APPROXIMATE METHOD FOR PREDICTING SUPERSONIC NORMAL FORCE

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APPROXIMATE METHOD FOR PREDICTING SUPersonic NORMAL FORCE COEFFICIENT OF VERY-LOW-ASPECT-RATIO LIFTING SURFACES

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ABSTRACT

A simple, empirical method has been developed for predicting at supersonic speeds the normal force coefficient, \( C_{N1} \) (including carryover) of very-low-aspect ratio lifting surfaces mounted on bodies of revolution. Predicted values of \( C_{N} \) using this method are shown to be in good agreement with test data obtained on both thick and thin surfaces, at Mach numbers from about 2.5 to 7.7 and angles of attack to 24°.

SYMBOLS AND NOMENCLATURE

\( A_{c}, A_{I}, A_{o} \) cross-sectional areas of the forebodies of the inlets, the inlets, and the freestream tube captured by the inlets of ramjet missiles, respectively

\( A.R. \) aspect ratio = \( b^2/S_E \) or \( b^2/S_W \)

\( b/2 \) exposed semi-span of a lifting surface mounted on a body of revolution

\( C_{D_c} \) cross-flow drag coefficient

\( C_{N} \) normal force coefficient, normal force/qS

\( C_{N\alpha} \) \( \partial C_N/\partial \alpha \)

\( \Delta C_{N_E}, \Delta C_{N_W} \) \( C_{N_E} - C_{N_B}, C_{N_W} - C_{N_B} \) at \( \alpha = 0^\circ \)

\( r \) root chord

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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d)</td>
<td>reference diameter; diameter of body on which lifting surfaces are mounted</td>
<td>in</td>
</tr>
<tr>
<td>(E)</td>
<td>complete elliptic integral of second kind with modulus ((1 - \beta^2 \cot^2 \Lambda)^\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(K_B, K_W)</td>
<td>Morikawa's interference factors</td>
<td></td>
</tr>
<tr>
<td>(M)</td>
<td>Mach number</td>
<td></td>
</tr>
<tr>
<td>(q)</td>
<td>dynamic pressure</td>
<td>lbs/in(^2)</td>
</tr>
<tr>
<td>(S)</td>
<td>reference area, (\pi d^2/4)</td>
<td>in(^2)</td>
</tr>
<tr>
<td>(S_E, S_W)</td>
<td>total planform area of housings (wings)</td>
<td>in(^2)</td>
</tr>
<tr>
<td>(t)</td>
<td>average thickness of lifting surface</td>
<td>in</td>
</tr>
<tr>
<td>(X)</td>
<td>body station; (X = 0) at nose tip of body</td>
<td>in</td>
</tr>
<tr>
<td>(X_{c.p.})</td>
<td>center-of-pressure location</td>
<td>in</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>angle of attack; angle between the velocity vector and the longitudinal axis of the body</td>
<td>deg</td>
</tr>
<tr>
<td>(\beta)</td>
<td>(\sqrt{M^2 - 1})</td>
<td></td>
</tr>
<tr>
<td>(\gamma)</td>
<td>ratio of specific heats; (\gamma = 1.4) used herein</td>
<td></td>
</tr>
<tr>
<td>(\Lambda)</td>
<td>leading edge sweep angle for delta wings</td>
<td>deg</td>
</tr>
<tr>
<td>(\varnothing)</td>
<td>aerodynamic roll angle; at (\varnothing = 0^\circ) the lifting surfaces are normal to the plane of (\alpha)</td>
<td>deg</td>
</tr>
</tbody>
</table>

**Subscripts**

- \(B\): body alone
- \(BE\): body-housing combination
- \(BW\): body-wing combination
- \(E\): housing
- \(I\): refers to inlet forebody and internal lift as in \(\Delta C_{N_1}\)
- \(W\): wing
INTRODUCTION

The requirement for compactness in U. S. Navy missile designs results frequently in configurations which incorporate thick lifting surfaces of very-low-aspect ratio. These surfaces are invariably thick, e.g., Figures 1 and 2, because they are used to house electronics and hydraulics or serve as ducts, as in the case of side-mounted inlets on ramjet missiles. Current requirements on missile speed have increased to regions where guidance for making aerodynamic estimates for these surfaces is not available, either from theory or experiment.

Empirical estimation of the normal force coefficient, $C_N$, and center-of-pressure location, $X_{c.p.}$, for these surfaces is difficult because the shapes are usually unique for each new missile design and, therefore, the limited test data available are invariably for shapes that are quite different from the proposed shape in a new missile design. Existing empirical methods\(^1\) have been derived for a specific class of surfaces and apply to the lower end of the Mach number range of interest in this presentation.

Simple theoretical methods that have been used (with limited success) do not take into account the effects of Mach number. Those that do, are not applicable at the very low values of aspect ratio inherent to these types of surfaces. These concerns have been expressed for some time.\(^2\)\(^3\)

A need exists, therefore, for either an empirical data base for a more general class of low-aspect-ratio lifting surfaces or a simple predictive method that is adequate in preliminary design for predicting $C_N$ and $X_{c.p.}$ of this class of surfaces in speed ranges from moderate supersonic to hypersonic.

A simple, empirical predictive method for estimating $C_N$ for very-low-aspect ratio surfaces is presented herein. It is shown to provide estimates of $C_N$ that are adequate for preliminary design for a variety of thicknesses and shapes and a wide range of Mach numbers ($M \approx 2.5$ to 7.7) and angles of attack ($\alpha$ to $24^\circ$).

OBJECTIVE

The objective of this study was to determine a simple method for estimating in preliminary design the aerodynamic normal force coefficient of very-low-aspect ratio lifting surfaces (and body-wing cer at moderate supersonic to hypersonic speeds and to moderate angles-of-attack.
METHODOLOGY

A. BACKGROUND

The method presented herein for predicting $C_N$ of very-low-aspect-ratio lifting surfaces evolved from observations of the experimental lifting characteristics of thick surfaces such as those depicted by the housings on the wind tunnel model shown in Figure 1. This model is representative of an Integral Rocket-Ramjet (IRR) missile. It was tested by APL/JHU in order to compile aerodynamic design information for components of this class of configurations since empirical methods for predicting $C_N$ and $X_{c.p.}$ for this type of configuration and combinations of components were not available. Hart's empirical curves had been shown to provide good predictions for low-aspect-ratio surfaces at $M \approx 3.0$, but these curves had been derived mostly for wings that were primarily thin surfaces. The applicability of this method to thick surfaces and to higher Mach numbers was therefore not known.

Simple theoretical methods that account for the Mach number variation of $\Delta C_N$ noted from test data are not applicable at the very-low-aspect ratios of interest herein; those derived for aspect ratios approaching zero do not account for the Mach number effects. This is demonstrated in Figure 3 wherein the test values of $\Delta C_N$ of the IRR E housings ($\Delta C_N$ minus internal momentum) are compared with two simple theoretical methods, viz: modified Newtonian theory, plus wing-body carryover, i.e.,

$$\Delta C_{NW} = \frac{\gamma + 3}{\gamma + 1} \left(1 - \frac{2}{\gamma + 3} \frac{1}{M^2}\right) \left(K_W + K_B\right) \frac{S_E}{S} \sin^2 \alpha$$

and slender wing theory plus cross flow as recommended by Flax and Lawrence, i.e.,

$$\Delta C_{NW} = \left[\frac{\pi}{2 \times 57.3} \alpha + C_{Dc} \sin^2 \alpha \right] \left(K_W + K_B\right) \frac{S_E}{S}$$

The value of $C_{Dc} = 1.0$ was used in these calculations following the recommendation of Flax and Lawrence for the case of rounded tips. This number, however, could be something other than 1.0 according to Hoerner. The Morikawa carryover factors' were used in Equations 1 and 2, and are used throughout this analysis.

B. APPROACH

Test data obtained on both thick and thin wings in various APL/JHU aerodynamic research and exploratory development programs were the primary source of data for the development of the empirical method presented herein. Selected
NASA data were also used. Sketches of the housing and wing configurations used in the analysis are given in Figures 4, 5, and 6; the sources for the test data\(^8-18\) are noted in the figures for each configuration.

In all cases, the wing (or housing) data were obtained from tests conducted with cruciform wing-body and with body alone configurations. The wing-body was roll oriented at \(\theta = 0^\circ\), i.e., one pair of wings in the angle-of-attack plane. The test data then are derived from \(\Delta C_{N_w} = C_{N_{BW}} - C_{N_B}\), and thus wing-body carryover is included in the wing lift.

The general approach in deriving and evaluating the present method using the test data discussed above, is:

1. Values of \(\beta C_{N_w}^{\alpha_w}\) were extracted from test data obtained on the wing configurations sketched in Figures 4, 5, and 6.

2. Correlation curves of \(\beta C_{N_w}^{\alpha_w}\) were deduced from the test values as follows:

   \[
   \beta C_{N_w}^{\alpha_w} = F (\beta \text{ A.R.}) \quad \text{for rectangular wings,}
   \]

   \[
   \beta C_{N_w}^{\alpha_w} = F (\beta \cot \Lambda) \quad \text{for delta wings, and}
   \]

   \[
   \beta C_{N_w}^{\alpha_w} = \text{constant for thick wings.}
   \]

A comparison of the derived curves with appropriate linear and slender wing theories is given.

3. These empirically derived curves were then used to calculate the values of \(\Delta C_{N_w}^{\alpha_w}\) for the 29 Mach number-configurational combinations used in the analysis. Comparisons with test data are given to demonstrate the adequacy of the present method.

RESULTS

A. PROCEDURE FOR EXTRACTING \(\beta C_{N_w}^{\alpha_w}\) FROM TEST DATA

Values of \(\beta C_{N_w}^{\alpha_w}\) that provide a good representation of the test data in the range of angle of attack tested were derived by first linearizing \(\Delta C_{N_w}^{\alpha_w}\) vs \(\alpha\), as demonstrated in Figure 7 and then extracting \(C_{N_w}^{\alpha_w}\) from the linearized values.
where \( \Delta C_{N_W} = C_{N_{BW}} - C_{N_B} \) and this includes mutual body-wing carryover. The carryover factors \( K_W \) and \( K_B \) were obtained from Morikawa's charts, Reference 7; Morikawa's values of \( K_W \) for rectangular wings were used for the configurations that are nearly rectangular. In the linearization of \( \Delta C_{N_W} \) vs \( \alpha \), more emphasis was given to obtaining a representation of \( \Delta C_{N_W} \) at the moderate to higher values of \( \alpha \) than at the lower values according to the objective of this investigation.

The \( E_1 \), \( E_2 \), and \( E_3 \) configurations of Figure 4 have flow through the inlet-duct system and thus \( \Delta C_{N_E} \) for these configurations include internal lift. The lift attributed to the inlet forebody and internal momentum was subtracted from the total lift of these housings in order to obtain \( \Delta C_{N_W} \) since we are only interested in the external lift. Thus, for these configurations,

\[
\Delta C_{N_W} = \Delta C_{N_E} - \Delta C_{N_I} = \Delta C_{N_E} - 2 \left( \frac{A_Q}{A_I} \frac{A_I}{S} + \frac{A_C}{S} \right) \sin \alpha
\]

A value of \( A_Q/A_I = 1.0 \) was used in these calculations since the internal shock was not expelled for the cases considered. \( A_I \) and \( A_C \) are the combined cross sectional areas of the inlets and inlet forebodies, respectively.

Finally, the derived slopes were expressed in the usual functional forms found in design charts, i.e.,

\[
\beta C_N = F (\beta \text{ A.R.}) \quad \alpha_W
\]

for rectangular wings, and

\[
\beta C_N = F (\beta \cot \alpha) \quad \alpha_W
\]

for delta wings.
B. CORRELATION CURVES OF $\beta C_{N_{\alpha_{w}}}$

1. Nearly-Rectangular Wings

The "best fit" values of $\beta C_{N_{\alpha_{w}}}$ deduced from the test data on the nearly-rectangular housings and wings of Figures 4 and 5 are plotted in Figure 8 as a function of $1/\beta$ A.R. For comparison, the values of $\beta C_{N_{\alpha_{w}}}$ predicted from linear and slender wing theories, Reference 5, for rectangular wings, are also shown in Figure 8, i.e.,

$$\beta C_{N_{\alpha_{w}}} = 4 (1 - \frac{1}{2 \beta \text{ A.R.}})$$
for $\beta \text{ A.R.} > 1$

$$\beta C_{N_{\alpha_{w}}} = \frac{4}{\pi} \left[ (2 - \frac{1}{\beta \text{ A.R.}}) \sin^{-1} \beta \text{ A.R.} + (\beta \text{ A.R.} - 2) \cosh^{-1} \frac{1}{\beta \text{ A.R.}} \right]$$
$$\quad + \left( 1 + \frac{1}{\beta \text{ A.R.}} \right) \sqrt{1 - (\beta \text{ A.R.})^2}$$
for $\frac{1}{2} < \beta \text{ A.R.} < 1$

and,

$$\beta C_{N_{\alpha_{w}}} = \frac{\pi}{2} (\beta \text{ A.R.})$$
for $\beta \text{ A.R.} < 1/2$

(Slender Wing)

It is seen, from Figure 8, that the difference between experiment and theory (given by these simple methods) is very large for $\beta \text{ A.R.} \approx 0.67$ [(1/\beta \text{ A.R.}) \approx 1.5].

Note that the theoretical values of $C_{N_{\alpha}}$ are lift curve slopes at $\alpha = 0^\circ$ whereas the test values are the mean values of $C_{N}/\alpha$ obtained from the full range of $\alpha$ tested. For the test cases where $C_{N}$ was linear with $\alpha (M \gtrsim 3.0)$, these two values should be the same. These theoretical methods are usually recommended in various handbooks and textbooks because of their success in predicting $C_{N_{\alpha_{w}}}$ at low values of $\alpha$. Their success has been demonstrated by several investigators at the low values of $\alpha$ and at low supersonic Mach numbers. The inadequacy of these theoretical methods for predicting $\Delta C_{N_{w}}$
without adding a non-linear term, such as cross-flow lift, was demonstrated by Flax and Lawrence\(^3\) in 1951. Cross-flow lift for wings is a concept, taken from cross-flow lift on cylinders, which attempts to account for the vortex lift. The cross-flow drag value used in determining cross-flow lift is basically an experimental value obtained for a limited class of wings.\(^3\,6\) More recent approaches use the concept of leading-edge and side-edge suction\(^1\,7\,18\) to account for non-linear lift. As far as can be established from the literature this approach is not applicable to the wing geometries of interest in this study.

Returning to the discussion of Figure 8, it is noted that the test values of \(\beta C_N\) for thick housings is generally lower than those for the "thin" wings. A separate R.M.S. curve for the thin wings demonstrates this. The value of \(\beta C_N = 4/3\) marked on the ordinate of Figure 8 will be shown later to provide a reasonable agreement with the majority of test values of \(\Delta C_N\) for the thick wings used in this study, 12 Mach number-configurational combinations. The solid points shown in Figure 8 are for test cases where \(M \leq 3.0\). In this region \(\Delta C_N\) is very non-linear with \(\alpha\) at low values of \(\alpha\). For these cases, it will be shown later that Hart's empirical method\(^1\) provides good predictions at the lower values of \(\alpha\) and for some cases at all values of \(\alpha\) tested.

2. Thin Delta Wings

A similar correlation plot of \(\beta C_N\) for the test data for delta wings is given in Figure 9 and is compared with linear theory for these wings. In this case \(\beta C_N\) is given as a function of \(\beta \cot \Lambda\) and plotted vs. \(1/\beta \cot \Lambda\).

The disagreement with linear theory is obvious. Note specifically that the test values of \(\beta C_N\) do not tend to 4 at \(\beta \cot \Lambda = 1\) as predicted by linear theory but rather they tend to 4 at \(\beta \cot \Lambda = \infty\) which is in agreement with predictions for rectangular wings.

3. Combined Correlation Curve for Very-Low-Aspect-Ratio Wings

A comparison of the R.M.S. curve of \(\beta C_N = F (1/\beta A.R.)\) for thin nearly-rectangular wings (Figure 8) with the R.M.S. curve of \(\beta C_N = F (1/\beta \cot \Lambda)\) for thin delta wings (Figure 9) shows that the two curves are essentially the same. Thus, one single curve is proposed for predicting \(\beta C_N\) for both thin rectangular (or nearly rectangular) and for delta wings. The curve has the same functional form for \(\beta C_N\) as shown in Figure 10. For

I-156
the thick wings, $\beta C_{N_{\alpha_w}} = 4/3$ is proposed for $(1/\beta \text{ A.R.}) \geq 1.5$. Data were
not found for thick surfaces for the region $(1/\beta \text{ A.R.}) \geq 1.5$ to determine the
trend of $\beta C_{N_{\alpha_w}}$ for this region. The effect of wing thickness for ratios,$\t/d$, between 0.2 and 0.1 also is not known; the thick wings used in the
analyses had $t/d \geq 0.2$; the average "thickness" for the thin wings used was$t/d \lesssim 0.1$.

In summary, the correlation curves of Figure 10 are proposed as a simple
empirical method for obtaining $\beta C_{N_{\alpha_w}}$ for very-low-aspect ratio wings. Since
in practice these surfaces are usually mounted on a body of revolution the
mutual body-wing interference should also be accounted for. Morikawa's
factors are recommended for accounting for this interference mainly because
they were used in deriving $\beta C_{N_{\alpha_w}}$ from test data. The adequacy of the pro-
posed method for providing good engineering estimates of $\Delta C_{N_{\alpha_w}} = C_{N_{\alpha_w}} - C_{N_B}$ at
$M \geq 2.5$ and $\alpha$ to about $24^\circ$ is demonstrated in the next section.

C. COMPARISON OF TEST VALUES OF $\Delta C_{N_{\alpha_w}}$ WITH EMPIRICAL PREDICTIONS
USING THE PRESENT METHOD

The predicted values of $\Delta C_{N_{\alpha_w}}$ are derived from the empirical curves of
Figure 10 as follows:

$$\Delta C_{N_{\alpha_w}} = \frac{\beta C_{N_{\alpha_w}}}{57.3} \left( K_{\omega} + K_{B} \right) \frac{S_{\omega}}{S} \alpha$$  \hspace{1cm} (6)

where $\beta C_{N_{\alpha_w}}$ is per radian and $\alpha$ is in degrees. These values are compared
in Figures 11 to 22 with the test data obtained from $C_{N_{\alpha_w}} - C_{N_B}$ for the 29
Mach number-configurational combinations used in the analysis. Calculated
values of $\Delta C_{N_{\alpha_w}}$ using Hart's method\(^1\) are also shown, for the cases where this
method is applicable, to demonstrate the adequacy of this method.

1. Thick Wings

Calculated values of $\Delta C_{N_{\alpha_w}}$ using the present method, given by Figure 10
and Equation 6, are compared with test data from the thick housings in
Figures 11 through 15. Values of $\Delta C_{N_{\alpha_w}}$ obtained from Hart's empirical corre-
lation curves\(^3\) are also shown. The comparisons show, in general, that the
present method with $\beta c_{N_w} = 4/3$ gives a good to excellent representation of $\alpha_w$
the test values of $\Delta c_N$ to $\alpha = 24^\circ$, $M \geq 2.5$, for the five housing configura-
tions of Figure 4. The predictions of the present method are especially good
at $M \geq 3.0$ where $\Delta c_{N_w}$ is nearly linear with $\alpha$.

At $M \leq 3.0$, the data are very non-linear with $\alpha$ at low angles-of-attack
and Hart's method gives a better prediction than the present method, see
Figures 11 and 14. At the higher Mach numbers the present method provides a
better prediction.

2. Thin Nearly-Rectangular Wings

The results of the evaluation of the present method for thin nearly-
rectangular wings are given in Figures 16 through 19. The test data are for
the configurations of Figure 5. The comparisons again show that the present
method provides good predictions. Hart's method also gives good predictions
in the region of applicability of his method, $\beta$ A.R. $\leq 0.8$, but this method
is not better than the present method.

3. Thin, Delta Wings

The present method provides excellent predictions for the test data for
the delta wings of Figure 6, Figures 20, 21 and 22. Hart's method was not
derived for delta wings and thus a comparison with this method is not made
for these wings.

CONCLUSIONS

An empirical method is derived herein for estimating the normal force
coefficient (plus wing-body carryover), $\Delta c_{N_w}$, of nearly-rectangular thick and
thin wings, and of thin delta wings, of very-low-aspect ratio. The method,
in combination with Morikawa's interference factors, gives good predictions
in the range of Mach numbers from 2.5 to 7.7 and angles of attack to 24°. For
near-rectangular wings at $M \geq 3.0$, Hart's empirical correlation curves of
Reference 1 are recommended for estimating $\Delta c_{N_w}$. 
REFERENCES


9. General Dynamics/Convair High Speed Wind Tunnel Test Report HST 258-0, 3-5 May 1968.


Fig. 2 Photo showing one type of very-low-aspect ratio, thick wings used in analysis.
Test data

- M = 2.0, \( \beta \text{A.R.} = 0.135 \)
- M = 2.5, \( \beta \text{A.R.} = 0.179 \)
- M = 3.5, \( \beta \text{A.R.} = 0.262 \)
- M = 4.5, \( \beta \text{A.R.} = 0.342 \)

Theory
- Slender wing plus cross flow
- Modified Newtonian

Fig. 3  Comparison of \( C_N \) data from IRR E₁ housing with two simple predictive methods \( \phi = 0^\circ \).
<table>
<thead>
<tr>
<th>Configuration designation</th>
<th>Max. cross section</th>
<th>Planform</th>
<th>A.R. (two wings)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRR E₁ ref. 8</td>
<td>0.363</td>
<td>0 2.09  7.04 11.63 13.53</td>
<td>0.078/8.5</td>
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<tr>
<td>E₂ ref. 9</td>
<td>0.407</td>
<td>0 5.10  13.78</td>
<td>0.096/8.75</td>
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<tr>
<td>E₃ ref. 10</td>
<td>0.438</td>
<td>0 11.28  17.15</td>
<td>0.150/6.541</td>
</tr>
<tr>
<td>W₁ ref. 11</td>
<td>0.219</td>
<td>0 2.44  11.56</td>
<td>0.044/5.534</td>
</tr>
<tr>
<td>W₂ ref. 12</td>
<td>0.250</td>
<td>0 1.68  8.90 10.00</td>
<td>0.075/4.234</td>
</tr>
</tbody>
</table>

Dimensions in body diameters

Fig. 4 Sketches of low A.R. wings (housings) used in analysis.
Fig. 5 Thin, low-aspect-ratio, nearly-rectangular wings.
Ref. 13

\[ \frac{W_x}{b/d} \quad \text{A.R.} \quad \frac{S_{w/S}}{d} \]

\[
\begin{array}{ccc}
8 & 0.33 & 85.6^\circ & 0.31 & 1.83 \\
9 & 0.67 & 81.3^\circ & 0.61 & 3.69 \\
10 & 1.33 & 72.9^\circ & 1.23 & 7.35 \\
\end{array}
\]

Ref. 16

\[ \frac{W_x}{b/2/d} \quad \text{A.R.} \quad \frac{S_{w/S}}{d} \]

\[
\begin{array}{ccc}
11 & 0.088 & 0.088 & 0.453 \\
12 & 0.206 & 0.204 & 1.057 \\
13 & 0.706 & 0.710 & 3.61 \\
\end{array}
\]

Dimensions in body diameters

Fig. 6 Thin, low-aspect-ratio, delta wings.
Let $\Delta C_{NW}$ be approximated, in range of $\alpha$ tested, by

$$\Delta C_{NW} = C_{Na} (K_W + K_B) (S_W/S) \alpha / 57.3$$

Then,

$$\beta C_{Na} = \frac{\Delta C_{NW}}{\alpha} \left( \frac{57.3 \beta}{(K_W + K_B) (S_W/S)} \right)$$

![Graph showing linear representation of test data to derive $\beta C_{Na}$](image)

**Fig. 7** Procedure used for linearization of test data to derive $\beta C_{Na}$
Fig. 8 Correlation of test data on nearly-rectangular wings with $\beta$ A.R.

Fig. 9 Correlation of test data on thin delta wings with $\beta \cot \alpha$. 

"Best" linear fit of test values of $\Delta C_{N_W}$ vs $\alpha$
- Thick wings
- Thin wings
- Data for M<3.0 is non-linear at low $\alpha$

R.M.S. of data on both thick and thin wings

Conical flow
$1 > \beta$ A.R. $> 1/2$

Slender wing, $1/2 > \beta$ A.R. $> 0$
Fig. 10 Proposed design charts for $\frac{\beta C_{N_a}}{\phi}$ of low-aspect-ratio wings $M \approx 2$, $\phi = 0^\circ$. 

Thin wings, $(t/d \lesssim 0.1)$

Thick wings, $(t/d \gtrsim 0.2)$
Fig. 11  Comparison of test and predicted values of $\Delta C_{N_w}$ of $E_1, \phi = 0^\circ, t/d = 0.305$. 
Fig. 12 Comparison of test and predicted values of $\Delta C_{NW}$ of $E_2$, $\phi = 0^\circ$, t/d = 0.407.
Fig. 13 Comparison of test and predicted values of $\Delta C_{N,W}$ of $E_3$, $M = 4.17$; $\phi = 0^\circ$, $\beta A.R. = 0.607$, $t/d = 0.312$. 

The graph shows the comparison between test data and predicted values of $\Delta C_{N,W}$ with the angle of attack, $a$, in degrees. The test data is represented by circles, the present method by a line, and Hart, ref. 1 by an X.
Fig. 14  Comparison of test and predicted values of $\Delta C_{N_W}$ of $W_1$, $\phi = 0^\circ$, $t/d = 0.352$ Ave.
Fig. 15  Comparison of test and predicted values of $\Delta C_{NW}$ of $W_{2}$, $\phi = 0^\circ$, $t/d = 0.200$. 

$I-174$
Fig. 16  Comparison of test and predicted values of $\Delta C_{NW}$ for thin rectangular wings $M = 2.96, \phi = 0^\circ$. 
Fig. 17 Comparison of test and predicted values of $\Delta C_{NW}$ for thin rectangular wings $M = 4.63$, $\phi = 0^\circ$. 
Fig. 18  Comparison of test and predicted values of $\Delta C_{N,w}$ for $W_6, M = 2.5, \phi = 0^\circ, \beta A.R. = 0.275.$

Fig. 19  Comparison of test and predicted values of $\Delta C_{N,w}$ for $W_7, M = 4.02, \phi = 0^\circ, \beta A.R. = 0.498.$
Fig. 20 Comparison of test and predicted values of $\Delta C_{NW}$ for thin delta wings, $M = 2.96$, $\phi = 0^\circ$. 

\[ W_8 \quad \beta \cot \theta = 0.214 \]

\[ W_9 \quad \beta \cot \theta = 0.426 \]

\[ W_{10} \quad \beta \cot \theta = 0.857 \]
Fig. 21  Comparison of test and predicted values of $\Delta C_{NW}$ for thin delta wings, $M = 4.63, \phi = 0^\circ$. 
Fig. 22  Comparison of test and predicted values of $\Delta C_{NW}$ of thin delta wings, $M = 4.37, \phi = 0^\circ$. 

### Graphs

- **$W_{11}$**
  - $\beta \cot \lambda = 0.092$
  - Test data
  - Present method

- **$W_{12}$**
  - $\beta \cot \lambda = 0.215$
  - Test data
  - Present method

- **$W_{13}$**
  - $\beta \cot \lambda = 0.737$
  - Test data
  - Present method

---

**Angle of attack, $\alpha$ (degrees)**