will have to resort to phenomena other than spall to explain surface wave anomalies from underground explosions. Several possibility have been proposed in the seismological literature, including effects of deviatoric prestress acting on the source volume, (e.g., Archambeau, 1972), explosion-triggered stress release on faults (e.g., Aki and Tsai, 1972), and explosion-driven block motion across rock joints (e.g., Bache and Lambert, 1976; Salvado and Minster, 1980). However, for none of these mechanisms is the source physics well understood. Our experience in studying spall shows that multi-dimensional numerical simulations, when carefully analyzed and interpreted, provide a useful tool for studying complex source physics and quantifying its effects on the seismic signal.

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