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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
Describes procedures for determining terminal effectiveness of high-explosive (HE) fragmenting projectiles against human targets. Includes methods of computing lethal area and fractional coverage.
1. **SCOPE.** This TOP presents techniques and analytical procedures for determining terminal effectiveness of high-explosive (HE) fragmenting projectiles against human targets. There are currently two methods of addressing antipersonnel effectiveness of fragmenting projectiles: lethal area ($A_L$) and fractional coverage ($F$). Both methods are accomplished by means of computer simulation. Methodology, input data, and examples of calculated results are included. The basic fragmentation data used in these calculations are obtained by methods described in TOP 4-2-813.1

2. **PREPARATIONS FOR COMPUTATIONS.**
   a. Obtain fragmentation test data in accordance with TOP 4-2-813.
   b. Select appropriate projectile terminal characteristics, e.g., velocity, angle of fall, height of burst, personnel targets to be evaluated, etc.
   c. Obtain projectile flight data in accordance with TOP/MTP 3-2-820 and TOP 3-2-825. To evaluate standard deviations in range and deflection ($\sigma_R$ and $\sigma_D$), see Appendix A. Flight and range dispersion data can be obtained from Ballistic Research Laboratory (BRL) firing tables.

3. **COMPUTATION PROCEDURES.**

   3.1 Lethal Area. Compute lethal areas by using the general full-spray mean area of effectiveness (MAE) computer program described in the Joint Munition Effectiveness Manual (JMEM) 61 JTCG/ME-70-6-2. This computer program calculates the effectiveness of fragmenting projectiles employed against human targets in prone, standing, or crouching-in-foxhole positions.

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*This TOP supersedes Materiel Test Procedure (MTP) 3-2-608, 5 October 1966.*

**Footnote numbers correspond to reference numbers in Appendix B.**

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Lethal area is a measure of the fragment casualty-producing potential of an exploding projectile when employed against human targets. It is defined such that the expected number of casualties \( N_c \) is equal to \( A_L \) times density of human targets \( \sigma_T \). \( A_L \) for a certain projectile will change as terminal conditions change.

The projectile terminal conditions (height of burst, velocity, and angle of fall) and fragmentation and blast characteristics are specified as input. The fragmentation characteristics are input using polar zones about the projectile centroid, each containing fragment weight groups. Associated with each fragment weight group in a zone is an average fragment weight and number of fragments. All fragments within a zone are assumed to have the same initial velocity. The fragment trajectories are assumed to be straight lines, but aerodynamic drag is applied. The target is represented as a point with associated vulnerability parameters and presented area.

Various options are available to calculate the effects of several terrain environments such as grass and forest. These effects are introduced by considering drag through layers of the medium and shielding afforded by trees. As an option, the projectile's fragment damage function can be presented in a rectangular matrix. The cells represent the average probability that a personnel target located within a particular grid cell will be incapacitated as defined by a preselected damage criterion. Damage criteria can be selected to simulate various tactical situations (assault, defense, or supply) and maximum times after wounding until incapacitation occurs (30-second, 5-minute, or half-day).

The expected number of casualties \( N_c \) is expressed by defining \( \sigma(\gamma,r) \) as the density of personnel in an element centered about the polar coordinate point \( (\gamma,r) \) and \( P(\gamma,r) \) as the probability that the personnel in that element will be incapacitated (unable to perform their tactical function after the maximum allowable time). (The center of the polar coordinates is at the point on the ground vertically below the point at which the shell bursts.) Thus:

\[
N_c = \int_0^\infty \int_0^{2\pi} \sigma(\gamma,r) P(\gamma,r)r \, d\gamma \, dr
\]

In mathematically determining lethal area, if it is assumed that personnel are uniformly distributed over the ground plane, \( \sigma(\gamma,r) \) can be represented by a constant \( \sigma \), and \( A_L \) is defined as:

\[
A_L = \frac{N_c}{\sigma} = \int_0^{2\pi} \int_0^{\infty} P(\gamma,r)r \, d\gamma \, dr
\]

Since a projectile is assumed to be symmetric about its longitudinal axis, \( A_L \) may be rewritten as:

\[
A_L = 2 \int_{-\pi/2}^{\pi/2} \int_0^{\infty} P(\gamma,r) \, d\gamma \, dr
\]

Applying the mean value theorem to the innermost integral, the mean probability of incapacitation \( P \) at ground range \( r \) from the projectile is defined as:
\[ P(r) = \frac{1}{\pi/2 - (-\pi/2)} \int_{-\pi/2}^{\pi/2} P(Y, r) dY \]

\[ P(r) = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} P(Y, r) dY \]

thus:

\[ \Lambda = 2 \pi \int_{0}^{\infty} P(r) rdr = 2 \pi \int_{0}^{R} P(r) rdr \]

in which \( R \) is the maximum effective fragment range, for the specified terminal conditions.

Since the effectiveness of a projectile is desired in simulated battlefield conditions, it is necessary to change the fragmentation data (collected in a static mode) to simulate the desired dynamic situation. This is accomplished by vectoring the fragment zone data by the terminal velocity of the projectile, projecting these dynamic zones onto the ground plane using the height of burst and angle of fall, and then grouping the vectored data into comparable dynamic zones. All subsequent calculations are accomplished with these dynamic data.

The probability of incapacitation at a ground range \( r \) is computed by:

\[ P(Y, r) = 1 - [1 - P_B(r)] [1 - P_F(Y, r)] \]

in which:

\( P_B(r) \) = probability of incapacitation due to blast alone

\( P_F(Y, r) \) = probability of incapacitation due to fragments alone

Since the blast effect is defined as a function of ground range rather than angle, the blast probability is \( P_B(r) \). Incapacitation from blast is determined from a two-step function based on the explosive type and weight in the subject projectile. From the ground plane burst point to an \( r=RB1 \), \( P_B(r)=1.0 \), and from \( RB1 \) to \( RB2 \), \( P_B(r) \) falls off linearly until \( RB2 \), when \( P_B(r)=0 \):

\[ P_B(r) = 1.0 \text{ when } r \leq RB1 \]

\[ P_B(r) = 1.0 - \frac{(r-RB1)}{(RB2-RB1)} \text{ when } RB1 < r < RB2 \]

\[ P_B(r) = 0.0 \text{ when } r \geq RB2 \]

in which: \( RB1 \) = radius about the burst point for which the probability of incapacitation due to blast is 1.

\( RB2 \) = radius about the burst point for which the probability of incapacitation due to blast is 0.
The probability of incapacitation due to fragments is computed by:

\[ P_F(y, r) = 1 - e^{-d(y, r) A_t} \]

in which \( d(y, r) \) is the mean density of lethal fragments and \( A_t \) the presented area of the target at \( r \) ground distance from the projectile burst point. Since there may be overlapping of vectored static zones in a dynamic simulation, the expected mean density of lethal fragments within a dynamic zone is:

\[ d(y, r) = \sum_{j=UZ}^{UZ} \sum_{i=1}^{k} \frac{N(i, j)}{S(j)} \frac{P_{I/H}(i, j)}{r^2} \]

in which:
- \( i \) = the fragment weight group
- \( j \) = the vectored static fragmentation zone
- \( UZ, LZ \) = the upper and lower static zones that contribute to the dynamic zone
- \( k \) = the number of lethal weight groups within \( j \)
- \( N(i, j) \) = the number of fragments in the \( i \)-th weight group within \( j \)
- \( S(j) \) = the number of steradians for \( j \) at a distance \( r \)
- \( r \) = distance from burst point
- \( P_{I/H} \) = the conditional probability of incapacitation, assuming a fragment hit. \( P_{I/H} \) is defined mathematically as:
in which: \( a, b, n \) = casualty constants that define the preselected tactical situation and maximum time after wounding until incapacitation occurs

\[ m = \text{fragment weight} \]

\[ V_r = \text{fragment remaining velocity at } r \]

3.1.1 Lethal Area Results. A typical example of lethal area information that can be provided is shown in Figures 5 through 7. These results indicate that, in addition to the fragmentation and blast characteristics, lethal area depends on projectile burst height, angle of fall, tactical situation, troop posture, and terrain environment. These parameters must be compatible with the particular projectile being evaluated and must be established before each computation. Since it is impractical to evaluate lethal area for all conditions (terminal ballistics, tactical situations, etc.), only those considered most appropriate for assessing the projectile of interest must be selected; e.g., lethal areas for certain ranges may be sufficient.

For some test projectiles, lethal area calculations are sufficient for judging the antipersonnel effectiveness. A direct comparison of the test projectile's lethal area to that of a reference item will yield the relative increase (or decrease) in its effectiveness. For others, an indirect use of lethal area in a more comprehensive comparison may be desirable, namely, a fractional coverage evaluation.

3.2 Fractional Coverage. This is defined as the fractional level that the target's capability has been degraded by a fragmenting projectile or volley thereof. In this case, a target is typically an individual, a squad, or a company of soldiers. Fractional coverage is a weapon systems evaluation approach to the solution of antipersonnel effectiveness. A fractional coverage computer evaluation combines the projectile antipersonnel effectiveness (using a lethal area probability of incapacitation damage matrix) with the accuracy of the weapon system of interest. A complete description of the computer program used to calculate fractional coverage is contained in JMEM Report 61 JTCG/ME-72-11. The matrix evaluator program provides a method of calculating the effectiveness of a single projectile or a volley of projectiles when employed against a rectangular target. The program is capable of calculating fraction coverage for as many as six rectangular targets, but the projectile trajectory to the impacts must be normal to one of the target dimensions. For targets considered, however, any number of impact patterns, mean point of impact (MPI) errors, and precision errors may be considered and the resulting fractional coverages computed for each combination.

The basic notion used in the matrix evaluator program is the expected coverage of two rectangles for a bivariate normal distribution which is described in the above-mentioned JMEM report. It is used for fractional coverage computations for a volley of projectiles against a given target, and it is used to adjust the damage matrix probabilities for precision errors.
Figure 1. Projectile fragment damage pattern.
The program requires as input a rectangular damage matrix (computed in the MAE program) that contains the expected damage level from a specific individual projectile to a target of known characteristics. The matrix elements \((D_{ij})\) denote the probability or level of damage to a target element located within the boundary of cell \(ij\). This matrix is built in the following manner. For a given projectile, the damage pattern and associated probability of incapacitation \((P_I)\) values are computed. The pattern is overlaid with a rectangular grid oriented in range and deflection directions of size sufficient to encompass the maximum effective range of any fragment. The dimensions of each cell are fixed by dividing a pattern dimension by the desired number of cells in that direction. For each cell, a \(P_I\) is determined so that it is a constant mean value within that area. These \(P_I\) values are stored in a matrix so that the matrix elements correspond to the grid squares. Figure 1 is a simplified illustration of a projectile fragment damage matrix. The damage pattern limit is defined by the outer curve. The shaded areas within the pattern are zones of equal \(P_I\) values. (The shaded areas are presented only to illustrate the typical damage pattern shape for three constant levels of \(P_I\), i.e., \(P_I = .9, .8,\) and \(.7\).) The overlaid grid represents the weapon damage matrix. Note that the pattern damage center is not necessarily the projectile impact point nor the center of the matrix. The discrepancy between the center of the matrix and the impact point is considered an offset distance and is accounted for in evaluating the damage matrix against the target. This offset is only required in the range direction since it is characteristic of impact patterns to be symmetrical with the range axis but not the deflection axis.

Precision errors are applied to the weapon damage matrix to generate a ballistic damage matrix. Precision errors are the round-to-round variation in the trajectory of a projectile that is attributed to random physical errors. This error is expressed in mils and is commonly stated as standard deviation in range \((\sigma_R)\) and deflection \((\sigma_D)\) (see Appendix A). The precision error of each projectile in a volley is assumed to be independent and follows a single bivariate normal distribution. This distribution is represented by a bell-shaped surface that approaches the range/deflection plane asymptotically in all directions. The probability that the projectile damage centroid will fall within a designated area is given by the volume over the area to the distribution surface. This probability decreases sharply as the distance from the origin increases. At distances beyond three standard deviations in range or deflection, the probability is essentially zero, and is so treated in the program. Thus, the projectile damage grid is effectively expanded in size to account for precision errors. The ballistic damage matrix and grid maintain the same number of cells as contained in the projectile damage matrix (the input matrix). Cell dimensions are generated to reflect the increase in pattern size due to precision errors. Likewise, the associated projectile damage matrix \(P_I\)'s must be adjusted to reflect this correction since the sum of products of a cell area and its associated \(P_I\) must remain constant, i.e., equal to the projectile's lethal area for the given set of terminal characteristics. The damage level for each cell in the ballistic matrix is computed by the relation:

\[
B_{xy} = \max_{i=\min} \max_{j=\min} \left( \sum D_{ij} A_{ij} \right) \frac{(XY)}{A_{xy}}
\]

in which: \(B_{xy}\) = damage level for ballistic grid cell \(XY\)

\(A_{xy}\) = area of ballistic grid cell \(XY\)
Figure 2. Volley damage grid.
\[ D_{ij} = \text{average damage level in projectile damage grid cell } ij \]

\[ A_{ij} (XY) = \text{expected area of coverage of ballistic cell } XY \text{ by the projectile damage cell } ij \]

The \( A_{ij} \)'s are calculated by using an algorithm developed for the expected coverage of two rectangles for a bivariate normal distribution.\(^5\) This algorithm is applied by considering each cell of the ballistic damage matrix as a target and computing the expected coverage of the weapon damage matrix on it. The combined damage effect of all projectiles in a volley is incorporated into a volley damage matrix. Using the mean impact coordinates for each projectile the smallest rectangle that would simultaneously cover the total associated ballistic grids is determined. This rectangle defines the perimeter of the volley damage. Figure 2 shows an example of a volley damage grid. Two equations are used to compute the expected damage level in each volley damage cell. The first computes the average damage level due to the effect of single projectile damage pattern.

\[ P_z(a, b) = \max \left( \sum_{i=\min}^{\max} \sum_{j=\min}^{\max} C_{ij} B_{ij} \right) \]

in which: \( P_z(a, b) = \text{average damage level over volley damage cell } ab \text{ due to the area coverage by ballistic grid } z \)

\[ C_{ij} = \text{proportion of the volley cell } ab \text{ covered by ballistic cell } ij \]

\[ B_{ij} = \text{expected damage level in ballistic cell } ij \]

The second equation considers the effect of overlapping weapon damage patterns (multiple coverage).

\[ S_{ij} = 1 - \prod_{z=1}^{N} (1 - P_z(ij)) \]

in which: \( S_{ij} = \text{expected damage level in volley damage cell } ij \)

\[ N = \text{number of projectiles in the volley} \]

\[ P_z(ij) = \text{coverage damage level in volley cell } ij \text{ due to area coverage by ballistic grid } z \]

The computations are performed for each volley damage cell overlapped by at least one ballistic grid. The probability of damage to a target located within the overlapping damage patterns of a number of projectiles is that the damage occurred from at least one. The damage \( (P_z) \) could have resulted from the first, second, \( N \)-th projectile, or any combination of projectiles.
\[ P_t = P (\text{any one projectile}) + P (\text{any two projectiles}) + \ldots + P (\text{all projectiles}) \]

also: \[ P_t + P (\text{no projectile-caused damage}) = 1 \]

or: \[ P_t = 1 - P (\text{no projectile-caused damage}) \]

The probability that no projectile caused any damage to the target is the probability that no damage resulted from the first, second, third, \ldots, or \( n \)-th projectile. This is known as the probability of survival and is expressed mathematically:

\[ P (\text{no projectile damage}) = (1-P_1)(1-P_2)(1-P_3) \ldots (1-P_n) \]

thus: \[ P_t = 1 - \prod_{i=1}^{N} (1-P_i) \]

in which: \( P_t = \text{probability of damage from at least one projectile} \)

\( (1-P_i) = \text{probability of no damage from projectile } i \)

\( N = \text{number of projectiles in the volley} \)

As many as six rectangular targets can be evaluated simultaneously by the matrix evaluator program. Other target configurations can be considered, provided they can be adequately approximated with rectangles. The fractional coverage is not computed according to an intuitive approach, namely, displacing the center of the volley damage grid from the intended aim point to the \( 3\sigma_R \)'s and \( 3\sigma_D \)'s. By this approach, the resulting damage grid would then define a new grid determined in a manner analogous to that used to define the ballistic damage grid. However, the approach used in the program is one in which the target center is displaced about the intended aim point, that is, the projectile is assumed to follow the correct trajectory, but the target is shifted a distance equal to the normally distributed aiming error. Figure 3 illustrates the effect of displacing the target center by \( 3\sigma_R \)'s and \( 3\sigma_D \)'s. The target center will fall anywhere within the shaded area. The displaced target perimeter defines the extent of an apparent target area (ATA). Fractional coverage is computed over the target area by using only those cells that overlap the ATA, i.e., by using only those cells within \( 3\sigma \)'s of the target perimeter. The center of the ATA is superimposed on offset distance, the distance between a projectile impact point and the center of its damage pattern, from the center of the volley damage grid (see Figure 4). Subsequent computations are limited to those volley damage cells that overlap the ATA. Fractional coverage (F) is found by the relation:

\[ F = \sum_{i=\text{min}}^{\text{Max}} \sum_{j=\text{min}}^{\text{Max}} \frac{S_{ij} C_{ij}}{A_t} \]

when: \( A_t = \text{area of the target} \)
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\[ C_{ij} = \text{expected area coverage of the apparent target by the volley damage cell } ij \]

\[ S_{ij} = \text{average damage level in volley damage cell } ij \]

3.2.1 Fractional Coverage Results. When conducting a weapon system analysis, it is necessary that lethal area input to the matrix evaluator program reflects an estimate of the system. This is accomplished by averaging individual lethal areas (and \( P_i \)'s) over the complete family of terminal conditions available to the system of interest. These parameters include burst height distribution, range usage rates, posture and terrain weighting factors, and type fuze (PD or VT) usage rates. These factors are used to generate the average damage matrix, which is combined with the system accuracy in the matrix evaluator program to compute the overall weapon system effect. Typically, the output from this program yields the fractional coverage as a function of the number of volleys for the selected rectangular targets. A direct comparison of these results with those of a standard would give the increase (decrease) in the number of volleys or rounds required to provide a predetermined level of incapacitation. An example of fractional coverage is shown below.

<table>
<thead>
<tr>
<th>Target Size, M</th>
<th>Fractional Coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of Volleys</td>
</tr>
<tr>
<td>2.3 x 2.3</td>
<td>1</td>
</tr>
<tr>
<td>10 x 50</td>
<td>2</td>
</tr>
<tr>
<td>100 x 100</td>
<td>3</td>
</tr>
</tbody>
</table>

NOTE: 2.3 x 2.3 represents an individual; 10 x 50 a squad; and 100 x 100, a company.
Figure 3. Apparent target area.
Figure 4. Superimposition of target and volley damage matrix.
Figure 5.

Lethal area vs burst height - Tactical situation A; troop posture A; terrain A
Figure 6.

Lethal area vs burst height - Tactical situation A; troop posture B; terrain A
Figure 7.
Lethal area vs burst height - Tactical situation A; troop posture C; terrain A
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APPENDIX A
EVALUATING PROJECTILE FLIGHT DATA

To evaluate the standard deviations in range and deflection ($\sigma_R$ and $\sigma_D$, respectively), the following model is used:

\[
\begin{align*}
\sigma_R^2 &= \text{Var}(R) = \text{Var}(\text{ammo + wpn}) + \text{Var}(\text{met}) + \text{Var}(\text{aiming}) \\
\sigma_D^2 &= \text{Var}(D) = \text{Var}(\text{ammo + wpn}) + \text{Var}(\text{met}) + \text{Var}(\text{aiming})
\end{align*}
\]

in which: $R$ = range  
$D$ = deflection

The center of impacts (or the center of projected airburst points) coincides with the center of the target. Range firings will provide the experimental data required by the ballistician for evaluation of the unit effects for major factors contributing to dispersion. These data include:

1. On-site corrections to be made by the gunner  
2. Probable errors in range, deflection, and height of burst
APPENDIX B
REFERENCES


3. TOP 3-2-825, Location of Impact or Airburst Positions, 2 November 1976.


6. BRL Memorandum Report 1203 (U).