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EVALUATION OF VARIANCE APPROXIMATIONS AND DEMAND FORECASTING TECHNIQUES

OPERATIONS ANALYSIS DEPARTMENT

NAVY FLEET MATERIAL SUPPORT OFFICE

Mechanicsburg, Pennsylvania 17055

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Report 147

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Evaluation of Variance Approximations and Demand Forecasting Techniques

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Report 147

Submitted: K. L. Hoff

K. L. HOFF
Operations Research Analyst

Approved: R. A. Lippert

R. A. LIPPERT, CDR, SC, USN
Director, Operations
Analysis Department

J. B. Whittaker

J. B. WHITTAKER, CAPT, SC, USN
Commanding Officer, Navy
Fleet Material Support Office

Date DEC 31 1981

Abstract

ALRAND Working Memorandum 365 of 26 September 1980 recommended two potential improvements to the current UICP (Uniform Inventory Control Program) demand forecasting techniques: (1) a more direct approximation for the variance of quarterly demand and (2) adaptive smoothing to forecast demand. This study used the 5A (Aviation Afloat and Ashore Allowance Analyzer) wholesale inventory simulator and actual demand observations to compare the suggested alternatives to the current method. The following criteria were used in the comparison: inventory investment, performance, workload, demand forecast accuracy and the required computer time. However, the primary criterion used was the change in performance per dollar invested. The study showed that (1) the more direct approximation of standard deviation of demand is not an improvement and (2) adaptive smoothing with filtering should be considered for implementation.

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EXECUTIVE SUMMARY

1. Background. ALRAND Working Memorandum 365 identified two potential improvements to the current UICP (Uniform Inventory Control Program) demand forecasting techniques. The first, a more direct approximation of the variance of quarterly demand (σ_D^2), would eliminate the current UICP practice of computing a MAD (Mean Absolute Deviation) and approximating the variance with the expression 1.57 X $(MAD)^2$. The second proposal, using adaptive smoothing to forecast the quarterly demand, would provide a more flexible and reactive smoothing weight determination than the current trend test.

2. Objective. To determine whether a more direct approximation of the variance of quarterly demand and adaptive smoothing will improve current UICP demand forecasting techniques.

3. Approach. The 5A (Aviation Afloat and Ashore Allowance Analyzer) wholesale inventory simulator and actual demand observations were used to determine the effect of using these two proposals. Simulations were run using both the current approximation of the variance of quarterly demand and the proposed approximation to determine if the proposed approximation would improve the variance forecast. To determine the best parameter values for the adaptive smoothing and moving average methods of demand forecasting, a sensitivity analysis was performed. Then, the adaptive smoothing and moving average methods were compared to the current method with the parameter values recommended by FMSO Report 146 to determine the best of these methods for demand forecasting. For both the variance comparison and the demand forecasting analysis, each method was evaluated by the four criteria of inventory investment, performance, workload and demand forecast accuracy. However, the primary criterion used was the change in performance per dollar invested. In addition, the computer time required for each method was considered.

4. Findings. Under the designated criteria, the proposed more direct approximation of the variance of quarterly demand shows no significant improvement over the current approximation and requires more computer time. Forecasting demand using both adaptive smoothing and the current method with recommended parameter values produces better results than using the moving average method. Adaptive smoothing appears to produce the best performance but at a higher cost. However, the changes in investment and performance are not statistically different when compared to the current method with recommended parameters. There is no significant difference in computer time between the two methods. Using the recommended parameter values requires a change of input values while adaptive smoothing would require changes to the UICP program. However, the current method requires periodic updating of the smoothing weights while under adaptive smoothing, the smoothing weight adapts to changes in demand and requires no manual intervention.

5. Conclusions. The proposed approximation of the variance of quarterly demand showed no significant improvement over the current approximation. In addition, the more direct approximation used more computer time. Therefore, the current approximation of the variance should be retained.

Both adaptive smoothing and the current method with recommended parameter values were better than the moving average method of demand forecasting. Adaptive smoothing was as effective as the current procedures. Programming changes would be necessary in order to use adaptive smoothing but the smoothing weight parameter values do not require the periodic re-evaluation by management that the current method requires. In conclusion, adaptive smoothing with filtering should be considered for implementation.

I. INTRODUCTION

Reference (1) identified two potential improvements to current UICP (Uniform Inventory Control Program) demand forecasting techniques. The first, a more direct approximation of the variance of quarterly demand (σ_D^2), would replace the current UICP method which approximates σ_D^2 with 1.57 $(MAD)^2$, where MAD is the Mean Absolute Deviation of demand. The second, adaptive smoothing, would replace the current parameter driven smoothing technique used to compute the demand forecast.

The method of calculating σ_D^2 is significant since σ_D^2 has two important uses in the UICP. First, σ_D^2 is one of several terms used to compute the procurement problem variance (DEN (Data Element Number) B019A) which is the variance in the attrition demand during leadtime plus a repair pipeline. Since the procurement problem variance is the basis of the safety level, using a more direct approximation to σ_D^2 should result in a more accurate safety level and a more responsive supply system per dollar invested. The second use of σ_D^2 is in demand forecasting. The square root of σ_D^2 (the standard deviation of demand, σ_D) is used, along with the mean demand, to establish a tolerance band around the mean. The tolerance band is expressed as MEAN $\pm$ $X\sigma_D$, where X is the number of standard deviations from the mean chosen to determine an acceptable range of quarterly demand observations. Extremely high or low observations which are outside of the tolerance band are then filtered out of the forecasting process.

Currently, the MAD for nonprogram-related items is computed by exponentially averaging the absolute value of the forecast error, where the forecast error is the difference between the observed and the previously forecasted quarterly demand. (Nonprogram-related items are those items with stock levels based solely on demand history, unlike program-related items which are based on both demand history and planned usage.) After the MAD of demand is computed, the variance

is approximated by 1.57 $(\text{MAD})^2$ and the standard deviation is approximated by 1.25 MAD. The approximations are based on the assumption that the forecasting errors are normally distributed. A more direct approach would be to exponentially average the square of the forecast errors as an approximation of the variance. Then, the variance of demand could be stored and available for use in computing the safety level without further modification. The standard deviation could be easily computed for use in demand forecasting. The proposed procedures to compute and store the variance of demand may require less computer time than the current method of computing and storing the MAD of demand and then approximating both σ_D and σ_D^2 as needed.

In addition to evaluating a more direct approximation for the variance of demand, the study also compared two alternative demand forecasting techniques: adaptive smoothing and moving average. The current method of demand forecasting (see reference (2) for a more complete explanation) consists of a sequence of three stages: (1) the filter check, (2) the trend test and (3) exponential smoothing. The first stage of demand forecasting is used to screen out "abnormal" demand observations. The second stage of demand forecasting, the trend test, detects steady increases or decreases in demand observations. The degree of change which must occur before the system recognizes a trend depends on two parameters which form a lower and upper bound. The outcome of the trend test determines what smoothing weight is applied in exponential smoothing. In the third stage of demand forecasting, exponential smoothing, different smoothing weights are applied to trending and nontrending demand observations.

An alternate approach to forecasting demand for nonstationary demand processes is to automatically adapt the weighting factor used in exponential smoothing to changes in level of demand or trends. This approach, known as adaptive smoothing,

reacts to changes in the demand pattern by automatically applying a higher smoothing weight when a change in the demand pattern occurs. Changes in the demand pattern are detected by tracking the forecast error. Higher forecast errors indicate that the forecasts and observations are diverging rather than converging as desired. Then a higher smoothing weight is used to place more emphasis on the more recent observations. Thus, adaptive smoothing continually obtains feedback concerning forecast accuracy and adjusts the smoothing weight accordingly. Unlike adpative smoothing, the current demand forecasting method requires management to periodically reevaluate the smoothing weights and the trend limits and adjust these parameter values if necessary.

The second alternate demand forecasting technique evaluated was moving average. The moving average method involves selecting an N (the number of observations to be included in the calculation of a moving average) and then calculating the mean demand for the latest N quarterly demand observations. As a new observation appears, the oldest observation is omitted from the mean calculation and replaced by the most recent. A smoothing weight is not required but N must be determined and periodically reviewed by management (since N does determine how much weight is applied to each demand observation) and adjusted if necessary.

II. TECHNICAL APPROACH

The 5A (Aviation Afloat and Ashore Allowance Analyzer) wholesale inventory simulator and actual demand observations were used to evaluate the differences in methods of approximating variance and forecasting demand. Specifically, the analysis was divided into four parts:

- A comparison of the proposed and the current approximations of variance.

- A sensitivity analysis of possible values for the smoothing weight used in adaptive smoothing.
- A sensitivity analysis of the base number of quarters for moving average computations.
- A comparison of the current method with current parameter values, the current method with parameter values recommended by reference (2) and the adaptive smoothing and moving average techniques.

A. SIMULATION MODEL. The 5A wholesale simulator, as described in references (2) and (3), replicates the inventory management operations of both ASO (Navy Aviation Supply Office) and SPCC (Navy Ships Parts Control Center). The demand forecasting routine of the 5A simulator, which calculates a quarterly demand forecast, includes the current sequence of filter check, trend test and exponential smoothing. The routine was modified to include the proposed methods of variance estimation and demand forecasting as discussed below in Sections D and E.

B. INPUT DATA. Information from two data bases, the THF (Transaction History File) and the SIG (Selective Item Generator) file, was combined to create the input to the 5A simulator. Actual demands were obtained from the THF while the majority of item information (e.g., leadtime, turn-around-time and unit price) was obtained from the SIG file. The SIG file provides a snapshot of the MDF (Master Data File). Six years (January 1974 through December 1979) of THF demand data was used for SPCC-managed material and four years (November 1975 through October 1979) of THF demand data was used for ASO-managed material.

The data was segmented into 1H, 2H, 1R and 2R cogs (cognizance symbols) and then cogs 1H, 2H and 1R were further divided into several systematic

random samples (see TABLE I). The division of 1H, 2H and 1R cogs into multiple samples was necessary not only to conserve computer time, but also to make the results more statistically sound. According to Tukey's Plan (see reference (4)), when results of the analysis of smaller samples are combined, the results are more representative of the universe than the results of one large sample. (The samples used in the analysis are the same samples which were used in reference (2).)

The majority of items in the 2R cog are program-related items. The study dealt with nonprogram-related items, which totaled 2,892 for the 2R cog. This total, also referred to as the universe of 2R nonprogram-related items, was small enough that the univepse could be simulated without sampling.

TABLE I
Input Categorization

Cog	# Items Universe	# Samples	# Items Sample I	# Items Sample II	# Items Sample III	# Items Sample IV
1H	125,797	4	1,572	1,572	1,571	1,571
2H	11,458	3	1,636	1,634	1,631	-
1R	103,201	4	1,587	1,587	1,587	1,587
2R	22,137	1	2,892	-	-	-

C. <u>OUTPUT DATA</u>. The simulator tabulates statistics and provides yearly averages of those statistics to evaluate accuracy of the forecast and effectiveness of the particular set of parameter values. The first two years of data were treated as a transition period and were not included in calculations of yearly averages. The following criteria were considered the most relevant in quantifying the effectiveness of the forecast.

1. <u>$OH + $DI - Dollar Value of Material On-Hand plus Dollar Value of Procurements Due-In</u> - dollar value of inventory investment at the end of the simulated year.

2. <u>SMA % - Supply Material Availability</u> - the sum of requisitions satisfied immediately divided by the total number of requisitions submitted. A requisition is considered satisfied only if the entire requisition is satisfied.

3. <u>ADD - Average Days Delay</u> - the time delay experienced by all backordered requisitions divided by the total number of requisitions submitted.

4. <u>#PI - Number of Procurements Initiated</u> - average number of procurement orders placed during a year.

5. <u>$RA - Number of Repair Actions</u> - average number of repair inductions made during a year.

6. <u>TMSE - Total Mean Square Error</u> - a statistic which measures the accuracy of the demand forecast by averaging the square of the forecast error and summing over all the items. The MSE was summed across all items and the total was used to compare forecasting errors.

$$TMSE_j = \sum_{i=1}^{n} \frac{\sum_{k=j+1}^{j+4} (d_{ki} - \overline{D}_{ki})^2}{4}$$

where

n = the number of items in a simulated sample

i = index of items in sample

j = index identifying the first quarter of each simulation year (0, 4, 8, 12, 16, 20)

k = index of the quarter being simulated

d = demand observation

$\overline{D}$ = quarterly demand forecast (DEN B022)

7. <u>TVAD MSE - Total Value of Annual Demand Weighted Mean Square Error</u> - a statistic which measures the accuracy of the demand forecast by weighting the square of the forecast error by the dollar value of annual demand. The VAD weighted MSE was summed across all items and the total was used to compare forecasting errors.

$$\text{TVAD MSE} = \sum_{i=1}^{n} \frac{\sum_{k=j+1}^{j+4} (d_{ki})(P_i)(d_{ki} - \overline{D}_{ki})^2}{\sum_{k-j+1}^{j+4} (d_{ki})(P_i)}$$

where

P = unit price (DEN B053)

8. <u>DWPE - Demand Weighted Percentage Error</u> - a statistic which measures the accuracy of the demand forecast by expressing the total absolute value of the forecast error as a percentage of the total observed quarterly demand.

$$\text{DWPE} = \frac{\sum_{i=1}^{n} \sum_{k=j+1}^{j+4} |d_{ki} - \overline{D}_{ki}|}{\sum_{i=1}^{n} \sum_{k=j+1}^{j+4} d_{ki}}$$

After all computer runs were made, means and standard deviations were calculated, under each of the preceding criteria. The eight criteria explained above were grouped into four major categories: inventory investment, performance, workload and demand forecast accuracy. The criteria \$OH + \$DI show the inventory

investment, SMA and ADD measure system performance, #PI and #RA measure the system workload and TMSE, TVAD MSE and DWPE determine forecast accuracy. All criteria were considered but the selection of a best technique was based on system performance per dollar invested. Due to inherent differences in the objective function, the consumable cogs' performance evaluation was based on ADD while the repairable cogs' performance evaluation emphasized SMA.

To test the hypotheses that the means of SMA, ADD and $OH + $DI are the same for the variance computations and demand forecasting methods evaluated, statistical tests using the Student-t distribution were performed. The tests were used to help determine parameter values for the sensitivity analysis performed for adaptive smoothing and moving average and then, to help choose the best method of variance computation and demand forecasting. Because the t-tests require at least two samples, t-tests were not applied to the 2R universe of nonprogram-related items. A sample calculation of the t-test appears in Appendix B. Equations and tables were supplied by reference (5).

D. VARIANCE APPROXIMATION COMPARISON. Two methods of variance approximation, the proposed more direct approximation and the current approximation, were compared. Simulations of each method used the current parameter values and parameter values recommended by reference (2) (and shown in TABLE II) to see if the recommended values had any effect on the variance calculation comparison.

Currently, the variance (σ_D^2) is approximated by the expression $1.57(\text{MAD})^2$ and the standard deviation is approximated by the expression 1.25(MAD). Neither the variance nor the standard deviation is stored in the MDF (Master Data File). The MAD is stored in the MDF and recomputed every quarter as follows:

$$\text{MAD}_t = \alpha\left|d_t - \overline{D}_t\right| + (1-\alpha)\ \text{MAD}_{t-1}$$

where

α = smoothing weight

t = index indicating time

The proposed direct approximation involves computing and retaining in the MDF the estimated variance but not the MAD. The MAD would no longer be required to compute σ_D^2 or σ_D, since the standard deviation equals the square root of the variance. The proposed calculation is: $\sigma_{D_t}^2 = \alpha(d_t - \bar{D}_t)^2 + (1-\alpha)\, \sigma_{D_{t-1}}^2$ Notice that the mathematical expression is very similar to the MAD computation except that the error term is squared.

Because the mathematical operations necessary for the current and proposed approximations were different, a study of the computer run times was also completed. For both methods of variance approximation, there are four different methods, or paths of computer code to compute the demand forecast: (1) a step increase or decrease, (2) forecast remains the same, (3) exponentially smoothed by a trending weight and (4) exponentially smoothed by a nontrending weight. The parameter values used to determine the appropriate paths are shown in TABLE II.

TABLE II

Current and Recommended Parameter Values for the Current and Proposed Variance Approximation Forecasting Methods

	Current		Recommended	
	SPCC	ASO	SPCC	ASO
Trend Significance Levels	1.1/.9	1.5/.99	1.1/.9	1.1/.9
Smoothing Weights	.3/.3/.1	.4/.4/.2	.4/.4/.2	.4/.4/.2
Filter Constants	6/2	3/15	9/15	6/25

All paths were examined to see how many different mathematical operations were used per path and how often each operation occurred. The filtering paths required different calculations for high and low demand items, so high and low demand items were considered separately for all paths. Time required for each mathematical operation was provided by a series of computer programs which calculated the total times required to perform the operations. Using the times provided, the total time per path could be calculated. The 5A wholesale simulator was modified to generate a table showing the number of observations processed on each path and the percent of time the path was used. Multiplying the percent of time each path was used by the total time per path, and then summing those values, produced a weighted average run time.

Average run times for the two methods of variance approximation were determined to see if the square root calculation required by the proposed approximation of variance was more time consuming than the multiplication required for the currently used approximation of the variance term. The calculations of run time were made for all samples. The results were summarized by cog and then by ICP (Inventory Control Point). The summarized results were used, along with other statistical data, to determine which method of variance approximation was better - the current or the proposed.

E. FORECASTING METHODS. Three methods of demand forecasting were examined: the current method with the recommended parameter values, adaptive smoothing and moving average. (The current method with current parameter values was included as a base case.) First, a sensitivity analysis determined the best parameter values for adaptive smoothing and moving average. Then, simulation runs using the selected parameters for adaptive smoothing and moving average were compared to simulations of the current method with recommended parameter values.

The current method of demand forecasting checks for demand observations that should be filtered out. If the present and previous observations should be filtered, the forecast is an average of the two observations. If only the present observation is to be filtered, the forecast remains the same. Observations not filtered are checked for trends and different smoothing weights are applied for nontrending and trending observations. The forecast then is determined by:

$$\overline{D}_{t+1} = \alpha d_t + (1-\alpha)\,\overline{D}_t$$

Adaptive smoothing uses a smoothing weight which equals the ratio of the smoothed forecast errors to the smoothed absolute value of the forecast errors. This allows the smoothing weight to adapt to changes in level of demand or trends. The forecast is determined by:

$$\overline{D}_{t+1} = \alpha_t\, d_t + (1-\alpha_t)\,\overline{D}_t$$

with $\alpha_t = \left|\frac{E_t}{M_t}\right|$

where

E_t = smoothed error = $\beta e_t + (1-\beta)\ E_{t-1}$

M_t = smoothed absolute error = $\beta|e_t| + (1-\beta)\ M_{t-1}$

e_t = error of the forecast = $d_t - \overline{D}_t$

β = constant interim smoothing weight (a value of .2 is recommended by reference (6))

NOTE: α_t is the tracking signal defined by Trigg and Leach in reference (7)

The following example demonstrates the use of the adaptive smoothing technique:

Let $d_4 = 197.5$, $\overline{D}_4 = 188.6$, $E_3 = -8.8$, $M_3 = 12.0$, $\beta = .2$ and $t = 4$.

Then $e_4 = 197.5 - 188.6 = 8.9$.

$E_4 = .2(8.9) + .8(-8.8) = 1.78 - 7.04 = -5.26$

$M_4 = .2(\ 8.9\ ) + .8\ (12) = 1.78 + 9.6 = 11.38$

$\alpha_4 = \left|\frac{-5.26}{11.38}\right| = .462$

So, $\overline{D}_5 = (.462)(197.5) + (.538)(188.6) = 91.2 + 101.5 = 192.7$.

The above example used a value of .2 for β. Because reference (6) gave no definite reason for this particular value, a sensitivity analysis was performed for β values ranging from .1 to .4. Simulations were also made using the adaptive smoothing technique combined with the filter values recommended by reference (2). The best value for β was chosen by comparing the output statistics discussed above, emphasizing the rate of change in system performance per dollar invested. Results of the hypothesis tests, which tested the hypothesis that the means of SMA, ADD and \$OH + \$DI were the same for the two methods being compared, were also considered.

The moving average method of demand forecasting involves choosing a base number of observations and then calculating a simple average of the observations.

As a new observation appears, the oldest observation is dropped from the calculations. The demand forecast is determined by:

$$\bar{D}_{t+1} = \frac{d_t + d_{t-1} + \ldots + d_{t-N+1}}{N}$$

where

N = base number of observations

A sensitivity analysis was performed to compare bases of four, six and eight quarters. Similar to the adaptive smoothing analysis, statistics generated by the simulator were analyzed with emphasis on examining the rate of change in system performance per dollar invested, and hypothesis tests were done using SMA, ADD and \$OH + \$DI.

After the sensitivity analysis was completed and parameter values chosen for adaptive smoothing and moving average method, the two methods were compared to the current method with recommended parameter values. Then, the two superior methods of demand forecasting were compared by a study of timed computer simulations, similar to the study which compared simulations using the two methods of variance approximation.

III. FINDINGS

A. VARIANCE APPROXIMATION COMPARISON. Comparison of the current and the proposed variance approximation methods, both with recommended parameter values, indicate that the proposed more direct approximation of the variance provides a less cost efficient demand forecast. Using the parameter values recommended by reference (2) produced superior results for both current and proposed

variance approximation so all comparisons of the two methods were based on the recommended parameter values. Both methods using current parameter values were included in the data as base cases.

Comparing the means of simulation runs using the proposed variance approximation to the current approximation (TABLEs III through VI) by cog, showed that ADD increased and \$OH + \$DI increased for 1H, both ADD and \$OH + \$DI decreased by a small amount for 1R, SMA decreased and \$OH + \$DI increased for 2H and SMA increased by a very small amount while \$OH + \$DI increased for 2R. Emphasis was placed on performance per dollars invested and the desired effect would be an increase of SMA for both 2H and 2R, a decrease of ADD for 1H and 1R and decreased \$OH + \$DI for all cogs. Under this criteria, the results appear to indicate that the current approximation of the variance with recommended parameter values was better than the proposed approximation of the variance with recommended parameter values.

However, statistical t-tests performed on the means and standard deviations of both methods using recommended parameter values showed that, for all factors (SMA, ADD, \$OH + \$DI) there was no significant difference between the two methods (for numerical results, refer to Appendix C, TABLE I). A sample t-test calculation appears in Appendix B. Because there is no significant difference between the current and proposed approximations of the variance term, the previous assumption of a normal distribution of forecast errors appears to be valid and the approximations of $1.57(\mathrm{MAD})^2$ and 1.25 MAD for σ_D^2 and σ_D are appropriate.

TABLE III

Variance Approximation Comparison
1H Means

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	8.217	59.7	71.71	936.6	N/A	8.410	.137	.786
Proposed Method Current Values	8.259	58.1	74.02	936.4	N/A	12.223	.138	.796
Current Method Recommended Values	8.427	65.5	59.77	832.9	N/A	10,072	.137	.762
Proposed Method Recommended Values	8.487	62.9	62.43	838.3	N/A	8.597	.137	.751

TABLE IV

Variance Approximation Comparison
2H Means

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	59.226	51.2	80.28	736.5	1426.0	1.604	6.491	.972
Proposed Method Current Values	60.662	50.7	79.20	734.3	1439.8	1.604	6.491	.967
Current Method Recommended Values	59.395	54.1	75.10	716.4	1395.4	1.584	6.488	.887
Proposed Method Recommended Values	60.184	51.4	77.74	728.9	1432.8	1.585	6.488	.887

TABLE V

Variance Approximation Comparison

IR Means

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	13.647	47.0	104.86	1290.4	N/A	32.201	.189	.814
Proposed Method Current Values	13.678	47.0	105.33	1290.9	N/A	33.050	.194	.823
Current Method Recommended Values	13.843	54.2	85.72	1239.4	N/A	20.428	.168	.798
Proposed Method Recommended Values	13.632	53.7	84.89	1233.0	N/A	20.713	.182	.795

TABLE VI

Variance Approximation Comparison
2R Values

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	97.580	59.6	54.81	1227.5	3446.0	11.388	12.609	1.050
Proposed Method Current Values	107.698	53.7	57.90	1234.0	3478.0	11.390	12.609	1.058
Current Method Recommended Values	93.751	64.0	45.17	1257.5	3709.0	7.935	12.588	.860
Proposed Method Recommended Values	97.383	64.5	47.72	1282.0	3726.5	10.505	12.600	1.011

The time studies showed that, for both SPCC and ASO, the percent of time that each path of the demand forecast routine was used was almost identical for both methods. This indicated that computing σ_D differently did not appear to affect the filter process because the simulator seemed to filter the same items the same number of times. The proposed approximation method used more square and square root computations, less absolute values and fewer multiplications. This difference in operations accounted for the differences in total average time for the two methods of variance approximation.

The number and type of operations per path, and the time required for each operation, depending on demand, are found in TABLE VII. For the filtering without averaging path with high demand items, there were two additions, no subtractions, four multiplications, no divisions, no squares, no square roots and no absolute values in the current approximation method. The proposed variance approximation method in the same category had two additions, no subtractions, two multiplications, no divisions, no squares, two square roots and no absolute values.

Multiplying the number of operations by their appropriate time requirements gave the total time per path, shown in TABLE VIII. For the high demand filtering without averaging path for the current method (TABLE VIII), the total time required was 318.76 nano seconds. The total time, per path, was multiplied by the percent of time the path was used to determine the contribution to mean run time.

This process was repeated for each path, for both methods of variance calculation for SPCC and ASO. The mean time per item per run, which is the sum of the paths' constributions to mean run time, is shown in TABLE IX. For SPCC, the current approximation method showed a mean time of 1,164 nano seconds per item while the proposed variance approximation method produced a mean time of 2,287 nano seconds per item, which is 1.96 times the value of the current method. For ASO, the proposed variance approximation method was 1.87 times slower.

Another change in calculations that influences the computer time requirement is the computation of the procurement problem variance. The proposed method stores σ_D^2 which is then used as the procurement problem variance. The current method, however, stores the MAD and then approximates the procurement problem variance by $1.57(\text{MAD})^2$. The approximation requires one multiplication and one square, 177 nano seconds more than the time required by the proposed method.

TABLE VII

Number of Operations Per Path

Current Method (Recommended Parameters)

Path	Demand	Addition	Subtraction	Multiplication	Division	Squares	Square Root	Abs. Value
Filtering w/o Averaging	High	2	0	4	0	0	0	0
	Low	0	0	2	0	0	0	0
Filtering with Averaging	High	4	0	5	2	1	0	0
	Low	2	0	3	2	1	0	0
No Trending	High	8	5	11	1	0	0	1
	Low	7	4	8	1	0	0	1
Trending	High	8	5	11	1	0	0	1
	Low	7	4	8	1	0	0	1

Proposed Method (Recommended Parameters)

Path	Demand	Addition	Subtraction	Multiplication	Division	Squares	Square Root	Abs. Value
Filtering w/o Averaging	High	2	0	2	0	0	2	0
	Low	0	0	2	0	0	0	0
Filtering with Averaging	High	4	0	5	2	2	2	0
	Low	2	0	5	2	2	0	0
No Trending	High	8	5	9	1	1	2	0
	Low	7	4	8	1	1	0	0
Trending	High	8	5	9	1	1	2	0
	Low	7	4	8	1	1	0	0

Time Required Per Operation (Nano Secs)

Operation	Time	Operation	Time
Addition (I+1)	46.88	Squares (I^2)	120.83
Subtraction (I-1)	47.92	Square Roots ($\sqrt{I}$)	1176.04
Multiplication (I*1)	56.25	Abs. Value (|I|)	40.63
Division (I/1)	55.20		

TABLE VIII

Contribution to Mean Run Time

Path	Demand	Current (Recommended Parameter Values)			Proposed (Recommended Parameter Values)		
		Time Per Path (Nano secs.)	SPCC % Time Path Used	ASO % Time Path Used	Time Per Path (Nano secs.)	SPCC % Time Path Used	ASO % Time Path Used
Filtering without Averaging	High	318.76	.017	.021	2558.34	.017	.022
	Low	112.50	.004	.004	112.50	.004	.004
Filtering with Averaging	High	700.00	.004	.006	3172.91	.004	.006
	Low	493.74	.001	.001	727.07	.001	.001
No Trending	High	1329.22	.337	.246	3649.00	.337	.245
	Low	1065.67	.401	.393	1145.87	.401	.394
Trending	High	1329.22	.108	.139	3649.00	.108	.138
	Low	1065.67	.128	.190	1145.87	.128	.190

TABLE IX

Average Run Time Per Item
(Nano Secs/Item)

	SPCC	ASO
Current Method (Recommended Parameter Values)	1164	1144
Proposed Method (Recommended Parameter Values)	2287	2143

SPCC - proposed variance approximation method is 1.96 times slower than the current method.

ASO - proposed variance approximation method is 1.87 times slower than the current method.

According to the t-tests that were done, the proposed approximation produced a demand forecast that is neither significantly better nor significantly worse than the current method with recommended parameter values produced (for numerical results refer to Appendix C, TABLE I). There is little difference, then, in the results of the two methods. The current method requires less computer time considering both calculation of the demand forecast and the procurement problem variance. Therefore, the analysis supports maintaining the current method.

B. FORECASTING METHODS.

1. Determination of β and N. A sensitivity analysis was performed for both adaptive smoothing and moving average techniques. Under adaptive smoothing β was varied from .1 to .4 to determine the best smoothing weight. Initial simulations using adaptive smoothing indicated a large increase in inventory investment ($OH + $DI) compared to the base case and current method. Because those increases may have been caused by extremely high demands that would have been filtered out by the current method, the adaptive smoothing process was combined with the filtering process, using the filter values recommended by reference (2). Combining the processes resulted in an inventory investment which was close to the inventory investment values of the base case and current method using the recommended parameter values.

The different values of β were compared to a value of .2 for β, the value recommended by reference (6). Examination of means, by cog, showed that a value of .1 for β produced a slight decrease in $OH + $DI but decreased SMA and increased ADD. A value of .3 for β increased SMA, decreased ADD, and slightly increased $OH + $DI. A value of .4 for β increased SMA, ADD and $OH + $DI.

Statistical t-tests indicated that no significant difference existed between values of .2 and .3 for β (for numerical results, refer to Appendix C, TABLE II). Wide variances in SMA, ADD and \$OH + \$DI influenced these tests and were considered when test results were evaluated. Examination of marginal differences of mean values (TABLE X) between β = .1, .2, .3, .4, focused on performance per dollar invested and used a value of .2 for β as the base case. A value of .1 for β showed a decrease in inventory investment for three cogs which was combined with increases in ADD and decreases in SMA. A value of .3 for β decreased ADD and increased SMA for either a minimal increase or a decrease in inventory investment. A value of .4 for β further decreased ADD but either slightly increased or greatly decreased SMA with an inventory investment increase two to six times as large as the increase for a β value of .3. Because SMA increased and ADD decreased for a slight increase in cost for β = .3 compared to β = .2, β = .3 was chosen as the best value.

TABLE X

Marginal Differences In β Values
For Adaptive Smoothing

β Values	SPCC				ASO			
	1H		2H		1R		2R	
	ADD	\$OH + \$DI (MIL)	SMA	\$OH + \$DI (MIL)	ADD	\$OH + \$DI (MIL)	SMA	\$OH + \$DI (MIL)
β = .1	+2.72	-.291	-2.5	- .951	+1.51	- .547	-1.6	+1.678
Base Value β = .2	57.74	9.510	55.8	60.465	80.37	16.367	62.6	99.580
β = .3	-3.52	+.282	+1.7	+ .439	-2.60	+ .246	+7.1	- 3.900
β = .4	-2.72	+1.710	+1.1	+ .968	-7.48	+1.304	-6.2	+ 5.517

Sensitivity analysis of the moving average method compared bases of eight, six and four quarters. Compared to an eight quarter base, the six quarter base increased SMA and decreased ADD for all cogs and slightly increased inventory investment for 1H and 1R while decreasing inventory investment for 2H and 2R. Comparison of six and four quarter bases showed that SMA increased and ADD decreased for the four quarter base. However, the improvement in performance was combined with an inventory investment two to 10 times as large as the inventory investment for the six quarter base. An examination of marginal differences of mean values (TABLE XI) of performance and inventory investment suggests that a base of six quarters is the best choice.

Statistical t-tests comparing bases of eight and six quarters showed no significant difference in the two bases for 1H and 2H but showed significant improvement in SMA and ADD for 1R with no significant change in inventory investment (for numerical results refer to Appendix C, TABLE III). A six quarter base seemed to show the most improvement in system performance per dollar invested.

TABLE XI

Marginal Differences in Base Number of Quarters
For Moving Average Method Means

	SPCC				ASO			
	1H		2H		1R		2R	
Number of Quarters	ADD	$OH + $DI (MIL)	SMA	$OH + $DI (MIL)	ADD	$OH + $DI (MIL)	SMA	$OH + $DI (MIL)
Base Value 8 quarters	67.45	8.303	51.1	61.147	96.41	12.907	58.9	77.885
6 quarters	-1.98	+.295	+1.0	-2.303	-9.71	+.563	+9.9	-4.615
4 quarters	-6.74	+2.143	+1.0	+4.088	-7.61	+.614	-7.8	+8.864

2. Comparison of Demand Forecasting Methods. After the best values for β in adaptive smoothing and the base of quarters in moving average were determined, the two methods were compared to the current method with recommended parameter values to choose the best method of demand forecasting. The current method with current parameter values was included as a base case. The means of all four cogs (TABLEs XII through XV) were examined under the previously explained criteria. The method with the best performance was adaptive smoothing with filters and β = .3. Adaptive smoothing showed the highest SMA and lowest ADD but had the highest inventory investment. The second best performing method was the current method with recommended parameter values which was compared to adaptive smoothing to see if the inventory investments of the two methods were significantly different.

TABLE XII

Forecasting Methods
1H Means

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	8.217	59.7	71.71	936.6	N/A	8.410	.137	.786
Current Method Recommended Values	8.427	65.5	59.77	832.9	N/A	10.072	.137	.762
Adaptive Smoothing with Filters $\beta = .3$	9.792	68.3	54.22	1216.5	N/A	12.903	.141	.797
Moving Average 6 quarter base	8.598	60.0	65.47	929.4	N/A	10.588	.139	.814

TABLE XIII

Forecasting Methodc
2H Means

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	59.226	51.2	80.28	736.5	1426.0	1.604	6.491	.972
Current Method Recommended Values	59.395	54.1	75.10	716.4	1395.4	1.584	6.488	.887
Adaptive Smoothing with Filters $\beta = .3$	60.954	57.5	72.69	883.8	1317.0	1.599	6.489	.887
Moving Average 6 quarter base	58.844	52.1	73.09	800.2	1594.0	1.598	6.491	.918

TABLE XIV

Forecasting Methods
1R Means

PARAMETER VALUES	$OR + $DI (MIL)	SMA %	ADO	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	13.647	47.0	104.86	1290.4	N/A	32.201	.189	.814
Current Method Recommended Values	13.843	54.2	85.72	1239.4	N/A	20.428	.168	.798
Adaptive Smoothing with Filters E = .3	16.613	62.9	77.77	1032.9	N/A	25.775	.179	.877
Moving Average 6 quarter base	13.470	49.6	86.70	1418.0	N/A	19.026	.168	.814

TABLE XV

Forecasting Methods
2R Values

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#BA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	97.580	59.6	54.81	1227.5	3446.0	11.388	12.609	1.050
Current Method Recommended Values	93.751	64.0	45.17	1257.5	3709.0	7.935	12.588	.860
Adaptive Smoothing with Filters $\beta = .3$	95.680	69.7	44.19	1295.0	3593.5	7.918	12.588	.874
Moving Average 6 quarter base	73.270	66.8	39.97	1293.0	4437.0	8.635	12.591	1.036

Statistical t-tests that compared the two methods showed no significant difference between the methods with the exception of the tests for IR. The tests for IR indicated that the SMA of the adaptive smoothing is significantly better while the cost is not significantly different (for numerical results, refer to Appendix C, TABLE IV). A time study (TABLEs XVI through XVIII), similar to the time study for variance computation comparison, showed that the current method was 1.03 times slower than adaptive smoothing for SPCC and 1.02 times slower for ASO - a nominal difference in time.

Analysis does not strictly determine the choice of a best method. The amount of work involved in implementing and maintaining the adaptive smoothing method should also be considered. The current method with recommended parameter values requires changing a few input parameter values, while adaptive smoothing requires changes to the UICP demand forecasting model. However, once adaptive smoothing is implemented, it requires no updating of parameter values, which may be necessary with the current procedure. If program changes are required for other reasons, such as resystemization, implementing adaptive smoothing will not require future updating or parameter value reevaluation and will give the same or better demand forecast.

TABLE XVI

Number of Operations Per Path

Path	Demand	Addition C*	Addition ASF*	Subtraction C	Subtraction ASF	Multiplication C	Multiplication ASF	Division C	Division ASF	Squares C	Squares ASF	Absolute Value C	Absolute Value ASF
Filtering without Averaging	High	2	2	0	0	4	4	0	0	0	0	0	0
	Low	0	0	0	0	2	2	0	0	0	0	0	0
Filtering with Averaging	High	4	4	0	0	5	5	2	2	1	1	0	0
	Low	2	2	0	0	3	3	2	2	1	1	0	0
No Trending	High	8	N/A	5	N/A	11	N/A	1	N/A	0	N/A	1	N/A
	Low	7	N/A	4	N/A	8	N/A	1	N/A	0	N/A	1	N/A
Trending	High	8	N/A	5	N/A	11	N/A	1	N/A	0	N/A	1	N/A
	Low	7	N/A	4	N/A	8	N/A	1	N/A	0	N/A	1	N/A
Adaptive Smoothing		N/A	5	N/A	2	N/A	10	N/A	1	N/A	1	N/A	2

*C - Current Recommended Parameter Values

*ASF - Adaptive Smoothing With Filters

Time Required Per Operation

Operation	Time (Nano seconds)
Addition (I+1)	46.88
Subtraction (I-1)	47.92
Multiplication (I*1)	56.25
Division (I/1)	55.20
Squares (I^2)	120.83
Absolute Value (|I|)	40.64

TABLE XVII

Contribution to Mean Run Time

Path	Demand	Current (Recommended Parameter Values)			Adaptive Smoothing w/Filtering		
		Time Per Path (Nano secs)	SPCC % Time Path Used	ASO % Time Path Used	Time Per Path (Nano secs)	SPCC % Time Path Used	ASO % Time Path Used
Filtering without Averaging	High	318.76	.017	.022	318.76	.019	.023
	Low	112.50	.004	.004	112.50	.004	.003
Filtering with Averaging	High	700.00	.004	.006	700.00	.004	.006
	Low	493.74	.001	.001	493.74	.001	.001
No Trending	High	1329.22	.337	.246	N/A	N/A	N/A
	Low	1065.67	.401	.392	N/A	N/A	N/A
Trending	High	1329.22	.108	.139	N/A	N/A	N/A
	Low	1065.67	.128	.190	N/A	N/A	N/A
Adaptive Smoothing	N/A	N/A	N/A	N/A	1150.03	.972	.967

TABLE XVIII

Average Run Time Per Item
(Nano Secs/Item)

	SPCC	ASO
Current Method (Recommended Parameter Values)	1164	1144
Adaptive Smoothing with Filtering	1128	1124

SPCC - current method with recommended parameter values is 1.03 times slower than adaptive smoothing with filtering

ASO - current method with recommended parameter values is 1.02 times slower than adaptive smoothing with filtering

IV. SUMMARY AND CONCLUSIONS

The 5A wholesale inventory simulator and actual demand observations were used to determine the effect of using a more direct approximation of the variance of quarterly demand and the advantage, if any existed, of using either adaptive smoothing or moving average techniques instead of the current method of demand forecasting. To determine the best parameter values for adaptive smoothing and moving average, a sensitivity analysis was performed. Methods of both variance computation and demand forecasting were evaluated by the four criteria of inventory investment, performance, workload and demand forecast accuracy with emphasis on performance per dollar invested. In addition, the computer time required for each method was considered.

A. VARIANCE APPROXIMATION COMPARISON. The current approximation of the variance term was compared to the proposed more direct approximation. Because the use of parameter values recommended by reference (2) produced better results overall, the determination of the best variance approximation method was based on the simulations using recommended parameter values. Comparison of mean values for SMA, ADD and $OH + $DI of both methods indicated that in most cases, the current method showed increased performance and decreased inventory investment (see TABLE XIX). But, statistical t-tests comparing the same means showed that the current method was not significantly better.

The variance calculation methods were further compared by a study of the change in the required computer time. For both high and low demand items, the demand forecast procedure was considered as four separate paths: (1) a step increase or decrease, (2) forecast remains the same, (3) exponentially smoothed by a trending weight and (4) exponentially smoothed by a nontrending weight. The simulator generated a table which showed the percent of time that each path was used, which turned out to be almost identical for the two methods. There were differences, however, in the time each path required for the different methods. These differences were due to the different mathematical operations used by each method; e.g., the current method required multiplication for one path where the proposed variance approximation method required a square root which is more time consuming. A total comparative time per observation per simulation was determined. The proposed variance approximation method was 1.96 times slower than the current method for SPCC and 1.87 times slower for ASO.

Even though the proposed variance approximation method is a more direct approximation, the proposed approximation produced no better results than the

current approximation. (This lends validity to the assumption of normal distribution of forecast errors which is used with the current method.) Therefore, the current method, with parameter values recommended by reference (2) is the better method of variance calculation.

TABLE XIX

	SPCC				ASO			
	1H		2H		1R		2R	
	ADD	$OH + $DI (MIL)	SMA	$OH + $DI (MIL)	ADD	$OH + $DI (MIL)	SMA	$OH + $DI (MIL)
Current Method*	59.77	8.427	54.1	59.395	85.72	13.843	64.0	93.751
Proposed Method*	62.43	8.487	51.4	69.184	84.89	13.632	64.5	97.383

*With parameter values recommended by FMSO Report 146

B. FORECASTING METHODS. Sensitivity analysis compared values from .1 to .4 for the smoothing constant, β, which is used in adaptive smoothing. Because results improved when adaptive smoothing was combined with the filtering process of the current method, only adaptive smoothing runs with the filters recommended by reference (2) were considered. Varying β between .1 and .4 and then examining the marginal differences of the means of the performance and investment indices, indicated that a value of .3 for β was the best choice. (T-tests comparing values of .2 and .3 for β indicated no significant difference between the two but this was due to wide variances in factors tested.) Thus, a value of .3 for β was chosen as the best value.

Similar analysis compared bases of four, six and eight quarters for the moving average method of demand forecasting to determine the best base number of quarters. Examination of marginal differences indicated an increase of SMA and a decrease in ADD and either a very small increase or a decrease in $OH + $DI when comparing six quarters to eight quarters. Marginal differences also showed a considerable decrease in $OH + $DI, increase in ADD and either a small decrease or considerable increase in SMA when comparing four quarters to six quarters. Six and eight quarter bases appeared to give the best forecast, having the best combinations of forecast accuracy, system performance and inventory investment. T-tests done comparing six quarters and eight quarters showed no significant differences in the two methods for cogs 1H and 2H but 1R showed a significant improvement in SMA and ADD for the six quarter base with no significant change in $OH + $DI. Therefore, a base of six quarters was chosen as the best value.

After parameter values were determined for adaptive smoothing and moving average, the two methods were compared to the current method with parameter

values recommended by reference (2) (see TABLE XX). Adaptive smoothing showed the best performance with the highest SMA and lowest ADD but also had the highest inventory investment. Adaptive smoothing was then compared to the current method with recommended parameter values, which showed the second best performance, to determine if the difference between the two methods of inventory investment was significant.

Statistical t-tests comparing the two methods indicated, with the exception of the 1R cog, that there was no significant difference in the two methods. The 1R cog showed the SMA of adaptive smoothing significantly better than that of current with recommended parameters with no significant change in cost.

Implementing adaptive smoothing would require program changes while using the current method with recommended parameter values requires changing a few input parameters. However, once adaptive smoothing is implemented, no further attention is required while the current method requires re-evaluation and updating of input parameters.

V. RECOMMENDATIONS

FMSO recommends that the current method of variance calculation be retained. FMSO also recommends that adaptive smoothing combined with filtering be implemented.

TABLE XX

Summary of Performance and Inventory Investment for Forecasting Methods

	SPCC				ASO			
	1H		2H		1R		2R	
Forecasting Methods	ADD	$OH + $DI (MIL)	SMA	$OH + $DI (MIL)	ADD	$OH + $DI (MIL)	SMA	$OH + $DI (MIL)
Current with Recommended Parameter Values	59.77	8.427	54.1	59.395	85.72	13.843	64.0	93.751
Adaptive Smoothing $\beta = .3$	54.22	9.792	57.5	60.954	77.77	16.613	69.7	95.680
Moving Average 6 quarter base	65.47	8.598	52.1	58.844	86.70	13.470	66.8	73.270

APPENDIX A: REFERENCES

1. ALRAND Working Memorandum 365
2. Operations Analysis Study Report 146 (Estimation of Parameter Values for UICP Demand Forecasting Rules)
3. Operations Analysis Study Report 128 (User's Manual for 5A (Aviation Afloat and Ashore Allowance Analyzer)
4. W. E. Demming, Some Theory of Sampling, John Wiley and Sons, Inc. 1950
5. Paul G. Hoel, Introduction to Mathematical Statistics, John Wiley and Sons, Inc. 1971
6. Makridukis and Wheelwright, Forecasting Methods and Applications, John Wiley and Sons, Inc. 1978
7. Exponential Smoothing with an Adaptive Response by D. W. Trigg and A. G. Leach, Operational Research Quarterly, Vol. 18, pp. 53-59

APPENDIX B: SAMPLE CALCULATION - HYPOTHESIS TEST

	NQTR = 8	NQTR = 6
SMA Mean	57.7%	60.0%
Standard Deviation	1.7%	2.5%
ADD Mean	67.45	65.47
Standard Deviation	6.11	7.09
\$OH + \$DI Mean	8.303 mil.	8.598 mil.
Standard Deviation	7.015 mil.	6.868 mil.

The data was taken from 1H cog, which had four separate samples, and the comparison was made using six quarter bases versus eight quarter bases for the moving average method. Let the six quarter base be sample X and the eight quarter base be sample Y.

Hypothesis: Mean of Sample X = Mean Sample Y
$\mu_X = \mu_Y$

Alternative: $\mu_X > \mu_Y$

$$\text{Let } T = \frac{|(\bar{X}-\bar{Y}) - (\mu_X-\mu_Y)|}{\sqrt{n_X(S_X)^2+n_Y(S_Y)^2}} \quad \sqrt{\frac{n_X\, n_Y\, (n_X + n_Y - 2)}{n_X + n_Y}}$$

where

$|\;|$ denotes absolute value

$\bar{X}$ is the actual mean of sample X

$\bar{Y}$ is the actual mean of sample Y

S_X and S_Y are the standard deviations of samples X and Y, respectively

n_X and n_Y are the number of elements in sample X and sample Y. These both qual 4.

Using the hypothesis that $\mu_X = \mu_Y$, we let $\mu_X - \mu_Y = 0$ and simplify calculations.

Then the degrees of freedom must be calculated:

$$\text{degrees of freedom} = n_X + n_Y - 2$$
$$= 4 + 4 - 2 = 6$$

Consider being 95% confident of the results of the test. Then, in a Student-t distribution table, look up t (1-.95, 6) = t (.05, 6) where .05 and 6 are used as column and row headings in the table.

$$t\ (.05,\ 6) = 1.943$$

If the T value that is calculated is less than 1.943, the test fails to reject the hypothesis. If the T value is greater than 1.943, the hypothesis of equality is rejected.

<u>SMA</u>

$$T = \frac{60.0 - 57.7}{\sqrt{4(2.5)^2 + 4(1.7)^2}} \sqrt{\frac{(4)(4)(6)}{8}}$$

$$= 1.318 < 1.943$$

Fail to reject the hypothesis

<u>ADD</u>

$$T = \frac{|65.47 - 67.45|}{\sqrt{4(7.09)^2 + 4(6.11)^2}} \sqrt{\frac{(4)(4)(6)}{8}}$$

$$= .366 < 1.943$$

Fail to reject the hypothesis

Inventory Investment ($OH + $DI)

$$T = \frac{|8.598 - 8.303|}{\sqrt{4(6.868)^2 + 4(7.015)^2}} \sqrt{\frac{4(4)(6)}{8}}$$

$$= .052 < 1.943$$

Fail to reject the hypothesis

Thus, under the categories of SMA, ADD and Inventory Investment ($OH + $DI) the test has failed to reject the hypothesis that there is no difference between a six quarter and eight quarter base for moving average calculations.

APPENDIX C: HYPOTHESIS TEST RESULTS

<u>Hypothesis</u>: The means of the two methods are the same for factors SMA, ADD and \$OH + \$DI.

Reject hypothesis if $T > t(d.f, 1-\alpha)$

d.f = degrees of freedom

α = percentage of confidence

TABLE I

Current versus Proposed Variance Approximation
(Both with Recommended Parameter Values)

Cog	d.f.	α	$t(d.f.,1-\alpha)$	T_{SMA}	T_{ADD}	$T_{\$OH+\$DI}$
1H	6	95	1.943	.990	.407	.010
2H	4	95	2.132	.870	.789	.029
1R	6	95	1.943	.137	.142	.088

TABLE II

Adaptive Smoothing With Filters β = .2 versus β = .3

Cog	d.f.	α	$t(d.f.,1-\alpha)$	T_{SMA}	T_{ADD}	$T_{\$OH+\$DI}$
1H	6	95	1.943	.560	.595	.049
2H	4	95	2.132	.538	.083	.018
1R	6	95	1.943	1.223	.609	.094

TABLE III

Moving Average 8 Quarter Base versus 6 Quarter Base

Cog	d.f.	α	$t(d.f., 1-\alpha)$	T_{SMA}	T_{ADD}	$T_{SOH+SDI}$
1H	6	95	1.943	1.318	.366	.052
2H	4	95	2.132	.382	.424	.089
1R	6	95	1.943	1.986*	2.408*	.199

*reject hypothesis for this value

TABLE IV

Adaptive Smoothing With Filters β = .3 versus Current Method
With Recommended Parameter Values

Cog	d.f.	α	$t(d.f., 1-\alpha)$	T_{SMA}	T_{ADD}	$T_{SOH+SDI}$
1H	6	95	1.943	1.340	.786	.236
2H	4	95	2.132	1.107	.405	.058
1R	6	95	1.943	2.650*	1.397	1.127

*reject hypothesis for this value

APPENDIX D: SAMPLE TABLES

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TABLE 1

Variance Calculation Comparison
1H Sample I

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	4.985	57.9	80.98	993.5	N/A	7.048	.290	.983
Proposed Method Current Values	5.024	56.9	85.59	1000.5	N/A	7.012	.290	.974
Current Method Recommended Values	5.381	65.6	67.14	889.2	N/A	7.218	.289	.953
Proposed Method Recommended Values	5.539	62.7	69.78	890.7	N/A	7.208	.289	.950

TABLE II

Variance Calculation Comparison
1H Sample II

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	4.119	63.5	68.06	927.3	N/A	5.170	.106	.730
Proposed Method Current Values	4.245	62.6	66.57	926.3	N/A	5.219	.106	.728
Current Method Recommended Values	4,469	68.7	51.08	825.0	N/A	5.172	.105	.705
Proposed Method Recommended Values	4.479	68.3	54.24	833.0	N/A	5.173	.105	.704

TABLE III

Variance Calculation Comparison
1H Sample III

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	4.575	57.2	71.01	917.2	N/A	20.115	.138	.664
Proposed Method Current Values	4.604	55.5	72.46	914.2	N/A	35.403	.140	.740
Current Method Recommended Values	4.787	62.5	68.55	797.5	N/A	26.760	.140	.672
Proposed Method Recommended Values	4.790	60.8	63.30	801.7	N/A	20.874	.139	.637

TABLE IV

Variance Calculation Comparison
1H Sample IV

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	T MSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	19.190	60.0	66.8	908.5	N/A	1.305	.014	.766
Proposed Method Current Values	19.163	57.4	71.47	904.5	N/A	1.259	.014	.741
Current Method Recommended Values	19.071	65.3	52.31	819.7	N/A	1.139	.013	.719
Proposed Method Recommended Values	19.147	59.9	62.4	827.7	N/A	1.134	.013	.713

TABLE V

Variance Calculation Comparison
2H Sample I

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	89.771	51.1	81.28	728.0	1397.7	4.269	19.403	1.162
Proposed Method Current Values	92.212	51.0	80.02	728.5	1418.5	4.270	19.403	1.168
Current Method Recommended Values	90.768	52.6	80.43	704.7	1368.7	4.243	19.403	1.072
Proposed Method Recommended Values	91.290	51.1	79.22	711.2	1408.7	4.244	19.403	1.073

TABLE VI

Variance Calculation Comparison
2H Sample II

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	46.835	48.2	86.7	746.7	1536.7	.373	.018	.798
Proposed Method Current Values	46.952	46.8	83.17	738.5	1528.5	.373	.018	.794
Current Method Recommended Values	46.356	52.6	74.88	721.2	1492.2	.382	.020	.762
Proposed Method Recommended Values	47.432	48.1	82.27	745.2	1545.7	.382	.020	.762

TABLE VII

Variance Calculation Comparison
2H Sample III

PARAMETER VALUES	SOH + SDI (MIL)	SMA %	ADD	ØPI	ØRA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	41.073	54.4	72.85	734.7	1343.7	.169	.052	.957
Proposed Method Current Values	42.822	54.3	74.40	736.0	1372.5	.169	.052	.940
Current Method Recommended Values	41.062	57.2	70.00	723.2	1325.2	.128	.042	.826
Proposed Method Recommended Values	41.831	54.9	71.74	730.2	1344.0	.128	.042	.827

TABLE VIII

Variance Calculation Comparison
IR Sample 1

PARAMETER VALUES	SOH + SDI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	11.594	48.1	97.44	1313.0	N/A	99.878	.252	1.026
Proposed Method Current Values	12.167	49.0	100.89	1328.5	N/A	99.958	.276	1.025
Current Method Recommended Values	13.398	52.9	89.52	1254.0	N/A	45.036	.168	.868
Proposed Method Recommended Values	12.543	54.9	85.23	1249.0	N/A	45.362	.167	.876

TABLE IX

Variance Calculation Comparison
IR Sample II

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	9.756	44.1	108.6	1341.0	N/A	5.018	.037	.757
Proposed Method Current Values	9.523	47.0	101.44	1344.5	N/A	5.684	.034	.792
Current Method Recommended Values	10.615	48.1	94.33	1286.5	N/A	5.323	.032	.789
Proposed Method Recommended Values	10.811	47.7	94.36	1286.5	N/A	5.195	.033	.781

TABLE X

Variance Calculation Comparison
IR Sample III

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	13.118	47.3	103.54	1205.0	N/A	18.206	.173	.696
Proposed Method Current Values	13.493	48.4	100.49	1195.5	N/A	20.820	.175	.699
Current Method Recommended Values	13.503	56.1	79.25	1155.5	N/A	25.612	.181	.757
Proposed Method Recommended Values	13.609	56.0	80.24	1149.5	N/A	26.523	.181	.750

TABLE XI

Variance Calculation Comparison
IR Sample IV

PARAMETER VALUES	\$OH + \$DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	20.121	48.5	109.86	1302.5	N/A	5.700	.292	.775
Proposed Method Recommended Values	19.527	43.5	118.51	1295.0	N/A	5.739	.291	.777
Current Method Recommended Values	17.854	59.7	79.79	1261.5	N/A	5.739	.289	.779
Proposed Method Recommended Values	17.565	56.2	79.74	1247.0	N/A	5.771	.345	.772

TABLE XII

Variance Calculation Comparison

2R Values

PARAMETER VALUES	SOH + $DI (MIL)	SHA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	97.58	59.6	54.81	1227.5	3446.0	11.388	12.609	1.050
Proposed Method Current Values	107.698	53.7	57.90	1234.0	3478.0	11.390	12.609	1.058
Current Method Recommended Values	93.751	64.0	45.17	1257.5	3709.0	7.935	12.588	.860
Proposed Method Recommended Values	97.383	64.5	47.72	1282.0	3726.5	10.505	12.600	1.011

TABLE XIII

Adaptive Smoothing Sensitivity Analysis
1H Sample 1

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	LWPC
Adaptive Smoothing No Filters β = .1	7.536	68.4	58.84	937.5	N/A	16.372	.290	1.350
Adaptive Smoothing No Filters β = .2	8.144	67.9	57.06	1287.5	N/A	17.142	.290	1.373
Adaptive Smoothing No Filters β = .3	8.505	68.1	56.27	1422.7	N/A	17.104	.289	1.359
Adaptive Smoothing with Filters β = .1	6.184	65.6	68.17	984.0	N/A	7.340	.289	.992
Adaptive Smoothing with Filters β = .2	6.445	66.3	62.78	1452.7	N/A	7.241	.288	.962
Adaptive Smoothing with Filters β = .3	6.903	66.3	69	1657.7	N/A	8.211	.286	1.006
Adaptive Smoothing with Filters β = .4	7.238	66.2	61.96	1817.5	N/A	8.179	.286	1.010

TABLE XIV

Adaptive Smoothing Sensitivity Analysis
LH Sample 11

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Adaptive Smoothing No Filters $\beta = .1$	19.595	72.6	44.77	800.2	N/A	9.737	.112	.950
Adaptive Smoothing No Filters $\beta = .2$	19.372	71.6	48.22	826.5	N/A	9.466	.109	.937
Adaptive Smoothing No Filters $\beta = .3$	20.101	75.8	41.6	1007.7	N/A	9.216	.108	.926
Adaptive Smoothing with Filters $\Gamma = .1$	5.379	67.8	54.59	839.0	N/A	5.806	.109	.728
Adaptive Smoothing with Filters $\Gamma = .2$	5.586	71.8	49.85	868.7	N/A	6.104	.106	.735
Adaptive Smoothing with Filters $\Gamma = .3$	5.657	72.0	46.20	1043.7	N/A	6.184	.106	.737
Adaptive Smoothing with Filters $\Gamma = .4$	5.822	72.7	47.82	1072.2	N/A	5.853	.105	.722

TABLE XV

Adaptive Smoothing Sensitivity Analysis
1H Sample III

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Adaptive Smoothing No Filters $\beta = .1$	8.035	63.3	162.72	758.2	N/A	34.603	.147	.815
Adaptive Smoothing No Filters $\beta = .2$	8.392	68.2	47.91	804.0	N/A	36.796	.151	.802
Adaptive Smoothing No Filters $\beta = .3$	8.670	68.5	47.67	953.7	N/A	36.293	.157	.780
Adaptive Smoothing with Filters $\beta = .1$	5.695	59.9	64.85	799.2	N/A	34.196	.147	.737
Adaptive Smoothing with Filters $\beta = .2$	5.946	62.3	63.53	794.7	N/A	36.438	.151	.725
Adaptive Smoothing with Filters $\beta = .3$	6.310	66.6	55.02	955.0	N/A	35.967	.157	.710
Adaptive Smoothing with Filters $\beta = .4$	6.305	68.4	48.24	1128.0	N/A	35.756	.168	.703

TABLE XVI

Adaptive Smoothing Sensitivity Analysis
1H Sample IV

PARAMETER VALUES	\$OH + \$DI (MIL)	SMA %	ADD	\$PI	\$RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Adaptive Smoothing No Filters $\beta = .1$	21.823	69.5	48.60	779.5	N/A	1.941	.014	.988
Adaptive Smoothing No Filters $\beta = .2$	22.032	68.3	50.78	915.7	N/A	2.000	.014	.991
Adaptive Smoothing No Filters $\beta = .3$	22.395	70.3	45.68	1079.0	N/A	1.921	.014	.966
Adaptive Smoothing with Filters $\beta = .1$	19.616	65.6	54.24	829.2	N/A	1.195	.014	.750
Adaptive Smoothing with Filters $\beta = .2$	20.061	66.3	54.78	970.0	N/A	1.273	.014	.752
Adaptive Smoothing with Filters $\beta = .3$	20.298	68.2	50.96	1209.5	N/A	1.249	.013	.736
Adaptive Smoothing with Filters $\beta = .4$	26.642	69.7	47.99	1456.7	N/A	1.248	.013	.734

TABLE XVII

Adaptive Smoothing Sensitivity Analysis
2H Sample I

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Adaptive Smoothing No Filters β = .1	99.531	58.8	67.42	735.2	1472.7	4.257	19.403	1.207
Adaptive Smoothing No Filters β = .2	101.579	59.9	61.35	709.2	1443.7	4.252	19.403	1.198
Adaptive Smoothing No Filters β = .3	101.065	62.6	61.61	793.0	1405.0	4.251	19.403	1.190
Adaptive Smoothing with Filters β = .1	91.433	53.4	77.39	728.0	1380.5	4.245	19.403	1.096
Adaptive Smoothing with Filters β = .2	91.202	55.3	75.40	786.0	1350.2	4.241	19.403	1.073
Adaptive Smoothing with Filters β = .3	91.478	55.4	76.17	903.5	1297.7	4.241	19.403	1.068
Adaptive Smoothing with Filters β = .4	91.372	55.7	73.15	986.5	1278.2	4.242	19.403	1.058

TABLE XVIII

Adaptive Smoothing Sensitivity Analysis
2H Sample II

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	IWPE
Adaptive Smoothing No Filters β = .1	62.010	52.1	70.73	750.5	1650.2	.390	.022	.901
Adaptive Smoothing No Filters β = .2	61.156	55.5	60.38	736.2	1622.0	.416	.023	.900
Adaptive Smoothing No Filters β = .3	54.990	59.7	64.66	832.0	1550.2	.440	.025	.885
Adaptive Smoothing with Filters Γ = .1	44.905	51.1	78.58	730.2	1531.2	.385	.022	.789
Adaptive Smoothing with Filters Γ = .2	46.942	53.2	76.09	743.7	1474.5	.413	.023	.795
Adaptive Smoothing with Filters Γ = .3	47.968	55.6	76.83	812.2	1405.5	.437	.025	.781
Adaptive Smoothing with Filters Γ = .4	49.468	58.5	72.89	890.7	1374.5	.481	.026	.804

TABLE XIX

Adaptive Smoothing Sensitivity Analysis
2H Sample III

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Adaptive Smoothing No Filters $\beta = .1$	50.608	58.9	58.96	679.5	1450.2	.155	.047	1.015
Adaptive Smoothing No Filters $\beta = .2$	51.579	61.4	56.25	747.5	1429.5	.131	.040	.968
Adaptive Smoothing No Filters $\beta = .3$	52.568	63.8	53.86	843.2	1364.2	.129	.038	.973
Adaptive Smoothing with Filters $\beta = .1$	42.204	55.4	71.80	731.2	1318.0	.145	.046	.861
Adaptive Smoothing with Filters $\beta = .2$	43.250	58.9	67.99	789.7	1299.7	.161	.052	.836
Adaptive Smoothing with Filters $\beta = .3$	43.417	61.4	65.08	935.7	1247.7	.120	.038	.811
Adaptive Smoothing with Filters $\beta = .4$	44.927	61.7	141.63	988.0	1191.2	.129	.039	.833

TABLE XX

Adaptive Smoothing Sensitivity Analysis
1R Sample 1

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	T MSE (MIL)	TVAD MSE (MIL)	DMPE
Adaptive Smoothing No Filters β = .1	16.273	60.6	83.87	1059.0	N/A	56.343	.171	1.022
Adaptive Smoothing No Filters β = .2	16.481	66.3	64.81	1032.0	N/A	52.693	.178	.998
Adaptive Smoothing No Filters β = .3	18.400	62.9	78.2	1014.5	N/A	53.193	.175	.978
Adaptive Smoothing with Filters β = .1	15.119	57.4	86.99	1129.0	N/A	50.104	.162	.939
Adaptive Smoothing with Filters β = .2	15.441	63.6	79.09	1070.0	N/A	53.034	.181	.929
Adaptive Smoothing with Filters β = .3	15.627	66.3	71.43	1054.0	N/A	52.765	.177	.924
Adaptive Smoothing with Filters β = .4	16.549	61.5	79.36	1625.5	N/A	59.283	.175	.913

TABLE XXI

Adaptive Smoothing Sensitivity Analysis
IR Sample II

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Adaptive Smoothing No Filters β = .1	14.207	58.1	72.49	1068.0	N/A	7.372	.029	1.036
Adaptive Smoothing No Filters β = .2	14.390	60.4	73.41	1039.5	N/A	8.004	.030	1.045
Adaptive Smoothing No Filters β = .3	15.527	58.7	92.51	1015.5	N/A	8.367	.033	1.039
Adaptive Smoothing with Filters β = .1	11.616	57.7	84.29	1148.5	N/A	4.994	.028	.841
Adaptive Smoothing with Filters β = .2	12.842	59.5	80.39	1093.5	N/A	7.609	.030	.909
Adaptive Smoothing with Filters β = .3	13.755	63.0	74.06	1080.0	N/A	7.809	.032	.912
Adaptive Smoothing with Filters β = .4	14.681	57.8	91.36	1045.5	N/A	7.676	.033	.898

TABLE XXII

Adaptive Smoothing Sensitivity Analysis
IR Sample III

PARAMETER VALUES	$OH + $DI (MIL)	SHA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	UWPE
Adaptive Smoothing No Filters $\beta = .1$	18.078	58.9	91.21	986.0	N/A	31.971	.186	.945
Adaptive Smoothing No Filters $\beta = .2$	18.119	60.7	76.78	951.0	N/A	40.141	.206	.950
Adaptive Smoothing No Filters $\beta = .3$	19.696	62.7	77.91	917.5	N/A	36.225	.204	.927
Adaptive Smoothing with Filters $\beta = .1$	15.300	57.0	87.86	1067.0	N/A	30.253	.181	.820
Adaptive Smoothing with Filters $\beta = .2$	16.190	60.5	85.22	986.0	N/A	38.729	.190	.831
Adaptive Smoothing with Filters $\beta = .3$	16.187	62.9	79.53	958.0	N/A	34.981	.190	.814
Adaptive Smoothing with Filters $\beta = .4$	17.308	62.0	83.52	941.5	N/A	31.002	.193	.795

TABLE XXIII

Adaptive Smoothing Sensitivity Analysis
IR Sample IV

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Adaptive Smoothing No Filters $\beta = .1$	21.334	61.5	71.74	1071.5	N/A	6.294	.238	.907
Adaptive Smoothing No Filters $\beta = .2$	21.577	64.8	69.11	1030.0	N/A	7.015	.238	.898
Adaptive Smoothing No Filters $\beta = .3$	23.123	64.9	69.77	1000.0	N/A	7.493	.272	.909
Adaptive Smoothing with Filters $\beta = .1$	21.243	62.2	68.37	1136.5	N/A	6.252	.310	.856
Adaptive Smoothing with Filters $\beta = .2$	20.995	58.7	76.76	1076.5	N/A	7.097	.285	.846
Adaptive Smoothing with Filters $\beta = .3$	20.881	59.2	86.04	1039.5	N/A	7.549	.315	.858
Adaptive Smoothing with Filters $\Gamma = .4$	23.130	61.1	86.77	1023.5	N/A	7.259	.292	.846

TABLE XXIV

Adaptive Smoothing Sensitivity Analysis
2R Values

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Adaptive Smoothing No Filters β = .1	105.966	68.4	36.99	1402.5	4519.5	25.005	12.563	1.627
Adaptive Smoothing No Filters β = .2	107.418	67.6	32.72	1378.0	4378.0	23.365	12.648	1.547
Adaptive Smoothing No Filters β = .3	120.933	65.9	38.19	1377.0	4376.5	21.887	12.644	1.508
Adaptive Smoothing with Filters β = .1	101.258	61.0	94.64	1259.5	3667.5	10.990	12.596	1.103
Adaptive Smoothing with Filters β = .2	99.580	62.6	49.95	1267.5	3545.0	7.947	12.586	.877
Adaptive Smoothing with Filters β = .3	95.680	69.7	44.19	1295.0	3593.5	7.918	12.588	.874
Adaptive Smoothing with Filters β = .4	101.197	63.5	47.91	1236.5	3564.0	7.916	12.589	.871

TABLE XXV

Moving Average Sensitivity Analysis
JH Sample 1

PARAMETER VALUES	SOH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPI
4 quarter base	5.575	64.3	67.03	911.0	N/A	8.304	.291	1.042
6 quarter base	5.156	61.2	71.04	981.5	N/A	7.796	.292	1.025
8 quarter base	4.905	58.1	76.16	1011.7	N/A	7.561	.292	1.028

TABLE XXVI

Moving Average Sensitivity Analysis
IH Sample II

PARAMETER VALUES	$OH + $DT (MIL)	SMA Z	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
4 quarter base	12.871	66.3	53.49	863.3	N/A	5.765	.104	.750
6 quarter base	5.779	62.2	59.39	932.5	N/A	5.706	.108	.746
8 quarter base	5.200	59.9	61.88	954.2	N/A	5.612	.109	.743

TABLE XXVII

Moving Average Sensitivity Analysis
10 Sample III

PARAMETER VALUES	SOH + SDI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
[illegible] quarter base	5.156	61.9	59.03	822.2	N/A	30.275	.160	.709
6 quarter base	4.582	56.5	72.16	892.0	N/A	27.704	.140	.716
8 quarter base	4.296	55.9	65.66	927.5	N/A	26.862	.153	[illegible]

TABLE XXVIII

Moving Average Sensitivity Analysis
LH Sample IV

PARAMETER VALUES	\$OH + \$DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
4 quarter base	19.361	63.3	55.35	845.0	N/A	1.126	.014	.781
6 quarter base	18.874	5[illegible].9	59.30	911.5	N/A	1.144	.014	.767
8 quarter base	18.810	56.[illegible]	66.11	940.5	N/A	1.157	.015	.775

TABLE XXIX

Moving Average Sensitivity Analysis
2H Sample I

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	$PI	$RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
4 quarter base	90.29	53.8	68.71	803.7	1474.7	4.241	19.403	1.081
6 quarter base	83.022	54.4	70.60	794.2	1565.2	4.242	19.403	1.093
8 quarter base	89.488	52.0	78.58	778.5	1556.2	4.246	19.403	1.102

TABLE XXX

Moving Average Sensitivity Analysis
2H Sample II

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
4 quarter base	55.933	47.9	79.52	812.7	1615.7	.387	.022	.784
6 quarter base	50.996	47.5	82.79	803.5	1674.2	.404	.023	.795
8 quarter base	53.220	47.8	79.78	786.0	1672.0	.365	.020	.790

TABLE XXXI

Moving Average Sensitivity Analysis
2H Sample III

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
4 quarter base	42.573	57.7	62.82	810.7	1458.5	.135	.043	.849
6 quarter base	37.514	54.5	65.87	803.0	1542.7	.149	.048	.866
8 quarter base	40.733	53.6	70.16	792.0	1555.0	.160	.051	.899

TABLE XXXII

Moving Average Sensitivity Analysis
IR Sample I

PARAMETER VALUES	SOH + SDI (MIL)	SMA Z	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
4 quarter base	12.750	55.7	72.81	1323.0	N/A	55.908	.189	.924
6 quarter base	12.963	49.8	91.68	1449.0	N/A	41.560	.182	.874
8 quarter base	11.144	49.7	89.93	1532.0	N/A	34.834	.176	.849

TABLE XXXIII

Moving Average Sensitivity Analysis
IR Sample II

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPI
4 quarter base	10.935	49.4	84.03	1337.5	N/A	5.586	.033	.825
6 quarter base	10.205	49.4	86.41	1467.0	N/A	5.202	.030	.807
8 quarter base	9.377	41.0	103.35	1542.5	N/A	5.086	.032	.796

TABLE XXXIV

Moving Average Sensitivity Analysis
JR Sample III

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PA	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWFE
4 quarter base	13.974	52.8	82.66	1218.0	N/A	26.950	.174	.809
5 quarter base	12.849	50.4	81.36	1321.0	N/A	22.732	.161	.775
8 quarter base	13.125	44.2	97.19	1422.5	N/A	24.228	.176	.805

TABLE XXXV

Moving Average Sensitivity Analysis
IR Sample IV

PARAMETER VALUES	SOH + SDI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	T VAD MSE (MIL)	DWPE
4 quarter base	18.677	56.0	76.86	1311.0	N/A	5.851	.254	.790
6 quarter base	17.862	48.9	87.35	1435.0	N/A	6.610	.300	.801
8 quarter base	17.980	46.4	95.18	1551.0	N/A	7.132	.344	.806

TABLE XXXVI

Moving Average Sensitivity Analysis

2R Values

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
4 quarter base	82.134	61.0	41.45	1417.0	4298.5	9.301	12.590	1.060
6 quarter base	73.270	66.8	39.97	1293.0	4437.0	8.635	12.591	1.036
8 quarter base	77.885	56.9	42.75	1240.5	4237.5	8.378	12.591	1.029

APPENDIX E: STANDARD DEVIATIONS

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TABLE I

Variance Calculation Comparison
1H Values S/$\overline{X}$

PARAMETER VALUES	SOH + SDI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	7.324 / 8.217	2.8 / 59.7	6.43 / 71.71	36.7 / 936.6	N/A	8.162 / 9.410	.115 / .137	.138 / .786
Proposed Method Current Values	7.275 / 8.259	3.1 / 58.1	8.13 / 74.02	43.7 / 936.4	N/A	15.639 / 12.223	.115 / .139	.119 / .796
Current Method Recommended Values	7.106 / 8.427	2.5 / 65.5	9.36 / 59.77	39.4 / 832.9	N/A	11.408 / 10.072	.115 / .137	.129 / .752
Proposed Method Recommended Values	7.121 / 8.487	3.8 / 52.9	6.37 / 62.43	37.5 / 838.3	N/A	8.565 / 8.597	.115 / .137	.137 / .751

TABLE II

Variance Calculation Comparison
2H Values S/X

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Current Method Current Values	26.609 / 59.226	3.1 / 51.2	6.98 / 80.28	9.5 / 736.5	99.6 / 1426.0	2.310 / 1.604	11.182 / 6.491	.182 / .972
Proposed Method Current Values	27.401 / 60.662	3.8 / 50.7	4.44 / 79.20	5.2 / 734.3	80.2 / 1439.8	2.311 / 1.604	11.182 / 6.491	.188 / .967
Current Method Recommended Values	27.298 / 59.395	2.7 / 54.1	5.22 / 75.10	10.2 / 716.4	86.6 / 1395.4	2.306 / 1.584	11.184 / 6.488	.164 / .887
Proposed Method Recommended Values	27.083 / 60.184	3.4 / 51.4	5.42 / 77.74	17.0 / 728.9	103.0 / 1432.8	2.307 / 1.585	11.184 / 6.488	.164 / .867

TABLE III

Variance Calculation Comparison
1R Values S/$\bar{X}$

PARAMETER VALUES	\$OH + \$DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWTE
Current Method Current Values	4.529 / 13.647	2.0 / 47.0	5.65 / 104.86	59.2 / 1290.4	N/A	45.524 / 32.201	.112 / .189	.146 / .814
Proposed Method Current Values	4.234 / 13.678	2.5 / 47.0	8.79 / 105.33	66.8 / 1290.9	N/A	45.170 / 33.050	.118 / .194	.141 / .823
Current Method Recommended Values	2.990 / 13.843	4.9 / 54.2	7.43 / 85.72	57.6 / 1239.4	N/A	18.942 / 20.428	.105 / .168	.048 / .798
Proposed Method Recommended Values	2.864 / 13.632	4.0 / 53.7	6.78 / 84.89	58.6 / 1233.0	N/A	19.195 / 20.713	.128 / .182	.036 / .795

TABLE IV

Adaptive Smoothing Sensitivity Analysis
1H Values S/$\bar{X}$

PARAMETER VALUES	\$CH + \$DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Adaptive Smoothing No Filters β = .1	7.519 / 14.247	3.9 / 68.5	56.31 / 78.73	80.9 / 818.9	N/A	13.936 / 15.663	1.072 / .718	.229 / 1.026
Adaptive Smoothing No Filters β = .2	7.261 / 14.485	1.7 / 69.0	4.24 / 50.99	224.6 / 958.4	N/A	14.966 / 16.351	.115 / .141	.245 / 1.026
Adaptive Smoothing No Filters β = .3	7.37[illegible] / 14.918	3.5 / 70.7	6.18 / 47.81	211.0 / 1115.8	N/A	14.801 / 16.134	.115 / .142	.247 / 1.008
Adaptive Smoothing with Filters β = .1	6.94[illegible] / 9.219	3.4 / 54.7	7.11 / 60.46	82.5 / 862.9	N/A	14.938 / 12.134	.114 / .140	.127 / .802
Adaptive Smoothing with Filters β = .2	7.04[illegible] / 9.510	3.9 / 66.7	6.58 / 57.74	296.3 / 1021.5	N/A	15.993 / 12.764	.114 / .140	.113 / .794
Adaptive Smoothing with Filters β = .3	7.022 / 9.792	2.6 / 68.3	7.86 / 54.22	312.5 / 1216.5	N/A	15.652 / 12.903	.114 / .141	.140 / .797
Adaptive Smoothing with Filters β = .4	10.[illegible] / 11.502	2.1 / 69.8	6.97 / 51.50	344.0 / 1368.6	N/A	14.128 / 12.009	.115 / .143	.146 / .792

TABLE V

Adaptive Smoothing Sensitivity Analysis
2H Values S/$\overline{X}$

PARAMETER VALUES	\$OH + \$DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Adaptive Smoothing No Filters β = .1	25.597 / 70.716	3.9 / 56.6	6.07 / 55.70	37.4 / 721.7	109.6 / 1524.4	2.303 / 1.601	11.182 / 6.491	.155 / 1.041
Adaptive Smoothing No Filters β = .2	26.538 / 71.438	3.1 / 58.9	2.71 / 59.33	19.7 / 731.0	107.3 / 1498.4	2.301 / 1.600	11.184 / 6.439	.156 / 1.022
Adaptive Smoothing No Filters β = .3	27.327 / 59.541	2.1 / 62.0	5.57 / 60.04	26.4 / 822.7	97.8 / 1439.8	2.295 / 1.607	11.184 / 6.439	.157 / 1.016
Adaptive Smoothing with Filters β = .1	27.676 / 59.514	3.2 / 53.3	3.62 / 75.92	1.6 / 729.8	109.6 / 1409.9	2.301 / 1.592	11.182 / 6.491	.161 / .915
Adaptive Smoothing with Filters β = .2	26.683 / 60.465	2.9 / 55.8	4.49 / 73.16	25.6 / 773.1	99.9 / 1374.8	2.286 / 1.605	11.181 / 5.493	.130 / .901
Adaptive Smoothing with Filters β = .3	26.332 / 60.954	3.4 / 57.5	6.60 / 72.69	64.1 / 883.8	80.6 / 1317.0	2.293 / 1.599	11.184 / 6.439	.158 / .857
Adaptive Smoothing with Filters β = .4	25.605 / 61.922	3.0 / 58.6	39.61 / 95.89	53.7 / 955.1	91.7 / 1281.3	2.280 / 1.617	11.184 / 6.439	.139 / .898

TABLE VI

Adaptive Smoothing Sensitivity Analysis

IR Values $S/\overline{X}$

PARAMETER VALUES	\$OH + \$DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
Adaptive Smoothing No Filters β = .1	3.021 / 17.473	1.6 / 59.8	9.40 / 79.83	40.4 / 1046.1	N/A	23.739 / 25.495	.089 / .156	.062 / .978
Adaptive Smoothing No Filters β = .2	3.035 / 17.642	3.0 / 63.1	5.20 / 71.03	41.6 / 1013.1	N/A	23.044 / 26.963	.092 / .163	.063 / .973
Adaptive Smoothing No Filters β = .3	3.148 / 19.177	2.6 / 62.3	9.45 / 79.60	46.8 / 985.9	N/A	22.353 / 26.345	.101 / .171	.058 / .963
Adaptive Smoothing with Filters β = .1	3.994 / 15.820	2.4 / 58.6	9.13 / 81.88	36.4 / 1120.3	N/A	21.540 / 22.901	.115 / .170	.052 / .864
Adaptive Smoothing with Filters β = .2	3.403 / 16.367	2.1 / 60.6	3.57 / 80.37	48.0 / 1056.5	N/A	22.999 / 26.617	.105 / .172	.048 / .879
Adaptive Smoothing with Filters β = .3	3.030 / 16.613	2.9 / 62.9	6.47 / 77.77	52.7 / 1032.9	N/A	22.122 / 25.775	.116 / .179	.051 / .877
Adaptive Smoothing with Filters β = .4	3.646 / 17.917	1.9 / 60.6	5.08 / 85.25	46.1 / 1099.0	N/A	24.627 / 26.305	.107 / .173	.054 / .963

TABLE VII

Moving Average Sensitivity Analysis
1H Values S/$\bar{X}$

PARAMETER VALUES	$CH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
4 quarter base	6.751 / 10.741	1.9 / 64.0	6.09 / 58.73	37.7 / 860.4	N/A	12.951 / 11.368	.116 / .142	.151 / .821
6 quarter base	6.868 / 8.598	2.3 / 60.0	7.09 / 65.47	38.5 / 929.4	N/A	11.744 / 10.588	.115 / .139	.143 / .811
8 quarter base	7.015 / 8.303	1.7 / 57.7	6.11 / 67.45	37.1 / 958.5	N/A	11.363 / 10.298	.115 / .142	.142 / [illegible]

TABLE VIII

Moving Average Sensitivity Analysis
2N Values $S/\bar{X}$

PARAMETER VALUES	$OH + $DI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
4 quarter base	24.616 / 62.932	4.9 / 53.1	8.47 / 70.35	4.7 / 809.0	86.5 / 1516.3	2.301 / 1.588	11.184 / 6.489	.156 / .905
6 quarter base	26.153 / 58.844	4.0 / 52.1	8.73 / 73.09	5.2 / 800.2	70.3 / 1594.0	2.293 / 1.598	11.182 / 6.491	.156 / .918
8 quarter base	25.326 / 61.147	3.0 / 51.1	5.28 / 76.15	6.8 / 785.5	67.2 / 1594.4	2.302 / 1.590	11.182 / 6.491	.158 / .930

TABLE IX

Moving Average Sensitivity Analysis
IR Values S/$\overline{X}$

PARAMETER VALUES	SCH + SDI (MIL)	SMA %	ADD	#PI	#RA	TMSE (MIL)	TVAD MSE (MIL)	DWPE
4 quarter base	3.307 / 14.084	3.1 / 53.5	5.21 / 73.09	54.0 / 1297.4	N/A	23.767 / 23.574	.093 / .163	.060 / .833
6 quarter base	3.193 / 13.470	.6 / 49.6	4.24 / 86.70	66.0 / 1418.0	N/A	16.998 / 19.026	.111 / .163	.042 / .814
8 quarter base	3.723 / 12.907	3.7 / 45.3	5.55 / 96.41	60.2 / 1512.0	N/A	14.224 / 17.820	.128 / .182	.024 / .814

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13. ABSTRACT

A RAND Working Memorandum 365 of 26 September 1980 recommended two potential improvements to the current UICP (Uniform Inventory Control Program) demand forecasting techniques: (1) a more direct approximation for the variance of quarterly demand and (2) adaptive smoothing to forecast demand. This study used the 5A (Aviation Afloat and Ashore Allowance Analyzer) wholesale inventory simulator and actual demand observations to compare the suggested alternatives to the current method. The following criteria were used in the comparison: inventory investment, performance, workload, demand forecast accuracy and the required computer time. However, the primary criterion used was the change in performance per dollar invested. The study showed that (1) the more direct approximation of standard deviation of demand is not an improvement and (2) adaptive smoothing with filtering should be considered for implementation.

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