TRANSMISSION OF ULTRASONIC WAVES THROUGH A SOLID LAYER IMMERSED—ETC(U)

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TRANSMISSION OF ULTRASONIC WAVES THROUGH A SOLID LAYER IMMERSED IN LIQUID

BY GEE IN GOO

RESEARCH AND TECHNOLOGY DEPARTMENT

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**Title:** Transmission of Ultrasonic Waves Through a Solid Layer Immersed in Liquid

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**ABSTRACT:**
This report presents a method using an existing theoretical expression, for determining the transmission coefficient of ultrasonic waves incident on a thin plastic plate at an arbitrary oblique angle. The theoretical results were substantiated by inhouse and published data. Frequencies investigated range from 100 to 1000 kHz and materials investigated were acrylic, plexiglas and low-density polyethylene.
FOREWORD

The design and development of liquid filled acoustic lenses necessitated a theoretical method which can accurately predict the transmission coefficient of an acoustic wave propagating through layered media at various angles to the normal. This report describes such a method for determining the transmission coefficients and demonstrates its accuracy against available published data. Funding for this effort was provided by the Naval Sea Systems Command, Task No. SO266001/U13CA.

The author would like to thank Dr. Bruce Hartmann for his guidance in this endeavor, Mr. Paul Huber for his assistance in verifying the accuracy of the method against published data, and Mr. W. Rust, Head, Electrical Design Branch, for reviewing this report.

IRA BLATSTEIN
By direction
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INTRODUCTION

Many materials are used in underwater acoustic systems for items such as transducer windows, acoustic concentrators, acoustic lens, acoustic tank linings, and reflectors. Their application depends strongly on their characteristic properties in transmission, reflection, and absorption. The continuous development in sonar domes and acoustic lens necessitated the investigation of ultra-sonic wave through single and multilayered thin plastic materials. Until recent years there has been limited experimental data and theoretical results. Often the transmission and reflection coefficients are measured instead of calculated. To date, there seems to be several methods available for theoretically approximating the transmission characteristics of the acoustic materials. This report presents one of the methods and its calculated results.

One of the many applications of acoustic materials in underwater acoustics is in the construction of acoustic lens. In particular, this program has an interest in a spherical liquid filled lens as diagrammed in Figure 1. A liquid-filled spherical lens basically consists of a thin plastic shell of the desired transmission coefficient and a liquid of the desired index of refraction \((n_2)\). It has the properties of converging the acoustic rays toward a focal point along the lens axis as shown in Figure 1. Its focal distance, \(f\), from center of the spherical lens is directly related to the index of refraction of the lens liquid.

To select the proper acoustic window for an acoustic lens, it is necessary to select the transmission characteristics of alternative materials. These characteristics are evaluated on the basis of determining the percentage of sound transmitted through and reflected from the sample material immersed in a liquid media. Through the transmission characteristic of a material, one can predict the approximate aperture of an acoustic lens. For a desired aperture, material of certain transmission characteristic will be selected. Having selected a desired aperture one would have defined the desired operational characteristic such as beam width of the acoustic lens.

The problem of transmission and reflection of ultrasonic waves through layered media has been treated by many authors. The theoretical treatment dates back to W. T. Thomson\(^1\) in 1950. In his paper, he treated the transmission and reflection problem as a boundary value problem. At the boundary, the stress and

particle velocity are assumed to be continuous. Brekhovskikh’s treatment paralleled that of Thompson’s and yields a similar expression which is valid only to limited cases, i.e., single-layer problem as indicated by Mr. D. Folds. In 1953, Haskell correctly formulated an expression which is applicable to multilayered cases. Since our interest is a single solid layer immersed in a liquid media and because Brekhovskikh’s solution is a widely used reference by many researchers, we have employed his expression for this analysis.


THEORY

Paralleling Brekhovskikh's development with Mr. P. Huber's assistance the same expressions were derived for a single solid layer surrounded by a liquid media as D. Folds and Barnard, et al.\(^5\) had indicated in their papers. The transmission (t) and reflection (r) coefficient expressions for a single solid layer surrounded by liquid media as diagrammed in Figure 2 are given by:

\[
T = \frac{2NZ}{M(Z_1 + Z_3) + i(N^2 - M^2)Z_1 + Z_3}
\]

\[
R = \frac{M(Z - Z) + i(N^2 - M^2)Z - Z}{1 \cdot \frac{1}{M(Z_1 + Z_3) + i(N^2 - M^2)Z_1 + Z_3}}
\]

where \(M, N,\) and \(Z\)'s are expressed as:

\[
M = \frac{Z \cos^2 2\gamma_2}{Z_1} \cot P + \frac{Z}{Z_1} \cdot \sin^2 \frac{Z \gamma_2 \cot Q}{Z_1}
\]

\[
N = \frac{Z \cos^2 2\gamma_2}{Z_1} \cdot \frac{Z}{Z_1} \cdot \sin^2 \frac{Z \gamma_2}{Z_1} \cdot \sin Q
\]

and

\[
Z = \frac{\rho \cdot c}{\cos \theta_1}
\]

where the subscripts 1, 2, and 3 indicate the medium of interest while 2t represents the shear (transverse) mode of propagation in solid layer, media 2. Also, ρ's are the densities and c's and b's are the velocities of the respective media. P and Q are two "dummy" variables which are related to the propagation constant for the longitudinal wave, a, shear wave, β, and thickness, d, of the sample material

\[ P = \frac{a}{d} \]
\[ Q = \frac{b}{d} \]

Since a represents the z-component of the propagational constant \( k_2 \) of the longitudinal wave while \( \beta \) is the like quantity \( k_2 \) of the shear wave, thus a and \( \beta \) can be expressed as

\[ a = -k_2 \cos \theta_2 \]
\[ \beta = -k_2 \cos \gamma_2 \]

where \( \theta_2 \) and \( \gamma_2 \) are related to the incident angle \( \theta_3 \) by the Snell's Law

\[ k_3 \sin \theta_3 = k_2 \sin \theta_2 = k_1 \sin \theta_1 = k_2 \sin \gamma_2 \]

\( \theta_3, \theta_2, \theta_1, \) and \( \gamma_2 \) are measured with respect to the normal to the plane of the solid layer and these quantities may be complex. Thus, the quantities P and Q can be rewritten as;

\[ P = -k_2 d \cos \theta_2 \]
\[ Q = -k_2 d \cos \gamma_2 \]

Note that \( k_2 \) and \( k_2 \) are wave numbers in the longitudinal and shear mode which are given respectively by

\[ k = \frac{\omega}{2c_2} \]
where $c_2$ and $b_2$ are respectively the longitudinal and shear sound velocities in the solid layer of thickness, $d$, and $f$ is the frequency of interest.

Since absorption is present in most solid materials and it is related to the sound velocity $c$ and $b$ in the solid by the expression below

$$c = \frac{c_0}{\sqrt{(1 + \phi_c)^2}} \exp \left( i \tan^{-1} \phi_c \right)$$

where

$$\phi_c = \frac{c_2}{W}$$

$\alpha$ and $\beta$ are redefined as the absorption constants expressed in neper/ft. The absorption can be related to the attenuation by

$$\alpha = \frac{a}{8.6858}$$

where $a$ is the attenuation in db/ft. Note that $c_2$ and $b_2$ are now complex quantities. It is indeed difficult to image physically that the sound velocity and reflected angles in the solid layer as complex quantities.
EXPERIMENTAL DATA

The experimental test set up at NAVSURFWPNCEN acoustical facility is basically that of References 3, 5, and 6, except for the collimator lens which was used as diagrammed in Figure 3. The collimator was used to insure that the impinging wave at the sample plate would be a plane wave (far-field effect). The receive hydrophone E-27 is located close to the sample plate to avoid edge effects; yet far enough to avoid standing waves between the hydrophone and the sample plate. The test sample plate was a 24-inch square and hydrophone locations are as shown in Figure 4.

Since only limited experimental measurements were made at NSWC and the sound velocities and test material are not known, experimental data used to verify calculated results were basically from References 3 and 6. Figure 5 to Figure 8 show results from Reference 3 versus NSWC calculated results. Figure 9 to Figure 16 show the data from Reference 3 versus NSWC calculated results. The material parameters from References 3 and 7 used in the theoretical calculations are listed in Table 1.

3See footnote 3 on page 9.
5See footnote 5 on page 11.
RESULTS

Brekhovskikh's expression was used in the theoretical calculation to predict the transmission coefficients. To better estimate the coefficients, two assumptions were made. First, the velocities c and b were allowed to be complex; thus, the wave numbers in the solid layer $k_2$ and $K_2$ are also complex. These complex quantities were used throughout the calculations. Secondly, instead of assuming the attenuations in the longitudinal and shear modes, $a$ and $\beta$ respectively are the same, they are assumed to have the relation of

$$\frac{a}{\lambda_s} = 2\beta \left( \frac{b}{c} \right)$$

Since $\lambda_L$ and $\lambda_S$, the respective wavelength, are defined as

$$\lambda_L = \frac{c}{f}$$
$$\lambda_S = \frac{b}{f}$$

thus, the above can be rewritten as

$$a = 2\beta \left( \frac{b}{c} \right)$$

and $\beta$ can be expressed as

$$\beta = \frac{a}{2} \left( \frac{c}{b} \right)$$

Throughout the development of the method of approximation it has been observed that the correct sound velocity measurements are very important. For example, in determining the theoretical transmission coefficient for Absonic-A at 19.1°C of Reference 5, there was no difficulty arriving at the same data as the measured data when measured parameters of Reference 5 were used. These results were shown in Figures 5 through 8. However, if other published

5See footnote 5 on page 11.
values were used for the Absonic-A sound velocities, the calculated results were very different from the measured data. In the case of Reference 3, using the values in Table 1, the calculated transmission coefficient was a good approximation of the published measured data for a single solid layer. This is shown in Figure 9 for LDPE; Figures 10 through 13 for plexiglas; and Figures 14 and 15 for ABS.

It is evident from this theoretical investigation that the material transmission coefficient is dependent on frequency, temperature and thickness of solid layer. For example, Figures 10 through 13 show the effect on the transmission coefficient of a 0.1 inch thick plexiglas plate as a function of frequency. A similar effect is shown on Figures 5 through 8 and Figures 10 through 18 for Absonic-A (ABS). This frequency effect is observed both theoretically and experimentally as shown in these plots. Since sound velocity changes as a function of temperature, one could expect some change in the transmission coefficient. In the case of the materials considered, the temperature effect is amplified greatly because sound velocity increases with increase of temperature in the liquid (water) while the contrary exists in the solid layer. The observed results are shown in Figures 19 through 23 for low-density polyethylene (LDPE). However, material with low thermal coefficients such as ABS has very little effect due to temperature change as shown in Figures 24 through 26. Thus careful selection of material could reduce the temperature effect on the transmission coefficient. Lastly, thickness of the shell also has some effect on the transmission coefficient as shown in Figures 27 through 36. From these plots one could observe the changes on transmission coefficient as a function of thickness.

Aperture shading is well known and has been used successfully in optics and underwater acoustics. In general, aperture shading is used to improve the directivity response of an acoustic system. The transmission coefficient on an acoustic lens shell has the same effect as that of a shading function. Thus, a lens of a certain operating characteristic would require a certain desired transmission coefficient on its shell. Through careful consideration of the material, thickness, frequency, and operating temperature range, a lens shell can be selected to exhibit a desired transmission coefficient with aperture.

\footnote{See footnote 3 on page 9.}
CONCLUSION

It is evident from this investigation that the acoustic transmission and reflection coefficient of a thin plate can be accurately predicted by employing the Brekhovskikh's expressions in conjunction with an accurately determined sound velocity in the thin plate material. Thus, this technique can be useful in assisting the selection of acoustic material for acoustic lens of the operating characteristic desired for the acoustic system.
FIGURE 6  TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE AT 100 kHz FOR ABS MATERIAL
Relative transmission loss as function of incident angle at 1000 kHz for the ABS material from Reference 6.

**FIGURE 8** TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE AT 1 mHz FOR ABS MATERIAL
Transmission coefficient versus incidence angle for 0.317-cm-thick low-density polyethylene (dashed line: Theoretical, solid line: Measured) from Reference 3.

FIGURE 9 TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE AT 500 kHz FOR LOW-DENSITY POLYETHYLENE MATERIAL
Transmission coefficient versus incidence angle for 0.254-cm-thick Plexiglas plate (dashed line: Theoretical, Solid Line: Measured) from Reference 3.

FIGURE 10  TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE AT 100 kHz FOR PLEXIGLAS MATERIAL
Figure 11. Transmission loss as a function of incident angle at 200 kHz for plexiglas material.
FIGURE 13  TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE AT 400 kHz FOR PLEXIGLAS MATERIAL

PLEX AT 20 DEGREES C
400000 FREQ. - HZ
0.1 INCHES THICK
24 ATTEN. IN DB/FT
FIGURE 14 TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE AT 100 kHz FOR ABS MATERIAL
FIGURE 15 TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE AT 200 kHz FOR ABS MATERIAL
FIGURE 16 TRANSmission LOSS AS FUNCTION OF INCIDENT ANGLE AT 300 kHz FOR ABS MATERIAL
ABS AT 20 DEGREES C
400000 FREQUENCY
0.1 INCHES THICK
29 ATTEN. IN DB/FT

FIGURE 17 TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE AT 400 kHz FOR ABS MATERIAL
Figure 18: Transmission loss as function of incident angle at 500 kHz for ABS material. ABS at 20 degrees C, 500000 freq- Hz, 0.1 inches thick, 33 atten. in db/ft.
FIGURE 19 TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE
AT 20°C FOR LOW-DENSITY POLYETHYLENE MATERIAL

LDPE AT 20 DEGREES C
300000 FREQ - HZ
0.1 INCHES THICK
96 ATTEN. IN DB/FT
Figure 21: Transmission Loss as Function of Incident Angle for Low-Density Polyethylene Material

LDPE at 18 Degrees C
300000 Hz
8.1 Inches Thick
91 Attenu. in dB/ft

Phase Angle
FIGURE 22 TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE AT 5°C FOR LOW-DENSITY POLYETHYLENE MATERIAL

LDPE AT 5 DEGREES C
300000 FREQ- HZ
0.1 INCHES THICK
89 ATTEN. IN DB/FT
FIGURE 23 TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE
AT 0°C FOR LOW-DENSITY POLYETHYLENE MATERIAL

LDPE AT 0 DEGREES C
300000 FREQ. HZ
0.1 INCHES THICK
86 ATTEN. IN DB/FT
ABS AT 20 DEGREES C
300000 FREQ. Hz
0.1 INCHES THICK
2S ATTEN. IN DB/FT

FIGURE 24 TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE AT 20°C FOR ABS MATERIAL
Figure 36: Transmission loss as function of incident angle at 0°C for ABS material.
Figure 27: Transmission loss as function of incident angle for .05 inch-thick ABS material.
FIGURE 28  TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE FOR .1 INCH THICK ABS MATERIAL
FIGURE 29 TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE FOR .125 INCH-THICK ABS MATERIAL
FIGURE 30 TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE FOR .2 INCH-THICK ABS MATERIAL
FIGURE 31 TRANSMISSION LOSS AS FUNCTION OF INCIDENT ANGLE FOR .25 INCH-THICK ABS MATERIAL

ABS AT 20 DEGREES C
100000 FREQ- Hz
.25 INCHES THICK
18 ATTEN. IN DB/FT
Figure 34: Transmission loss as function of incident angle for 0.5 inch-thick ABS material.
Figure 35 Transmission loss as a function of incident angle for .75 inch-thick ABS material.
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<th>Material</th>
<th>Longitudinal velocity c (m/sec)</th>
<th>Shear velocity b (m/sec)</th>
<th>Longitudinal absorption a (dB/m 100 kHz)</th>
<th>Shear absorption β (dB/(kHz))</th>
<th>Density (g/cm³)</th>
<th>( \frac{dc}{dt} ) (m/sec deg)</th>
<th>( \frac{db}{dt} ) (m/sec deg)</th>
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\(^3\) See footnote 3 on page 9.
\(^7\) See footnote 7 on page 15.
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