The final report for Air Force Grant No. F-49620-79-C-0193 consists of the annual progress reports dated November 21, 1979 and August 25, 1980, as well as the following report for 1980-81:

During 1980-81 various numerical experiments were made on one-dimensional model inverse problems modelled by a method that is extendable to higher dimensions. The underlying problem is to recover the speed of propagation or the shape of an object from a scattered field.

The one-dimensional problem that was investigated was based on the wave equation with potential

\[ u_{tt} - u_{xx} + q(x)u = 0 \]

where the function to be recovered is the potential \( q(x) \).

The model initial conditions were \( u = 0 \), \( u_t = -2\delta'(x) \).

The model boundary condition was \( u = 0 \) on \( x = 0 \). And the "extra" condition which determines \( q(x) \) is either (a) \( \delta u/\delta x \) on \( x = 0 \) for all time or (b) the scattered field \( u\delta(t-x) + R(t-x) \) at \( x = +\infty \). The free space (q=0) solution is \( u = \delta(x-t) - \delta(x+t) \). The distorted plane wave \( P(x,t) \) is defined by the condition that it is a solution of the equation and at \( x = -\infty \), \( t = +\infty \),
it becomes $\delta(t-x)$ (exactly if $q$ has compact support) and $P$ is written as $\delta(t-x) + P_{\text{scat}}$.

The linearized equation based on the assumption that $q << 1$ and that terms of order $q^2$ may be neglected, yields for (a)

$$q = + 2 \frac{d}{dx} \left[ H(2x) - P_{\text{scat}}(0,-2x) \right]$$

with

$$\left. \frac{du}{dx} \right|_{x=0} = H(t) \text{ for } t > 0 ;$$

$$q = -2 \frac{d}{dx} R(2x)$$

for (b).

The numerical method is based on the well-known formula that follows from the propagation of singularities

$$q = 2 \frac{d}{dx} \lim_{t \to x^+} u(x,x)$$

and then representing $u(x,x)$ by means of distorted plane waves. (All these have analogous formulas in higher dimensions.)

The analogues of the above equation are for (b)

$$q = -2 \frac{d}{dx} R(2x) + \frac{d}{dx} \int_0^\infty (P_{\text{scat}}(x,x-s) - P_{\text{scat}}(-x,x-s)) R(s) \, ds$$

and for (a) we insert in this formula...
The idea is to use one of these formulas as part of an iteration. We guess q and P and recompute q using the above formulas for cases (a) or (b).

Note that if the given data vanishes in case (b) we recover \( q \equiv 0 \) but in case (a) we do not.

We have been unable to implement (a) as planned. The reason is that we must operate in a finite region and use a radiation condition at a finite distance. In our case for example with q a simple quadratic with support in \( |x| < 2 \), over \(-8 \leq x \leq 8\) and \(0 \leq t \leq \ldots\). The errors produced are compounded in case (a). The potential could only be recovered to 40% of its original value with an error of about 5% of the maximum potential. However, the potentials were well beyond the range of the linear theory.

In case (b) even with a coarse mesh the results are surprisingly worse.

The results are being written up as a technical report to be issued at the Courant Institute.
BIBLIOGRAPHY


This report summarizes progress made in various numerical experiments made on one-dimensional model inverse problems.