FOREIGN TECHNOLOGY DIVISION

EVALUATION OF PARAMETERS OF THE OSCILLATORY MOTION OF DIGITAL SERVO SYSTEM IN A STEADY-STATE MODE

by

V.P. Strakhov, Ye.G. Kuznetsova

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EDITED TRANSLATION

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**U.S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM**

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*ye initially, after vowels, and after ь, ь; е elsewhere.
When written as ö in Russian, transliterate as ý or Ö.

**RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS**

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i
Examine is a digital servo system (TsSS) characterized by the quantization of a signal with respect to level and time. The steady-state mode in the system is characterized by the emergence of an oscillating motion. The dependence of parameters of the oscillating motion on parameters of the system is given.

A TsSS with a three-positional relay characteristic of the control element was examined in reference [2]. Shown is the possibility of analysis of the dynamics of the TsSS by means of the representation of its motion on a phase surface in the presence of quantization of the signal with respect to level and time.

The purpose of this work is to evaluate parameters of the oscillatory motion in the transient and steady-state modes.

The trajectory of the representing point, which characterizes the behavior of the TsSS on the phase plane, will consist of segments each of which can be described by the equation of trajectories:

\[ X = X_0 + \dot{X}_0 - \Phi(t) \]  

with the appropriate values of \( \Phi(t) \). [Translator's note: see footnote on next page.]
Let us examine the case when the representing points $M_1$ and $M_2$ during time $T_4$ occupy the extreme positions $H$ and $C$, respectively, (Fig. 1).

![Diagram](image)

Fig. 1. Determination of parameters of the oscillatory motion in the transient and steady-state modes.

The maximum overswing in the transient mode with respect to the position of equilibrium corresponds to $X_{MAX}^{'}$ for the trajectory of motion of the representing point $M_1$ and $X_{MAX}^{''}$ for the trajectory of motion of the representing point $M_2$. $X_{MAX}^{'} = X_1 + a_1 \phi_1$.

Having taken in equation (1)

$$\Phi(0) = 0; \quad X_0 = 0.5 \Lambda, \quad Y_0 = 1,$$

we get

$$X_i = 0.5 \Lambda + 1 + Y.$$

Having substituted into equation of line ND: $Y = \frac{1}{e^{\Delta t}} X - \frac{0.5 \Lambda}{e^{\Delta t}}$

[2] the value $X_s$, after conversions, we obtain $Y_s = \frac{1}{e^{\Delta t}}$, then

$$X_s = \frac{e^{\Delta t} - 1}{e^{\Delta t}} + 0.5 \Lambda.$$

*The change in the velocity and coordinate are described, correspondingly, by these equations:

$$V = V_0 e^{-\phi} + \Phi(t) (1 - e^{-\phi}), \quad (2) \quad \dot{X} = X_0 + Y_0 (1 - e^{-\phi}) + \Phi(t) (1 - (1 - e^{-\phi})).$$
From equation (1), having assumed that \( X_0 = 0; \ Y_0 = 0; \ Y_s = Y; \ X = x_{10}, \)
we get
\[
\begin{align*}
  a_i b_i & = -\frac{1}{e^u} - \ln \left( \frac{1}{e^u} + 1 \right), \\
  X_{\text{wave}}^* & = \frac{e^u - 1}{e^u} \cdot 0.5A_x \cdot -\frac{1}{e^u} \cdot \ln \left( 1 - e^u \right),
\end{align*}
\]
or
\[
\begin{align*}
  X_{\text{wave}}^* & = T_n + 1 + 0.5A_x \cdot \ln \left( 1 - e^u \right), \\
  X_{\text{wave}}^* & = X_c + a_2 b_2.
\end{align*}
\]

The segment BC is numerically equal to the time of the comparison of codes \( T_u \), then \( X_v = T_n - 0.5A_x \).

From equation (1), having assumed that \( \Theta(\delta) = -1; \ X_0 = 0; \ Y_0 = Y_c = 1; \ X = a_2 b_2; \ Y = 0, \) we obtain \( a_2 b_2 = 1-\ln 2, \) and then
\[
X_{\text{wave}}^* = 1-\ln 2+T_n-0.5A_x \quad \text{or}
\]
\[
X_{\text{wave}}^* = T_n - 0.5A_x \cdot \ln 0.3069.
\]

From equations (4) and (5) it is seen that the maximal over-swing of the system in the transient mode is determined by the quantity \( T_u \). The mutual position of of points \( X_{\text{MAKC}}' \) and \( X_{\text{MAKC}}'' \) varies from the magnitude \( T_u \).

Having equated equations (4) and (5), after transformations, we obtain the value \( T_u \), for which \( X_{\text{MAKC}}' = X_{\text{MAKC}}'' \):
\[
T_u = \ln (1 - 2e^u).
\]

The presence of regions of switching of function \( \Theta(\delta) \) lying in the quadrants I and III of plane XY, is the reason for the emergence in the TsSS of the undamped oscillating motion in the steady-state mode. Each trajectory from any initial point with the lapse of a definite time interval enters into a defined region and from this moment no longer emerges from it.

Two kinds of closed trajectories of the representing point are possible in the steady-state mode. Figure 1 gives closed trajectories of types I and II. The trajectory of type I consists of two segments of the integral curve which correspond to the controlling action
Trajectory II consists of segments of the integral curve with values \( \Phi(\delta) = 1; \Phi(\delta) = 0 \) and \( \Phi(\delta) = -1 \). Segments \( mn \) and \( m'n' \) correspond to the absence of the controlling action, i.e.,
\[ \Phi(\delta) = 0. \]

The time of the movement of the representing point from \( m \) to \( n \) (or from \( m' \) to \( n' \)) is equal to \( \Delta t = T_u \).

From Fig. 1 it is evident that the region of switching CHE (closed trajectory of type I) is considerably larger than the switching region MBHE (closed trajectory of type II). To determine parameters of the oscillating motion in the steady-state mode, let us examine the maximal possible closed trajectory, i.e., the trajectory of type I.* The period of the closed trajectory \( T_0 \) should consist of a finite even number of time segments \( T_u \) (sampling is considered). The time of the half-cycle \( T_0/2 \), i.e., the time between the switching points \( p \) and \( p' \) should correspond to
\[ \frac{T_u}{2} = mT_u. \tag{7} \]
where \( m = 1, 2 \) and \( 3 \).

The maximal value of the velocity with a steady-state oscillating motion (at the moment of switching of the controlling action) can be found by assuming that in equation (2) \( Y_0 = Y_\alpha \); \( Y = -Y_\alpha \); \( \Phi(\delta) = -1 \); and \( \tau = mT_u \). By examining the movement from point \( p \) to \( p' \), we find
\[ Y_s = \frac{1 - e^{-mT_u}}{1 + e^{-mT_u}}. \tag{8} \]

According to equation (8), several limiting closed trajectories can be established in the system. It is necessary to determine the parameters of the maximally possible closed trajectory.

The period of the maximally possible closed trajectory \( T_{max} = 2mT_u \) must be less than the period of the closed trajectory, the switching points of which lie on lines \( AF \) and \( A'F' \).**

From the condition of point transformation of line \( AF \) into line

*The presence of several closed trajectories of the representing point is possible in the region [1].

**Lines \( AF \) and \( A'F' \) are limits of regions of switching with the two-position characteristic of \( \mu \) the controlling element.
AF', it is possible to find the dependence of $Y^*$ on $T_q$, where $Y^*$ is the maximum velocity of the closed trajectory, the switching points of which lie on the indicated line AF and A'F'.

The period of the steady-state oscillations in the system of maximal amplitude can be determined from condition $T_{\text{osc}} < T_q$. The dependence of $Y^*$ on $T_q$ is given on Fig. 2.

![Fig. 2. Limit of the region of the existence of the limiting closed trajectory with the largest parameters.](image)

Having used equation (1), let us define the doubled amplitude of oscillations of the outlet coordinate $2A_x$ in the steady-state mode

$$2A_x = \ln \frac{1}{(1-Y_{\text{max}})(1+Y_{\text{max}})}.$$

(9)

Dependences $Y_{\text{max}}$, $T_q$ and $2A_x$ define the basic relations of the oscillating motion from parameters of the system.

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Submitted

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