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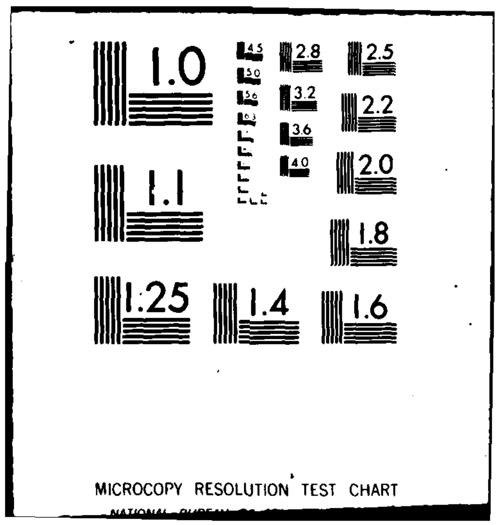
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Technical Report 598

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Application of Adaptive Estimation to Target Tracking

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FOR THE COMMANDER

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APPLICATION OF ADAPTIVE ESTIMATION TO TARGET TRACKING

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ABSTRACT

In this report, we present a survey of problems and solutions in the area of target tracking. The discussion includes design trade-offs, performance evaluation, and current issues.

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1. INTRODUCTION

For nearly two decades the target tracking-trajectory estimation problem has been a fruitful applications area for state estimation. Many problems have been solved yet new and diversified applications still challenge systems engineers. In this report, we provide a review of the subject and discuss some current issues.

An earlier excellent reference on tracking-trajectory estimation [1] deals with tracking algorithms for ballistic re-entry vehicles. Most of these algorithms are also applicable to a more general class of targets such as tactical missiles and airplanes. The emphasis of this paper will therefore not be on algorithmic details but rather on discussing problem areas and approaches. References containing estimation algorithms are cited for more interested readers. Furthermore, discussions contained in this report will be oriented from the practitioner's point of view and consequently rigorous mathematical terms are only of secondary interest.

This report is organized as follows. The fundamental problem of target tracking, approaches and some design tradeoffs are reviewed in Section 2. Four approaches for tracking targets. with sudden maneuvers are presented in Section 3. The discussion includes a comparison of their relative merits. The problem of tracking with passive sensors (measurements containing only line-of-sight angles) is discussed in Section 4. In some applications, one is confronted with large scale system issues, i.e., the existence of many sensors operating in a multiple target environment. In Section 5, several algorithms for processing multiple sensor data and an algorithm for correlating measurements from multiple sensors are presented. The problem of tracking in a multiple target environment is discussed in Section 6. In each tracking application, one may be interested in performance evaluation without resorting to Monte Carlo simulations. Covariance analysis techniques are outlined in Section VII for this purpose, namely, the polynomial analysis, the Riccati Equation, and the Cramer-Rao Bound.

Since this report focuses discussions on design considerations, performance trade-offs and approaches to given problems, several related subjects discussing algorithmic details such as the square root [53] or U-D factorization algorithm [54] are not included. One may notice that certain subjects are given more attention than others, this is due primarily to personal preferences. In each case however, we try to provide an adequate list of references to allow a indepth study for interested readers.

2. FUNDAMENTALS

2.1 Problem Definition

The tracking problem is a state estimation problem, i.e., assuming the state of a target evolves in time according to the equation

$$\dot{\underline{x}} = \underline{f}(\underline{x}) + \underline{w} \tag{2.1}$$

and the corresponding (discrete) measurement vector* is given by

$$\underline{z}_k = \underline{h}(\underline{x}_k) + \underline{v}_k \tag{2.2}$$

where $\underline{w}$ and $\underline{v}_k$ are input and measurement noise processes, respectively, one is interested in estimating the target states $\underline{x}_k$ based upon all measurements $\underline{z}_\ell$, $\ell=1,...,k$.

We make these following remarks.

(1) Eq. (2.1) is a mathematical model representative of the target dynamics. The state vector $\underline{x}_k$ usually contains target position, velocity, and sometimes acceleration as state variables. In some situations, key parameters characterizing important target properties are also included as state variables. The filter designer usually has the option of choosing among several models with different level of complexity. The trade-off is performance versus real-time computational requirement.

(2) Eq. (2.2) is the measurement equation relating state variables to measurement variables. When a radar is used, $\underline{z}_k$ has at least three components, i.e., range and two angles. If a passive sensor (such as a telescope) is used, $\underline{z}_k$ only contains two angle measurements.

*The measurement device can be a radar, sonar, telescope (passive), and others. In most cases, these measurements are taken in discrete times.

(3) The system (input) and measurement noise processes $\underline{w}$ and $\underline{v}_k$, respectively, are assumed to be zero mean white noise processes. The covariance of $\underline{w}$, Q is selected to compensate for modeling errors (discrepencies between (2.1) and the actual process). The statistics of the measurement noise process $\underline{v}_k$ should also be selected to represent all possible excursions such as measurement biases, false measurements, etc.

2.2 Basic Approaches

The basic tracking filter is a recursive algorithm. During time t_k to t_{k+1}, the state estimate is computed by integrating

$$\dot{\underline{x}} = \underline{f}(\underline{x}) \tag{2.3}$$

from t_k to t_{k+1} using $\hat{\underline{x}}_{k/k}$ as the initial state. At time t_{k+1}, a new measurement $\underline{z}_{k+1}$ is obtained, the state estimate is updated by

$$\hat{\underline{x}}_{k+1/k+1} = \hat{\underline{x}}_{k+1/k} + K_{k+1}(\underline{z}_{k+1} - \underline{h}(\hat{\underline{x}}_{k+1/k})) \tag{2.4}$$

where $\hat{\underline{x}}_{i/j}$ is the estimate of $\underline{x}_i$ based upon measurements $\underline{z}_k$, k=1,...,j. Two questions arise:

(1) How does one choose $\underline{f}(\;)$?

(2) How is the filter gain K_{k+1} computed?

These two questions which appear different initially are actually intimately related and are discussed below.

2.2.1 Target Dynamics

For some targets, a constant velocity (CV) model is sufficient, i.e., the state vector contains six variables

$$\underline{x} = [x_1,x_2,x_3,x_4,x_5,x_6]^T = [x,\dot{x},y,\dot{y},z,\dot{z}]^T \tag{2.5}$$

where (x,y,x) are coordinates of a Cartesian coordinate system used to describe the target dynamics. The state equations are

$$\left.\begin{aligned} \dot{x}_i &= x_{i+1} \\ \dot{x}_{i+1} &= w_i \end{aligned}\right\} i=1,3,5 \tag{2.6}$$

where w_i is a *process noise* term used to characterize modeling errors.

If the target being tracked is maneuvering (accelerating), a constant acceleration (CA) model is usually used, i.e.,

$$\underline{x} = [x,\dot{x},\ddot{x},y,\dot{y},\ddot{y},z,\dot{z},\ddot{z}] \tag{2.7}$$

The state equations can be written accordingly, i.e.,

$$\left.\begin{aligned} \dot{x}_i &= x_{i+1} \\ \dot{x}_{i+1} &= x_{i+2} \\ \dot{x}_{i+2} &= w_i \end{aligned}\right\} i=1,4,7 \tag{2.8}$$

These two state equations are also referred to as the first and second order polynomial dynamics, respectively.

Note also that equations (2.6) and (2.8) assume complete decoupling between x,y, and z. A commonly used Cartesian system has the coordinate centered at the sensor location with the x-axis pointing east, the y-axis pointing north, and the z-axis perpendicular to the local horizontal plane.

The target dynamic is sometimes described in the sensor coordinates. For example, if a dish radar is used for tracking, the measurement variables include range (r), azimuth (a), and elevation (e). A constant velocity target dynamic model decoupled in r, a, and e results in the following equations,

$$\begin{aligned} \ddot{r} &= w_r \\ \ddot{a} &= w_a \\ \ddot{e} &= w_e \end{aligned} \tag{2.9}$$

Where w_r, w_a, and w_e are process noise terms representing modeling errors in r, a, and e directions, respectively. The advantage of using (2.9) is that one is required to construct three two-dimensional filters instead of one six-dimensional filter. Considerable computational savings result. This model however, is inconsistent with the assumption the target dynamics is decoupled in the Cartesian coordinate, eqs. (2.6), and (2.8). A more appropriate model is to retain the coupling among r, a and e. This results in the following set of equations.

$$\ddot{r} = r(\dot{e}^2 + \dot{a}^2 \cos^2 e)$$

$$\ddot{a} = -2\frac{\dot{r}}{r}\dot{a} + 2\,\dot{a}\,\dot{e}\tan e \qquad (2.10)$$

$$\ddot{e} = -2\frac{\dot{r}}{r}\dot{e} - \frac{\dot{a}^2}{2}\sin 2e$$

We note that using measurement variables as state variables <u>may</u> result in more accurate state estimates because this makes the measurement equations linear (see for example, the discussion of [1]). The Cartesian coordinates however, appeal intuitively and provide easier interpretation of target motion.

These models are often used for tracking airplanes and tactical missiles and are also used for tracking ballistic missiles. In some re-entry (RV) vehcile applications, however, the vehicle aerodynamic parameters such as the ballistic and lifting coefficients must be estimated in real time. This requires a set of nonlinear differential equations to describe RV motion. A simplified* model is

$$\vec{a}(t) = \frac{1}{2}\rho v^2(t)[\frac{1}{\beta}\vec{u}_d + \alpha_t\vec{u}_t + \alpha_c\vec{u}_c] + \vec{g} \qquad (2.11)$$

where $\vec{a}(t)$ is the total acceleration applied on the vehicle

*This is a simplified model neglecting Coriolis and Centrifugal forces. They are usually included using the vector sum as in (2.11).

ρ is the air density

$v(t)$ is the magnitude of the vehicle velocity

β is the ballistic coefficient

$\vec{u}_d$ is the unit vector along the drag force direction which is opposite to the velocity vector

$\vec{u}_t, \vec{u}_c$ are two orthogonal unit vectors defining a plane perpendicular to $\vec{u}_d$ for modeling lifting forces

$\vec{g}$ is the gravity force vector.

A state model including β, α_t, and α_c as state variables results in a 9-state state vector. Further discussion of this problem is found in Refs. [7], [8].

2.2.2 Filter Gain Computation

The filter update equation (2.4) gives the weighted sum of the one-step predicted state and the difference of the new and predicted measurement (this difference is called the filter residual process and in more rigorous situations, the innovation process). The filter gain provides the weighting of this update procedure. When the filter gain is very small, the estimator becomes insensitive to the new measurements and this may result in large bias errors. When the filter gain is very large, the estimator is discarding past measurements resulting in larger random errors. A balance can be achieved if one has sufficient knowledge regarding the accuracy of the state model and the measurement process. Such knowledge is embedded in the filter gain computation. Four approaches for treating this problem are discussed.

2.2.2.1 The Extended Kalman Filter (EKF)

If an extended Kalman filter is used, the filter gain is

$$K_k = P_{k/k} H_k^T R_k^{-1} \tag{2.12}$$

where H_k is the measurement Jacobian matrix, R_k is the measurement noise covariance, and $P_{k/k}$ is the covariance of the state estimate $\hat{\underline{x}}_{k/k}$ obtained by solution of the matrix Riccati equations, i.e.,

(1) From t_k to t_{k+1}, compute $P_{k+1/k}$ by integrating

$$\dot{P} = FP + PF^T + Q \quad , P_{k/k} \tag{2.13a}$$

(2) At t_{k+1}

$$P_{k+1/k+1} = P_{k+1/k}[I-H_{k+1}^T(H_{k+1}P_{k+1/k}H_{k+1}^T+R_{k+1})^{-1}H_{k+1}P_{k+1/k}] \tag{2.13b}$$

where F is the system Jacobian matrix and Q is the process noise covariance matrix. The matrix Q must be chosen to be representative of modeling errors. For example, if a CV model is chosen while the target may actually be accelerating, Q should be chosen proportional to the expected magnitude of target acceleration. A method for selecting Q such that the filter computed covariance is an upper bound of the actual performance is suggested in [9]. Depending upon specific applications, the value of Q can also be obtained empirically using Monte Carlo simulation, Ref. [8].

Modeling error may become a significant contributor to estimation error when the Riccati equation reaches a very small steady state solution (the P matrix). In this case the filter gain, eq. (2.12), becomes nearly equal to zero and the filter is essentially running open-loop, eq. (2.4). The use of a large Q forces the steady state P matrix to stay "significantly" large, so that the estimate is sensitive to the most recent measurements.

The above discussion presents a heuristic argument on the use of a process noise covariance Q. Besides methods using covariance upper bounds and Monte Carlo studies suggested in [8] and [9], there also exists considerable literature which discusses methods of estimating Q, R, (and sometimes K directly) in realtime based upon the statistics of the innovation process. One notable paper in this area is by Mehra [13]. We briefly outline Mehra's method below.

It is well-known that if both the system model and noise statistics are true representations of the actual physical process, the filter innovation process

$$\underline{\gamma}_k = \underline{z}_k - \underline{h}(\hat{\underline{x}}_{k/k-1}) \qquad (2.14)$$

is white Gaussian with zero mean and covariance*

*This is clearly not true for nonlinear systems. In most applications using an extended Kalman filter, this expression appears to be a very close approximation.

$$P_{\gamma_k} = H_k P_{k/k-1} H_k^T + R_k \tag{2.15}$$

and the filter achieves optimal performance. In [13], a method for testing filter optimality based upon the aforementioned property of the innovation process was presented. If the test indicates that the filter does not attain optimal performance, one then proceeds to adjust Q and R so that the covariance of the innovation process will be consistent with that of filter prediction. This method first computes a sampled correlation function assuming that $\underline{\gamma}_k$ is ergodic over a certain time interval, one therfore has

$$\begin{aligned} C_j \triangleq E[\underline{\gamma}_i\ \underline{\gamma}_{i-j}^T] &= HP_\infty H^T + R; \quad j=0 \\ &= H[\Phi(I-KH)]^{j-1}\Phi[P_\infty H^T - KC_0]; \ j>0 \end{aligned} \tag{2.16}$$

and

$$\hat{C}_j = \frac{1}{N}\sum_{i=1}^{N} \underline{\gamma}_i \underline{\gamma}_{i-j}^T \tag{2.17}$$

where P_∞ denotes the steady state error covariance matrix and Φ is the transition matrix of the system dynamics.

Using the above equations and the steady state Riccati equation, Mehra gives a procedure for solving for Q and R. There are situations in which there is no sufficient number of independent equations for solving them; Mehra then gives a recursive procedure for solving for the filter gain K directly.

We emphasize that the above technique is important because a systematic procedure for identifying the noise covariance or filter gain is established. It may not be very useful for realtime applications because of its computational requirement. For non-realtime applications, however, this method is useful for simulations and post-mission data analysis studies.

There are situations in which the dynamic process takes large and sudden changes (such as target maneuvering), such that either a large Q must be used all the time to account for the maximum expected deviation or else a quick and easy method must be utilized for realtime identification of Q. This subject is covered in Section 3.1.

There exist other methods for preventing the P matrix from becoming too small. These including the finite memory filter [1], [12], and the fading memory filter [1], [10], [11].

2.2.2.2 The Finite Memory Filter

The finite memory filter applies a sliding window to the data and computes the state estimate based only upon data in that time span. The window width is selected such that the system model is an adequate approximation to the actual process over the time interval. During this time interval, the system is assumed to be noise free. This assumption is related to the selection of the time interval in that the variation of the unknown parameter over this interval is small. The filter to be discussed is due to Jazwinski [12]. We briefly outline it below.

Let the measurement sequence be denote by

$$\underline{z}_1, \underline{z}_2, \ldots, \underline{z}_m, \underline{z}_{m+1}, \ldots, \underline{z}_k$$

and N=k-m where N is the total number of measurements desired in the finite memory filter. Let the finite memory state estimate and covariance be denoted by $\hat{\underline{x}}_{k/m,k}$ and $P_{k/m,k}$, respectively; they can be computed using the following equations

$$\hat{\underline{x}}_{k/m,k} = P_{k/m,k}\,(P_{k/k}{}^{-1}\hat{\underline{x}}_{k/k} - P_{k/m}{}^{-1}\hat{\underline{x}}_{k/m})$$
$$P_{k/m,k}{}^{-1} = P_{k/k}{}^{-1} - P_{k/m}{}^{-1} \tag{2.18}$$

where $\hat{\underline{x}}_{k/k}$ is the state estimate at time k based upon all data up to and including $\underline{z}_k$ and $\hat{\underline{x}}_{k/m}$ is the state estimate at time k based upon all data up to and including $\underline{z}_m$. One can interpret the above equations to say that the finite memory estimate is obtained by subtracting an estimate based on all data prior to the time window from the estimate based upon all data. We note that the above equations can be re-organized to obtain a computationally more efficient and numerically more stable algorithm. For more discussions, see [12].

Notice that the finite memory filter requires that a batch of N measurement vectors be stored (for updating $\hat{x}_{k/m}$). Furthermore, the computational burden is much larger than that of the EKF. Typically, the cost of added computation outweighs the benefit when compared with other methods.

2.2.2.3 The Fading Memory Filter

The fading memory filter (sometimes referred to as the aging filter) weights recent data exponentially higher than past data. A derivation of this filter can be found in [10]-[11]. The resulting algorithm turns out to be very simple. Let $P^*_{k+1/k}$ denote the covariance of the one-step predicted estimate of the aging filter, it is related to the Kalman filter covariance (solution of Eq. (2.13a)) by

$$P^*_{k+1/k} = \alpha P_{k+1/k} \tag{2.19}$$

where α is a scalar quantity greater than unity. The $P^*_{k+1/k}$ is then used in Eq. (2.13b) to obtain the update covariance $P_{k+1/k+1}$. The scalar α is the exponential weighting factor. This is accomplished by changing the measurement noise covariance matrix to

$$R^*_i = R_i(\alpha)^{k-i} \tag{2.20}$$

for $i=1,\ldots,k$ where k is the current time. With measurement noise covariance, data rate and an assumed modeling error, one can find an optimum α for minimizing the mean square error.

2.2.2.4 A Constant Gain Filter: The α-β-γ Tracker

In some cases, because of computational constraints, it may be impractical to compute the filter gain in real-time. Under such conditions one must use either a set of pre-computed filter gains or a constant gain filter. One commonly used constant gain filter is based upon the steady state solution of the Riccati equation. In order to minimize the modeling error effect, a sufficiently large process noise covariance Q must be artfully selected.

Another popular constant gain filter is the α-β-γ filter, (or α-β filter when using a CV model), [5], [6]. The main difference between the α-β-γ filter and the steady state gain filter is that the former assumes the complete independence of the three spatial coordinates in the filter update equation. Notice that the state space and the measurement space may be related through a nonlinear function (Eq. (2.2)). In using the α-β-γ filter, this assumption is not allowed. Let (r,a,e) denote the radar range, azimuth and evaluation, respectively; the state vector becomes,

$$\underline{x} = [r,\dot{r},\ddot{r},a,\dot{a},\ddot{a},e,\dot{e},\ddot{e}]^T \tag{2.21}$$

Let $\underline{r}$ denote $[r,\dot{r},\ddot{r}]^T$, then the filter update equation becomes

$$\hat{\underline{r}}_{k+1/k+1} = \hat{\underline{r}}_{k+1/k} + K_{k+1}[\underline{r}_{k+1} - \hat{\underline{r}}_{k+1/k}] \tag{2.22}$$

where $\underline{r}_{k+1}$ is the range measurement at time k+1. The gain matrix

K_{k+1} for the α-β-γ filter is

$$K_{k+1} = [\alpha, \frac{\beta}{t_{k+1}-t_k}, \frac{2\gamma}{(t_{k+1}-t_k)^2}] \quad (2.23)$$

The update equations for azimuth and elevation can be obtained accordingly.

Notice in Eq. (2.12), when an extended Kalman filter is used, the gain matrix K_k has the dimension (9x3) while the gain matrix for the α-β-γ filter consists of three (3x1) vectors.

There is a wide range of methods for choosing the values of α, β , and γ. One method is to compute the steady state Kalman gain in the above chosen coordinate. A more reliable method is to conduct extensive Monte Carlo simulation studies to define α, β, γ over a variety of cases for a given application.

2.3 A Batch Filter

In Section 2.2.2.2, we discussed a finite memory filter developed by Jazwinski. The input elements of that algorithm are the outputs of the recursive Kalman filter. The estimate is based upon data from a finite time interval and a batch of N measurement vectors corresponding to that time interval must be stored. Another assumption of that filter is that the system model is noise free and this assumption is reasonable because the finite time interval is selected to reflect the tolerance on model errors. The Jazwinski algorithm is optimum for linear systems under the given conditions.

In this section, we present an algorithm for nonlinear systems. This algorithm iteratively solves for the optimum estimate based upon a noise free system model and a batch of N measurement vectors $\underline{z}_1$, $\underline{z}_2,\ldots,\underline{z}_N$. This filter is fundamentally different from the approach of Jazwinski. For linear systems, this approach is the same as the Kalman filter, For nonlinear systems this approach gives a better estimate than that of the extended Kalman filter.

Let the system and measurement equations be the same as eqs. (2.1) and (2.2) except that $\underline{w}$ is now zero. The batch of N measurement vectors is denoted by $\{\underline{z}_1, \underline{z}_2,\ldots,\underline{z}_N\}$. Let $\hat{\underline{x}}^k_{N/N}$ denote the k-th iteration of the estimate of $\underline{x}_N$, then

$$\hat{\underline{x}}^{k+1}_{N/N} = \hat{\underline{x}}^k_{N/N} + P^k_{N/N}\left[\sum_{n=1}^{N} G_n^{k^T} H_n^{k^T} R_n^{-1}(\underline{z}_n - \underline{h}(\hat{\underline{x}}^k_{n/N}))\right] \tag{2.24}$$

$$P^k_{N/N} = \left[\sum_{n+1}^{N} G_n^{k^T} H_n^{k^T} R_n^{-1} H_n^k G_n^k\right]^{-1} \tag{2.25}$$

where

$$G_n^k = \Phi_n^{k^{-1}} G_{n+1}^k \quad ; \quad n=N-1,\ N-2,\ldots,1$$

G^k_N = I (an identity matrix)

Φ^k_n = $e^{\int F^k_s ds}$

F^k_s = the Jacobian matrix of $f(\hat{\underline{x}}^k_{s/N})$

H^k_n = the Jacobian matrix of $h(\hat{\underline{x}}^k_{n/N})$

$\hat{\underline{x}}^k_{n/N}$ = result of integrating $\dot{\underline{x}}=f(x)$ backward from t_N to t_n using $\underline{x}_N = \hat{\underline{x}}^k_{N/N}$.

The iteration is terminated when $\hat{\underline{x}}^{k+1}_{N/N}$ and $\hat{\underline{x}}^k_{N/N}$ are sufficiently close.

We make the following remarks:

1) The above algorithm is a realization of the maximum likelihood estimator with Gaussian measurement noise process. It is well-known that the maximum likelihood estimate is asymptotically efficient and Gaussian and approaches the Cramer-Rao bound.

2) The $P_{N/N}$ of (2.25) is an approximate expression for the covariance of $\hat{\underline{x}}_{N/N}$. The $P_{N/N}$ evaluated at the true state is the Cramer-Rao lower bound on the covariance of $\hat{\underline{x}}_{N/N}$. Since $\hat{\underline{x}}_{N/N}$ approaches the true state with probability one, $P_{N/N}$ also approaches the Cramer-Rao bound with probability one.

3) Notice that the inverse of $P_{N/N}$ is Fisher's information matrix. The invertibility of the information matrix is tied with the observability of the system, see for example [58].

4) For linear systems a closed form solution can be found and the iterative procedure becomes unnecessary. An interesting exercise is to derive a batch filter using the polynominal dynamics with linear measurements equations (see for example [49]).

There are many application areas for this algorithm. For example, in tracking space objects where the target dynamics can be modeled very accurately, the algorithm of this section is particularly suitable. This method has been used for ballistic trajectory tracking with angle-only measurements [30] and tracking of deep space satellites, [31]. Another application is for <u>track initiation</u>. Since the initial covariance and state estimates are not generally given a priori, the above algorithm can obtain the best estimates based on the first N measurement vectors and then proceed to use $\hat{\underline{x}}_{N/N}$ and $P_{N/N}$ as the initial state and covariance estimates, respectively. This method is sometimes referred to as the information matrix approach for filter initiation.

2.4 Summary

In the above, we have discussed various algorithms for addressing the basic tracking problem. These approaches employ simple to sophisticated system models for target dynamics and attempt to compensate for modeling errors in a variety of ways. Several well-known nonlinear estimation algorithms were not discussed. These include the second order filter [14] and the single stage iterative filter [15]. They are not included because of their excessive computational requirement although these algorithms can indeed improve the estimation accuracy.

Before closing this section, a brief algorithm comparison can be stated. If the objective of tracking is to obtain precision information about the target dynamics, then one should use the most accurate target model and apply the EKF, (or even more sophisticated algorithms). If the dynamic model is sufficiently accurate so that the process noise term is negligible, then the algorithm considered in the Section 2.3 is a good choice provided that the computation time and data storage requirements are not excessive.

If the objective is just to maintain the target in track, then one may use the simplest track algorithm such as the α-β-γ tracker. One exception to this case is when tracking in a dense target environment where precision tracking may be necessary for target correlation. This subject will be discussed later.

The finite and fading memory filters are usually the secondary choices (especially the finite memory filter) because the same purpose (reducing sensitivity to model errors) can be achieved by adjustment of the process noise covariance, Q. They are nevertheless included here because (1) they reflect the historical development of adaptive filtering techniques and (2) they still provide an option for readers to choose for their applications.

3. TARGETS WITH SUDDEN MANEUVERS

Targets with sudden maneuvers can be modeled as systems with abrupt changes. We modify equation (2.1) to become two sets of equations, one representing the pre-maneuver dynamics and the other incorporating the maneuver feature

$$\dot{\underline{x}} = \underline{f}(\underline{x},\underline{x}_m) + \underline{w} \qquad (2.1a)$$

where $\underline{x}_m$ is the vector representing maneuvering force and satisfies

$$\underline{x}_m = \underline{0} \quad , \quad \text{for } t > t_m \qquad (3.1)$$

and

$$\dot{\underline{x}}_m = \underline{f}_m(\underline{x}_m) + \underline{w}_m, \text{ for } t \geq t_m \qquad (3.1a)$$

where t_m is the time the maneuver begins, $\underline{f}_m(\cdot)$ is the maneuvering dynamics and $\underline{w}_m$ is the system noise for $\underline{f}_m(\cdot)$. For targets with sudden maneuvers, t_m is unknown, $\underline{f}_m(\cdot)$ and $\underline{w}_m$ may be unknown or partially known.

In tracking airplanes for example, the target may first be flying in a straight line with constant speed. A CV model is adequate for the airplane dynamics in this case. A sudden maneuver of this airplane implies that the airplane is accelerating unexpectedly and the acceleration is time-varying and following an unknown profile. The instantaneous acceleration vector is therefore the maneuvering vector $\underline{x}_m$ defined above.

In tracking a ballistic re-entry vehicle, the target dynamics is the equation (2.11) with α_c and α_t equal to zero. A sudden aerodynamic maneuver means α_c and α_t become nonzero and follow unknown time-varying profiles. The maneuvering vector in this case contains α_c and α_t as its elements.

Four approaches to this problem are discussed individually below.

3.1 Filter Compensation Using Process Noise Covariance

In this first method, one simply ignores the maneuver vector $\underline{x}_m$ and lumps the system errors introduced by $\underline{x}_m$ with the process noise term $\underline{w}_m$. If the estimator's only concern is to maintain the target in track (adequate position estimation accuracy), this method can work quite well.

Basically, it examines the "regularity" of the filter residual vector

$$\underline{\gamma}_k = \underline{z}_k - \underline{h}(\hat{\underline{x}}_{k/k-1}) \tag{3.2}$$

against its covariance matrix

$$P_{\gamma_k} = H_k P_{k/k-1} H_k^T + R_k \tag{3.3}$$

using (the Chi-square variable)

$$\ell_k = \underline{\gamma}_k^T P_{\gamma_k}^{-1} \underline{\gamma}_k \; . \tag{3.4}$$

When ℓ_k becomes too large one suspects that the target is maneuvering and the covariance of w, Q is increased so that ℓ_k is reduced to a reasonable value. This method therefore has the combined feature of maneuver detection and filter compensation.

This method is based upon the adaptive filter of Jazwinski, [16] and shares great similarities with the method of Mehra, [13] (see also section 2.2.2.1). The main difference is that the above method tests the filter regularity using Chi-square statistics, and stresses the simplicity (not the optimality) of the approach.

A thorough discussion of this method and its performance against maneuvering re-entry vehicles can be found in Refs. [7] [8].

3.2 State Augmentation

The second method is straightforward, computationally more costly, but with substantially better performance than the previous method. This method is to include $\underline{x}_m$ as part of the state vector, i.e., the augmented state consists of

$$\underline{x}_a = [\underline{x}^T, \underline{x}_m{}^T]^T \tag{3.5}$$

In the case when the target maneuvering dynamic is completely unknown, one uses

$$\dot{\underline{x}}_m = \underline{w}_m \tag{3.6}$$

where $\underline{w}_m$ is modeled as a zero mean white noise process with covariance Q_m. If bounds on the magnitudes of maneuvers are known, a method for choosing Q_m such that the actual filter performance is bounded by the computed filter covariance can be found in [9]. As a rule of thumb, the values of the entries in Q_m should be a fraction of the expected magnitude squared of the maneuver force and proportional to the measurement time interval. A method for selecting Q_m for estimating ballistic and lifting coefficients of reentry vehicles is discussed in [7], [8].

Notice that Eq. (3.6) assumes $\underline{x}_m$ uncorrelated in time. This assumption is sometimes not very realistic. A model often used in airplane tracking is

$$\begin{aligned} \dot{\underline{x}}_m &= \alpha \underline{x}_m \\ \dot{\alpha} &= w_\alpha \end{aligned} \tag{3.7}$$

where α is the correlation constant to be estimated and w_α is a noise process. Methods for selecting statistics for w_α and the performance against airplane tracking can be found in Ref. [17].

The maneuvering state $\underline{x}_m$ is usually not influenced by the state vector $\underline{x}$ (Eqs. (2.1a) and (3.1a)). With this assumption, one can compute the state and maneuver estimates separately to obtain

$$\hat{\underline{x}}_{k/k} = \tilde{\underline{x}}_{k/k} + A_k \hat{\underline{x}}_{m,k} \tag{3.8}$$

where $\tilde{\underline{x}}$ is the estimate assuming $\underline{x}_m$ is zero, $\hat{\underline{x}}_{m,k}$ is the maneuver estimate, A_k is a gain matrix, and $\hat{\underline{x}}_{k/k}$ is the final state estimate. The advantage of this decoupled implementation is a saving in computation. It can be shown that the decoupled estimator is optimum for linear systems when t_m is known and the maneuver state model is known and deterministic; see for example [18], [19], and [21]. Suboptimal designs for the case when the above assumptions are not true can be found in [9].

Since the maneuver time t_m is unknown, a trival application of the above method is to use it throughout the entire track. Inevitably, the filter performance is degraded when the target is non-maneuvering, Refs. [8], [9]. An ideal extension is to use a maneuver detector for switching the tracking filter from nonmaneuver to maneuver mode, this subject is discussed in the following subsection.

3.3 Manuever Detection

The detection problem is the problem of discriminating the following two hypotheses based upon filter residuals:

$$\begin{aligned} H_1:\ \tilde{\underline{\gamma}}_k &= \underline{\gamma}_k + g_k(x_m)\ ; \text{ maneuvering target hypothesis} \\ H_0:\ \tilde{\underline{\gamma}}_k &= \underline{\gamma}_k \qquad\qquad ; \text{ non-maneuvering hypothesis} \end{aligned} \tag{3.9}$$

for $k = k_0, \ldots, K$ where $\underline{\gamma}_k$ is the residual vector before the maneuver starts, and g( ) is a known function relating the maneuvering vector $\underline{x}_m$ to the residual vector. A generalized likelihood ratio test is

$$\Lambda = \frac{\max\limits_{\underline{x}_m} p(\tilde{\underline{\gamma}}_k; k_0, \ldots, K/H_1)}{p(\tilde{\underline{\gamma}}_k; k_0, \ldots, K/H_0)} \gtrless \lambda \tag{3.10}$$

For a given application (target dynamics, sensor type, etc.), the above equation can be further simplified, [7], [18], [20], [21].

The use of a generalized likelihood ratio test for detecting maneuvers is discussed in [7], [8] for re-entry vehicles and in [20] for airplanes. General discussions on the detection of sudden changes in linear systems can be found in Refs. [18] and [21].

Drawbacks of using a detector-directed tracking include (1) detection delay and (2) large transient errors during filter switch. Numerical results in Ref. [8] show large estimation errors immediately after filter switch. This is partially due to the maneuvering filter going through its transient period.

Notice that the maneuver detection relys on the fact that target maneuvering generates residual bias in a non-maneuvering tracker. Once the filter is switched to maneuvering, the above detection scheme can not be used to discriminate if the target has returned to non-maneuvering status.

All the above problems can be alleviated if one employs the adaptive multiple model estimator, [22]-[29], discussed in the next subsection.

3.4 Multiple Model Estimator

Let H_1 denote the hypothesis that the target is maneuvering and H_0 the hypothesis that the target is non-maneuvering. One may construct two filters, the first one uses a maneuvering dynamic while the other one uses a non-maneuvering model. Let these respective state be denoted by $\underline{x}^1{}_k$ and $\underline{x}^0{}_k$; the optimum estimate $\hat{\underline{x}}_{k/k}$ is obtained using

$$\hat{\underline{x}}_{k/k} = P_1(k)\hat{\underline{x}}^1_{k/k} + P_0(k)\hat{\underline{x}}^0_{k/k} \tag{3.11}$$

where $P_i(k)$ is the a posteriori probability that H_i is true at time t_k. The computation of $P_i(k)$ can be very complicated. If one uses a simplifying assumption that hypotheses at time t_k are independent of hypotheses before t_{k-1}, i.e., a first order Markov process, then one obtains a much simplified expression for $P_i(k)$.

$$P_i(k) = \frac{\sum_{j=0}^{N} p_{ij}(k)P_{ij}P_j(k-1)}{\sum_{i=0}^{N}\sum_{j=0}^{N} p_{ij}(k)P_{ij}P_j(k-1)} \tag{3.22}$$

where $p_{ij}(k)$ is the residual density assuming that the i-th hypothesis is true at time t_k and the j-th hypothesis is true at time t_{k-1}, P_{ij} is the transition probability, and $P_j(k-1)$ is the a posteriori probability that H_j is true at time t_{k-1}. Notice that we have made the above equation slightly more general by assuming-that there are total of N+1 hypotheses. The $\hat{\underline{x}}^i_{k/k}$ is computed using

$$\hat{\underline{x}}^i_{k/k} = \sum_{j=0}^{N} \hat{\underline{x}}^{ij}_{k/k}\; P(H_j(k-1)/H_i(k),Z_k) \tag{3.23}$$

$$P(H_j(k-1)/H_i(k),Z_k) = \frac{p_{ij}(k)P_{ij}P_j(k-1)}{\sum_{j=0}^{N} p_{ij}(k)P_{ij}P_j(k-1)} \tag{3.24}$$

where $\hat{\underline{x}}^{ij}_{k/k}$ is the state estimate using $\hat{\underline{x}}^{j}_{k-1/k-1}$ updated with the i-th hypothesis and Z_k denotes all the measurements up to time t_k.

Derivations and discussion of the above results in a more general context are given in [22], [23].

The advantage of the above approach is that when the target switches between maneuvering and non-maneuvering modes, the hypothesis probability values change to provide a smooth transition of the final estimates $\hat{\underline{x}}_{k/k}$.

A higher level multiple model approach is to use several hypotheses to model different maneuvering force levels. This approach enhances the estimation performance at the cost of more computation resources.

We would like to emphasize that the multiple model estimator is a general adaptive estimation technique. It was first derived by Magill [27] for the time invariant hypothesis case, i.e.,

$$P_{ij} = \begin{cases} 1 , & \text{when } i = j \\ 0 , & \text{otherwise} \end{cases} \tag{3.25}$$

Its extension to the switching hypothesis case was the subject of [22]. References [28] and [29] stated the above concept in the continuous domain and gave a representation theorem known as the partition theorem.

A word of caution about the multiple model method; it is a tightly tuned algorithm (for the case of modeling various maneuvering levels) and therefore very vulnerable to the situation where none of the models match with the actual dynamics. In this case, strange filter behavior may appear and this behavior is not yet completely understood from a theoretical point of view, [23], [24].

Discussions applying this method to target tracking can be found in [25]-[26]. Its extension to adaptive control was applied to the flight control of a F-8C experimental aircraft [24]. A tutorial treatment of this method for state estimation is given in [23].

3.5 Summary

In this section, we have discussed four approaches to tracking targets with sudden maneuvers. The first method (Section 3.1) provides a way for adjusting the process noise covariance level through use of the filter residual "regularity" to compensate for the modeling error induced by target maneuver. This method uses the least computation but does not give very precise velocity and parameter estimates. The second method (Section 3.2) is to augment the state vector with maneuvering variables. The dimension of the state vector is enlarged and the filter is therefore computationally more costly. It does however provide more accurate state estimates. The drawback to this method besides the higher computational burden is that the filter performance is degraded when the target is non-maneuvering. A method to circumvent this problem is to use a maneuver-detector-directed filter (Section 3.3). This method suffers from large errors occurring during filter switch over. A compromise to this problem is to contruct two filters (Section 3.4) with one using a maneuvering dynamic model and the other a non-maneuvering dynamic model. The final estimate is a weighted sum of outputs of these two filters using the a posteriori hypothesis probabilities as weighting factors. This method has the advantages of all the above approaches but at a cost of a much larger computational burden. One may also use a bank of filters to model different maneuver levels and apply the multiple model adaptive estimation method. This method may work very well, it is however, also very sensitive to model mismatch errors and system nonlinearites.

4. TRACKING WITH ANGLE-ONLY MEASUREMENTS

In this section, the problem of tracking with a sensor measuring only the target line-of-sight angle is discussed.

There may be two separate objectives of tracking. The first one is simply trying to maintain targets in track in the angle domain. In this case, techniques discussed in Section 2 are applicable. The only difference is that in the angle only case the target dynamic equations are described in two-dimensional coordinates. Polynomial equations decoupled in two orthogonal angular directions are often used. The second tracking objective is to obtain estimates of the complete state vector as defined in a three-dimensional coordinate system. This objective may not always be achievable since tracking with angle-only measurements may constitute an unobservable system. Physically, it can be explained as follows. In radar tracking systems, each measurement vector determine the instantaneous target position to within a finite uncertainty volume, i.e., uncertainties in both range and angles can be expressed with finite standard deviations. In an angle-only tracking system, the uncertainty volume of each measurement vector is infinite (due to the inability to measure range). Such a system may be observable only for certain types of target dynamics. For example, when a telescope is used to track a satellite, this constitutes an observable system and target range can be estimated (with large errors however) because the satellite trajectory is influenced by the earth gravity [30]. If a telescope is used to track an airplane traveling with constant speed, this system is unobservable, i.e., the estimation of the 3-dimensional dynamics is impossible.

There are means available for enhancing the observability of the system. For example, one may use two passive sensors at separate locations simultaneously tracking the same object. The intersection of two angular beams gives the total measurement uncertainty which now has a finite uncertainty volume. One complication of this approach occurs when tracking in a multiple target environment. Here, one is confronted with the problem of recognizing the same target at both sensors. The subject of sensor-to-sensor correlation and efficient methods of processing multiple sensor measurements are discussed in the next section. Another method of enhancing observability is to incorporate other types of measurements in addition to angle measurements. One such applications is in passive sonar tracking [32] where target Doppler measurement is included. A passive sonar system with Doppler measurements makes the system completely observable.

We note that orbit estimation using angle only measurements is an ancient problem (see [33] for a discussion of classical approaches and a list of references). Most emphasis of [33] however, was placed on the planetary mechanics and dynamic modeling and very little effort was applied from the estimation point of view. For example, the Gauss method for orbit determination ensures the solution of the entire orbit with only three angle measurements. This is true if noise free angle measurements can be made. With even slight errors in angle measurements, the target range estimation error can be very large.

When one is tracking a ballistic object (a satellite or a long range ballistic missile) using a passive sensor, one is confronted with two questions: 1) how to initiate a Kalman type recursive tracking filter; and 2) since the range estimation error will inevitably be larger, will the extended Kalman filter provide adequate performance. In [30], these issues were studied in detail. It was shown that the batch filter described in Section 2.3 can be used to provide initial conditions for the extended Kalman filter. Other discussions include methods for computing an initial guess for the iterative procedure, applications of the batch filter recursively for tracking, and techniques for incorporating trajectory a priori knowledge (bounds on heading angles, velocity, energy, etc.) for improving the state estimation accuracy. An important conclusion of [30] is that the performance of the batch filter asymptotically approaches the Cramer-Rao bound (see Section 7.3 of this paper) for the covariance of state estimates while the performance of the extended Kalman filter generally does not have this property.

Finally, we remark that recent developments in target tracking on the focal plane using a infrared sensor are in the problem areas of both signal processing and target tracking. The approach discussed in [55] is to first process focal plane data with a prescribed model, then establish two-dimensional target state estimates using a Kalman filter.

5. TRACKING WITH MULTIPLE SENSORS

There are a number of applications in which multiple sensors operate in a multiple target environment. There are two problems which confront a multiple sensor tracking system: 1) how to efficiently update the tracking filter with multiple sensor measurements and 2) how to identify the same target for all sensors in a multiple target environment. The first question is discussed in Section 5.1. The second question is a special case of tracking in a dense target environment. It fits however, conveniently in the context of sensor-to-sensor correlation and will therefore be discussed in Section 5.2.

5.1 Filter Update Algorithm Considerations

Consider the situation in which there are several sensors simultaneously observing a single target. If these measurements are <u>not</u> time synchronized, one can formulate the problem as that of state estimation with non-uniform measurement times. It should be noted that in this particular formulation the functional transformation from state space to observation space depends upon the geometry, hence one can be confronted with a situation in which the measurement equations change from observation to observation.

If the measurements <u>are</u> time synchronized (such as in a multi-static radar system), then one is interested in seeking the most efficient way of processing these measurements. Let $\underline{z}_{k,i}$ denote the measurement at time t_k from the i-th sensor, then

$$\underline{z}_{k,i} = \underline{h}_i(\underline{x}_k) + \underline{v}_{k,i}, \quad i = 1,\ldots, I. \tag{5.1}$$

Three processing methods are discussed by Willner, et al., [34].

1. Parallel filter: The set of measurement vectors are concatenated to form a new measurement vector

$$\underline{z}_k = [\underline{z}_{k,1}^T, \underline{z}_{k,2}^T, \ldots, \underline{z}_{k,I}^T]^T \tag{5.2}$$

   which is processed by the filter at once.

2. Sequential filter: Each measurement is processed sequentially by the filter with zero prediction time between measurements.

3. Data compression filter: Prior to processing, measurement vectors are transformed to a common coordinate system and compressed to a single measurement vector. Let $\underline{z}_{k,i}$ denote the measurement of i-th sensor and all $\underline{z}_{k,i}$'s have been transformed to a common coordinate system. The compressed measurement vector $\underline{z}_k$ is

$$\underline{z}_k = R_k \left[\sum_{i=1}^{I} R_{k,i}^{-1}\, \underline{z}_{k,i}\right] \tag{5.3}$$

$$R_k = \left[\sum_{i=1}^{I} R_{k,i}^{-1}\right]^{-1} \tag{5.4}$$

The above three methods can be shown to be equivalent and optimum for linear systems, [34]. The data compression method is computationally the most efficient for a wide range of cases [34].

It is possible to use decentralized processing wherein each sensor produces its own estimate. These are then combined to form a single estimate. The major drawback to such an approach is that the computational burden is significantly higher than any of the methods described above. Furthermore, when the system model is stochastic, the state estimates obtained by each sensor are correlated. It is uncertain as to whether the estimate combination method is optimum even if the correlation matrics in the combination procedure are included (see for example, the Gauss-Markov theorem stated in [35]).

Algorithms for data compression with non-synchronized data vectors is a straightforward extension of the work described above. This may involve integration of measurement vectors to achieve time alignment followed by an application of the data compression equations. Polynomial data smoothing may be used for data compression in very high data rate systems as is discussed in [36]. One can also apply polynomial smoothers to the non-synchronized data case.

5.2 Sensor-to-Sensor Correlation

One important problem facing a multiple sensor tracking system in a multiple target environment is the unique identification of the same target as observed by each sensor. There are two approaches to this problem. The first approach attempts to correlate the existing track files (state estimate of a given string of measurements) with measurements. This method is the same as the track continuation mode of tracking in the dense target environment. We will address this approach in the next section. The second method is to directly correlate the set of measurements from the i-th sensor with that of the j-th sensor. A brief discussion of the second method is given below.

Consider the case when there are two sensor simultaneously tracking multiple targets. The results which follow can be extended to that of multiple sensors. Let $\{\underline{y}_i;\ i=1,\ldots,N\}$ and $\{\underline{z}_j;\ j=1,\ldots,N\}$ denote N measurements obtained by the first and second sensors, respectively* and respective covariances are given by P_i and E_j. Consider now the question of which $\underline{y}_j$ corresponds to a particular $\underline{z}_i$. This is an N-array hypothesis decision problem. That is, for a given $\underline{z}_i$, one asks which of the following hypotheses is true.

*We have obviously neglected the problem of imperfect detection at the sensor and unequal coverage problem by assuming that each sensor observes the same number of targets and that there is a one-to-one assignment between them. For dicussions which include these factors, consult Ref. [37].

$$\begin{cases} H_1 : \underline{z}_i = \underline{y}_1 + \underline{n}_i \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ H_N : \underline{z}_i = \underline{y}_N + \underline{n}_i \end{cases} \tag{5.5}$$

where $\underline{n}_i$ is a random noise vector with zero mean and covariance E_i. Using the likelihood ratio test procedure, it can be shown that the $\underline{y}_j$ is choosen as the one which maximizes the joint density of $\underline{z}_i$ and $\underline{y}_j$. That is, decide that H_j is true when

$$p(\underline{z}_i, \underline{y}_j) = \max_{\underline{y}_\ell = \underline{y}_j} p(\underline{z}_i, \underline{y}_\ell) \tag{5.6}$$

If $\underline{z}_i$ and $\underline{y}_j$ are Gaussian random vectors, then one obtains the following equivalent procedure.

Decide H_j is true when $\underline{y}_j$ minimizes

$$w_{i\ell} = (\underline{z}_i - \underline{y}_\ell)^T (P_i + E_\ell)^{-1} (\underline{z}_i - \underline{y}_\ell) \tag{5.7}$$

for all $\ell = 1, \ldots, N$. Notice that (5.7) is a Chi-square random variable or, a weighted distance measure.

The above discussion gives a procedure for selecting a $\underline{y}_j$ vector for a given $\underline{z}_i$ vector. When one considers all $\underline{z}_i$ vectors, this procedure can not be extended without modificantions. This is because if one repeats the above procedure for all $\underline{z}_i$'s, one may obtain the correlation of the same measurement from one sensor to several measurements of the other sensor. One therefore has to impose the constraint that each measurement can only be assigned (correlated) once while optimizing some performance index (the problem of multiple correlation due to limited sensor resolution is discussed in [37]). One such performance index is the sum of all Chi-squares.* That is, the N correlated pairs of $\underline{z}_i$ and $\underline{y}_j$ are those achieving the minimum of

$$\sum_{i,j} w_{ij} \tag{5.8}$$

under the constraint that each $\underline{z}_i$ and $\underline{y}_j$ can only be used once.

We note that the above problem is the same as the assignment problem in operations research. The optimum answer may be obtained by exhaustively searching for all combinations which results in searching through N! possibilities. A procedure called the Hungarian method [38] (or the Munkre's method for a specific implementation procedure [39]) requiring at most $(11\,N^3 + 12\,N^2 + 31\,N)/6$ operations is often used in this type of problems.

In the case when the target density is not very high so that the w_{ij} of a mismatched pair may attain large values, a threshold may be first applied to examine all w_{ij}'s. For Gaussian measurement vectors, this threshold can be selected to provide a given probability of leakage. Those pairs exceeding the threshold are first rejected, one is therefore only required to correlate on a subset of measurement pairs which do not exceed the threshold.

*This criterion is corresponding to the minimum error probability for Gaussian measurement errors.

6. TRACKING IN A DENSE TARGET ENVIRONMENT

Tracking in a dense target environment was the subject of a recent review article [40] and several invited sessions in recent IEEE-CDC conferences. Representative work in this area can be found in [40]-[48]. This problem is sometimes referred to as scan-to-scan correlation or tracking data association. In this section, we offer some very general discussion on this problem and the interested reader should consult the references for details.

This problem can be divided into two phases. The first phase is track initiation and the second phase is track maintenance. They are discussed individually below.

6.1 Track Initiation

Consider the case of a scanning sensor. The first and second scan produce N_1 and N_2 detections, respectively. The problem is to associate the two sets of detections to form min (N_1,N_2) number of track files. Notice that we have assumed that $N_1 \neq N_2$. This can be caused by (1) imperfect detection (2) emergence of new targets in the second scan, and (3) targets leaving the sensor field of view before the second scan. In the following, an approach for track initiation with k scans of data is described.

Let Z_k denote all the measurements (N) collected during k-th scan, i.e.,

$$Z_k = \{\underline{z}_1(k),\ \underline{z}_2(k),\ldots,\underline{z}_N(k)\} \tag{6.1}$$

Let Z^k denote the set of measurements up to and including the k-th scan, i.e.,

$$Z^k = \{Z_i;\ i=1,\ldots,\ k\} \tag{6.2}$$

Furthermore to simplify discussion, assume that N is the number of detections for all Z^k's. Assume also that sensors have perfect target detection. When this is not true, one has to enumerate more hypotheses to account for all possibilities. With Z^k, there can be N^k combinations of measurement sequences and each measurement sequence represents a possible track. Let each possible combination be denoted by a hypthesis, $H_{m_k}(k)$ which is defined by

$$H_{m_k}(k) = \{\underline{z}_{n_1}(1),\ \underline{z}_{n_2}(2),\ldots,\underline{z}_{n_k}(k)\} \tag{6.3}$$

Suppose that a tracking filter is applied to process each possible measurement sequence. The a posteriori hypothesis probability of $H_{m_k}(k)$ being true can be computed recursively using

$$P(H_{m_k}(k)/Z^k) = \frac{p(\underline{z}_{n_k}(k)/H_{m_{k-1}}(k-1),Z^{k-1})}{p(\underline{z}_{n_k}(k)/Z^{k-1})}\ P(H_{m_{k-1}}(k-1)/Z^{k-1}) \tag{6.4}$$

where $p(z_{n_k}(k)/H_{m_{k-1}}(k-1),\ Z^{k-1})$ is the probability density of the residual from the tracking filter using $H_{m_{k-1}}(k-1)$ and $z_{n_k}(k)$*. The above equation can be derived as a special case of the results presented in [22],[23]. The final set of tracks (total N) can be chosen as those N feasible hypotheses with the largest hypothesis probabilities, i.e.,

$$\max_{\{N;\ H_{m_k}(k)\in\mathscr{F}\}} \{P(H_{m_k}(k)/Z^k);\ m_k = 1,\ldots, N^k\} \qquad (6.5)$$

where the feasible set, $\mathscr{F}$, is the restriction that each measurement at a given time can be used only once, i.e.,

$$\mathscr{F} = \{H_{m_k}(k):\ H_{i_k}(k) \cap H_{j_k}(k) = \phi \text{ for } i \neq j\} \qquad (6.6)$$

The computational requirement of the above method is clearly non-trivial. In fact, the above optimization problem defines a <u>N-dimensional</u> <u>assignment</u> problem. To the best of the authors' knowledge, the N-dimensional extension of the Hungarian algorithm is not yet available. In many applications, one may be able to pre-cluster the detections so that the search over the entire set of detections is not necessary. Other physical constraints can sometimes be imposed to reduce the search requirements depending upon given systems and application.

A similar approach using a maximum likelihood method

*A more parametric approach for modelling this probability density function is given in Refs. [40],[42] and [46] in which situations including a priori traget distribution and the probability of a given number of detections were also considered.

was described in [42] in which the multidimension search problem was reduced to a 0-1 integer programming problem. Discussions on other track initiation techniques can be found in [41], [46],[48].

6.2 Track Maintenance

Once track files have been established, the computational requirement is greatly reduced. This is because for each track file one is only required to search the "admissible" region dictated by the covariance of the filter residual process.

We note that a slightly modified method of the track initiation algorithm discussed in 6.1 can be applied to the track maintenance problem. That is, one establishes a new hypothesis for each detection resident in the admissible region. This procedure results in an exponentially growing number of track files. One can inhibit the growing memory and computational requirement by selecting a tree depth and conducting a global search for a set of feasible tracks having the highest hypothesis probabilities (eqs. (6.5), (6.6)). Another approach is to combine a set of "most likely hypothesis" growing out of the same track file using the weighted sum of state estimates with the hypothesis probabilities as weighting factors. This second approach is the basis of the Bayesian tracker presented by Singer et al.[43], [44]. If the depth is equal to one, i.e., one combines all admissible detections at each scan, then one obtains the probabilistic data association filter of Bar-Sahlom and Tse [45]. We emphasize however, that the approaches of [43]-[45] are suitable for tracking in a cluttered environment (see also [47]) and do not directly address the multiple target tracking issue.

The above discussion did not include problems of track termination, imperfect sensor detection, false measurements, etc. Reid [46] provides an interesting paper which discusses extensively features of both track initiation and track maintenance.

We note that the subject of tracking in a dense target environment is of current interest. Although all quoted references present general approaches to this problem each application typically has its own unique features which impose certain restrictions resulting in a variety of modifications. Reference [48] represents such a situation in which practical constraints were considered for tracking algorithm design with a passive sensor.

7. ANALYSIS TECHNIQUES

In system design and trade-off studies involving target tracking, one is often required to evaluate the performance of trajectory estimation (tracking and prediction, etc.) for specific situations. Although Monte Carlo simulation is a frequently used method, it is nevertheless time consuming and costly. For a quick and general performance evaluation particularly in system trade-off studies covariance analysis techniques are most appropriate. The following discussion treats the application of polynomial analysis, the Riccati Equation and the Cramer-Rao bound.

7.1 Polynomial Analysis

Let p_0, v_0, and a_0 denote at t=0, the position, velocity and acceleration, respectively, of a moving object projected along a given coordinate (it may be either range or angles) then its position at an arbitary time t is

$$p(t) = p_0 + v_0 t + \frac{1}{2} a_0 t^2. \tag{7.1}$$

Let $\tilde{p}(t_n)$, n=1,..., N denote a set of noisy measurements of $p(t_n)$. For the purpose of convenience, let the time samples be taken uniformly with spacing T and let the total time interval be centered about t=0. Under these conditions, the estimates of target position, velocity, and acceleration at an arbitrary time t are given by

$$\begin{bmatrix} \hat{p}(t) \\ \hat{v}(t) \\ \hat{a}(t) \end{bmatrix} = \Phi(t) S \underline{y} \tag{7.2}$$

where

$$\Phi(t) = \begin{bmatrix} 1 & t & t^2/2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix} \tag{7.3}$$

$$S = \begin{bmatrix} \frac{3(3N^2-7)}{4(N-2)N(N+2)} & 0 & \frac{-30}{T^2(N-2)N(N+2)} \\ 0 & \frac{12}{T^2(N-1)N(N+1)} & 0 \\ \frac{-30}{T^2(N-2)N(N+2)} & 0 & \frac{720}{T^4(N-2)(N-1)N(N+1)(N+2)} \end{bmatrix} \tag{7.4}$$

$$\underline{y} = \begin{bmatrix} \sum \tilde{p}(t_n) \\ \sum T(n-1 - \frac{(N-1)}{2})\tilde{p}(t_n) \\ \sum T^2(n-1 - \frac{(N-1)}{2})^2 \tilde{p}(t_n)/2 \end{bmatrix} \tag{7.5}$$

T = time between measurements

The covariance of the state estimate (Eq. (7.2)) is

$$P = \sigma^2 \Phi(t) S \; \Phi(t)^T \tag{7.6}$$

where σ^2 is the variance of the measurement noise.

We note that the above result, especially (7.6), is very useful because of its simplicity and applicability to a wide range of problems. Specific application of the above analysis to re-entry vehicle tracking problems and discussions on polynomial analyses are found in [49].

7.2 The Use of the Riccati Equation

The above analysis is straightforward and its calculation can be carried out using desk top (or pocket) calculators. The drawback is that it becomes overly optimistic for long track intervals ((N-1)T). This results since the target dynamic model (Eq. (7.1)) does not include process noise. To circumvent this problem, one may compute the covariance matrix using the Riccati equation (Eq. (2.13)). This will require a computer when nonlinear dynamics and/or measurement equations are involved. It is nevertheless a convenient method since the Monte Carlo simulation is not required.

We note that the use of Riccati equation for nonlinear problems only represents an approximate error analysis. If one choses a sufficiently large process noise covariance, then the result of the Riccati equation is an upper bound to the actual performance, [9]. The tightness of this bound is however, rather difficult to assess.

7.3 The Use of The Cramer-Rao Lower Bound

There exist some situations in which the process noise is negligible. The techniques of Section 7.1 are therefore applicable to a limited degree although they do ignore the coupling between coordinates and assume a linear measurement system. When the coupling between coordinates becomes significant one must use the Riccati equation to compute the covariance. When the process noise term is indeed negligible (e.g., the exo-atmospheric trajectory estimation, Section 5.1), the solution of the Riccati equation using Jacobian matrices evaluated along the true trajectory becomes the Cramer-Rao lower bound on the covariance of the state estimates, [49], [50], [51]. Furthermore, the Riccati equation has a closed form solution. As an example, for the discrete case,

$$P_N = \left[\sum_{n=1}^{N} F_n^T H_n^T R_n^{-1} H_n F_n \right]^{-1} \tag{7.7}$$

$$F_n = \begin{cases} \prod_{n=1}^{N-1} \Phi_j^{-1} & \text{for } n \leq N-1 \\ I & \text{for } n = N \end{cases} \tag{7.8}$$

where P_N is the Cramer-Rao bound for the covariance of $\hat{\underline{x}}_{N/N}$, I is an identity matrix and Φ_n and H_n are system (discrete) and measurement Jacobian matrices evaluated at x_n, respectively.

The significance of the above fact is that the actual performance is lower bounded by the solution of the Riccati equation. An extended Kalman filter generally does not perform as well while a properly designed filter can asymptotically achieve this bound, see [30], [57].

Finally we remark that the corresponding Cramer-Rao bound for systems with process noise is very difficult to compute. Versions of lower bounds to the Cramer-Rao bound have been proposed [51], [52], these bounds are tight only for very large signal to noise ratios.

8. CONCLUDING REMARKS

In this report, we have presented a survey of problems and solutions which deal with target tracking. Although this problem has been of active concern to practitioners in both military and civilian applications for many years, new problems still emerge and challenge systems engineers. It is felt that the basic approach to tracking filter design is no longer an issue. However, the problem of integrating the tracking system into the overall command, control, and communication structure to achieve improved performance while minimizing data processing requirements represent the nature of problems that require current attention.

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In this report, we present a survey of problems and solutions in the area of target tracking. The discussion includes design trade-offs, performance evaluation, and current issues.

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