IDENTIFICATION OF WHEELSET/RAIL CREEP COEFFICIENTS
FROM DYNAMIC RESPONSE DATA USING THE MAXIMUM
LIKELIHOOD PARAMETER IDENTIFICATION TECHNIQUE-

by

William N. Herzog
Princeton University
School of Engineering and Applied Science
Department of Mechanical and Aerospace Engineering

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**Identification of Wheelset/Rail Creep Coefficients from Dynamic Response Data Using the Maximum Likelihood Parameter Identification Technique**

**William N. Herzog**

**AFIT STUDENT AT: Princeton University**

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This thesis explores the application of the maximum likelihood parameter identification technique to determine the wheel/rail creep coefficients using dynamic response data. The equations of motion for the dynamically scaled wheelset are presented and the reduced form of the maximum likelihood equations as applicable to the dynamically scaled wheelset model are developed. The maximum likelihood equations were formulated into a maximum likelihood algorithm which was implemented in Fortran IV. Using simulated wheelset data, the effects of a random input representation of the track versus a deterministic input with uncertainty representation are determined. The effects of various levels of measurement noise are also examined. This preliminary analysis indicates that the deterministic representation of the track input yields better results. Representing the track as a random track input requires further investigation into the effects of longer data records and smaller time steps on the performance of the maximum likelihood algorithm.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>i</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>ii</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>iii</td>
</tr>
<tr>
<td>List of Figures</td>
<td>v</td>
</tr>
<tr>
<td>List of Tables</td>
<td>viii</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>ix</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Chapter 1. Maximum Likelihood Theory</td>
<td>4</td>
</tr>
<tr>
<td>1.1 General Form of the Maximum Likelihood Equations</td>
<td>4</td>
</tr>
<tr>
<td>1.2 Application of the Maximum Likelihood Equations to the Wheelset</td>
<td>11</td>
</tr>
<tr>
<td>1.3 Maximum Likelihood Algorithm</td>
<td>15</td>
</tr>
<tr>
<td>1.4 Test Data Generation</td>
<td>19</td>
</tr>
<tr>
<td>Chapter 2. Experimental Program</td>
<td>21</td>
</tr>
<tr>
<td>2.1 Objectives</td>
<td>21</td>
</tr>
<tr>
<td>2.2 Experimental Facility</td>
<td>21</td>
</tr>
<tr>
<td>2.3 Transducers</td>
<td>22</td>
</tr>
<tr>
<td>2.4 Experimental Procedure</td>
<td>34</td>
</tr>
<tr>
<td>2.5 Measurement Noise</td>
<td>35</td>
</tr>
<tr>
<td>2.6 Filtering</td>
<td>36</td>
</tr>
<tr>
<td>2.7 Digitization</td>
<td>36</td>
</tr>
<tr>
<td>Chapter 3. Results</td>
<td>39</td>
</tr>
<tr>
<td>3.1 Outline of Test Cases</td>
<td>39</td>
</tr>
</tbody>
</table>
### 3.1.1 Explanation of Test Case Data .......................... 39
### 3.1.2 Purpose of Each Test Case ............................... 41
### 3.2 Method of Testing the Maximum Likelihood Program ............ 43
### 3.3 Test Case Results ........................................ 46
  **3.3.1 Likelihood Function** .................................. 46
  **3.3.2 Gradient and Fisher Information Matrix** .................. 67
### 3.4 Application of Results to Actual Wheelset Data .................. 72
### References ..................................................... 76
### Appendix A. Derivation of $\frac{3E(t)}{3\theta}$ ....................... 77
### Appendix B. Solution to $\frac{3F(t_1)}{3\theta}$ ...................... 78
### Appendix C. Definitions of Variables in F and G Matrices ............ 81
### Appendix D. Simplification of $\frac{3F(t_1)}{3\theta}$ for Wheelset Problem ..... 82
### Appendix E. Listing of the Maximum Likelihood Program ............... 83
### Appendix F. List of APL Functions Used to Generate Wheelset Simulated Data .. 112
### Appendix G. List of the Program Used to Operate the A/D Converter ... 116
### Appendix H. Plots of the Likelihood Function, Observation Term and Bias Term for Test Cases 1 through 9 ................ 119
LIST OF FIGURES

Chapter 1

Figure 1.1 Maximum Likelihood Algorithm
Figure 1.2 Development Form of Maximum Likelihood Algorithm

Chapter 2

Figure 2.1 Side View of Wheelset Idler-Carriage Relationship
Figure 2.2 Top View of Wheelset Idler-Carriage Relationship
Figure 2.3 Wheelset Instrumentation
Figure 2.4 Transducer Signal Conditioning
Figure 2.5 Definition of Lateral Displacement
Figure 2.6 Calibration of Number One Displacement Transducer
Figure 2.7 Calibration of Number Two Displacement Transducer
Figure 2.8 Sample Responses of Wheelset Model to Random Track Inputs
Figure 2.9 Sampling Interval

Chapter 3

Figure 3.1 Representations of Track Input
Figure 3.2 Computer Output for the Last Iteration of Test Case Number 8
Figure 3.3 Plot of the Likelihood Function for Test Case 2
Figure 3.4 Plot of the Observation Term for Test Case 2
Figure 3.5 Plot of the Bias Term for Test Case 2
Figure 3.6 Plot of the Observation Term for Test Case 4
Figure 3.7 Plot of the Observation Term for Test Case 6
Figure 3.8 Plot of the Bias Term for Test Case 4
Figure 3.9 Plot of the Bias Term for Test Case 6
Figure 3.10 Plot of the Likelihood Function for Test Case 4
Figure 3.11 Plot of the Likelihood Function for Test Case 6
Figure 3.12 Plot of the Likelihood Function for Test Case 7
Figure 3.13 Average Slope Method for Checking the Gradient and Second Partial of the Likelihood Function
Figure 3.14 Plot of the Gradient for Test Case 4
Figure 3.15 Plot of the Gradient for Test Case 6
Appendix H
Figure H.1 Plot of the Likelihood Function for Test Case 1
Figure H.2 Plot of the Observation Term for Test Case 1
Figure H.3 Plot of the Bias Term for Test Case 1
Figure H.4 Plot of the Likelihood Function for Test Case 2
Figure H.5 Plot of the Observation Term for Test Case 2
Figure H.6 Plot of the Bias Term for Test Case 2
Figure H.7 Plot of the Likelihood Function for Test Case 3
Figure H.8 Plot of the Observation Term for Test Case 3
Figure H.9 Plot of the Bias Term for Test Case 3
Figure H.10 Plot of the Likelihood Function for Test Case 4
Figure H.11 Plot of the Observation Term for Test Case 4
Figure H.12 Plot of the Bias Term for Test Case 4
Figure H.13 Plot of the Likelihood Function for Test Case 5
Figure H.14 Plot of the Observation Term for Test Case 5
Figure H.15 Plot of the Bias Term for Test Case 5
Figure H.16 Plot of the Likelihood Function for Test Case 6
Figure H.17 Plot of the Observation Term for Test Case 6
Figure H.18 Plot of the Bias Term for Test Case 6
Figure H.19 Plot of the Likelihood Function for Test Case 7
Figure H.20  Plot of the Observation Term for Test Case 7
Figure H.21  Plot of the Bias Term for Test Case 7
Figure H.22  Plot of the Likelihood Function for Test Case 8
Figure H.23  Plot of the Observation Term for Test Case 8
Figure H.24  Plot of the Bias Term for Test Case 8
Figure H.25  Plot of the Likelihood Function for Test Case 9
Figure H.26  Plot of the Observation Term for Test Case 9
Figure H.27  Plot of the Bias Term for Test Case 9
LIST OF TABLES

Chapter 2
Table 2.1 Filter Cutoff Frequencies

Chapter 3
Table 3.1 Summary of Maximum Likelihood Program Testing
Table 3.2 Q and R Matrices In Test Cases 1 Through 5
Table 3.3 Summary of Results for Test Cases
NOMENCLATURE

A  a general matrix
B  covariance matrix of V
E[f]  expected value of f
F  system dynamics matrix for dynamically scaled wheelset model
G  system input matrix for dynamically scaled wheelset model
H  measurement matrix for dynamically scaled wheelset model
K  Kalman gain matrix
$K_y, K_{\psi}$  lateral (yaw) spring constants, n/m (n-m/rad)
L  likelihood function
M  Fisher information matrix
M  mass of dynamically scaled wheelset model, kg
N  end of summation index for likelihood function
N(x,y)  Gaussian distribution with mean x and covariance matrix y
P  error covariance of Kalman Filter state estimates
Q  covariance matrix of random input disturbance
R  covariance matrix of measurement noise
T  period of a signal
V  translational velocity of the dynamically scaled wheelset model
Y  transformation used to solve matrix Riccati equation and \( DF/\partial \theta \) using a Kalman-Engler solution technique
a  element of a general matrix
b  bias in the yaw potentiometer mounted on the dynamically scaled wheelset model
Lateral damping coefficient, n-sec/m

Electrical bias added to the yaw potentiometer output by the onboard analog computer

Creep coefficient, n

Summation index, product index

Imaginary part of a complex number

Distance from wheelset c.g. to contact point between wheel and rail, m

Slope of the straight line function which describes the characteristics of the displacement transducers

Probability density

Wheel rolling radius, m

Time

Deterministic input into a dynamic system

Measurement noise vector

Random input into a dynamic system

Vector of state variables for a dynamic system

Vector of estimates of the state variables of a dynamic system

Lateral displacement of wheelset, m

Wheel conicity, rad

Factor which multiplies the nominal values for the creep coefficients to get the true values for the creep coefficients

Matrix which multiplies the deterministic input in the Kalman Filter equation for the propagation of the state variable estimates

Transformation used to derive \( \partial P / \partial \theta \) using the Kalman-Engler solution technique

Small increment

Distance from rail to inertial reference
\( \delta_1, \delta_2 \) distance of left and right rails from fixed reference, m

\( \overline{\delta} \) distance of track centerline from reference \((= (\delta_1 + \delta_2)/2)\), m

\( \varepsilon \) error criterion

\( \xi \) term of the likelihood function

\( \eta \) yaw potentiometer signal after processing on the on-board analog computer

\( \theta \) vector of unknowns to be identified using the maximum likelihood parameter identification technique

\( \Theta \) matrix which multiplies the measurement vector in the Kalman Filter equation for the propagation of the state variable estimates

\( \Lambda \) matrix which multiplies the state variable estimates in the equation for the propagation of the sensitivity vector \( \partial \dot{x} / \partial \theta \)

\( \Pi \) product

\( \Sigma \) summation

\( \Theta \) matrix which multiplies the measurement vector in the equation for the propagation of the sensitivity vector \( \partial \dot{x} / \partial \theta \)

\( \Phi \) state transition matrix

\( \Psi \) matrix which multiplies the deterministic input in the equation for the propagation of the sensitivity vector \( \partial \dot{x} / \partial \theta \)

\( \Omega \) axle rotational velocity \((= V/r_o), \text{sec}^{-1}\)

\( \Lambda \) transformation used in solving the matrix Riccati equation using the Kalman-Engler solution technique

\( \nu \) the innovation \( z - H\hat{x} \)

\( \sigma \) eigenvalues of the dynamically scaled wheelset model system dynamics matrix
\( \phi \)  element of \( \Phi \)
\( \psi \)  yaw angle of dynamically scaled wheelset model

**Subscripts**

- \( i \)  discrete point in time
- \( L \)  left side
- \( n \)  last of a sequence of discrete value
- \( r \)  right side
- \( y \)  lateral
- \( ss \)  steady state
- \( \psi \)  yaw
- \( o \)  starting point
- \( 11 \)  subscript of longitudinal creep coefficient
- \( 22 \)  subscript of lateral creep coefficient

**Sub-subscripts**

- \( o \)  nominal value

**Superscript**

- \( * \)  true value
INTRODUCTION

The dynamic response of railroad vehicles to track irregularity inputs is determined by interactions between the vehicle body and suspension components, kinematic constraints dependent on the profiles of the wheels and rails, and friction forces generated at the wheel/rail interface. These latter forces, known as creep forces, result from small differential velocities between wheel and rail in directions tangential and lateral to the wheel. Linearized dynamic models for rail vehicles represent creep forces using the slope of the creep force versus relative velocity function at the origin; the slopes are known in the literature as creep coefficients.

The values of these coefficients has been determined from Hertzian contact theory by Kalker (4) and by measurements under steady-state conditions (9). However, previous analysis (12) of dynamic response data has indicated that better agreement between theory and experiment is obtained if values for creep coefficients are used that are up to 50% lower than determined in (4) and (9). The "best" values for creep coefficients under dynamic conditions have not been determined with precision; such determination would be of great value to the rail research technical community.

The objective of this thesis is the estimation of creep coefficients from experimental vehicle response data using advanced parameter identification techniques. To focus the effort on estimation of creep coefficients, data used are measurements of the response of a simplified scale model vehicle, whose characteristics and parameters are well-understood, exclusive of the
creep coefficients. Use of the simplified model, in this case a wheelset (single axle with two wheels and lateral and yaw suspension, capable of two degrees-of-freedom motion), permits examination of the fundamental kinematic hunting and instability characteristic of rail vehicles, without the effects of more complex vehicle configurations obscuring the results. The use of dynamically scaled models (9) allows better control of experimental conditions than expected in full scale.

The method used to estimate the creep coefficients is the maximum likelihood parameter identification technique. This technique has been used to identify aircraft stability derivatives successfully for several years, and represents the state-of-the-art in parameter estimation. The specific objective of this thesis is to develop and implement the maximum likelihood method for dynamically scaled wheelset models. In addition some insight into the best way to conduct dynamic experiments involving the dynamically scaled wheelset model was desired.

This research program was divided into two phases. The first was the experimental phase during which the dynamic wheelset experiments were conducted. The purpose of the experiments was to obtain data that could be processed using a maximum likelihood processor. The second stage of the research project was a developmental one. During this part of the research program the maximum likelihood algorithm was developed and implemented into a Fortran IV computer program. The final phase of research was the test phase during which simulated wheelset data was generated using a computer model of the dynamically scaled wheelset model. The maximum likelihood computer program was then checked out using this simulated data.

The experimental phase was conducted first so that as much information as possible about the wheelset model could be obtained prior to the development
of the maximum likelihood program. The actual wheelset data taken was not analyzed using the maximum likelihood program. The results for the simulated data suggest that more research needs to be done with simulated data so that more conclusive results can be drawn when the actual wheelset data is processed.

The body of this thesis is divided into three chapters. The first presents the generalized maximum likelihood equations and then develops the reduced equations that apply to the dynamically scaled wheelset model. The second chapter discusses the experimental phase of the research program. The knowledge gained from these initial experiments will be used to develop an improved experimental program for the next series of data. The final chapter presents the results obtained for the simulated data, and the interpretation of these results in terms of actual wheelset data.
1.1 General Form of the Maximum Likelihood Equations

The maximum likelihood parameter identification technique is used to identify unknown parameters of a dynamic system given measurements of some or all of the system's state variables. This method maximizes the probability of the given measurements conditioned on the vector of unknowns. The following development of the maximum likelihood equations will be for a general case, linear, time-varying system. Except where otherwise noted the equations in this section were taken from References (8) and (3).

The state-space form of the differential equations for a linear dynamic system is

\[ \dot{x}(t) = F(t)x(t) + L(t)u(t) + G(t)w(t) \]  

(1.1)

where \( x(t) \) = state variable vector

\( u(t) \) = known input vector

\( w(t) \) = input disturbance vector

The measurement process is described by the equation

\[ z(t) = H(t)x(t) + v(t) \]  

(1.2)

where \( z(t) \) = measurement vector

\( x(t) \) = state variable vector

\( v(t) \) = measurement noise vector

The foundation of the maximum likelihood method is maximizing the conditional probability of the vector for unknowns given the measurements of the states.
maximize \( p(\theta | z_n) \)

where \( p \) denotes the probability density

\( \theta \) = vector of unknowns to be identified

\( z_n \) = sequence of measurements

Maximizing \( p(\theta | z_n) \) is equivalent to maximizing the log likelihood function.

\[ L = \ln[p(\theta | z_n)] \] (1.3)

where \( L \) is log likelihood function

Sequential application of Baye's Rule to the conditional probability of \( \theta \) given \( z_n \) yields:

\[
p(\theta | z_n) = p(z_n | \theta) \frac{1}{p(\theta | z_{n-1})}
= p(z_n | \theta) \frac{1}{p(\theta | z_{n-1})} \frac{1}{p(\theta | z_{n-2})}
= \prod_{i=1}^{N} p(z_i | \theta) \frac{1}{p(\theta | z_{i-1})} \frac{1}{p(\theta | z_{i-2})} \frac{1}{p(\theta | z_{i-3})}
\] (1.4)

Because the state vector is dependent upon the vector of unknowns the equation

\[
p(\theta | z_n) = \prod_{i=1}^{N} p(z_i | \theta) \frac{1}{p(\theta | z_{i-1})} \frac{1}{p(\theta | z_{i-2})} \frac{1}{p(\theta | z_{i-3})}
\] (1.5)

can be re-written as

\[
p(\theta | z_n) = \prod_{i=1}^{N} p(z_i | x_i(\theta)) \frac{1}{p(\theta | z_{i-1})} \frac{1}{p(\theta | z_{i-2})} \frac{1}{p(\theta | z_{i-3})}
\] (1.6)

Assuming the input disturbance \( (w(t)) \) and the measurement noise \( (v(t)) \) are gaussian, then the formula for multivariate gaussian conditional distribution can be applied because \( p(z_i | x_i(\theta)) \) is gaussian.

\[
p(z_i | x_i(\theta)) = \frac{1}{(2\pi)^{N/2} |B_i|^{1/2}} \cdot e^{-\frac{1}{2}(z_i - E[z_i])^T B_i^{-1}(z_i - E[z_i])}
\] (1.7)
In equation 1.7 \( E[z_1] = H_1 \hat{X}_1 \) where \( \hat{X} \) is the output of a Kalman Filter. The matrix \( B_1 \) is defined as follows:

\[
B_1 = E[(z_1 - E[z_1])(z_1 - E[z_1])^T]
\]

\[
B_1 = E[(z_1 - H_1 \hat{X}_1)(z_1 - H_1 \hat{X}_1)^T]
\]

\[
B_1 = H_1 P_1 H_1^T + R_1
\]  \( (1.8) \)

Combining equations 1.3, 1.6 and 1.7 yields:

\[
L = \ln\left[ \prod_{i=1}^{N} \frac{1}{2\pi(\pi)^{n/2}} |B_1|^{1/2} e^{-\frac{1}{2}(z_1 - H_1 \hat{X}_1)^T B_1^{-1}(z_1 - H_1 \hat{X}_1)} \right] + \ln|p(0)|  
\]

\[
L = -\frac{1}{2} \sum_{i=1}^{N} \left\{ (z_1 - H_1 \hat{X}_1)^T B_1^{-1}(z_1 - H_1 \hat{X}_1) + \ln|B_1| \right\} - \frac{1}{2} \ln(2\pi) + \ln|p(0)|  
\]

\[
L = -\frac{1}{2} \sum_{i=1}^{N} \left\{ (z_1 - H_1 \hat{X}_1)^T B_1^{-1}(z_1 - H_1 \hat{X}_1) + \ln|B_1| \right\} + \text{constant}  
\]  \( (1.9) \)

The maximum likelihood problem involves maximizing equation 1.11 with respect to \( \theta \), subject to the constraints of the Kalman Filter equations. The Kalman Filter equations must be satisfied because the state estimate \( \hat{X}_1 \) and the state covariance \( P_1 \) are outputs of the Kalman Filter. A summary of the continuous time, time varying Kalman Filter equations are given below.

**System Model:**

\[
\dot{x}(t) = F(t)x(t) + L(t)u(t) + G(t)w(t)  
\]  \( (1.12) \)

**Measurement Model:**

\[ z(t) = H(t)x(t) + v(t)  
\]  \( (1.13) \)

**State Estimates:**

\[ \hat{x}(t) = F(t)\hat{x}(t) + L(t)u(t) + K(t)z(t) - H(t)\hat{x}(t)  
\]  \( (1.14) \)
Error Covariance Propagation:

\[ \dot{P}(t) = F(t)P(t) + P(t)F^T(t) + G(t)Q(t)G^T(t) - P(t)H^T(t)R^{-1}(t)H(t)P(t) \]  \hspace{1cm} (1.15)

Kalman Gain Matrix:

\[ K(t) = P(t)H^T(t)R^{-1}(t) \]  \hspace{1cm} (1.16)

where \( \nu(t) \sim \text{N}(0,R(t)) \)  \hspace{1cm} (1.17)

and \( \nu(t) \sim \text{N}(0,Q(t)) \)  \hspace{1cm} (1.18)

The notation used in equation 1.17 denotes that \( \nu(t) \) is a random sequence with a gaussian probability density with a mean value and variance defined below.

\[ E[\nu(t)] = 0 \]  \hspace{1cm} (1.19)

\[ E[(\nu(t) - E[\nu(t)])(\nu(t) - E[\nu(t)])^T] = R(t) \]  \hspace{1cm} (1.20)

Equation 1.14 is solved on the digital computer for discrete values of \( t \) using equations 1.21 through 1.24.

\[ \dot{x}(t_0 + \Delta t) = \Phi(t_0, \Delta t)\dot{x}(t_0) + \Theta(t_0)\nu(t_0) + \Gamma(t_0)u(t_0) \]  \hspace{1cm} (1.21)

where

\[ [F(t_o) - K(t_o)H(t_o)]\Delta t \Phi(t, \Delta t) = e \]  \hspace{1cm} (1.22)

\[ \Theta(t_o) = \Phi(t_o, \Delta t)[F(t_o) - K(t_o)H(t_o)]^{-1} \cdot [I - \Phi^{-1}(t_o, \Delta t)]K(t_o) \]  \hspace{1cm} (1.23)

\[ \Gamma(t_o) = \Phi(t_o, \Delta t)[F(t_o) - K(t_o)H(t_o)]^{-1} \cdot [I - \Phi^{-1}(t_o, \Delta t)]L(t_o) \]  \hspace{1cm} (1.24)

The error covariance propagation equation (1.15) is solved using the Kalman-Engler solution.

\[ P(t_0 + \Delta t) = [\phi_{\nu\nu}(t_o, \Delta t) + \phi_{\nu\lambda}(t_o, \Delta t)P(t_o)] \cdot [\phi_{\nu\nu}(t_o, \Delta t) + \phi_{\nu\lambda}(t_o, \Delta t)P(t_o)]^{-1} \]  \hspace{1cm} (1.25)

where

\[
\begin{bmatrix}
\dot{y}(t_0 + \Delta t) \\
\dot{\lambda}(t_0 + t)
\end{bmatrix} =
\begin{bmatrix}
\phi_{\nu\nu}(t_0, \Delta t) & \phi_{\nu\lambda}(t_0, \Delta t) \\
\phi_{\lambda\nu}(t_0, \Delta t) & \phi_{\lambda\lambda}(t_0, \Delta t)
\end{bmatrix}
\begin{bmatrix}
y(t_0) \\
\lambda(t_0)
\end{bmatrix}
\]  \hspace{1cm} (1.26)
and

$$\Phi(t, \Delta t) = \begin{bmatrix} \phi_{yy}(t, \Delta t) & \phi_{y\lambda}(t, \Delta t) \\ \phi_{\lambda y}(t, \Delta t) & \phi_{\lambda\lambda}(t, \Delta t) \end{bmatrix}$$

(1.27)

The equations for $\dot{y}(t)$ and $\dot{\lambda}(t)$ are given in equation 1.28.

$$\begin{bmatrix} \dot{y}(t) \\ \dot{\lambda}(t) \end{bmatrix} = \begin{bmatrix} -F^T(t) & H^T(t)R^{-1}(t)H(t) \\ G(t)Q(t)G^T(t) & F(t) \end{bmatrix} \begin{bmatrix} y(t) \\ \lambda(t) \end{bmatrix}$$

(1.28)

Therefore

$$\Phi(t, \Delta t) = \exp \left[ \begin{bmatrix} -F^T(t_o) & H^T(t_o)R^{-1}(t_o)H(t_o) \\ G(t_o)Q(t_o)G^T(t_o) & F(t_o) \end{bmatrix} \Delta t \right]$$

(1.29)

The linear system of equations in 1.29 is derived in Reference (3).

The likelihood function can be expanded into a Taylor series with only three terms because the likelihood function for Gaussian conditional probability distribution is quadratic.

$$L(\theta) = L(\theta_o) + \frac{\partial L}{\partial \theta} |_{\theta_o} (\theta - \theta_o) + \frac{1}{2} \frac{\partial^2 L}{\partial \theta^2} |_{\theta_o} (\theta - \theta_o)^T (\theta - \theta_o)$$

(1.30)

Taking the partial derivative of $L(\theta)$ with respect to $\theta$ and setting it equal to zero to find the maximum:

$$\frac{\partial L}{\partial \theta} |_{\theta^*} = \frac{\partial L}{\partial \theta} |_{\theta_o} + \frac{\partial^2 L}{\partial \theta^2} |_{\theta_o} (\theta^* - \theta_o) = 0$$

(1.31)

where $\theta^*$ is the value of $\theta$ that results in the maximum of the likelihood function. Solving for $\theta^*$ yields:

$$\theta^* = \theta_o - \left[ \frac{\partial^2 L}{\partial \theta^2} |_{\theta_o} \right]^{-1} \left[ \frac{\partial L}{\partial \theta} |_{\theta_o} \right]$$

(1.32)

In equation 1.32 $\Delta \theta$, or the step in theta is equal to:
\[ \Delta \theta = - \left[ \frac{\partial^2 L}{\partial \theta^2} \right]^{-1} \left[ \frac{\partial L}{\partial \theta} \right]^T \]  

(1.33)

\[ \frac{\partial L}{\partial \theta} \] is defined as the gradient and is equal to:

\[ \frac{\partial L}{\partial \theta} = \sum_{i=1}^{N} v^T(t_i) B^{-1}(t_i) \frac{\partial \nu(t_i)}{\partial \theta} \]

\[ - \frac{1}{2} v^T(t_i) B^{-1}(t_i) \frac{\partial B(t_i)}{\partial \theta} B^{-1}(t_i)^T v(t_i) \]

\[ + \frac{1}{2} \text{tr} \left[ B^{-1}(t_i) \frac{\partial B(t_i)}{\partial \theta} \right] \]

(1.34)

where

\[ v(t_i) = z(t_i) - H(t_i) \hat{x}(t_i) \]

\[ \frac{\partial \nu(t_i)}{\partial \theta} = -H(t_i) \frac{\partial \hat{x}(t_i)}{\partial \theta} - \frac{\partial H(t_i)}{\partial \theta} \]

(1.35)

and

\[ \frac{\partial B(t_i)}{\partial \theta} = H(t_i) \left[ P(t_i) \frac{\partial (H(t_i))^T}{\partial \theta} + \frac{\partial P(t_i)}{\partial \theta} H(t_i)^T \right] \]

\[ + \frac{\partial H(t_i)}{\partial \theta} P(t_i) H(t_i)^T + \frac{\partial H(t_i)}{\partial \theta} \]

(1.36)

\[ \frac{\partial H(t_i)}{\partial \theta} \]

The term \( \frac{\partial^2 L}{\partial \theta^2} \) is defined as the Fisher information matrix.

\[ \frac{\partial^2 L}{\partial \theta^2} = \sum_{i=1}^{N} \frac{\partial \nu^T(t_i)}{\partial \theta} B^{-1}(t_i) \frac{\partial \nu(t_i)}{\partial \theta} \]

\[ - (2) v^T(t_i) B^{-1}(t_i) \frac{\partial B(t_i)}{\partial \theta} B^{-1}(t_i)^T v(t_i) \]

\[ - \frac{1}{2} \text{tr} \left[ B^{-1}(t_i) \frac{\partial B(t_i)}{\partial \theta} B^{-1}(t_i) \frac{\partial B(t_i)}{\partial \theta} \right] \]

(1.37)

The term \( \frac{\partial \nu(t_i)}{\partial \theta} \) in equation 1.36 is referred to as the sensitivity term and is derived from the Kalman Filter state estimate equation (1.14).
\[
\frac{\partial x(t)}{\partial \theta} = F(t) \frac{\partial x(t)}{\partial \theta} + \frac{\partial F(t)}{\partial \theta} x(t) + \frac{\partial L(t)}{\partial \theta} u(t) - K(t)H(t) \frac{\partial x(t)}{\partial \theta} \\
- K(t) \frac{\partial H(t)}{\partial \theta} x(t) + \frac{\partial K(t)}{\partial \theta} \frac{\partial x(t)}{\partial \theta} \frac{\partial x(t)}{\partial \theta} - \frac{\partial K(t)}{\partial \theta} H(t) x(t) \quad (1.39)
\]

See Appendix A for the derivation of equation 1.39.

Equation 1.39 can be solved for discrete values of \( \frac{\partial x(t)}{\partial \theta} \) on the digital computer using the following equations.

\[
\frac{\partial x(t)}{\partial \theta} \Delta = \frac{\partial x(t) + \Delta t}{\partial \theta} = \Phi(t_o, t) \frac{\partial x(t)}{\partial \theta} + \Lambda(t_o) x(t_o) \\
+ \Psi(t_o) u(t_o) + T(t_o) z(t_o) \quad (1.40)
\]

where

\[
\Phi(t_o, t) = e^{-\int_{t_o}^{t} \frac{\partial F(t)}{\partial \theta} dt} \left( K(t_o) g(t_o) - \frac{\partial H(t_o)}{\partial \theta} \right) \\
\Lambda(t_o) = \Phi(t_o, t) \left( K(t_o) g(t_o) - \frac{\partial H(t_o)}{\partial \theta} \right)^{-1} \\
[1 - \Phi^{-1}(t_o, \Delta t)] \left[ \frac{\partial F(t_o)}{\partial \theta} - K(t_o) \frac{\partial H(t_o)}{\partial \theta} - \frac{\partial K(t_o)}{\partial \theta} H(t_o) \right] \quad (1.42)
\]

\[
\Psi(t_o) = \Phi(t_o, \Delta t) [F(t_o) - K(t_o) H(t_o)]^{-1} \\
[1 - \Phi^{-1}(t_o, \Delta t)] \left[ \frac{\partial L(t_o)}{\partial \theta} \right] \quad (1.43)
\]

\[
T(t_o) = \Phi(t_o, \Delta t) [F(t_o) - K(t_o) H(t_o)]^{-1} \\
[1 - \Phi^{-1}(t_o, t)] \left[ \frac{\partial K(t_o)}{\partial \theta} \right] \quad (1.44)
\]

The term \( \frac{\partial F(t)}{\partial \theta} \) in equation 1.37 is solved using equation 1.45.

\[
\frac{\partial F(t_o + \Delta t)}{\partial \theta} = \left[ \Phi_{yy}(t_o, \Delta t) + \frac{\partial F(t_o, \Delta t)}{\partial \theta} \right] \left[ \Phi_{yy}(t_o, \Delta t) + \frac{\partial F(t_o, \Delta t)}{\partial \theta} \right]^{-1} \quad (1.45)
\]
Equation 1.45 is an iterative solution to $\frac{\partial P(t+t)}{\partial \theta}$ and is derived in Appendix B.

The final unknown term in the sensitivity equation is $\frac{\partial K(t)}{\partial \theta}$. This term is derived by differentiating equation 1.16 with respect to theta

$$\frac{\partial K(t)}{\partial \theta} = P(t) \left[ \frac{\partial H^T(t)}{\partial \theta} R^{-1}(t) + H^T(t) \frac{\partial R^{-1}(t)}{\partial \theta} \right] + \frac{\partial P(t)}{\partial \theta} H^T(t) R^{-1}(t)$$  \hspace{1cm} (1.46)

1.2 Application of the Maximum Likelihood Equations to the Wheelset

The equations developed in the last section apply to any linear dynamic system. In this section those equations will be tailored to the dynamically scaled wheelset problem.

The wheelset equations of motion are given in equations 1.47 through 1.50.

$$\dot{x}(t) = Fx(t) + Gw(t)$$  \hspace{1cm} (1.47)

where

$$F = \begin{bmatrix} -2\alpha_{22} + cV + \frac{2\alpha_{22}}{M} & \frac{-K_y}{M} & \frac{-\gamma V}{M} \\ 0 & \frac{-K_{yy}}{2\alpha_{11}} & \frac{-\alpha V}{\alpha_{11}} \\ 1 & 0 & 0 \end{bmatrix} \hspace{1cm} x = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \gamma - \delta \end{bmatrix}$$  \hspace{1cm} (1.48)

$$G = \begin{bmatrix} c \\ T \\ 0 \\ \dot{\delta} \\ -1 \end{bmatrix}$$  \hspace{1cm} (1.49)

and

$$w(t) \sim N(0,Q)$$  \hspace{1cm} (1.50)

See Reference (11) for a detailed development of the wheelset equations of motion. Appendix C contains definitions for the variable of the F and G matrix.
The wheelset measurement equation is given in (1.51)

\[ z(t) = Hx(t) + v(t) \]  

where

\[ H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  

and

\[ v(t) \sim N(0, R) \]  

The system dynamics matrix \( F \), the system random input matrix \( G \), and the system measurement matrix \( H \), the covariance matrix for the random input disturbance \( Q \) and the covariance matrix for the measurement noise \( R \) are all time-invariant.

The creep coefficients \( f_{11} \) and \( f_{22} \) in the system dynamics matrix are the parameters to be identified by the maximum likelihood method.

For the purposes of simplifying the maximum likelihood equations the actual creep coefficients will be defined as a multiple of their nominal values.

\[ f_{22} = \beta_{22} f_{22} \]  

\[ f_{11} = \beta_{11} f_{11} \]  

where, for simplicity \( \beta_{22} = \beta_{11} = \beta \)  

The nominal values for the creep coefficients have been identified in other research on railroad vehicle stability \( \beta \).

Combining equations 1.46 and 1.54 through 1.56 the system dynamics matrix is:

\[ F = \begin{bmatrix} -2\beta f_{22} + cV & 2\beta f_{22} & -K_x \\ \frac{N}{M} & -K_y & 0 \\ 0 & \frac{N}{M} & -\alpha \frac{V}{k_r} \\ 1 & 0 & 0 \end{bmatrix} \]  

(1.57)
Since the system dynamics matrix contains the only parameter to be identified the vector of unknowns $\bar{\theta}$ reduces to a scalar.

$$\bar{\theta} = \theta$$  \hspace{1cm} (1.55)

The maximum likelihood equations of the last section can be greatly simplified for two reasons:

1) the wheelset is a time-invariant system.

2) the system dynamics matrix ($F$) contains the only parameter to be identified.

A summary of the time-invariant Kalman Filter Equations for the wheelset system is given below

State Estimates:

$$\dot{x}(t) = Fx(t) + Kz(t) - KH(t)$$ \hspace{1cm} (1.59)

Error Covariance Propagation:

$$\dot{P}(t) = FP(t) + P(t)F^T + QG^T - P(t)H^TH - 1HP(t)$$ \hspace{1cm} (1.60)

Kalman Gain Matrix:

$$K = P_{ss}H^TR^{-1}$$ \hspace{1cm} (1.61)

$$\mathbf{v}(t) \sim N(0,R)$$ \hspace{1cm} (1.62)

$$\mathbf{w}(t) \sim N(0,Q)$$ \hspace{1cm} (1.63)

Equation 1.21, the solution to the Kalman filter estimates equation (1.14), reduces to

$$\ddot{x}(t_o+\Delta t) = \phi(\Delta t) \ddot{x}(t_o) + \Theta \bar{z}(t_o)$$ \hspace{1cm} (1.64)

where

$$\phi(\Delta t) = e^{[F-KH]\Delta t}$$ \hspace{1cm} (1.65)

$$\Theta = \phi(\Delta t)[F-KH]^{-1} \cdot [I - \phi^{-1}(\Delta t)]K$$ \hspace{1cm} (1.66)

$P_{ss}$ in equation 1.61 is the steady state solution to equation 1.67

$$P(t_o+\Delta t) = [\phi_{xx}(\Delta t) + \phi_{\lambda\lambda}(\Delta t) P(t_o)] \cdot [\phi_{yy}(\Delta t) + \phi_{\gamma\gamma}(\Delta t) P(t_o)]^{-1}$$ \hspace{1cm} (1.67)

This equation is the same as equation 1.25 except that $\phi$ is no longer dependent
on \( t_o \), but is a function only of \( \Delta t \), as shown in equation 1.68.

\[
\Phi(\Delta t) = \exp \begin{bmatrix} -F^T & H^T R^{-1} H \\ \text{QG}^T & F \end{bmatrix} \Delta t
\] (1.68)

Because all of the elements of the matrix in equation 1.68 are time invariant, the iterative solution for \( P \) has to be solved only once. This value of \( P_{S0} \) is used to calculate the Kalman gain matrix according to equation 1.C1. The Kalman gain matrix for a time invariant system such as the wheelset remains constant for the entire Kalman gain problem.

Taking into account the two simplifications listed above, the maximum likelihood equations reduce to:

\[
L = \frac{1}{2} \left[ \sum_i v_i(t_i)B^{-1}v(t_i) + \ln|B| \right] 
\] (1.69)

The constant term in equation 1.68 can be dropped because it shifts the likelihood function up or down but does not change the location of the maximum of the likelihood function.

\[
\frac{\partial L}{\partial \theta} = \sum_i v_i(t_i)B^{-1} \frac{\partial v(t_i)}{\partial \theta} - \frac{1}{2} v_i(t_i)B^{-1} \frac{\partial B}{\partial \theta} B^{-1} v(t_i) + \frac{1}{2} \text{tr}[B^{-1} \frac{\partial B}{\partial \theta}]
\] (1.70)

\[
\frac{\partial v(t_i)}{\partial \theta} = -H \frac{\partial \theta(t_i)}{\partial \theta}
\] (1.71)

\[
\frac{\partial B}{\partial \theta} = H \frac{\partial \theta(t_i)}{\partial \theta} H^T
\] (1.72)

\[
\frac{\partial^2 L}{\partial \theta^2} = \sum_i v_i(t_i)B^{-1} \frac{\partial v(t_i)}{\partial \theta} - \frac{1}{2} v_i(t_i)B^{-1} \frac{\partial B}{\partial \theta} B^{-1} v(t_i) + \frac{1}{2} \text{tr}[B^{-1} \frac{\partial B}{\partial \theta} B^{-1} \frac{\partial B}{\partial \theta}]
\] (1.73)

\[
\frac{\partial \theta(t_i)}{\partial \theta} = F \frac{\partial \theta(t_i)}{\partial \theta} + \frac{\partial F}{\partial \theta} \delta(t_i) - KH \frac{\partial \theta(t_i)}{\partial \theta} \delta(t_i) - \frac{\partial K}{\partial \theta} \delta(t_i) - \frac{\partial H}{\partial \theta} \theta(t_i)
\] (1.74)
\[
\frac{\partial \xi(t)}{\partial \theta} = \frac{\partial \xi(t + \Delta t)}{\partial \theta} = \Phi(\Delta t) \frac{\partial \xi(t)}{\partial \theta} + \Lambda \xi(t) + T \xi(t) \quad (1.75)
\]

where

\[
\Phi(\Delta t) = e^{[(F-K)H] \Delta t} \quad (1.76)
\]

\[
\Lambda = \Phi(\Delta t) [F-K]^\dagger \cdot \left( I - \Phi^{-1}(\Delta t \right) \left[ \frac{\partial \xi}{\partial \theta} \right] \left[ \frac{\partial \xi}{\partial \theta} \right] \frac{\partial \xi}{\partial \theta} H \quad (1.77)
\]

\[
T = \Phi(\Delta t) [F-K]^\dagger \cdot \left( I - \Phi^{-1}(\Delta t) \right) \left[ \frac{\partial \xi}{\partial \theta} \right] \frac{\partial \xi}{\partial \theta} \quad (1.78)
\]

The term \( \frac{\partial \xi}{\partial \theta} \) is the steady state solution to equation 1.79.

\[
\frac{\partial \xi(t + \Delta t)}{\partial \theta} = \left[ \phi_{yy}(\Delta t) + \phi_{yy}(\Delta t) \frac{\partial \xi(t) - 1}{\partial \theta} \right] \cdot \left[ \phi_{yy}(\Delta t) + \phi_{yy}(\Delta t) \frac{\partial \xi(t)}{\partial \theta} \right] \quad (1.79)
\]

Equation 1.79 is a time invariant form of equation 1.45 and since \( \frac{\partial \xi}{\partial \theta} \) is time invariant for the wheelset problem, equation 1.79 is solved only once during each iteration of the maximum likelihood processor. Equation 1.79 is derived in Appendix D.

Equation 1.46 which defines \( \frac{\partial K(t)}{\partial \theta} \) reduces to:

\[
\frac{\partial K(t)}{\partial \theta} = \left[ \frac{\partial P(t)}{\partial \theta} \right] H^{-1} \quad (1.50)
\]

1.3 Maximum Likelihood Algorithm

The equations of the preceding section were implemented into a Fortran IV computer program. This section discusses the algorithm used to solve the wheelset maximum likelihood problem on the digital computer.

The maximum likelihood method is a batch processor in that the entire set of measurements is used for each iteration of the maximum likelihood equations (8). Each iteration of the maximum likelihood algorithm produces an estimate of the value of the parameter to be identified which will maximize
the likelihood function given in equation 1.69. This estimate $\theta^*$ then replaces $\theta_0$ within the program and another iteration is made. When the following condition is met, iteration is stopped.

$$\theta^* - \theta_0 = \Delta \theta < \epsilon$$  \hspace{1cm} (1.81)

During the development phase of a maximum likelihood processor it is helpful to calculate the value of the likelihood function ($L$), the gradient ($\frac{\partial L}{\partial \theta}$), and the Fisher information matrix ($\frac{\partial^2 L}{\partial \theta^2}$) for a range of $\theta$ where the maximum of the likelihood function is expected to lie. Changes in the flowchart of Figure 1.1 are shown in Figure 1.2. The flowchart at the bottom of the dotted box replaces the $\Delta \theta$ test at the end of the flowchart in Figure 1.1. This procedure is beneficial in that the maximum likelihood program can be checked out without the risk of an infinite loop being set up because of a convergence problem.

No mention has been made so far of the method used to calculate the state transition matrix (STM). On a digital computer, a very fast and accurate way to calculate the STM is through a power series expansion.

$$\Phi(\Delta t) = I + A \Delta t + \frac{1}{2!} A^2 (\Delta t)^2 + \frac{1}{3!} A^3 (\Delta t)^3 + \ldots + \frac{A^i (\Delta t)^i}{i!}$$  \hspace{1cm} (1.52)

The series is truncated when the following condition is satisfied

$$\left( \sum_{i=1}^{N+1} A^i (\Delta t)^i \frac{1}{i!} + I \right) - \left( \sum_{i=1}^{N} A^i (\Delta t)^i \frac{1}{i!} + I \right) < \epsilon$$  \hspace{1cm} (1.53)

where $\epsilon$ is some error criterion chosen by the user. A value of

$$\epsilon = 1.0 \cdot 10^{-10}$$  \hspace{1cm} (1.54)

is usually considered to be sufficient. A listing of the maximum likelihood program is contained in Appendix E.
Figure 1.1 Maximum Likelihood Algorithm (8).

A

READ IN PARAMETERS

B

CALCULATE P FROM EQU. 1.67

C

CALCULATE 2

D

CALCULATE 2

E

CALCULATE 2

F

CALCULATE 2

G

CALCULATE 2

H

CALCULATE q(t)

I

CALCULATE THE KALMAN GAIN MATRIX K FROM EQU. 1.51

J

CALCULATE THE STATE ESTIMATES (x)

K

CALCULATE THE SENSITIVITY SOLUTION FROM EQU. 1.70 TO 1.76

L

CALCULATE THE INNOVATION (t-

M

CALCULATE THE LIKELIHOOD FUNCTION FROM EQU. 1.69

N

FORM THE MULTIPLE SOLUTION TO THE KALMAN FILTER PROBLEM FROM EQU. 1.68

O

CALCULATE THE STATE TRANSITION MATRIX FOR THE MATRIX ABOVE FROM EQU. 1.68

P

END TO 11

Q

NO

R

YES

S

END

T

NO

U

YES

V

END TO 11

W

END TO 11

X

END TO 11

Y

END TO 11

Z

END TO 11
Figure 1.2 Development Form of Maximum Likelihood Algorithm.
1.4 Test Data Generation

In order to ensure that the maximum likelihood computer program implement the maximum likelihood algorithm properly, it was tested using simulated wheelset data. The data was generated using the state transition matrix approach to solve the following equation.

\[ \dot{X} = FX + GW \]  

where \( F \) and \( G \) are specified by equations 1.57 and 1.49 respectively. Due to limitations in computer storage a total of 1000 data points for each state was all that could be generated.

The time step (\( \Delta t \)) was chosen by examining the eigenvalues of the \( F \) matrix. Typical values for the poles of the wheelset model are:

\[ \sigma_1 = -210.7 \text{ sec}^{-1} \]

\[ \sigma_{2,3} = -0.423 \pm j 5.982 \text{ sec}^{-1} \]

These poles indicate that transient responses are characterized by a lightly damped sinusoid (\( \omega_n = 6.0 \text{ sec}^{-1}; \zeta = 0.07 \)) plus a rapidly converging exponential decay (\( \tau = \frac{1}{\omega_n} = 4.7 \text{ msec} \)). In the frequency domain a sharp resonant peak occurs at the natural frequency, with an additional breakpoint at \( \omega = 1/\tau \).

As the estimates of the creep coefficients change, the damping ratio of the low frequency resonance changes, as does the location of the higher frequency real pole. In selecting the time interval \( \Delta T \) and the record length \( N \Delta T \) it is desirable for information on both high and low frequency modes to be present in the data. To obtain six data points per cycle at the higher frequency \( f = \sigma_1/2\pi \) a \( \Delta T \) is chosen to be 0.005 sec. Since computer storage constraints limited the record length to \( N = 1000 \) points, the record length is 5 seconds, which corresponds to 4.8 cycles of the low frequency mode.
Large values for $\Delta t$ were tried but the state transition matrix approach used in the maximum likelihood program had convergence problems. A $\Delta t = .005$ was the largest time step that could be used without causing numerical problems.

In addition to the random track input $w$ in equation 1.85, there was also measurement noise added to the state vectors.

$$z = Hx + v$$  \hspace{1cm} (1.86)

$w$ and $v$ were generated by summing 10 random number vectors which were generated by an APL* operator. The random vectors generated by successive runs of this APL operator are not correlated. Ten of these random vectors were summed to force the distribution of $u$ and $w$ to be approximately Gaussian \( \frac{1}{2} \). Once these gaussian random vectors were formed they could be manipulated to force their means and mean squared values to equal what the user desired.

Although maximum likelihood theory requires $w$ and $v$ to be white noise because these vectors were of finite length they were bandwidth limited. Also since only 10 random vectors were summed, the probability distributions of $w$ and $v$ were not exactly gaussian, only approximately gaussian. A listing of the APL functions used to generate the simulated wheelset data is contained in Appendix F.

* APL: A Programming Language. Implemented on the Princeton University IBM 370/158 time-sharing system.
Chapter 2
EXPERIMENTAL PROGRAM

2.1 Objective

The experimental program was designed to make measurements of the wheelset state variables. The third order wheelset model has the following state variables:

\[ \dot{y} = \text{lateral velocity} \]
\[ \psi = \text{yaw angle} \]

\[ (y-\overline{y}) = \text{wheelset to rail centerline relative lateral displacement} \]

See Reference (11) for a detailed development of the wheelset equations of motion.

2.2 Experimental Facility

The entire experimental program was carried out on the Princeton Dynamic Model Track. Previous research at the Dynamic Model Track involved the development and validation of the dynamically scaled wheelset model and the measurement, restraint and propulsion system for the wheelset (9).

The experimental apparatus consisted of the wheelset model and the idler carriage. Both of these components rode on the 400 foot LEXAN track. The idler carriage surrounded the wheelset but was dynamically isolated from it. The idler carriage provided restraints to prevent the wheelset from completely derailing. The idler carriage was also part of the linkage that connects the wheelset to the propulsion unit. Figures 2.1 and 2.2 diagram the wheelset-idler carriage relationship.
The propulsion system was a hydraulically operated drive unit that rode along a steel I-beam guideway above the LEXAN track. The hydraulic drive unit had a feedback control system for maintaining a constant, user-selected velocity.

Suspended from the propulsion unit on instrument racks were the on-board analog computer, the bridge amplifiers, and the portable four track FM analog cassette recorder. The bridge amplifiers were used to amplify the output of the lateral accelerometer.

2.3 Transducers

Figure 2.3 is a diagram of transducer placement on the wheelset and Figure 2.4 traces the output of the transducers through the signal conditioning equipment to the recorder.

Lateral velocity could not be measured directly, therefore an accelerometer was used to measure lateral acceleration. A bridge amplifier was used to excite the differential type accelerometer and to amplify the output of the accelerometer. The lateral acceleration signal will have to be integrated on a digital computer to obtain lateral velocity.

The yaw angle was measured directly using a potentiometer. The equation relating potentiometer output (v) to the yaw angle (ψ) is:

\[ ψ = mv + b \]  \quad (2.1)

where m is the slope of the straight line described by this function and b is the y-intercept. The intercept b varied from day to day due to amplifier drift. A pre-run and post-run procedure was developed so that the value of b could be determined at the start of every day.
Figure 2.2 Top View of Wheelset Idler-Carriage Relationship.
Figure 2.4 Transducer signal conditioning.
The variations in yaw angle during a run were smaller in magnitude than the value of the \( y \)-intercept. In order to obtain the best resolution on the cassette tape an electrical bias was summed with the yaw potentiometer output on the on-board analog computer. The value of the electrical bias was adjusted from day to day so that the output of the analog computer circuit equalled zero when the yaw angle was zero. The equation that describes the analog computer output is:

\[
\eta = \frac{y-b}{m} + e \tag{2.2}
\]

where \( \eta \) = the output of the analog computer

\( e \) = the electrical bias

A displacement transducer (DCDT) was mounted on each side of the wheelset. Each displacement transducer measured the distance of the wheelset from the inside edge of the track. Figure 2.5 diagrams the relationship between the variable measured by the DCDT's and the state variable \( (y-\delta) \). The equations for the DCDT's are:

\[
V_r = m_r (y-\delta_r) \tag{2.3}
\]

\[
V_L = m_L (y-\delta_L) \tag{2.4}
\]

where \( V_r \) = voltage output of the right DCDT

\( V_L \) = voltage output of the left DCDT

\( m_r \) = slope of the straight line function

\( m_L \) = slope of the straight line function

Multiplying \( V_L \) by \( \frac{m_r}{m_L} \) and adding \( V_r \) to \( V_L \):

\[
V_r + \frac{m_r}{m_L} V_L = m_r (y-\delta_r) + m_r (y-\delta_L) \tag{2.5}
\]

This is the signal that was recorded during the experiment. The on-board
Figure 2.5 Definition of Lateral Displacement.
analog computer was used to multiply the left DCDT signal by \( \frac{m_r}{m_L} \). The state variable \((y - \delta)\) can be derived from the recorded signal by dividing \( V_r + \frac{m_r}{m_L} V_L \) by \( 2m_r \).

\[
\frac{V_r + \frac{m_r}{m_L} V_L}{2m_r} = \frac{2m_r y - m_r (\delta_L + \delta_r)}{2m_r}
\]

(2.6)

\[(y - \delta) = y - \frac{(\delta_L + \delta_r)}{2} = y - \delta \]

(2.7)

The DCDT's have a linear range of operation which is much smaller than their total range of operation. The linear range of operation is slightly different for each DCDT as shown in Figures 2.6 and 2.7. Each DCDT was positioned so that its output was the center of its linear range when the wheelset was centered on the track. Because the DCDT's were not mounted symmetrically about the wheelset's longitudinal centerline, a bias was introduced into the measuring process.

\[
V_L = m_L (y - \delta_L) + b
\]

(2.8)

\[
V_r = m_r (y - \delta_r)
\]

(2.9)

where \( b = \) position bias

The same algebraic operations as previously yield:

\[
\frac{m_r}{m_L} V_L = \frac{m_r}{m_L} m_L (y - \delta_L) + \frac{m_r}{m_L} b
\]

(2.10)

\[
V_r = m_r (y - \delta_r)
\]

(2.11)

\[
V_r + \frac{m_r}{m_L} V_L = m_r (y - \delta_L) + \frac{m_r}{m_L} b + m_r (y - \delta_r)
\]

(2.12)

\[
= 2m_r y - m_r (\delta_L + \delta_r) + \frac{m_r}{m_L} b
\]

(2.13)
Figure 2.6 Calibration of number one displacement transducer.
Figure 2.7 Calibration of number two displacement transducer

Voltage Output

Relative Displacement
Figure 2.8 Typical transient response of wheelset to track inputs. Wheelset has lateral and yaw linear spring suspensions.
\[
\frac{V_r + m_r V_L}{2m_r} = y - \frac{(\delta_L + \delta_r)}{2} + \frac{b}{2m_L}
\]

\[
= (y-\delta) + \frac{b}{2m_L}
\]

(2.44)

(2.45)

In addition to the three variables mentioned above, the wheelset axle angular velocity \( \lambda = v/r_o \) in the rolling direction was also measured. Although the angular velocity is not a state variable, it is necessary to construct the velocity dependent state matrix \( F \). The output of the tachometer was recorded directly with no processing on the on-board analog computer as with the lateral displacement and yaw angle signals. Sample responses from the above transducers are given in Figure 2.8.

2.4 Experimental Procedure

All of the above mentioned signals were recorded on a portable, four track FM analog cassette type recorder* which was carried on board the drive unit. The recorder was started and stopped manually before and after each run.

The bridge amplifier that powered the accelerometer was balanced at the beginning of each day to eliminate the drift in amplifier output that occurred from day to day.

The on-board analog computer had a built-in calibration circuit for the accelerometer bridge amplifier. This calibration circuit produced a known voltage that appeared to the accelerometer as an acceleration load. This acceleration load remained constant from day to day. In order to determine the input-output relationship of the accelerometer, the accelerometer output

* Philips Mini Log 4 Portable Analog Cassette Recorder.
was recorded during the calibration. This calibration test was performed before every data run to account for amplifier drift between data runs.

2.5 Measurement Noise

There were two sources of uncertainty in the data. The first of these was electrical noise. The bridge amplifier, analog computer and hydraulic pump for the propulsion unit all operate on 400 cycle alternating current.

Vibration of the cassette recorder during data runs was another source of noisy data. The steel I-beam guideway that supported the hydraulic drive unit had gaps between the I-beams. The vibration that resulted when the drive unit wheels hit these gaps was transmitted to the recorder, and showed up as noise on the data tapes. As mentioned earlier, the recorder was suspended from the drive unit on an instrument rack which had shock absorbers to help isolate the recorder from vibration. Nonetheless, some vibrations still affected the recorder and added noise to the data.

In addition to the two definite sources of noise, there is a third possible source of noise. It is known that there are small gaps between lengths of LEXAN rail. These small gaps are the result of contraction and expansion of the LEXAN material due to temperature changes. The 800 foot building which houses the Princeton Dynamic Model Track does not have a completely controlled environment; therefore, temperature variations of as much as 50°F are possible. Although no quantitative analysis has been carried out to verify this hypotheses, it is possible for the wheelset structural modes to be excited when the wheels hit these gaps. The transducers mounted on the wheelset, especially the lateral accelerometer, could detect this vibration, thereby adding noise to the data.
Finally to insure the validity of the data, the LEXAN track and wheels were cleaned before every run. Dirt and dust on the track or wheels could alter rail/wheel adhesion, in which case the wheelset model would be invalid. Absolute methanol was used as the cleaning agent because it left no residue on the track.

2.6 Filtering

To remove the noise due to the electrical system and vibration of the wheelset structural modes all of the signals were filtered off-line. The acceleration signal was filtered with a second order low-pass filter. The yaw angle and lateral displacement signals were filtered using seventh or eighth order band-pass filters, and the rotational velocity signal was filtered using a first order low-pass filter. The cutoff frequencies for the filters are given in Table 2.1.

Table 2.1
Filter Cutoff Frequencies

<table>
<thead>
<tr>
<th>Signal</th>
<th>Filter Type</th>
<th>High Pass Cutoff</th>
<th>Low Pass Cutoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral Acceleration</td>
<td>low-pass</td>
<td>N/A</td>
<td>49.9 Hz</td>
</tr>
<tr>
<td>Yaw Angle</td>
<td>band-pass</td>
<td>.1 Hz</td>
<td>60 Hz</td>
</tr>
<tr>
<td>Rolling Angular Velocity</td>
<td>low-pass</td>
<td>N/A</td>
<td>20 Hz</td>
</tr>
<tr>
<td>Wheelset to Rail Relative</td>
<td>band-pass</td>
<td>.1 Hz</td>
<td>60 Hz</td>
</tr>
<tr>
<td>Lateral Displacement</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.7 Digitization

After filtering the data had to be digitized so it could be stored and analyzed on a digital computer. As mentioned in the previous section, the
lowest permissible sampling rate was determined when the low-pass filter cutoff frequencies were chosen. It was decided to use a sampling rate of 120 samples/second per channel. The A/D converter used for digitization was a 12 bit sample and hold A/D converter.* This A/D converter was controlled by a minicomputer system.** The minicomputer had two memories for storing data (memory A and memory B) each of which had the capacity to store 512 data points. When the digitizing process started, the computer stored the digitized data in memory A. Once memory A was full the computer began filling memory B while at the same time writing the data in memory A to magnetic tape. Since the A/D converter was a sample and hold type, there was only a several nanosecond time delay between the sample for each channel. However, the switching process from memory A to memory B took 3 msec., therefore the sampling interval between the 512th data point and the 513th data point was 11.33 msec. The sampling interval between all other sets of four data points was 8.33 msec. Figure 2.8 depicts this shift in sampling interval. Appendix G is a listing of the program that controlled the A/D converter during the digitizing process.

* Preston GMAD-1 Analog to Digital Conversion System.
** Hewlett Packard HP 1000 System.
Figure 2.9 Effect of memory switching on sampling rate.
Chapter 3

RESULTS

3.1 Outline of Test Cases

In this chapter the results obtained from the maximum likelihood computer program using the simulated data will be presented. As mentioned in Chapter 1, the AFL program which generated the simulated data had provisions for any combination of deterministic input, random input, initial condition and measurement noise. Table 3.1 is a summary of the maximum likelihood program testing.

3.1.1 Explanation of Test Case Data

The LEXAN track lateral alignment is ideally a white noise velocity input, with a gaussian probability density. Because of these characteristics the track input does not have to be measured but can be described as a random input into the wheelset model. In this case, it is the Q matrix which gives the maximum likelihood processor all of its information about the track input. It is also possible to measure the track input, although there would be some uncertainty involved. If the track input were measured, then the measured values for the track input would become a deterministic input and the uncertainty in the track measurements would be described as a random track input.

For the first seven test cases the track input is treated as a random track input, with the only information describing it contained in the Q matrix.

In test cases 7 and 8 the track input is treated as a deterministic input. The random track input for these cases represents the uncertainty in the deterministic track input.
Table 3.1
Summary of Maximum Likelihood Program Testing

<table>
<thead>
<tr>
<th>Test Case</th>
<th>Deterministic Track Input (u)</th>
<th>Initial Condition</th>
<th>Random Track Input (w) Mean Squared Value</th>
<th>Measurement Noise (v) Mean Squared Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>$x(1)=x(2)=x(3)=10$</td>
<td>0 0</td>
<td>0 0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$x(1)=x(2)=x(3)=0$</td>
<td>0 1.0</td>
<td>0 0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$x(1)=x(2)=x(3)=10$</td>
<td>0 0</td>
<td>0 .01</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$x(1)=x(2)=x(3)=0$</td>
<td>0 1.0</td>
<td>0 .01</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>$x(1)=x(2)=x(3)=10$</td>
<td>0 1.0</td>
<td>0 .01</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>$x(1)=x(2)=x(3)=0$</td>
<td>0 100.0</td>
<td>0 1.0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>$x(1)=x(2)=x(3)=0$</td>
<td>0 100.0</td>
<td>0 .01</td>
</tr>
<tr>
<td>8</td>
<td>$E[u] = 0$</td>
<td>$x(1)=x(2)=x(3)=0$</td>
<td>0 .1</td>
<td>0 .1</td>
</tr>
<tr>
<td>9</td>
<td>$E[u] = 0$</td>
<td>$x(1)=x(2)=x(3)=0$</td>
<td>0 10.0</td>
<td>0 .1</td>
</tr>
</tbody>
</table>

Group 1
Group 2
Group 3

* $X = \begin{bmatrix} y \\ \psi \\ (y-\delta) \end{bmatrix}$
In terms of generating the simulated wheelset data, the two representations of the track input are handled in the following way. For the case when the track input is considered to be a totally random input, a random vector is used as the input to the system model which generates the data. When this data is processed, the maximum likelihood program (specifically the Kalman Filter) is not told what the random vector was, the only information it is given is the covariance matrix for the vector Q.

When the track input is considered to be deterministic, the system model which generates the data has two random vectors as inputs. One of these random vectors represents the deterministic input; the other represents the uncertainty in the deterministic input. This time when the data generated by the system model are analyzed by the maximum likelihood processor, the Kalman Filter is given the vector that represents the deterministic input, but it is given only the covariance matrix for the vector that represents the uncertainty in the deterministic input. Figure 3.1 diagrams the generation of the simulated data for both representations of the track input. This figure is not designed to represent the entire process for generating simulated data (here can be measurement noise) nor is it designed to represent all of the inputs that go into the maximum likelihood processor. Its only purpose is to demonstrate the variations in representing the track input.

3.1.2 Purpose of Each Test Case

As stated previously, test cases 8 and 9 are different from the other seven test cases in how the track input is represented. Test case 9 is different from test case 8 in that the level of uncertainty in the deterministic track input is much higher. These two cases give some indication of how well the real track input would have to be measured to significantly increase the
Figure 3.1 Representations of Track Input.
performance of the maximum likelihood processor over the case when the track is considered to be a random track input.

Test cases 1 through 5 were run to find out how various combinations of initial condition, random track input and measurement noise affect the maximum likelihood processor. Test case 1 through 3 were also run to find out how well that maximum likelihood processor performs when it is given incorrect information about the random track input and the measurement noise. In other words the simulated data for test cases 1 through 3 was generated with a random track input and measurement noise that had statistics as given in Table 3.1. However, when this data was analyzed using the maximum likelihood processor, the Kalman Filter was given incorrect information about the statistics of the $w$ and $v$ used to generate the data. This incorrect information took the form of an incorrect $Q$ and $R$ matrix. Table 3.2 shows the true $Q$ and $R$ matrices for $w$ and $v$ versus the $Q$ and $R$ matrices actually used by the Kalman Filter.

3.2 Method of Testing the Maximum Likelihood Parameter

Essentially the maximum likelihood program has 3 main parts. The first of these calculates the likelihood function based on the equation

$$L = -\frac{1}{2} \sum_{i=1}^{N} (v(t_i)B^{-1}v(t_i) + \ln |B|)$$

(3.1)

The likelihood function has two main components as can be seen in equation 3.1.

$$\varepsilon_{\text{observation}} = -\frac{1}{2} \sum_{i=1}^{N} v(t_i)B^{-1}v(t_i)$$

(3.2)

and

$$\varepsilon_{\text{bias}} = -\frac{1}{2} \sum_{i=1}^{N} \ln |B|$$

(3.3)
### Table 3.2

Q and R Matrices for Test Cases 1 Through 5

<table>
<thead>
<tr>
<th>Test Case</th>
<th>Random Track Input ( \mathbf{w} )</th>
<th>Value of ( Q ) Actually Given to Maximum Likelihood Program</th>
<th>Measurement Noise ( \mathbf{v} )</th>
<th>R Matrix Actually Given to Maximum Likelihood Processor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Squared Value</td>
<td>Correct Value of ( Q ) for ( \mathbf{w} )</td>
<td>Mean Squared Value</td>
<td>Correct R Matrix for ( \mathbf{v} )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.0</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>0</td>
</tr>
</tbody>
</table>
The second part of the maximum likelihood equation calculates \( \frac{\partial L}{\partial \theta} \) which is referred to as the gradient, and the third part of the program calculates \( \frac{\partial^2 L}{\partial \theta^2} \), known as the Fisher Information Matrix. The inverse of the Fisher Information Matrix is defined as the Cramer-Rao Lower Bound on the covariance of the \( \theta \) estimates. The maximum likelihood approaches the lower bound asymptotically (\( \theta^* \)). The gradient and the Fisher Information Matrix are used to calculate the estimate of the value of the parameter to be identified, according to the equation

\[
\theta^* = \theta_0 - \Delta \theta
\]

where \( \Delta \theta = M^{-1} [\frac{\partial L}{\partial \theta}]^T \) and \( M = \frac{\partial^2 L}{\partial \theta \partial \theta^T} \)

As stated in Chapter 1, the maximum likelihood technique involves iterating through equations for \( L, \frac{\partial L}{\partial \theta}, \frac{\partial^2 L}{\partial \theta^2} \), and \( \theta^* \) until \( \Delta \theta \) becomes smaller than some error criterion (\( \varepsilon \)). According to maximum likelihood theory as \( \varepsilon \to 0 \), \( \theta^* \) approaches the true identity of the unknown parameter. At the true value for \( \theta^* \), the likelihood function has its maximum, and according to maximum likelihood theory, for the technique to converge the likelihood function must have a maximum. Because it was not certain that the likelihood function for all the test cases had a maximum, the maximum likelihood processor was not allowed to converge. Instead the likelihood function was given specific values of theta for which to calculate \( L, \frac{\partial L}{\partial \theta}, \frac{\partial^2 L}{\partial \theta^2} \). Theta ranged from .50 to 1.45 in increments of .05. In all of the simulated data generated, the following value for the unknown parameter was used:

\[
\beta = 1.00
\]

therefore the peak in the likelihood function for all cases should be at

\[
\theta = \beta = 1.00
\]

For each value of theta in the range specified above, the maximum likelihood
program iterated through its equations once calculating:

(1) \( L \)
(2) \( \xi_{bias} \)
(3) \( \xi_{observation} \)
(4) \( \frac{\partial L}{\partial \theta} \)
(5) \( \frac{\partial^2 L}{\partial \theta^2} \)
(6) \( \Delta \theta \)
(7) \( \theta^* \)

Figure 3.2 is a sample of the computer program output for one iteration. After the maximum likelihood program finished the iteration for \( \theta = 1.45 \) the five quantities listed above are plotted for the range of theta.

3.3 Test Case Results

A summary of the results of the maximum likelihood program for test cases 1 through 9 are given in Table 3.3. These results will be discussed in three sections. The first section will discuss the likelihood function for each test case. The second section will discuss the results for the gradient and the Fisher Information Matrix and the third will examine the application of the test data results to the analysis of real wheelset data.

3.3.1 Likelihood Function

In this section the test cases will be discussed in three groups as indicated on the right hand side of Table 3.1.

Examining test cases 8 and 9 first, Table 3.1 shows that a deterministic track input as well as a random track input was used to generate the simulated
STEP  =  20
THETA =  1.4499313

F MATRIX
0.21115749E 03  0.61370366E 03  -0.12370279E 03  
  0.0  -0.82806154E 00  -0.12261953E 02  
  0.10000000E 01  0.0  0.0

Z MATRIX
0.21115749E 03  0.0  -0.10000000E 01  0.10000003E 02  0.0  0.0
-0.61370366E 03  0.0  0.0  0.0  0.0  0.0
0.12370279E 03  0.0  0.0  0.0  0.0  0.0
0.61805696E 01  0.0  -0.80115962E 00  -0.21115749E 03  0.61370366E 03  -0.12370279E 03
0.0  0.0  0.0  0.0  0.0  0.0
-0.80115962E 00  0.0  0.99999944E 01  0.10000000E 01  0.0  0.0

19 ITERATIONS FOR STM.

STM DIFF MATRIX
0.0  -0.33881318E-20  0.0  0.06736174E-18  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  3.0  0.0  -0.86736174E-18  0.0  0.0
0.0  0.0  0.67762636E-20  -0.54210109E-19  -0.27755576E-16  0.0
0.33881318E-20  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0

KALMAN FILTER STM (PHI)
0.287256697D 01  -0.22740329E-03  -0.89864147E-02  0.59778821E-01  0.84016501E-01  -0.18788868E-01
-0.58562479D 01  0.10235310D 01  0.11381150D-01  -0.84137010D-01  -0.32933510D-01  0.16613497D-01
0.95935641D 00  0.61336246D-01  0.97970346D-00  0.15378138D-01  0.17091651D-01  0.46560944D-01
0.39001072D-01  -0.98344142E-04  -0.26964110D-02  0.34764025D 00  0.18924628D 00  -0.45000951D 00
-0.17458080D-03  -0.31859230E-06  -0.15429495D-04  -0.10080300D-03  0.99778093D 00  -6.1228081D-01
-0.68188948D-02  0.15442454D-06  0.40691631E-03  0.29803268E-02  0.54457854E-02  0.99879297D 00

200 ITERATIONS. RICCATI SOLUTION DOES NOT CONVERGE.

RICCATI DIFFERENCE MATRIX.
0.76749468D-04  0.22176855E-06  -0.2039317D-04
0.22717685D-04  0.49067202D-05  -0.61998646D-05
-0.2039317D-04  0.61998646D-05  0.55693854D-05

STATE COVARIANCE MATRIX (P)
0.42935962E 03  0.1366687CE 00  -0.19712276E-01
0.1366687CE 00  0.45515712E-01  -0.10566846E-01
-0.19712276E-01  -0.10566846E-01  0.15621197E-01

Figure 3.2: Computer output for the last iteration of test case number 8.
Figure 3.2 (continued)

**Kalman Gain Matrix (RKM)**

\[
\begin{pmatrix}
0.42905970 & 0.11646736 & 0 & -0.19712286 & 0 & 0.16461206 & 0 \\
0.13606878 & 0.84577129 & 0 & -0.10056684 & 0 & 0.16461206 & 0 \\
0 & 0.13606878 & 0.84577129 & 0 & -0.10056684 & 0 & 0.16461206 \\
\end{pmatrix}
\]

**19 Iterations for STM**

**STM Diff Matrix**

\[
\begin{pmatrix}
-0.27755576 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.67362646 & -0.20 & 0 & 0 & 0 & 0 & 0 \\
0.58251019 & -0.06736174 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

**System State Transition Matrix (EHY)**

\[
\begin{pmatrix}
0.14704362 & 0 & 0.18917807 & 0 & -0.44986582 & 0 & 0 \\
-0.11093843 & 0 & 0.97739328 & 0 & -0.52187031 & 0 & 0 \\
0.30843516 & 0 & 0.55599247 & -0.02 & 0.39875748 & 0 & 0 \\
\end{pmatrix}
\]

**Kalman Filter System Matrix (FKF)**

\[
\begin{pmatrix}
-0.21544007 & 0 & 0.61024102 & 0 & -0.12310506 & 0 & 0 \\
0.13606878 & 0 & -0.07933224 & 0 & -0.12153862 & 0 & 0 \\
0.13571124 & 0 & 0.10056684 & 0 & -0.16461206 & 0 & 0 \\
\end{pmatrix}
\]

**19 Iterations for STM**

**STM Diff Matrix**

\[
\begin{pmatrix}
-0.27755576 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
\end{pmatrix}
\]

**STM for Kalman Filter System (FHM)**

\[
\begin{pmatrix}
0.14704362 & 0 & 0.18917807 & 0 & -0.44986582 & 0 & 0 \\
-0.11093843 & 0 & 0.97739328 & 0 & -0.52187031 & 0 & 0 \\
0.30843516 & 0 & 0.55599247 & -0.02 & 0.39875748 & 0 & 0 \\
\end{pmatrix}
\]

**Measurement Covariance Matrix (E)**

\[
\begin{pmatrix}
0.52905959 & 0 & 0.13606878 & 0 & -0.19712286 & 0 & 0 \\
0.13606878 & 0 & 0.97739328 & 0 & -0.52187031 & 0 & 0 \\
0.39122768 & 0 & 0.10056684 & 0 & -0.16461206 & 0 & 0 \\
\end{pmatrix}
\]

**Natural Log of the Determinant of B**

\[
\ln |B| = -0.59115900e-01
\]

**Value of the Likelihood Function (RLIKE)**

\[
RLIKE = 0.16259472e+01
\]

**Partial of F WRT Theta (DFHW)**

\[
\begin{pmatrix}
-0.13010138 & 0.24714603 & 0 & 0 & 0 & 0 & 0 \\
0.0 & 0.24714603 & 0 & 0 & 0 & 0 & 0 \\
0.0 & 0.0 & 0.0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

**Z Matrix for Solution to Partial (W) / Partial (Theta)**

\[
\begin{pmatrix}
0.23454070 & 0.13366919 & 0 & -0.11971286 & 0 & 0.0 & 0.0 \\
-0.13010138 & 0.87941422 & 0 & -0.10056684 & 0 & 0.0 & 0.0 \\
0.13571124 & 0.12181146 & 0 & -0.14661206 & 0 & 0.0 & 0.0 \\
-0.50425720 & 0.93331470 & -0.13010138 & 0.23454070 & 0 & 0.12343028 & -0.12338056 & 0.03
\end{pmatrix}
\]
19 I T E R A T I O N S F O R S T M.

<table>
<thead>
<tr>
<th>S T M DIFF MATRIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
</tr>
<tr>
<td>0.0</td>
</tr>
<tr>
<td>0.0</td>
</tr>
<tr>
<td>-0.50010709-19</td>
</tr>
<tr>
<td>0.0</td>
</tr>
</tbody>
</table>

STM FOR SOLUTION TO PARTIAL (P)/PARTIAL (THETA)

| 0.2910193601 | 0.11886654-18 | -0.12513381-01 | 0.0 |
| -0.5491885801 | 0.9492059D00 | 0.1532659300 | 0.0 |
| 0.96643133D0 | 0.67823132-06 | 0.9791642000 | 0.0 |
| -0.30804797D0 | 0.47647496-06 | 0.20641942D-02 | 0.0 |
| 0.2935789D00 | -0.12325492D-06 | -0.4289117D-02 | 0.0 |
| 0.11635022D-01 | 0.1956478E-06 | -0.1723599D-04 | 0.0 |


<table>
<thead>
<tr>
<th>RICCATI DIFFERENCE MATRIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.637457955D-05</td>
</tr>
<tr>
<td>0.29625280D-05</td>
</tr>
<tr>
<td>-0.9123054D-06</td>
</tr>
</tbody>
</table>

PARTIAL OF P WRT THETA (DELTA P)

| 0.4533927E-01 | 0.2048747E-01 | 0.40192576E-03 | 0.0 |
| 0.2014874E-01 | 0.67489929E-02 | -0.16518242E-02 | 0.0 |
| 0.40192576E-03 | -0.16518242E-02 | 0.22276950E-02 | 0.0 |

PARTIAL OF P WRT THETA (DELTA T)

| 0.4533927E-03 | 0.2048747E-02 | 0.40192576E-02 | 0.0 |
| 0.2014874E-03 | 0.67489929E-01 | -0.16518242E-01 | 0.0 |
| 0.40192576E-02 | -0.16518242E-01 | 0.22276950E-01 | 0.0 |

GRADIENT - PARTIAL OF THE LIKELIHOOD FUNCTION WRT THETA

DELTA P = 0.404175674E-02
DELTA T = 0.48205394E-02

P ISHPK INFORMATION MATRIX

DELTA P = 0.404175674E-02
DELTA T = 0.48205394E-02

DTTHETA = 0.30261946E+01
PHI TTHETA = 0.42277852E+00
Table 3.3

Summary of Results for Test Cases

<table>
<thead>
<tr>
<th>Test Case</th>
<th>Likelihood Function Maximum $\theta$ =</th>
<th>Observation Term Maximum $\theta$ =</th>
<th>Bias Term Maximum $\theta$ =</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.95</td>
<td>.95</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>N/A</td>
<td>.95</td>
<td>N/A</td>
</tr>
<tr>
<td>3</td>
<td>.95</td>
<td>.95</td>
<td>N/A</td>
</tr>
<tr>
<td>4</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>5</td>
<td>.95</td>
<td>.95</td>
<td>N/A</td>
</tr>
<tr>
<td>6</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>7</td>
<td>.65</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>8</td>
<td>.85</td>
<td>1.10</td>
<td>N/A</td>
</tr>
<tr>
<td>9</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

NOTE: N/A = not applicable, no maximum occurs or only a local maximum occurs.
wheelset data. As presented in Chapter 1, the wheelset equations of motion in state vector form are:

\[ \dot{x} = Fx + Gw \]  \hspace{1cm} (3.5)

where \( w \) = random track input

The Kalman Filter state estimates for this case are given by the equation

\[ \bar{x} = (F-KH)\bar{x} + Kz \]  \hspace{1cm} (3.6)

To incorporate deterministic and random track inputs the following changes in the above two equations are necessary.

\[ \dot{x} = Fx + Gu + Gw \]  \hspace{1cm} (3.7)

where \( u \) = deterministic track input

\[ \bar{x} = (F-KH)\bar{x} + Kz + Gu \]  \hspace{1cm} (3.8)

Although \( u \) is a white noise input it is called deterministic because it was an input into the system model which generated the data (equation 3.7) and it was an input into the Kalman Filter of the maximum likelihood program (equation 3.3). \( w \) is called a random track input because it appears as an input into the wheelset system (equation 3.7) but it is not an input into the Kalman Filter (equation 3.3). The Kalman Filter in the maximum likelihood program has the \( Q \) matrix as its only source of information about \( w \).

The difference between test cases 8 and 9 is the size of the random track input as compared to the deterministic track input. Case 9 has a larger random track input than case 8, which signifies that case 9 has more uncertainty in the deterministic track input than case 8. As Table 3.3 shows, for case 8 the likelihood function had a maximum but for case 9 the likelihood function did not have a maximum. This would suggest that the maximum likelihood method needs a well-defined deterministic track input if it is to identify a peak in the likelihood function. However, maximum likelihood theory has no restrictions.
This suggests that the maximum likelihood processor may need more measurements—a longer data record—when the deterministic track input has a large amount of uncertainty. In reference (6) an increase in the number of observations is shown to cause a more pronounced maximum in the likelihood function. Although there are no quantitative results which prove more measurements will produce a peak in the likelihood function for the dynamically scaled wheelset case, this is an area in which further research should be done.

For all of the test cases in group 1, the Kalman Filter in the maximum likelihood program was given the following values for $Q$ and $R$, as shown in Table 3.1 and Table 3.2.

$$Q = 1.0$$

$$R = \begin{bmatrix}
0.01 & 0 & 0 \\
0 & 0.01 & 0 \\
0 & 0 & 0.01
\end{bmatrix}$$

These five test cases were run for diagnostic purposes but have been included here because they exhibit some interesting trends.

Case 2 is called the perfect observation case in the literature. In reference (7) the likelihood function is shown to reduce to equation 3.9 for this case.

$$L = \prod_{i=1}^{n} \frac{T(t_i)Q^{-1}T^T(t_i)}{i}$$  \hspace{1cm} (3.9)

where

$$v(t_i) = z(t_i) - \Phi z(t_{i-1})$$  \hspace{1cm} (3.10)

when $Q$ is known. Essentially equation 3.9 says that the likelihood function simplifies to the observation term for the perfect measurement case. The maximum likelihood processor did not analyze this limiting case because equations 3.9 and 3.10 were not incorporated into the maximum likelihood
algorithm for test case number 2. The results of test case 2 do provide insight into the performance of maximum likelihood processor when it has incorrect or imprecise information about $w$ or $v$. Figures 3.3, 3.4 and 3.5 contain plots of the output of the maximum likelihood program for test case 2.

Test case 2 has a random track input only which makes it a limiting case for the effect of $u$ versus $w$ that was examined in cases 8 and 9. Test case 2 suggests that the more randomness there is in the track input, the more difficulty the maximum likelihood processor has in identifying maximum for the likelihood function. There are no theoretical limitations of this type for maximum likelihood. As in test cases 8 and 9 one practical aspect of implementing the maximum likelihood technique that should be investigated is data record length. Although several assumptions were made in the generation of the simulated data as discussed in section 1.4 it is believed that these assumptions will not seriously affect the performance of the maximum likelihood processor.

Case number 3 is a special case of maximum likelihood identification because there is no random track input. The likelihood function reduces to the observation term for this case also; however, the weighting matrix is different than in the no measurement noise case.

$$L = \sum_{i=1}^{N} v(t_i) R^{-1} v(t_i) \quad \text{(3.11)}$$

where

$$v(t_i) = z(t_i) - H R(t_i) \quad \text{(3.12)}$$

when $R$ is known. In this limiting case a Kalman Filter is used by the maximum likelihood processor as shown in equation 3.12, however the weighting matrix is not dependent on $P$ but only on $R$. The observation term calculated by the maximum likelihood program for test case 3 was
\[ \xi = -\frac{1}{2} \sum_{i=1}^{N} v(t_i)^T B^{-1} v(t_i) \]  
(3.13)

where
\[ B = HPH^T + R \]  
(3.14)

and
\[ v(t_i) = z(t_i) - H\xi(t_i) \]  
(3.15)

The state estimates as calculated by the maximum likelihood program are not as accurate as they could be because the Kalman Filter was given an incorrect value for \( Q \) (Table 3.2). Also the weighting matrix used in equation 3.13 is very different from the weighting matrix used in equation 3.11. Because the maximum likelihood algorithm for test case 3 did not incorporate equation 3.11 and because \( Q \) was incorrect, the limiting case of no process noise was not analyzed correctly.

Test case 1 is a combination of the two limiting cases presented above.

The likelihood function reduces to:
\[ L = \prod_{i=1}^{N} v(t_i)^T v(t_i) \]  
(3.16)

where
\[ v(t_i) = z(t_i) - \phi z(t_{i-1}) \]  
(3.17)

As equation 3.17 shows a Kalman Filter would not be used in this case because there is no random track input and no measurement noise. As for the other test cases in group 1 the values for \( Q \) and \( R \) used by the maximum likelihood program for test case 1 were:
\[ Q = 1.0 \]
\[ R = \begin{bmatrix} .01 & 0 & 0 \\ 0 & .01 & 0 \\ 0 & 0 & .01 \end{bmatrix} \]

Because of this discrepancy, the results obtained from test case 1 are not necessarily those that would be obtained if the no random track input-no
measurement noise case were implemented correctly using equations 3.16 and 3.17.

Test case 4 and test case 5 are the only members of group 1 for which the maximum likelihood program was given correct values for Q and R. The results of test cases 4 and 5 are given. As demonstrated in other test cases the maximum likelihood processor did not identify a peak in the likelihood function when the only input driving the wheelset system is a random track input. From the test cases performed as part of this research program no conclusive explanations can be given for the above problem. As suggested before, a longer data record in these cases may be necessary however there is no direct evidence to support this conclusion.

Test cases 6 and 7 were run to see if the ratio of Q to R had any effect on the performance of the maximum likelihood processor. For test case 6 the ratio of Q to R is the same as that for test cases 4 and 5. According to maximum likelihood if the values of Q and R are changed but their ratio remains constant then the observation term will remain the same but the bias term will change. Graphs of the observation term for test case 4 and test case 6 are presented in Figure 3.6 and 3.7 respectively. Comparing these two plots shows that the observation term did not change between these two cases. Comparing Figures 3.8 and 3.9 it can be seen that the bias term did change its range of values, but not its general shape. The likelihood functions for case 4 and case 6 are given in Figures 3.10 and 3.11. The likelihood function has the same general shape for each case, however, the location of the local maximum does change. This agreement between test cases 4 and 6 further supports the validity of the maximum likelihood algorithm given in Chapter 1, and the computer program which implements it.
Figure 3.6 Plot of the observation term for test case h.
Figure 3.8 Plot of the bias term for test case $h$. 
Figure 3.9 Plot of the bias term for test case 6.
Test case 6 has as its only input a random input disturbance and in accordance with the results of other similar test cases, its likelihood function does not have a maximum. Test case 7 also has a random disturbance as the only input to the wheelset model; however, the likelihood function in the case 7 results does have a maximum at \( \theta = 0.65 \). A plot of this likelihood function is given in Figure 3.12. The results of test case 7 indicate that the test data record was long enough for the maximum likelihood processor to identify a peak in the likelihood function, although the peak occurred at \( \theta = 0.65 \) instead of \( \theta = 1.00 \). There is no indication that a longer record will shift the peak from \( \theta = 0.65 \) to \( \theta = 1.00 \), although this is an area which needs further research. The fact that the measurement noise in case 7 is much smaller than in case 6 may be the reason the maximum likelihood processor was able to identify a peak in the likelihood function.

There is another possible explanation for the problems the likelihood function had in identifying a peak in the likelihood function. As discussed in section 1.4 a \( \Delta t = 0.05 \) sec was used to generate the test data. This time step resulted in 6 data points per cycle of the highest frequency response of the wheelset. The maximum likelihood processor may need more information about this particular mode of response of the wheelset. This could be accomplished by using a smaller \( \Delta t \). However, because the present method used to generate the simulated data is restricted to 1000 data points per state variable, a smaller \( \Delta t \) would produce data with fewer cycles of the low frequency response of the wheelset. In order to investigate the effects of a smaller \( \Delta t \) a new method for generating the simulated data will have to be developed.

Since all of the simulated wheelset data was generated with the same \( \Delta t \), and for several cases a peak in the likelihood function was identified there
does not seem to be a need for a smaller $\Delta t$. However, in those cases where the simulated data was generated using a random track input, the maximum likelihood processor was given minimal information about the input into the wheelset model that generated the data—only the covariance matrix for the random track input. To make up for this limited information about the input the maximum likelihood processor may need better information in other areas. This "better" information includes more data points per cycle of the high frequency response mode of the wheelset—obtained by a smaller $\Delta t$—and more observations of the low frequency response of the wheelset—obtained with a larger data record.

3.3.2 Gradient and Fisher Information Matrix

The task of evaluating the performance of the maximum likelihood processor in calculating $\frac{\partial L}{\partial \theta}$ and $\frac{\partial^2 L}{\partial \theta^2}$ is considerably more difficult than evaluating the calculation of $L$ itself. There are two methods that can be used to check the gradient and second partial. The first method involves curve fitting likelihood function and gradient data points with $n^{th}$ order polynomials. The derivatives can then be calculated analytically using the polynomial equation. The problem experienced with this method is that the data points do not represent a smooth function and inflection points which do not exist in the data are created. These inflection points cause large errors in the calculation of the derivative. The second method, and the one used to analyze the test data results, is explained in Figure 3.13. From Figure 3.13

$$m_{32} = \frac{y_3 - y_2}{x_3 - x_2}$$

$$m_{43} = \frac{y_4 - y_3}{x_4 - x_3}$$

(3.18) (3.19)
Figure 4.4 Average slope method for checking the gradient and second partial of the likelihood function.
Equations 3.18 through 3.22 were used to calculate the derivatives of the likelihood function and the gradient. The slope at the first data point was set equal to the slope between the first two data points. The slope at the last data point was set equal to the slope between the last two data points. This method of averaging slopes works very well when the data points form a smooth function. In many cases the likelihood function data points and the gradient data points do not form smooth functions and the derivatives calculated using this second method can have large errors. Since the only objective was to check the reasonableness of the gradient and second partial data, the second method of averaging slopes was used. In general the difference between the gradient and the derivative of L as determined by the average slope method is on the order of 5%. The difference between the second partial and the slope of the gradient as determined by the averaging method was considerably larger—on the order of 35%. This difference for the second partial was considerably larger for test cases 4 and 6. The reason the difference between the second partial and the slope of the gradient may be so large for test cases 4 and 6 is that determining derivatives by averaging slopes produces large errors in data points which do not form smooth functions. The data points for the cases 4 and 6 are not as smooth as in other cases. See Figures ... discrepancy is most probably due to errors in the Information Matrix rather than in the calculation of the gradient and the second partial was
Figure 3.15 Plot of the gradient for test case 6.
not carried out for test cases 6 and 9 because based on the results for test cases 1 through 7 it was concluded that the quasilinearization section of the maximum likelihood algorithm was implemented correctly in the maximum likelihood computer program.

3.4 Application of Results to Actual Wheelset Data

The test data cases were analyzed by the maximum likelihood program for the following reasons:

(1) to ensure that the computer program was working properly.
(2) to identify any potential problems that may arise when the actual wheelset data is analyzed.
(3) to make recommendations for further research, based on the conclusions reached in (2) above.

The first objective was attained as documented in the previous sections. The purpose of this section is to discuss items (2) and (3).

In every case except one where the only input into the wheelset model was the random track input the likelihood function did not have a maximum. Several times in the previous section the printout suggested that the likelihood function failed to have a maximum because the data record needed to be longer. The test cases run as part of this research program do not form a large enough experimental base to state the above point with any certainty. It is suggested that further research in this area be directed to find out how the length of the data record affects the performance of the maximum likelihood processor. As mentioned in Chapter 1, the present method of generating simulated data is limited to 1000 observations because of limitations in computer storage, therefore another method of generating simulated data will
have to be designed.

Another area of research which should be examined is the effect of a smaller $\Delta t$ in generating the test data. A smaller $\Delta t$ would provide more information about the high frequency response mode of the wheelset. In order not to lose information about the low frequency mode, data records longer than 1000 points per state variable will have to be used. As pointed out above this requires a new method of generating simulated data.

Appendix H contains plots of the likelihood function, observation time and bias term for test cases 1 through 9.
CONCLUSION

As stated in the Introduction, the objective of this research program was the development and implementation of a maximum likelihood parameter identification algorithm applicable to a dynamically scaled wheelset model. The following is a summary of the conclusions that can be drawn from the results of this research program.

1) The maximum likelihood parameter identification equations can be tailored to the problem of identifying creep coefficients for the dynamically scaled wheelset model.

2) The reasonable results obtained for many of the test cases proves that the implementation of the maximum likelihood algorithm in Fortran IV was performed correctly.

3) More research involving simulated wheelset data is necessary before actual wheelset data can be processed. It is recommended that the following areas be investigated:

   a) the effect of data record length on the performance of the maximum likelihood processor

   b) advantages in representing the track input as a deterministic input with uncertainty as compared to representing the track input as a random track input

   c) the effect of a smaller $\Delta t$ on the performance of the maximum likelihood processor.

The primary motivation for the above three recommendations is that it seems reasonable to assume the maximum likelihood processor works better
when it has more information about the system and about the inputs into the system. If the track input is a random track input then the covariance matrix is the only information the maximum likelihood processor receives about this random track input. In order to make up for this limited information about the input into the wheelset system, the maximum likelihood processor should receive better information in other areas. A smaller $\Delta t$ and a longer data record provide more information about the wheelset high frequency response mode and low frequency model respectively. This increase in the quantity of information could be considered better from the maximum likelihood processor point of view.
LIST OF REFERENCES


Appendix A

DERIVATION OF \( \frac{\partial \mathbf{x}(t)}{\partial \theta} \)

\[
\dot{x}(t) = F(t)x(t) + L(t)u(t) + K(t)[z(t) - H(t)x(t)] \quad (A.1)
\]

\[
\frac{\partial x(t)}{\partial \theta} = \frac{\partial F(t)}{\partial \theta} x(t) + \frac{\partial L(t)}{\partial \theta} u(t) + \frac{\partial K(t)}{\partial \theta} [z(t) - H(t)x(t)] - K(t)H(t) \frac{\partial x(t)}{\partial \theta} \quad (A.2)
\]

Since the input disturbance is deterministic \( \frac{\partial u(t)}{\partial \theta} = 0 \). The measurements \( z \) are independent of \( \theta \) therefore \( \frac{\partial z}{\partial \theta} = 0 \). Therefore equation A.3 reduces to:

\[
\frac{\partial x(t)}{\partial \theta} = F(t) \frac{\partial x(t)}{\partial \theta} + \frac{\partial F(t)}{\partial \theta} x(t) + \frac{\partial L(t)}{\partial \theta} u(t) + \frac{\partial K(t)}{\partial \theta} \frac{\partial x(t)}{\partial \theta} x(t) - K(t)H(t) \frac{\partial x(t)}{\partial \theta} \quad (A.3)
\]
Appendix B

SOLUTION TO $\frac{\partial P(t)}{\partial \theta}$

The matrix Riccati equation is

$$\dot{P}(t) = F(t)P(t) + P(t)F^T(t) + G(t)Q(t)G^T(t) - P(t)H^T(t)R^{-1}(t)H(t)P(t)$$ (B.1)

Taking the partial derivative of both sides with respect to $\theta$:

$$\frac{\partial P(t)}{\partial \theta} = F(t)\frac{\partial P(t)}{\partial \theta} + \frac{\partial F(t)}{\partial \theta} P(t)$$

$$P(t) \frac{\partial (F^T(t))}{\partial \theta} + \frac{\partial F(t)}{\partial \theta} F^T(t)$$

$$+ G(t)Q(t) \frac{\partial (G^T(t))}{\partial \theta} + G(t) \frac{\partial G(t)}{\partial \theta} G^T(t) + \frac{\partial G(t)}{\partial \theta} Q(t)G^T(t)$$

$$- F(t)H^T(t)R^{-1}(t)H(t) \frac{\partial (H^T(t))}{\partial \theta} - F(t)H(t) \frac{\partial (R^{-1}(t))}{\partial \theta} R^{-1}(t)H(t)P(t)$$

$$- P(t)H^T(t) \frac{\partial (R^{-1}(t))}{\partial \theta} R^{-1}(t)H(t)P(t) - \frac{\partial P(t)}{\partial \theta} H^T(t)R^{-1}(t)H(t)F(t)$$ (B.2)

Post-multiply both sides of (B.2) by $\mathbf{v}$. The time notation will be dropped in further equations however all matrices are still time dependent.

$$\frac{\partial \mathbf{P}}{\partial \theta} \mathbf{v} = F \frac{\partial \mathbf{P}}{\partial \theta} \mathbf{v} + \frac{\partial F}{\partial \theta} \mathbf{P} \mathbf{v} + \mathbf{P} \frac{\partial F^T}{\partial \theta} \mathbf{v} + \frac{\partial G(t)}{\partial \theta} \mathbf{Q} \mathbf{v} + G(t) \frac{\partial \mathbf{Q}^T}{\partial \theta} \mathbf{v} + \mathbf{G} \frac{\partial \mathbf{Q}^T}{\partial \theta} \mathbf{v}$$

$$+ \mathbf{Q} \frac{\partial \mathbf{Q}^T}{\partial \theta} - PH^T R^{-1} H \frac{\partial \mathbf{P}}{\partial \theta} \mathbf{v} - PH^T R^{-1} H \frac{\partial \mathbf{P}}{\partial \theta} \mathbf{v} - PH^T R^{-1} H \frac{\partial \mathbf{P}}{\partial \theta} \mathbf{v}$$

$$\mathbf{v} - P \frac{\partial H^T}{\partial \theta} R^{-1} \mathbf{H} \mathbf{v} - \mathbf{P} \frac{\partial H^T}{\partial \theta} R^{-1} \mathbf{H} \mathbf{v}$$ (B.3)

Apply the transformations:

$$\mathbf{Y} = \frac{\partial \mathbf{P}}{\partial \theta} \mathbf{v}$$ (B.4)(3)
By reviewing the order of differentiation for the term \( \frac{\partial^2 y}{\partial \theta^2} \) in equation (B.3), the equation (B.5) can be substituted in equation (B.3).

\[
\dot{\ddot{y}} = \frac{\partial}{\partial \theta} \dot{y} + \left( \frac{\partial}{\partial \theta} \right)^2 y \tag{B.5}
\]

Apply the transformation

\[
\dot{\tilde{y}} = -P^T \dot{y} + HR^{-1}HPy \tag{B.7}(3)
\]

\[
\dot{\tilde{y}} = F\dot{y} + P \frac{\partial}{\partial \theta} \dot{y} + GQ \frac{\partial}{\partial \theta} y + G \frac{\partial}{\partial \theta} G^T \dot{y} + \frac{\partial}{\partial \theta} QT \dot{y} - PH^T R^{-1}HPy \tag{B.8}
\]

Equation (B.7) and (B.8) are a linear system and are written in matrix form in B.9.

\[
\begin{bmatrix}
\dot{x} \\
\dot{\tilde{y}}
\end{bmatrix} =
\begin{bmatrix}
\omega_{11} & \omega_{12} \\
\omega_{21} & \omega_{22}
\end{bmatrix}
\begin{bmatrix}
x \\
\tilde{y}
\end{bmatrix}
\tag{B.9}
\]

where

\[
\omega_{11} = -P^T + HR^{-1}HP \tag{B.10}
\]

\[
\omega_{12} = 0 \tag{B.11}
\]

\[
\omega_{21} = \frac{\partial}{\partial \theta} P + P \frac{\partial}{\partial \theta} F^T + GQ \frac{\partial}{\partial \theta} G + G \frac{\partial}{\partial \theta} G^T + \frac{\partial}{\partial \theta} QT \dot{y} - PH^T R^{-1}HP - P \frac{\partial}{\partial \theta} R^{-1}HP \tag{B.12}
\]

\[
\omega_{22} = F - PH^T R^{-1}H \tag{B.13}
\]
Equation (B.9) can be solved using a state transition matrix approach.

\[
\begin{bmatrix}
Y(t_o + \Delta t) \\
Y(t_o + \Delta t)
\end{bmatrix} =
\begin{bmatrix}
\phi_{yy}(t_o, \Delta t) & \phi_{y}(t_o, \Delta t) \\
\phi_{y}(t_o, \Delta t) & \phi_{yy}(t_o, \Delta t)
\end{bmatrix}
\begin{bmatrix}
Y(t_o) \\
Y(t_o)
\end{bmatrix}
\]  \hspace{1cm} (B.14)

where

\[
\phi(t_o, \Delta t) = \exp\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix} \Delta t
\]  \hspace{1cm} (B.15)

From equation (B.14)

\[
Y(t_o + \Delta t) = \phi_{yy}(t_o, \Delta t) Y(t_o) + \phi_{yy}(t_o, \Delta t) Y(t_o)
\]  \hspace{1cm} (B.16)

\[
Y(t_o + \Delta t) = \phi_{yy}(t_o, \Delta t) Y(t_o) + \phi_{yy}(t_o, \Delta t) Y(t_o)
\]  \hspace{1cm} (B.17)

Applying equation (B.4) and combining (1.16) and (1.17):

\[
\frac{\partial P(t + \Delta t)}{\partial \theta} = [\phi_{yy}(t_o, \Delta t) + \phi_{yy}(t_o, \Delta t) \frac{\partial P(t)}{\partial \theta}]^{-1}
\]

\[
[\phi_{yy}(t_o, \Delta t) + \phi_{yy}(t_o, \Delta t) \frac{\partial P(t)}{\partial \theta}]^{-1}
\]  \hspace{1cm} (5.15)
Appendix C

DEFINITIONS OF VARIABLES USED IN $F$ AND $G$ MATRICES

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<th>Parameter</th>
<th>Symbol</th>
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<td>kg</td>
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Appendix D

SIMPLIFICATION OF $\frac{\partial P(t)}{\partial \theta}$ FOR WHEELSET PROBLEM

Simplifying equations (B.11) through (B.14) from Appendix E, equation (B.10) reduces to:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} -F^T + H^T R^{-1} HP & 0 \\ \frac{\partial F}{\partial \theta} P + P \frac{\partial F^T}{\partial \theta} & F - PH^T R^{-1} HP \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

Equation (D.1) is solved using the state transition matrix approach.

$$\begin{bmatrix} Y(t_0 + \Delta t) \\ Y(t_0 + \Delta t) \end{bmatrix} = \begin{bmatrix} \phi_{yy}(\Delta t) & \phi_{yy}(\Delta t) \\ \phi_{yy}(\Delta t) & \phi_{yy}(\Delta t) \end{bmatrix} \begin{bmatrix} Y(t_0) \\ Y(t_0) \end{bmatrix}$$

$$\phi(\Delta t) = \exp \left[ \begin{bmatrix} -F^T + H^T R^{-1} HP & 0 \\ \frac{\partial F}{\partial \theta} P + P \frac{\partial F^T}{\partial \theta} & F - PH^T R^{-1} HP \end{bmatrix} \Delta t \right]$$

Using the same methodology as in Appendix E

$$\frac{\partial P(t_0 + \Delta t)}{\partial \theta} = \left[ \phi_{yy}(\Delta t) + \phi_{yy}(\Delta t) \frac{\partial F}{\partial \theta} \right]$$

$$\left[ \phi_{yy}(\Delta t) + \phi_{yy}(\Delta t) \frac{\partial F}{\partial \theta} \right]^{-1} \frac{\partial P(t_0)}{\partial \theta}$$

Equation (D.4) is an iterative equation and is solved for $\left. \frac{\partial P}{\partial \theta} \right|_{ss}$.

Because the wheelset is a time invariant system, $\left. \frac{\partial P}{\partial \theta} \right|_{ss}$ is solved for only once for each iteration of the maximum likelihood equations.
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1967 A
Appendix E

LISTING OF THE MAXIMUM LIKELIHOOD PROGRAM
CALL INDUMP

THIS PROGRAM IS THE CONTROLLING PROGRAM FOR A SERIES OF
SUBROUTINES WHICH FORM THE MAXIMUM LIKELIHOOD PARAMETER
IDENTIFICATION PROCESSOR FOR THE SIMULATED WHEELSET DATA.

COMMON P(3,3), G(3,1), H(3,3), P(3,3), C(1,1), K1F, K1G, K2G, K1H, K2H, K1Q,
$ K1K, PGKHM (3,3), K1PHKM, K2PHKM, K1PHIZ, K2PHIZ,
$ K1KGM, K2KGM, RMASS, P11, P22, RRT, RKS1, RIE, ALPHA, RZERO, DT, NRECIP,
$ NREC, IROSTA, IROX, IROD, IROST, IDATS, DATEN, NDATA, NRAC2,
$ DATEN (4,3), RVAL, TVEL, GRCAL, NPOINT, SD1, S P(3,3), K1B, K1P,
$ RLIKE, K1DPH, K2DPH, K1DPRG, K2DPRG, NPT1, XIC (3,1), BZERO, K1DELP,
$ K2DELP, K1DELP, K2DELP, K2DELP, FFLAG

DIAMENSIONS DLTAP(3,3)
DIMENSION DLTAK(3,3)
DIMENSION DFHIX(3,3)
DIMENSION ZMEAS (3,1001)
DIMENSION ZMEAS1 (3,1001)
DIMENSION XHAT (3,1001)
DIMENSION RNU(3,1001)
DIMENSION ZHAT (3,1001)
DIMENSION DXHAT (3,1001)
DIMENSION XMEAS1 (3,1001)
DIMENSION XMEAS (3,1001)
DIMENSION XERROR(3,1001)
DIMENSION XVECT1(1001)
DIMENSION XVECT2(1001)
DIMENSION O (1,101)

READ (5,10) K1F, K1G, K2G, K1H, K2H, K1Q, K1P

10 FORMAT (I3)

WRITE (6,15) K1F, K1G, K2G, K1H, K2H, K1Q, K1P

15 FORMAT (I3,5X,'K1F=',I3,5X,'K1G=',I3,5X,'K2G=',I3,5X,'K1H=',I3,5X,'K2H=',I3,5X,'K1Q=',I3,5X,'K1P=',I3)

K1DFL = K1F
K2DFL = K1P
K1DFL = K1F
K2DFL = K1P
K1DFL = K1F
K2DFL = K1P
K1DFL = K1F
K2DFL = K1P

READ (5,25) IDATS, IDATN

25 FORMAT (I4)

NPOINT = (IDATN-IDATS) + 1
NPT = NPOINT + 1

IFLAG = 0

WRITE (6,45) IDATS, IDATN, NPOINT, NPT

45 FORMAT (I4,5X,'IDATS=',I4,5X,'IDATN=',I4,5X,'NPOINT=',I4,5X,'NPT=',I4)

CALL DRIVER (ZMEAS, ZMEAS1, XHAT, RNU, ZHAT, DLTAP, DLTAK, DXHAT,
$ DPATX, DPATX, XERROR, XMEAS, XMEAS1, XVECT1, XVECT2, 0)

STOP
END
DOUBLE PRECISION PHIX(6,6)
DIMENSION PHIX(3,3)
DOUBLE PRECISION PHIXDP(3,3)
DIMENSION ZMEAS(3,NPOINT)
DIMENSION ZMEAS1(3,NPT1)
DIMENSION DXHAT(3,NPOINT)
DIMENSION RNU(3,NPOINT)
DIMENSION ZHAT(3,NPOINT)
DIMENSION DELTAP(K1DELP,K2DELP)
DIMENSION DELTA(K1DELP,K2DELP)
DIMENSION DPHI(K1DPH,K2DPH)
DIMENSION DFMTX(K1DF,K2DF)
DOUBLE PRECISION PHIK(3,3)
DIMENSION ITI1(10)
DIMENSION ITI2(10)
DIMENSION JAB1(10)
DIMENSION JAE2(10)
DIMENSION JAB3(10)
DIMENSION ITI3(10)
DIMENSION VDELJ(20)
DIMENSION VDELJ2(20)
DIMENSION VR1KE(20)
DIMENSION VTHETA(20)
DIMENSION ITI5(10)
DIMENSION ITI4(10)
DIMENSION JAB4(10)
DIMENSION JAB5(10)
DIMENSION ITI6(10)
DIMENSION JAB6(10)
DIMENSION JAB7(10)
DIMENSION ITI7(10)
DIMENSION VCONST(20)
DIMENSION VDELT(20)
DIMENSION U(1,KPT1)
TVL=3.021
CALL PARAM(TBTHETA)
WRITE(6,20) RMASS,F11,F22,RKY,RKSI,REL,ALPHA,RZERO,THETA,BZERO,
$DT,SDT
20 FORMAT(//,'RMASS=',F12.5,2X,'F11=',F12.5,2X,'F22=',F12.5,
$2X,'RKY=',F12.5,2X,'RKSI=',F12.5,2X,'REL=',F12.5,
$2X,'ALPHA=',F12.5,2X,'RZERO=',F12.5,2X,'THETA=',
$F12.5,2X,'BZERO=',F12.5,2X,'DT=',F12.5,2X,'SDT=',F12.5)
WRITE(6,40)
40 FORMAT(//,'G MATRIX')
CALL OUTPUT(G,K1G,K2G)
WRITE(6,60)
60 FORMAT(//,'H MATRIX')
CALL OUTPUT(H,K1H,K2H)
WRITE (6, 80)
80 FORMAT(/,' ',12X,'Q MATRIX')
CALL OUTPUT(Q,K10,K10)
WRITE (6,100)
100 FORMAT(/,' ',12X,'R MATRIX')
CALL OUTPUT(R,K1R,K1R)
DO 120 I=1,NPT1
READ (5,140) (ZMEAS1(J,I),J=1,3)
120 CONTINUE
140 FORMAT(3F15.8)
DO 160 J=1,3
I=IDATS1-1
XIC(J,1)=ZMEAS1(J,I)
160 CONTINUE
WRITE (6,180)
180 FORMAT(/,' ',12X,'XIC')
CALL OUTPUT(XIC,3,1)
DO 200 J=1,3
DO 200 I=IDATS1,IDATEN
H=I-1
ZMEAS(J,H)=ZMEAS1(J,I)
200 CONTINUE
READ (5,210) IT11
210 FORMAT(10A4)
READ (5,210) JLAB1
READ (5,210) IT12
READ (5,210) JLAB2
READ (5,210) IT13
READ (5,210) JLAB3
READ (5,210) IT14
READ (5,210) JLAB4
READ (5,210) IT15
READ (5,210) JLAB5
READ (5,210) IT16
READ (5,210) JLAB6
READ (5,210) IT17
READ (5,210) JLAB7
DO 212 J=1,NPT1
212 READ (5,213) U(I,J)
213 FORMAT(E15.8)
DO 800 ISTEP=1,20
WRITE (6,215) ISTEP
215 FORMAT(/,' ',1X,'ISTEP=',I3)
WRITE (6,216) THETA
216 FORMAT(/,' ',2X,'THETA=',F15.8)
CALL SYSTMX (THETA)
WRITE (6,220)
220 FORMAT(/,' ',12X,'F MATRIX')
CALL OUTPUT(F,K1F,K1F)
CALL SMTRX(Z,K1Z)
WRITE (6,240)
240 FORMAT(/,' ',12X,'Z MATRIX')
CALL OUTPUT(Z,K1Z,K1Z)
CALL SMZ(Z,DT,PHIZ,K1Z,K1PHIZ,K2PHIZ)
IF(IPLAG .EQ. 1) GO TO 1000
WRITE (6,260)
260 FORMAT(/,' ',12X,'KALMAN FILTER SIM (PHIZ)')
CALL OUTDP(PHIZ,6,6)
CALL RICATI(PHIZ,K1PHIZ,P,K1P)
IF(IPLAG .EQ. 1) GO TO 1000
WRITE (6, 280)
280 FORMAT (/ * * 12X, 'STATE COVARIANCE MATRIX (P)' *)
CALL KGMTRX
WRITE (6,300)
300 FORMAT (/ * * 12X, 'KALMAN GAIN MATRIX (RKG)' *)
CALL STMF (P, DT, PHI, X, K, K1PHI, K2PHI)
CALL DPTOSP (PHI, X, K1PHI, K2PHI)
WRITE (6, 320)
320 FORMAT (/ * * 12X, 'SYSTEM STATE TRANSITION MATRIX (PHI)' *)
CALL STMF (PHI, X, K1PHI, K2PHI)
CALL STATE (Z, X, PHI, K1PHI, K2PHI, U)
339 CALL RINV W (X, Z, PHI, K1PHI, K2PHI)
CALL ZCOV
WRITE (6, 340)
340 FORMAT (/ * * 12X, 'MEASUREMENT COVARIANCE MATRIX (B)' *)
CALL RMLPI (Z, S, SC, SD)
WRITE (6, 360)
360 FORMAT (/ * * 12X, 'VALUE OF THE LIKELIHOOD FUNCTION (RLIKE)' *)
WRITE (6, 380)
380 FORMAT (' *, 12X, 'RLIKE=', E15.8)
CALL PARP (THETA, STM)
WRITE (6, 400)
400 FORMAT (/ * * 12X, 'PARTIAL OF P WRT THETA (DFMTX)' *)
CALL RMLPI (STM, X, THETA, K1DF, K2DF)
CALL PDP (DFMTX, DELTAP, DELTAK)
WRITE (6, 440)
440 FORMAT (/ * * 12X, 'PARTIAL OF P WRT THETA (DELTAP)' *)
WRITE (6, 460)
460 FORMAT (/ * * 12X, 'PARTIAL OF RKG WRT THETA (DELTAK)' *)
CALL RMLPI (DELTAK, K1DELK, K2DELK)
CALL XSE (PHI, K1PHI, STM, K1DELP, K2DELP)
CALL GRADE (STM, X, THETA, Z, DELTAP, DELTAK)
WRITE (6, 480)
480 FORMAT (/ * * 12X, 'GRADIENT - PARTIAL OF THE LIKELIHOOD FUNCTION', 
* ' WRT THETA' *)
WRITE (6, 500)
500 FORMAT (' *, 12X, 'DELTAP=', E15.8)
CALL FIESR (STM, STM, DELTAP, DELJ2)
WRITE (6, 520)
520 FORMAT (/ * * 12X, 'FIESER INFORMAATION MATRIX' *)
WRITE (6, 540) DELJ2
540 FORMAT (' *, 12X, 'DELTJ2=', E15.8)
CALL STEP (THETA, DELTAP, DELJ2, DTHETA, THETAN)
WRITE (6, 550) DTHETA
550 FORMAT (' *, 12X, 'DTHETA=', E15.8)
WRITE (6, 560) THETAN
560 FORMAT (' *, 12X, 'THETAN=', E15.8)
WRITE (6, 580)
580 CONTINUE
WRITE (6, 810) ITIT3
FORMAT (11, 40X, 10A4)
CALL WPLOT1(VTHETA, VRLIKE, ISTEP, 40, JIAE3)
WRITE (6, 810) ITIT1
CALL WPLOT1(VTHETA, VCONST, ISTEP, 4C, JIAE1)
WRITE (6, 810) ITIT6
CALL WPLOT1(VTHETA, VDELB, ISTEP, 40, JIAE6)
WRITE (6, 810) ITIT4
CALL WPLOT1(VTHETA, VDELJ, ISTEP, 40, JIAE4)
WRITE (6, 810) ITIT5
CALL WPLOT1(VTHETA, VDELJ2, ISTEP, 40, JIAE5)
900 CONTINUE
1000 RETURN
END

SUBROUTINE PARAM (THETA)
SUBROUTINE PARAM READS IN THE PARAMETERS NECESSARY TO FORM
THE SYSTEM MATRIX
COMMON F (3, 3), G (3, 3), H (3, 3), R (3, 3), C (1, 1), K1F, K1G, K2G, K1H, K2H, K1Q,
$K1a, RKGm (3, 3), K1PHIX, K2PHIX, K1PHIZ, K2PHIZ,$
$K1aRGM, K2RGM, RMASS, F11, F22, RKY, RKS1, RLE, ALPHA, BZERO, DT, KPECE,$
$SNRC, IROST4, IR1ST4, IR1ST4, IDATS4, IDATS4, NDATS4, NPCAL,$
$SATEN (4, 3), RVCAJ, TVEL, GRCAL, NPONIT, SET, P (3, 3), B (3, 3), K1B, K1P,$
$SLIKE, K1DHP, K2DHP, K1DRK, K2DRK, NPT1, XIC (3, 1), BZERO, K1DELP,$
$K2DELP, K1DELK, K2DELK, K1EP, K2DP, IFLAG
10 FORMAT (F15. 8)
READ (5, 10) RMasse, F11, F22, RKY, RKS1, RLE, ALPHA, BZERO, THETA, BZERO, DT,
$SM4$$
DO 20 T = 1, K1G
DO 20 J = 1, K2G
20 READ (5, 10) G (I, J)
DO 30 I = 1, K1H
DO 30 J = 1, K2H
30 READ (5, 10) H (I, J)
DO 40 I = 1, K1Q
DO 40 J = 1, K1Q
40 READ (5, 10) D (I, J)
DO 50 I = 1, K1R
DO 50 J = 1, K1R
50 READ (5, 10) ATTEN (I, J)
70 CONTINUE
RETURN
END

SUBROUTINE SYSTX (THETA)
SUBROUTINE SYSTX FORMS THE SYSTEM MATRIX - P MATRIX
COMMON F (3, 3), G (3, 3), H (3, 3), R (3, 3), C (1, 1), K1F, K1G, K2G, K1H, K2H, K1Q,
$K1a, RKGm (3, 3), K1PHIX, K2PHIX, K1PHIZ, K2PHIZ,$
$K1aRGM, K2RGM, RMASS, F11, F22, RKY, RKS1, RLE, ALPHA, BZERO, DT, NRBCR,$
$SNRC, IROST4, IR1ST4, IR1ST4, IDATS4, IDATS4, NDATS4, NPCAL,$
$SATEN (4, 3), RVCAJ, TVEL, GRCAL, NPONIT, SDT, P (3, 3), B (3, 3), K1B, K1P,$
$SLIKE, K1DHP, K2DHP, K1DRK, K2DRK, NPT1, XIC (3, 1), BZERO, K1DELP,$
$K2DELP, K1DELK, K2DELK, K1EP, K2DP, IFLAG
F (1, 1) = - (((2.0 * F22 * THETA) + (BZERO * TVEL)) / (RMASS * TVEL))
F (1, 2) = 2.0 * F22 * THETA / RMASS
\[ P(1, 3) = (-RKY) / RMASS \]
\[ P(4, 1) = 0.0 \]
\[ P(4, 2) = (-RKSI) * IVEL / (2.0 * F11 * THETA * (RIE * 2.0)) \]
\[ P(4, 3) = (-ALPHA) * TVL / (RIE * ZZERO) \]
\[ F(3, 1) = 1.0 \]
\[ F(3, 2) = 0.0 \]
\[ F(3, 3) = 0.0 \]

RETURN

SUBROUTINE ZMT1X(Z, K11)

SUBROUTINE ZMT1X FORMS THE Z MATRIX FOR THE KALMAN EQUATION

COMMON P (3, 3), G (3, 3), H (3, 3), R (3, 3), Q (1, 1), K1F, K1G, K2G, K1H, K2H, K1Q, K2Q, KRM (3, 3), KPHIX, K2PHIX, KPHIZ, K2PHIZ, K1RM, K2RM, RMASS, F11, F22, RY, RKSI, RIE, ALPHA, RZERO, DT, NREC, SNREC, IP, IPOSTA, IROEND, IF1STA, IREND, ILATSA, IDATEN, HDATA, NRCA, SATEN (4, 3), RVCA, TVEL, GFCAL, NPOINT, SEI, P (3, 3), B (3, 3), K15, K1P, SRLIKE, KIDPH, K2LPH, K1DRG, K2DRG, NFT1, XIC (3, 1), BZERO, K1DELP, K2DELP, K1DELK, K2DELK, K1DP, K2DP, IFLAG

DIMENSION Z (6, 6)
DIMENSION P (3, 3)
DIMENSION Z11 (3, 3)
DIMENSION R1 (3, 3)
DIMENSION H1 (3, 3)
DIMENSION F1H (3, 3)
DIMENSION Z12 (3, 3)
DIMENSION GT (1, 3)
DIMENSION OGT (3, 3)
DIMENSION Z21 (3, 3)
DIMENSION Z22 (3, 3)

DOUBLE PRECISION WAREA (9)
DOUBLE PRECISION RDP (3, 3)
DOUBLE PRECISION RDP (3, 3)

CALL TRANS (P, PT, K1F, K1F, K1FT, K2FT)
CALL N3G (PT, Z11, K1FT, K2FT, K111, K211)

IDT = 0
CALL SPT0DP (R, K1R, K1R, RIP)
CALL LINV1F (RDP, K1R, K1R, RDP, IDGT, WAREA, TFP)
CALL DPT0SP (RDP, K1R, K1R, RI)
CALL TRANS (H, HT, K1H, K2H, K1HT, K2HT)
CALL MULT (H, H, RIH, K1R, K1R, K1H, K2H, K1RIH, K2RIH)
CALL MULT (HT, R1Z1, Z12, K1HT, K2HT, K1RIH, K2RIH, K1Z12, K2Z12)
CALL TRANS (G, GT, K1G, K2G, K1GT, K2GT)
CALL MULT (G, GT, Z21, K1Q, K1Q, K1GT, K2GT, K1QGT, K2QGT)
CALL MULT (G, GT, Z21, K1Q, K1Q, K1QGT, K2QGT, K1Z21, K2Z21)
CALL SCALAR (F, 1.0, Z22, K1P, K1P, K1Z22, K2Z22)

DO 10 I = 1, K1Z1
DO 10 J = 1, K2Z1
M = I
N = J

10 Z (I, N) = Z11 (I, J)
DO 20 I = 1, K1Z12
DO 20 J = 1, K2Z12
M = I
N = J + K2Z11

20 Z (I, N) = Z12 (I, J)
DO 30 I = 1, K1Z21
DO 30 J = 1, K2Z21
M=I+K1211
N=J
30 Z(J,M)=Z21(I,J)
DO 40 I=1,K1Z22
DO 40 J=1,K2Z22
M=I+K122
N=J+K2Z12
40 Z(J,M)=Z22(I,J)
K1Z=K1Z11+K1221
RETURN
END

SUBROUTINE STM(F,LT,PHI,K1F,K1PHI,K2PHI)
SUBROUTINE STM DIMENSIONS THE NECESSARY ARRAYS FOR SUBROUTINE STM. IT ACTS AS A DRIVER FOR SUBROUTINE STM.
DOUBLE PRECISION FDP(3,3)
DIMENSION F(K1F,K1F)
DOUBLE PRECISION PHI(K1F,K1F)
DOUBLE PRECISION FDT(3,3)
DOUBLE PRECISION PRD(3,3)
DOUBLE PRECISION PRD1(3,3)
DOUBLE PRECISION TERM(3,3)
DOUBLE PRECISION PHI1(3,3)
DOUBLE PRECISION DIFF(3,3)
DOUBLE PRECISION RM(3,3)
CALL SPTODP(F,K1F,K1F,FDP)
CALL STM(FDP,PHI,FDT,PRD,PRD1,TERM,PHI1,DIFF,RM,DT,K1F,$K1PHI,K2PHI,IFLAG)
RETURN
END

SUBROUTINE STMZ(Z,DT,PHI,K1Z,K1PHIZ,K2PHIZ)
SUBROUTINE STMZ DIMENSIONS THE NECESSARY ARRAYS FOR SUBROUTINE STMZ. IT ACTS AS A DRIVER FOR SUBROUTINE STMZ.
DIMENSION Z(K1Z,K1Z)
DOUBLE PRECISION PHI(6,6)
DOUBLE PRECISION FDT(6,6)
DOUBLE PRECISION PRD(6,6)
DOUBLE PRECISION PRD1(6,6)
DOUBLE PRECISION TERM(6,6)
DOUBLE PRECISION ZDP(6,6)
DOUBLE PRECISION PHI1(6,6)
DOUBLE PRECISION DIFF(6,6)
DOUBLE PRECISION RM(6,6)
CALL SPTODP(Z,K1Z,K1Z,ZDP)
CALL STM(ZDP,PHI,FDT,PRD,PRD1,TERM,PHI1,DIFF,RM,DT,K1Z,$K1PHIZ,K2PHIZ,IFLAG)
RETURN
END

SUBROUTINE STM(F,PHI,FDT,PRD,PRD1,TERM,PHI1,DIFF,RM,DT,K1F,$K1PHI,K2PHI,IFLAG)
SUBROUTINE STM CALCULATES THE STATE TRANSITION MATRIX.
DOUBLE PRECISION F(K1F,K1F)
DOUBLE PRECISION FDT(K1F,K1F)
DOUBLE PRECISION PHI(K1F,K1F)
DOUBLE PRECISION PRD(K1F,K1F)
DOUBLE PRECISION PHI1(K1F,K1F)
DOUBLE PRECISION PRED1(K1F,K1F)
DOUBLE PRECISION DIFF(K1F,K1F)
DOUBLE PRECISION TERM(K1F,K1F)
DOUBLE PRECISION RM(K1F,K1F)
DOUBLE PRECISION B
DOUBLE PRECISION COEF
DOUBLE PRECISION CT
DOUBLE PRECISION CHK
DOUBLE PRECISION DTDP
DOUBLE PRECISION FACTDP

01 DTDP=DBLE(DT)
  CT=DBLE(1.0E-16)
  IFLAG=0
  B=DBLE(0.0)
CALL SCALDP(F,B,RM,K1F,K1F,K1RM,K2RM)
DO 20 I=1,K1RM!
  RM(I,I)=DBLE(1.0)
DO 320 I=1,200
  IF(I .GT. 1)GO TO 120
CALL SCALDP(F,PRED,FDT,K1F,K1F,K1FD1,K2FD1)
CALL ADDDP(PRED,FM,PHI,K1RM,K2RM,K1PHI,K2PHI)
  B=DBLE(1.0)
CALL SCALDP(F,E,PBD,K1P,K1PRD,K2PBD)
GO TO 300
120 CALL MULTDP(F,FRD,PRED1,K1F,K1F,K1PRD,K2PRD,K1PRD1,K2PRD1)
  CHK=DBLE(1.0E-71)
CALL OVERDP(PRED1,CHK,K1PRD1,K2PRD1,IFLAG)
  IF(IFLAG .EQ. 0)GO TO 160
WRITE (6,125)
125 FORMAT(//,' ',2X,'PRED1 EXCEEDS EXPONENT OVERFLOW.',$1X,'CURRENT VALUE OF PHI USED. EXECUTION CONTINUES.')
GO TO 420
160 B=DBLE(1.0)
CALL SCALDP(PHI,B,PHI1,K1PHI,K2PHI)
  CHK=DBLE(1.0E-76)
  IF(COEF .LE. CHK)GO TO 380
CALL SCALDP(PR1,COEF,TERM,K1PRD,K2PRD,K1TERM,K2TERM)
CALL ADDDP(PHI,TERM,PHI,K1PHI,K2PHI)
CALL SUBDP(PHI,PHI1,DIFF,K1PHI,K2PHI)
DO 260 J=1,K1PHI
  DO 260 K=1,K2PHI
    IF(DABS(DIFF(J,K)) .GT. CT)GO TO 280
260 CONTINUE
GO TO 420
280 IF(I .LT. 300)300,340,340
300 B=DBLE(1.0)
CALL SCALDP(PHI,B,PHI1,K1PHI,K1PHI1)
320 CONTINUE
340 WRITE (6,360)
360 FORMAT(//,' ',5X,'200 ITERATIONS. NO CONVERGENCE')
GO TO 450
380 WRITE (6,400)
400 FORMAT(//,' ',2X,'COEF .LE. 1.0E-76. ITERATION STOPPED.',$1X,'CURRENT VALUE OF PHI USED. EXECUTION CONTINUES.')
420 WRITE (6,440)
440 FORMAT(//,' ',2X,'13,1X,ITERATIONS FOR STM.')
450 WRITE (6,460)
460 FORMAT(//,' ',12X,'STM DIFF MATRIX')
SUBROUTINE PTCATI(PHIZ,K1PHI,PSP,K1P)

SUBROUTINE PTCATI CALCULATES THE STATE COVARIANCE MATRIX I.E.

THE P MATRIX.

DOUBLE PRECISION PHI11(I,J)
DOUBLE PRECISION PHI12(I,J)
DOUBLE PRECISION PHI21(I,J)
DOUBLE PRECISION PHI22(I,J)
DOUBLE PRECISION W(I,J)
DOUBLE PRECISION X(I,J)
DOUBLE PRECISION Y(I,J)
DOUBLE PRECISION DIFF(I,J)
DOUBLE PRECISION PHI(I,J)
DOUBLE PRECISION PHI2(I,J)
DOUBLE PRECISION PHI1(I,J)
DOUBLE PRECISION PHI2(I,J)
DOUBLE PRECISION F(I,J)
DOUBLE PRECISION PSUM(I,J)
DOUBLE PRECISION PHIZ(K1PHI,K1PHI)
DOUBLE PRECISION PHIZ(K1PHI,K1PHI)
DOUBLE PRECISION F(I,J)

DIMENSION PSP(3,3)
DOUBLE PRECISION CONST
DOUBLE PRECISION ERROR
DOUBLE PRECISION WKAREA(36)
DOUBLE PRECISION A,B

ER2OP=DELE(1.0E-16)
K11=K1PHI/2.0
DO 20 I=1,K11
   DO 20 J=1,K11
20   PHI11(I,J)=PHIZ(I,J)
DO 40 I=1,K11
   DO 40 J=1,K11
   M=I
   N=K11+J
40   PHI12(I,J)=PHIZ(M,N)
   DO 60 I=1,K11
      DO 60 J=1,K11
      M=I+N
      N=K11+J
60   PHI21(I,J)=PHIZ(M,N)
   DO 80 I=1,K11
      DO 80 J=1,K11
      M=K11+I
      N=K11+J
80   PHI22(I,J)=PHIZ(M,N)
A=DOUBLE(0.0)
CALL SCALDP(PHI11,A,P2,K11,K11,K1P2,K2P2)
ICJUNT=1
85 B=DOUBLE(1.0)
CALL SCALDP(P2,B,P1,K1P2,K2P2,K1P1,K2P1)
CALL MULTDP(PHI22,P2,W,K11,K11,K1P2,K2F2,K1W,K2W)
CALL ADDDP(PHI11,B,X,K11,K11,K1X,K2X)
CALL MULTDP(PHI12,P2,X,K11,K11,K1P2,K2P2,K1Y,K2Y)
CALL ADDDP(PHI11,Y,Z,K11,K11,K1Z,K2Z)
IDGT=0
CALL LINV1F(Z,K1Z,K11,Z1,IDGT,WKAREA,IER)
CALL MULTDP (X, Z, F2, K1X, K2X, K1Z, K2Z, K1P2, K2P2)
CALL TRANS (P2, F2T, K1P2, K2P2, K1P2T, K2P2T)
CALL ADDDP (P2, F2T, PSUM, K1P2, K2P2, K1PSUM, K2PSUM)
CALL =ST=DELE (500)
CALL SCALDP (PSUM, CONST, F2, K1PSUM, K2PSUM, K1P2, K2P2)
CALL SUBDP (P2, P1, DIFF, K1P2, K2P2, K1DIFF, K2DIFF)
DO 100 I=1, K1DIFF
DO 100 J=1, K2DIFF
100 CONTINUE
GO TO 150
110 ICOUNT=ICOUNT+1
IF (ICOUNT .GE. 200) GO TO 115
GO TO 85
115 WRITE (6, 120)
120 FORMAT (//, '5X, '200 ITERATIONS. RICCATI SOLUTION DOES NOT', ' CONVERGE. *)
B=J3LE (1, 0)
150 CALL SCALDP (P2, B, P, K1F2, K2P2, K1P, K2F)
CALL DPTOSP (P, K1P, K1P, P, K1P)
IF (ICOUNT .GE. 200) GO TO 165
WRITE (6, 160) ICOUNT
160 FORMAT (//, '5X, '13, 1X, 'ITERATIONS FOR RICCATI SOLUTION TO', ' CONVERGE. *)
165 WRITE (6, 170)
170 FORMAT (//, '5X, '200, 'RICCATI DIFFERENCE MATRIX. ')
CALL OUTDP (DIFF, K1DIFF, K2DIFF)
END

SUBROUTINE KGMTRI
SUBROUTINE KGMTRI CALCULATES THE KALMAN GAIN MATRIX.
COMMON F (3, 3), G (3, 1), H (3, 3), R (3, 3), Q (1, 1), K1P, K1G, K2G, K1H, K2H, K1Q, K2Q, K1PGM, K2PGM, MASS, P11, P22, RXY, PKSI, RIE, ALPHEL, RSZEL, DT, NFECE, SNP, JCD, ISTOA, IREND, IRASTA, IRENDS, ITATS, IDAT2, NDATA, M3CAL, SASHE (4, 3), RICAL, TVEL, 3RCAL, NPOINT, SIT, P (3, 3), B (3, 3), K1B, K1P, RLIKE, K1IDP, K2DPH, K1DPG, K2DPG, NPT1, XIC (3, 1), BZERO, K1DELP, K2JLP, K1DELK, K2FUK, K1FLP, K2DF, IFLAG
DIMENSION HTFI (3, 3)
DIMENSION HT (3, 3)
DIMENSION R1 (3, 3)
DOUBLE PRECISION WKAREA (9)
DOUBLE PRECISION RIDP (3, 3)
DOUBLE PRECISION RDP (3, 3)
01 IDST=0
CALL SPOTDP (R, K1R, K1R, FDP)
CALL LINVP (RDF, K1R, K1R, RIDP, IEGT, WKAREA, IER)
CALL DPTOSP (RIDE, K1R, K1R, RI)
CALL TRANS (H, HT, K1H, K2H, K1HT, K2HT)
CALL MULT (H, R1, HTRI, K1HT, K2HT, K1R, K1HTRI, K2HTRI)
CALL MULT (P, HTRI, RKG, K1P, K1P, K1HTRI, K2HTRI, K1RKG, K2RKG)
RETURN
END

SUBROUTINE STAHEL
COMMON F (3, 3), G (3, 1), H (3, 3), R (3, 3), Q (1, 1), K1P, K1G, K2G, K1H, K2H, K1Q, K2Q, K1PGM, K2PGM, MASS, P11, P22, RXY, PKSI, RIE, ALPH, RSZEL, DT, NFECE, SNP, JCD, ISTOA, IREND, IRASTA, IDATA, IDAT2, NDATA, M3CAL, SASHE (4, 3), RICAL, TVEL, 3RCAL, NPOINT, SIT, P (3, 3), B (3, 3), K1B, K1P, RLIKE, K1IDP, K2DPH, K1DPG, K2DPG, NPT1, XIC (3, 1), BZERO, K1DELP, K2JLP, K1DELK, K2FUK, K1FLP, K2DF, IFLAG
DIMENSION HTFI (3, 3)
DIMENSION HT (3, 3)
DIMENSION R1 (3, 3)
DOUBLE PRECISION WKAREA (9)
DOUBLE PRECISION RIDP (3, 3)
DOUBLE PRECISION RDP (3, 3)
01 IDST=0
CALL SPOTDP (R, K1R, K1R, FDP)
CALL LINVP (RDF, K1R, K1R, RIDP, IEGT, WKAREA, IER)
CALL DPTOSP (RIDE, K1R, K1R, RI)
CALL TRANS (H, HT, K1H, K2H, K1HT, K2HT)
CALL MULT (H, R1, HTRI, K1HT, K2HT, K1R, K1HTRI, K2HTRI)
CALL MULT (P, HTRI, RKG, K1P, K1P, K1HTRI, K2HTRI, K1RKG, K2RKG)
RETURN
END
SUBROUTINE STATE(ZMEAS , XHAT, PHIK1, PHIK2)
    THIS SUBROUTINE CALCULATES THE STATE ESTIMATES OF THE
    KALMAN FILTER ACCORDING TO THE CCONTINUOUS FORM OF THE
    KALMAN FILTER.

    COMMON P(3,3),Q(3,3),H(3,3),R(3,3),K1P,K1G,K2G,K1H,K2H,K1Q,
        SK1,SK2,RKGM,HKGM,FMAS,PI1,P22,FKY,RKSI,RIE,ALPHA,BZERO,DT,WEPR,
        SH&CD,IROSTA,IR0END,IR1STA,IR1END,ICATS,ICATSA,IDATEN,MDATA,NRCAL,
        SATLEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $SK2DELP,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
        $PLKE,K1DELP,K2DELP,K1DRKGM,K2DRKGM,NPT1,XIC(3,1),BZERO,K1DELP,
DIMENSION PRD1(3,3)
DIMENSION PRD2(3,3)
DIMENSION XM1(3,1)
DIMENSION XE1(3,1)
DIMENSION XE2(3,1)
DIMENSION GAMMA(3,3)
DIMENSION X1(3,1)
DIMENSION X2(3,1)
DOUBLE PRECISION FKFPDP(3,3)
DOUBLE PRECISION FKFPIDP(3,3)
DOUBLE PRECISION PHIKID(3,3)
DIMENSION U(1,NPT1)
DIMENSION U1(1,1)
DIMENSION A1(3,1)
DIMENSION A2(3,1)
DIMENSION LAMEDA(3,1)
DIMENSION XEP(3,1)
CALL NEG(H,HNEG,K1H,K2H,K1NEG,K2HNEG)
CALL MULT(FKGM,HNEG,RH1,K1FKGM,K2FKGM,K1HNEG,K2HNEG,K1RH,K2EH)
CALL ADD(F, RH, FKFK, K1F, K1FK, K2FK)
WRITE(6,5)
5 FORMAT(//,'(12X,'KALMAN FILTER SYSTEM MATRIX (FKF)')
CALL OUTPUT(FKF,K1FKF,K2FKF)
CALL STMF(KF,F1,FHIK1,K1F1,K2F1,FHIK2,F2F1)
FACT=1.0D0
CALL SCALDP(FHIK1,FAC1,FHIK2,FAC2,FHIK3,FAC3)
WRITE(6,10)
10 FORMAT(//,'(12X,'F matrices for kalman filter system (PHIK)')
CALL OUTPUT(PHIK1,K1PHIK,K2PHIK)
IDST=0
CALL LINVF(FHIK,K1PHIK,K2PHIK,PHIK1,HDGT,WKAR2A,IER)
CALL DPTOSP(FHIK,K1PHIK,K2PHIK,PHIK1)
IDST=0
CALL SPTDSP(FKF,K1FKF,K2FKF,FKFPDP)
CALL LINVF(FKFPDP,K1FKF,K2FKF,FKFPIDP,HDGT,WKARPA,IER)
CALL DPTOSP(FKFPDP,K1FKF,K2FKF,FKFPDP)
CALL SCALAR(F,0.0,RM1,K1F1,K1F2,K1RM,K2RM)
DO 20 I=1,K1RM
RM(I,I)=1.0
20 CONTINUE
CALL SUB(RM,PHIK1,T2,K1RM,K2RF,K1T2,K2T2)
CALL MULT(T2,FKGM,PRD1,K1T2,K2T2,K1FKGM,K2FKGM,K1PRD1,K2PRD1)
CALL MULT(FKFI,ERD1,PRD2,K1FKF,K2FKF,K1PFD1,K2PFD1)
CALL MULT(PHIKSP,PRD2,SK2AM,K1PHIK,K2PHIK,K1PFD2,K2PFD2)
CALL MULT(T2,G,A1,K1T2,K2T2,K1G,K2G,K1A1,K2A1)
CALL MULT(FKFI,A1,A2,K1KP1,K2KP1,K1A1,K2A1,K1A2,K2A2)
CALL MULT(PHIKSP,A2,LBDAA,K1PHIK,K1PHIK,K1A2,K2A2,K1LAMB,K2LAMB)
CALL SCALAR(XIC,0.0,XEI,3,1,K1XE1,K2XE1)
DO 1000 I=1,NPOINT
DO 1000 J=1,3
1000 CONTINUE
XM1(J,1)=2MEAS(J,I)
100 CONTINUE
U1(1,1)=U(1,1)
CALL MULT(PHIKSP,XE1,X1,K1PHIK,K2PHIK,K1XE1,K2XE1,K1X1,K2X1)
CALL MULT(GAMMA,XM1,X2,K1GAM,K2GAM,3,1,K1X2,K2X2)
CALL MULT(LAMDAO,U1,X3,K1LAMB,K2LAMB,1,1,K1X3,K2X3)
CALL ADD(X1,X2,XEP,K1X1,K2X1,K1XEP,K2XEP)
CALL ADD(XEP, X3, XE2, K1XEP, K2XEP, K1XE2, K2XE2)
DO 200 J=1,3
XHAT(J, I) = XE2(J, I)
200 CONTINUE
CALL SCALAP(XE2, 1.0, XE1, K1XE2, K2XE2, K1XE1, K2XE1)
1000 CONTINUE
RETURN
END

SUBROUTINE ZCOV
SUBROUTINE ZCOV CALCULATES THE P MATRIX FOR THE MAXIMUM
LIKELIHOOD PARAMETER IDENTIFICATION PROCESSOR.
COMMON P(3,3), G(3,1), H(3,3), R(3,3), C(1,1), K1F, K1G, K2G, K1H, K2H, K1Q,
$K1a, RKGM(3,3), K1PHIX, K2PHIX, K1PHIZ, K2PHIZ,
$K1RKG, K2RKG, RMASS, F11, F22, RKY, RKS1, RLE, ALPHA, RZERO, DT, NRECR,
$NRECD, IPOSTA, IPOEND, IRISTA, IRIEND, IDATS, IDATN, NDATA, NRCAL,
$ATTEN(4,3), RVCAL, TVEL, GRCAL, NPOINT, SDT, P(3,3), B(3,3), K1F, K1P,
$SLIKE, K1DPH, K2DPH, K1DRKG, K2DRKG, NPT1, XIC(3,1), BZERO, K1DELP,
$K2DELP, K1DELK, K2DELK, K1DF, K2DF, IFLAG
DIMENSION HT(3,3), HHT(3,3), HPHT(3,3)
01 CALL TRANS(H, HT, K1H, K2H, K1HT, K2HT)
CALL MULT(P, HT, K1F, K1H, K1HT, K2HT, K1PHT, K2PHT)
CALL MULT(P, HPHT, K1H, K2H, K1PHT, K2PHT, K2PHT)
CALL ADD(HPHT, B, K1HPHT, K2HPHT, K1E, K2E)
RETURN
END

SUBROUTINE RINCOV(XHAT, ZMEAS, ZHAT, RNU)
SUBROUTINE RINCOV CALCULATES THE INNOVATION FOR THE NLPI
PROCESSOR.
COMMON P(3,3), G(3,1), H(3,3), R(3,3), C(1,1), K1F, K1G, K2G, K1H, K2H, K1Q,
$K1a, RKGM(3,3), K1PHIX, K2PHIX, K1PHIZ, K2PHIZ,
$K1RKG, K2RKG, RMASS, F11, F22, RKY, RKS1, RLE, ALPHA, RZERO, DT, NRECR,
$NRECD, IPOSTA, IPOEND, IRISTA, IRIEND, IDATS, IDATN, NDATA, NRCAL,
$ATTEN(4,3), RVCAL, TVEL, GRCAL, NPOINT, SDT, P(3,3), B(3,3), K1E, K1P,
$SLIKE, K1DPH, K2DPH, K1DRKG, K2DRKG, NPT1, XIC(3,1), BZERO, K1DELP,
$K2DELP, K1DELK, K2DELK, K1DF, K2DF, IFLAG
DIMENSION XHAT(3, NPOINT)
DIMENSION ZMEAS(3, NPOINT)
DIMENSION RNU(3, NPOINT)
DIMENSION ZHAT(3, NPOINT)
01 CALL MUL(H, XHAT, ZHAT, K1H, K2H, 3, NPOINT, K1ZHAT, K2ZHAT)
CALL SUB(ZMEAS, ZHAT, RNU, 3, NPOINT, K1NU, K2NU)
RETURN
END

SUBROUTINE RMLPI(RNU, SC, SD)
SUBROUTINE RMLPI CALCULATES THE VALUE OF THE LIKELIHOOD Func-
TION.
COMMON P(3,3), G(3,1), H(3,3), R(3,3), C(1,1), K1F, K1G, K2G, K1H, K2H, K1Q,
$K1a, RKGM(3,3), K1PHIX, K2PHIX, K1PHIZ, K2PHIZ,
$K1RKG, K2RKG, RMASS, F11, F22, RKY, RKS1, RLE, ALPHA, RZERO, DT, NRECR,
$NRECD, IPOSTA, IPOEND, IRISTA, IRIEND, IDATS, IDATN, NDATA, NRCAL,
$ATTEN(4,3), RVCAL, TVEL, GRCAL, NPOINT, SDT, P(3,3), B(3,3), K1B, K1P,
$SLIKE, K1DPH, K2DPH, K1DRKG, K2DRKG, NPT1, XIC(3,1), BZERO, K1DELP,
$K2DELP, K1DELK, K2DELK, K1DF, K2DF, IFLAG
DIMENSION RNU(3, NPOINT)
DIMENSION C (3, 1)
DIMENSION AT (1, 1)
DIMENSION AT (1, 3)
DIMENSION A (3, 1)
DIMENSION BI (3, 3)
DOUBLE PRECISION EX
DOUBLE PRECISION WKAREA (9)
DOUBLE PRECISION EIDP (3, 3)
DOUBLE PRECISION DETBDP
DOUBLE PRECISION EDP (3, 3)
DOUBLE PRECISION VECTOR (3)
DOUBLE PRECISION D1DP
DOUBLE PRECISION E2DP
DOUBLE PRECISION BDP1 (3, 3)
DOUBLE PRECISION FACTOR

01 SUM = 0.0
SC = 0.0
SD = 0.0
ID = 0
CALL SPTODP (B, K1B, K1B, BDP)
FACTOR = 1.00D 00
CALL SCALDP (BDP, FACTOR, BDP1, K1B, K1B, K1EDP, K2BDP01)
CALL LINV1P (BDE, K1B, K1B, BDP, IDGT, WKAREA, IER)
CALL DPTOSP (BDP, K1B, K1B, BI)
IJ = 4
D1DP = 0.00D 00
CALL LINV3F (BDEP1, VECTOR, IJOB, K1B, K1B, D1DP, D2DP, WKAREA, IER)
IF (D2DP .GT. C.C.C 00) GO TO 50
D2DP = -D2DP
EX = 2.00D 00 ** D2DP
DETB = D1DP / EX
GO TO 60
50 EX = 2.00D 00 ** D2DP
DETB = D1DP * EX
GO TO 60
60 DETB = DETBDP
SI = ALOG (DETB)
WRITE (6, 65) SI
65 FORMAT (//, ' ', 12X, 'NATURAL LOG OF THE DETERMINANT OF B=', $1X, E15.8)
DO 100 I = 1, NPCINT
DO 80 J = 1, 3
A (J, 1) = RNU (J, 1)
80 CONTINUE
CALL MULT (BI, A, C, 3, 3, 1, K1C, K2C)
CALL TRANS (A, AT, 3, 1, K1AT, K2AT)
CALL MULT (AT, C, ATC, K1AT, K2AT, K1C, K2C, K1ATC, K2ATC)
CONST = ATC (1, 1)
SC = SC + CONST
SD = SD + ALOG (DETB)
SUM = SUM + (CONST + ALOG (DETB))
100 CONTINUE
SD = (-1.0 / 2.0) * SC
SC = (-1.0 / 2.0) * SC
RLIKE = (-1.0 / 2.0) * SUM
RETURN
END

SUBROUTINE PARPHI (DPMTX, DPHIX)
SUBROUTINE PARPHI CALCULATES THE PARTIAL DERIVATIVE OF THE
STATE TRANSITION MATRIX (PHI) WITH RESPECT TO THETA.

COMMON P(3,3),G(3,1),H(3,3),F(3,3),C(1,1),K1P,K1G,K2G,K1H,K2H,K1Q,
SK1D,RKG(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
$K1PKGH,K2PKGM,MMASS,PI1,P22,KRY,RSI,RL,ALPHA,RZERO,DT,NCER,
$NK1CD,IRTSTA,IR1END,IR1STA,IR1END,IDATS2,IDATEN,NDATA,NCAL,
$SATE(4,3),RVCAL,TVEL,GRCAL,NCGLT,SDT,P(3,3),B(3,3),K1B,K1P,
$RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
$K2DELP,K1DELK,K2DELK,K1DP,K2DP,IFLAG

DATA ERROR,.000001/
DIMENSION PFD(3,3)
DIMENSION PFD1(3,3)
DIMENSION TERM(3,3)
DIMENSION SUM(3,3)
DIMENSION DFMTX(K1DP,K2DP)
DIMENSION DPFX(K1DPH,K2DPH)
DIMENSION PFD2(3,3)
DIMENSION DIFF(3,3)

01 CALL SCALAR(F,0.0,DPFX,K1P,K1DPH,K2DPH)
CALL SCALAR(F,1.0,PFD,K1P,K1PRD,K2ERD)
DO 100 I=1,200

COF=(1/FACT(I))*(DT**I)*I
CALL MULT(PFD,DFMTX,PFD2,K1PFL,K1DPH,K2DPH,K1PRD2,K2PRD2)
CALL SCALAR(PFD2,COEF,TERM,K1PFL,K1DPH,K2DPH,K1TERM,K2TERM)
CALL SCALAR(DPFX,1.0,TERM,K1DPH,K2DPH,K1SUM,K2SUM)
CALL ADD(TERM,DPFX,DPFX,K1TERM,K1DPH,K2DPH)
CALL SUB(DPFX,TERM,DIFF,K1DPH,K2DPH,K1DIFF,K2DIFF)
CALL MULT(F,FF1,PFD1,K1F,K1PFL,K2PFL,K1PFD1,K2PFD1)
CALL SCALAF(PFD1,1.0,PFD1,K1PFL,K2PFL,K1PFD1,K2PFD1)
DO 50 J=1,K1DIFF
DO 50 K=1,K2DIFF

IF(ABS(DIFF(J,K))-ERROR)50,50,100

50 COFINUF
GO TO 200

100 COFINUF
WRITE(6,110)
110 FORMAT(/'*10X,'200 ITERATIONS. NC CONVERGENCE.')
200 REIDPH
END

SUBROUTINE PARF(THETA,DFMTX)
SUBROUTINE PARF CALCULATES THE PARTIAL DERIVATIVE OF THE SYSTEM
MATRIX WITH RESPECT TO THETA.

COMMON P(3,3),G(3,1),H(3,3),F(3,3),C(1,1),K1P,K1G,K2G,K1H,K2H,K1Q,
$SK1D,RKG(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
$K1PKGH,K2PKGM,MMASS,PI1,P22,KRY,RSI,RL,ALPHA,RZERO,DT,NCER,
$NK1CD,IRTSTA,IR1END,IR1STA,IR1END,IDATS2,IDATEN,NDATA,NCAL,
$SATE(4,3),RVCAL,TVEL,GRCAL,NCGLT,SDT,P(3,3),B(3,3),K1B,K1P,
$RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
$K2DELP,K1DELK,K2DELK,K1DP,K2DP,IFLAG

DIMENSION DFMTX(K1DP,K2DP)
DIMENSION A(3,3)
A(1,1)=((-2.0)*P22)/MMASS TVEL
A(1,2)=(2.0*P22)/MMASS
A(1,3)=0.0
A(2,1)=0.0
A(2,2)=(RKS*I+TVEL)/2.0*(THETA**2.0)*PI1*(B**2.0)
A(2,3)=0.0
A(3,1)=0.0
A(3,2)=0.0
A (j, 3) = 0.0
CALL SCALAR (A, 1.0, DFMTX, 3, 3, K1DF, K2DF)
RETURN
END

SUBROUTINE PDP (DFMTX, DELTAP, DELTAK)
COMMON F (3, 3), G (3, 1), H (3, 3), R (3, 3), C (1, 1), K1F, K1G, K2G, K1H, K2H, K1Q,
K1R, K2R, K1M, K2M, RM, F11, F22, R11, R12, ALPHA, BZERO, DT, NPT, CR,
SR, CD, IROA, ITA, IDAT, IDATJ, INDA, INDM, NSCAL,
SATEN (4, 3), RCAL, TEL, ECAL, NPOINT, LST, P (3, 3), B (3, 3), K1B, K1P,
& RLIKE, K1DPH, K2DPH, K1DKG, K2DKG, NPT1, XIC (3, 1), BZERO, K1DEL,
K2DELK, K1DELK, K2DELK, K1DF, K2DF, XPIAG
DIMENSION RI (3, 3)
DIMENSION HRI (3, 3)
DIMENSION DELTAK (3, 3)
DIMENSION DFMTX (3, 3)
DIMENSION HT (3, 3)
DIMENSION DMFX (3, 3)
DIMENSION HPR (3, 3)
DIMENSION RIHF (3, 3)
DIMENSION PRD1 (3, 3)
DIMENSION PT (3, 3)
DIMENSION A11 (3, 3)
DIMENSION PRD2 (3, 3)
DIMENSION PRD3 (3, 3)
DIMENSION A21 (3, 3)
DIMENSION RH (3, 3)
DIMENSION RFH (3, 3)
DIMENSION RHIDP (3, 3)
DIMENSION PRD4 (3, 3)
DIMENSION A22 (3, 3)
DIMENSION A12 (3, 3)
DIMENSION DELTAP (3, 3)
DOUBLE PRECISION PHI (6, 6)
DIMENSION ZDELP (6, 6)
DOUBLE PRECISION RHIDP (3, 3)
DOUBLE PRECISION RHDP (3, 3)
DOUBLE PRECISION WKAREA (9)
NAME LIST/DIMEN/
C1
CALL TRANS (F, PT, K1F, K1F, K1PT, K2PT)
CALL SPTODP (R, K1R, K1R, REP)
IDT = 0
CALL LINVF (PDF, K1P, K1R, BDP, IDAT, WKAREA, IFP)
CALL UPTOSP (RIE, K1F, K1F, RI)
CALL TRANS (H, HT, K1H, K2H, K1HT, K2HT)
CALL TRANS (DFMTX, DFMTXT, K1P, K1F, K1DFM, K2DFM)
CALL MULT (H, P, BP, K1H, K2H, K1F, K1HP, K2HP)
CALL MULT (RI, HP, RIHP, K1R, K1R, K1HP, K2HP, K1RIHP, K2RIHP)
CALL MULT (HT, RIHF, PRD1, K1HT, K2HT, K1RIHP, K2RIHP, K1PRD1, K2PRD1)
CALL SUB (PRD1, PT, A11, K1PRD1, K2PRD1, K1A11, K2A11)
CALL MULT (P, DFMTXT, PRD2, K1P, K1P, K1DFM, K2DFM, K1PPD2, K2PRD2)
CALL MULT (DFMTX, P, PRD3, K1DFM, K2DFM, K1P, K1PRD3, K2PRD3)
CALL ADD (PRD2, PRD3, A21, K1PRD2, K2PRD2, K1A21, K2A21)
CALL MULT (RI, H, RIH, K1R, K1R, K1H, K2H, K1RIH, K2RIH)
CALL MULT (HT, RIH, RIHT, K1HT, K2HT, K1RIHT, K2RIHT)
CALL MULT (P, RIHT, PRD4, K1P, K1P, K1RIHT, K2RIHT, K1PRD4, K2PRD4)
CALL SUB (P, PRD4, A22, K1P, K1P, K1A22, K2A22)
CALL SCALAR (P, 0.0, A12, K1P, K1P, K1A12, K2A12)
DO 20 I = 1, 3
DO 20 J=1,3
ZDELP(I,J)=A11(I,J)
20 CONTINUE
DO 40 I=1,3
DO 40 J=1,3
H=I
N=J+K2
1
ZDELP(M,N)=A12(I,J)
40 CONTINUE
DO 60 I=1,3
DO 60 J=1,3
H=I+K11
N=J
ZDELP(M,N)=A21(I,J)
60 CONTINUE
DO 80 I=1,3
DO 80 J=1,3
M=I+K1
N=J+K2
ZDELP(I,J)=A22(I,J)
80 CONTINUE
WRITE(6,100)
100 FORMAT(//' ', 'Z MATRIX FOR SOLUTION TO PARTIAL(P)/'
', 'PARTIAL(THETA)'
', 'CALL OUTPUT(ZDELP,M,M)
WRITE(6,DIMEN)
CALL STMZ(ZDELP,CT,PHI,M,K1PHI,K2PHI)
WRITE(6,120)
120 FORMAT(//' ', 'STM FOR SOLUTION TO PARTIAL(P)/'
', 'PARTIAL(THETA)'
', 'CALL OUTDP(PHI,K1PHI,K2PHI)
CALL RICATI(PHI,K1PHI,DELTA,K1DELP)
CALL TRANS(H,H1,K1H,K2H,K1HT,K2HT)
CALL MULT(H1,K1HT,K2HT)
CALL MULT(DELTA,H1,K1DELP,M1DELP,K2DELP,K1HTRI,K2HTRI,K1DELK,
K2DELK)
RETURN
END
SUBROUTINE XSEN(PHI,K1PHI,K2PHI,DELTAK,XHAT,ZMEAS,DXHAT)
SUBROUTINE XSEN CALCUALTES THE PARTIAL DERIVATIVE OF XHAT
WITH RESPECT TO THETA(THETA). THE VECTORS OF UNKNOWNS.
COMMON P(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
K13,RKGM(3,3),K1PHI,K2PHI,K1PHIZ,K2PHIZ,
$K1RKGM,K2RKGM,RMASS,P11,P22,PKY,RKSI,RIE,ALPHA,RZ,PQ,LT,NREC,
$NECD,DROG,INTMA,LRT,IEND,IR1END,IDATN,NDATA,HRCAL,
$ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SIT,P(3,3),B(3,3),K1B,K1P,
$BILKE,K1DPH,K1DPH,K1DRKG,K2DRKG,NPI1,XIC(3,1),BZERO,K1DELP,
$K2DELP,K1DELK,K2DELK,K1DP,K2DP,IFLAG
DOUBLE PRECISION PHI(3,3)
DOUBLE PRECISION FKAREA(9)
DOUBLE PRECISION REL(3,3)
DOUBLE PRECISION DFK(3,3)
DIMENSION PHI(3,3)
DIMENSION DFK(3,3)
DIMENSION XHAT(3,NPOINT)
DIMENSION ZMEAS(3,NPOINT)
DIMENSION DHAT(3,100)
DIMENSION RM(3,3)
DIMENSION PBD1(3,3)
DIMENSION XINPT(3,3)
DIMENSION PHIK(3,3)
DIMENSION RHG(3,3)
DIMENSION FKH(3,3)
DIMENSION PRD2(3,3)
DIMENSION PRD3(3,3)
DIMENSION PRD4(3,3)
DIMENSION PRD5(3,3)
DIMENSION PRD6(3,3)
DIMENSION PKH(3,3)
DIMENSION VXHAT(3,1)
DIMENSION VDIHAT(3,1)
DIMENSION VZ(3,1)
DIMENSION TERM1(3,1)
DIMENSION TERM2(3,1)
DIMENSION TERM3(3,1)
DIMENSION SUM1(3,1)
DIMENSION VXSEN(3,1)
DIMENSION LAMELA(3,3)
DIMENSION LAMELA(3,3)
CALL MULT (DELTAK, RM, PBD1, K1DELT, K2DELT, K1H, K2H, K1PRD1, K2PRD1)
CALL SUB (PBD1, XINPT, K1F, K2F, K1XINP, K2XINP)
CALL DPTOSP (PHIK, 3,3)
FACTOR=0.0
CALL SCALAR (P, FACTOR, RM, K1F, K1H, K1F, K2F)
DO 40 I=1, K1F
RM(I,1)=1.0
40 CONTINUE
CALL LINVIF (PHIK, K1PHIK, K1PHIK, PHIK, 10, WKAREA, IE2)
CALL DPTOSP (PHIK, K1PHIK, K1PHIK, PHIK)
CALL MULT (RHG, RM, K1RHG, K2RHG, K1H, K2H, K1PRDG, K2PRDG)
CALL SUB (RM, RHG, K1F, K1F, K1FKH, K2FKH)
CALL SPPTOSP (FKH, K1FKH, K2FKH, DFKH)
IDT=0
CALL LINVIF (DFKH, K1FKH, K2FKH, DFKH, K1EG1, WKAREA, IE2)
CALL SUB (RM, SPH, K1RD, K2RD, K1RD, K2RD)
CALL MULT (K1RD, XINPT, K2RD, K1PRD, K2PRD, K1PRD, K2PRD)
CALL MULT (K1RD, DELTAK, K1RD, K2RD, K1DELT, K2DELT, K1PRD, K2PRD)
CALL MULT (SPH, K1PHIK, K1PHIK, K2PHIK)
CALL MULT (DELTAK, K1DELT, K2DELT, K1PRD, K2PRD)
CALL MULT (S2RH, DELTAK, K1DELT, K2DELT, K1PRD, K2PRD)
DO 200 J=1,3
VDIHA(T,1)=0.0
200 CONTINUE
DO 500 I=1, NPCINT
DO 220 J=1,3
VXIAT(J,1)=XHAT(J,1)
VZ(J,1)=ZMEAS(J,1)
220 CONTINUE
CALL MULT (SPH, VDIHAT, TERM1, K1PHIK, K2PHIK, 3, 1, K1TER1, K2TER1)
CALL MUL(T(GAMMA, VXHAT, TERM2, K1GAMA, K2GAMA, 3, 1, K1TER2, K2TER2)
CALL MUL(T(LAMBDA, V2, TERM3, K1LAMB, K2LAMB, 3, 1, K1TER3, K2TER3)
CALL ADD(TERM1, TERM2, SUM1, K1TER1, K2TER1, K1SUM1, K2SUM1)
CALL ADD(SUM1, TERM3, VXSEN, K1SUM1, K2SUM1, K1VXSN, K2VXSN)
DO 400 J=1,3
400 DXHAT (J, 1) = VXSEN (J, 1)
400 CONTINUE
CALL SCALAR (VXSEN, 1, C, VXHAT, K1VXSN, K2VXSN, K1VDHT, K2VDHT)
500 CONTINUE
RETURN
END

SUBROUTINE GRADE (DXHAT, RNU, DELTAP, DELTAP)
SUBROUTINE GRADE COMPUTES THE GRADIENT OF THE LIKELIHOOD FUNCTION.
COMMON F (3, 3), G (3, 1), H (3, 3), R (3, 3), Q (1, 1), K1F, K1G, K2G, K1H, K2H, K1Q,
$K1R, K2R, PMASS, F11, P22, RPY, RRSI, RLE, ALPHA, ZER0, DT, WREC,
$NRCD, IPOSTA, IR1STA, IR1END, IDATA1, IDATA2, NDATA, NFCAL,
$SATIN (4, 3), NVCAL, TVEL, GACAL, NPOINT, SET, P (3, 3), B (3, 3), KIB, K1P,
$LIKE, K1DPH, K2DPH, K1DRKG, K2DRKG, NPT1, XIC (3, 1), BZER0, K1DEL2P,
$K2DEL2P, K1DELP, K2DELP, K1DEL, K2DELP, IFLAG
DIMENSION DXHAT (3, NPOINT)
DIMENSION DELTAP (K1DEL2P, K2DELP)
DIMENSION RNU (3, NPOINT)
DIMENSION A1 (3, 1)
DIMENSION A1T (1, 3)
DIMENSION B1 (3, 3)
DIMENSION HT (3, 3)
DIMENSION A2 (3, 3)
DIMENSION A3 (3, 3)
DIMENSION DELTAE (3, 3)
DIMENSION A4 (3, 3)
DIMENSION A5 (3, 3)
DIMENSION A6 (3, 1)
DIMENSION A6T (3, 1)
DIMENSION A7 (3, 1)
DIMENSION A8 (3, 1)
DIMENSION TERM1 (1, 1)
DIMENSION TERM2 (1, 1)
DOUBLE PRECISION WKAREA (9)
DOUBLE PRECISION BDP (3, 3)
DOUBLE PRECISION BIDP (3, 3)
DIMENSION DELTAV (3, 1)
01 SUB=0, 0
IDST=0
CALL SPTODP (B, K1B, K1B, BDP)
CALL LINVP (BDP, K1B, K1B, BIDP, IDST, WKAREA, IER)
CALL DPTOSP (BILP, K1B, K1B, BI)
CALL TRANS (H, HT, K1H, K2H, K1HT, K2HT)
CALL MUL(T (DELTAP, HT, A2, K1DELP, K2DELP, K1HT, K2HT, K1A2, K2A2)
CALL MUL(T (H, A2, DELTAB, K1H, K2H, K1A2, K2A2, K1DELB, K2DELB)
CALL MUL(T (DELTAE, BI, A3, K1DELB, K2DELE, K1B, K1B, K1A3, K2A3)
CALL MUL(T (BI, A3, A4, K1B, K1B, K1A3, K2A3, K1A4, K2A4)
CALL TRACE (A5, T3, K1A5)
DO 100 I=1, NPOINT
DO 20 J=1, 3
A1 (J, 1) = RNU (J, I)
A6(J, I) = DIXHAT(J, I)

CONTINUE
CALL TRANS(A1, A1T, 3, 1, K1A1T, K2A1T)
CALL MULT(H, A6, HA6, K1H, K2H, 3, 1, K1HA6, K2HA6)
CALL NEG(HA6, DELTAV, K1HA6, K2HA6, K1DELV, K2DELV)
CALL MULT(B1, DELTAV, A7, K1B, K1ELV, K2DELV, K1A7, K2A7)
CALL MULT(A1T, A7, TERM1, K1A1T, K2A1T, K1A7, K2A7, K1TER1, K2TER1)
CALL MULT(A4, A1, A8, K1A4, K2A4, 3, 1, K1A8, K2A8)
CALL MULT(A1T, A8, TERM2, K1A1T, K2A1T, K1A8, K2A8, K1TER2, K2TER2)
T1 = TERM1(1, 1)
T2 = TERM2(1, 1)
SUd = SUM + T1 * (1.0 / 2.0) * (13 - T2)

CONTINUE
DELTA3 = SUM
RETURN
END

SUBROUTINE FISHER(RNU, DXHAT, DELTAP, DELJ2)
SUBROUTINE FISHER COMPUTES THE FISHER INFORMATION MATRIX.
THE FISHER INFORMATION MATRIX IS THE SECOND PARTIAL DERIVATIVE
OF THE LIKELIHOOD FUNCTION WITH RESPECT TO THETA.
COMMON F(3, 3), G (3, 1), H (3, 3), R(3, 3), G(1, 1), K1P, K1G, K2G, K1H, K2H, K1Q,
$K1A, RKG(3, 3), K1PHI, K2PHI, K1PHI2, K2PHI2,
$K1KGM, K2KGM, RMG(3, 3), P11, P22, RKY, RK1, RLE, ALPHA, PZERO, DT, NREC,
$NRECD, IPOSTA, ITOEND, IF1STA, IR1END, IDATS, IDATEN, NDATA, NRCAL,
$SAT(4, 3), RVAL, TVEL, GRCAL, NPOINT, SIT, P(3, 3), B(3, 3), K1B, K1P,
$SELIKE, K1DPH, K2DPH, K1RKG, K2RKG, NPT1, XIC(3, 1), BZERO, K1DELP,
$K2DELP, K1DELP, K2DELP, IFLAG
DIMENSION RNU (3, POINT)
DIMENSION DXHAT (3, NPOINT)
DIMENSION DELTAP (K1DELP, K2DELP)
DIMENSION DELTAE (3, 3)
DIMENSION NAG (3, 3)
DIMENSION HT (3, 3)
DIMENSION DELMU (3, 1)
DIMENSION DELNUT (1, 3)
DIMENSION DXHAT1 (3, 1)
DIMENSION RNU1 (3, 1)
DIMENSION RN1T (1, 3)
DIMENSION TERM1 (1, 1)
DIMENSION TERM2 (1, 1)
DIMENSION TERM3 (1, 1)
DIMENSION A1 (3, 3)
DIMENSION PRD11 (3, 1)
DIMENSION PRD21 (3, 1)
DIMENSION PRD22 (3, 1)
DIMENSION PPD23 (3, 1)
DIMENSION PRD31 (3, 1)
DIMENSION PRD32 (3, 1)
DIMENSION PRD33 (3, 1)
DIMENSION PRD41 (3, 3)
DIMENSION PRD42 (3, 3)
DIMENSION PRD43 (3, 3)
DIMENSION BI(3, 3)
DOUBLE PRECISION WKAREA (9)
DOUBLE PRECISION BDP (3, 3)
DOUBLE PRECISION EDP (3, 3)

01 SUM = 0.0
10 ST = 0
CALL SPTODP (E, K1B, K1B, BDF)
CALL LINVF (BDF, K1B, K1B, BIDP, IDGT, WKAPIA, IER)
CALL DPTOSP (BDF, K1B, K1B, BI)
CALL TRANS (H, HT, K1H, K2H, K1HT, K2HT)
CALL NEG (H, HNEG, K1H, K2H, K1HNEG, K2HNEG)
CALL MULT (DeltAE, HT, A1, K1DELP, K2DELP, K1HT, K2HT, K1A1, K2A1)
CALL MULT (H, A1, DELTAB, K1H, K2H, K1A1, K2A1, K1DELB, K2DELB)
DO 100 I = 1, NPCINT
DO 20 J = 1, 3
DXHAT1 (J, 1) = DXHAT (J, I)
RNJ1 (J, 1) = RNJU (J, I)
CONTINUE
C COMPUTE THE 1ST TERM OF THE INFORMATION MATRIX
CALL MULT (HNEG, DXHAT1, DELNU, K1HNEG, K2HNEG, 3, 1, K1DNU, K2DNU)
CALL MULT (BI, DELNU, PRD11, K1B, K1E, 3, 1, K1PD11, K2PD11)
CALL TRANS (DELNU, DELNU1, K1DNU, K2DNU, K1DNUT, K2DNUT)
CALL MULT (DELNU, PRD11, TERM1, K1DNUT, K2DNUT, K1PD11, K2PD11, K1TER1, $K2TER1)
C COMPUTE THE 2ND TERM OF THE INFORMATION MATRIX
CALL MULT (BI, DELNU, PRD21, K1B, K1B, K1DNU, K2DNU, K1PD21, K2PD21)
CALL MULT (DELTAE, PRD21, PRD22, K1DELB, K2DELB, K1PD21, K2PD21, K1PL22, $K2PL22)
CALL MULT (BI, PRD22, PRD23, K1B, K1E, K1PL22, K2PD22, K1PD23, K2PD23)
CALL TRANS (ENU1, EN1T, 3, 1, K1FNN1T, K2FNN1T)
CALL MULT (RN1T, FRD23, TERM2, K1FNN1T, K2FNN1T, K1PD23, K2PD23, K1TER2, $K2TER2)
C COMPUTE THE 3RD TERM OF THE INFORMATION MATRIX
CALL MULT (BI, DELNU, PRD31, K1B, K1B, K1DNU, K2DNU, K1PD31, K2PD31)
CALL MULT (DELTAE, PRD31, PRD32, K1DELB, K2DELB, K1PD31, K2PD31, K1PL32, $K2PL32)
CALL MULT (BI, PRD32, PRD33, K1E, K1E, K1PL32, K2PD32, K1PD33, K2PD33)
CALL MULT (FN1T, FRD33, TERM3, K1FNN1T, K2FNN1T, K1PD33, K2PD33, K1TER3, $K2TER3)
C COMPUTE THE 4TH TERM OF THE INFORMATION MATRIX
CALL MULT (BI, DELTAB, PRD41, K1E, K1B, K1DELB, K2DELB, K1PD41, K2PD41)
CALL MULT (DELTAB, PRD41, FRD42, K1DELB, K2DELB, K1PD41, K2PD41, K1PD42, $K2PD42)
CALL MULT (BI, PRD42, PRD43, K1B, K1B, K1PD42, K2PD42, K1PD43, K2PD43)
CALL TRACE (PRD43, T4, K1PE43)
T1 = TERM1 (1, 1)
T2 = TERM2 (1, 1)
T3 = TERM3 (1, 1)
PSJM = T1 - T2 - T3 - (1.0/2.0)*T4
SUM = SUM + PSJM
CONTINUE
DELF2 = SUM
RETURN
END
C
C SUBROUTINE STIF (THETA, DELTAJ, DELJ2, LTHETA, THETAN)
SUBROUTINE STIF CALCULATES THE NEW VALUE OF THETA (THETAN).
DTHETA = (-1.0/DELF2)*DELTAJ
THETAN = THETA + DTHETA
RETURN
END
C
C SUBROUTINE MULT (A, B, D, K1A, K2A, K1B, K2B, F1D, K2D)
DIMENSION A (K1A, K2A), B (K1B, K2B), D (K1A, K2B)
IF(K2A .NE. K1B) GC TO 40
DO 20 I=1,K1A
DO 20 J=1,K2B
L=J
DO 20 K=1,K2A
L=L+1
IF(L .NE. 1) GO TO 20
D(I,J)=0
20 D(I,J)=D(I,J) + (A(I,K)*B(K,J))
K1J=K1A
K2J=K2B
GO TO 80
40 WRITE(6,60)
60 FORMAT('MATRICES A AND B ARE NOT CONFORMABLE')
RETURN
END

SUBROUTINE SCALAR(A,B,C,K1A,K2A,K1C,K2C)

DIMENSION A(K1A,K2A),C(K1A,K2A)
DO 10 I=1,K1A
DO 10 J=1,K2A
10 C(I,J)=B*A(I,J)
K1C=K1A
K2C=K2A
RETURN
END

SUBROUTINE TRANS(A,B,K1A,K2A,K1B,K2B)

DIMENSION A(K1A,K2A),B(K2A,K1A)
DO 20 I=1,K1A
DO 20 J=1,K2A
20 B(J,I)=A(I,J)
K1B=K2A
K2B=K1A
RETURN
END

SUBROUTINE NEG(A,B,K1A,K2A,K1B,K2B)

DIMENSION A(K1A,K2A),B(K1A,K2A)
DO 10 I=1,K1A
DO 10 J=1,K2A
10 B(I,J)=0-A(I,J)
K1B=K1A
K2B=K2A
RETURN
END

SUBROUTINE ADD(A,B,C,K1A,K2A,K1C,K2C)

DIMENSION A(K1A,K2A),B(K1A,K2A),C(K1A,K2A)
DO 20 I=1,K1A
DO 20 J=1,K2A
20 C(I,J)=A(I,J) + B(I,J)
K1C=K1A
K2C=K2A
RETURN
END
SUBROUTINE SUB (A, B, C, K1A, K2A, K1C, K2C)
DIMENSION A(K1A, K2A), B(K1A, K2A), C(K1A, K2A)
DO 20 I = 1, K1A
DO 20 J = 1, K2A
C(I,J) = A(I,J) - B(I,J)
K1C = K1A
K2C = K2A
RETURN
END

FUNCTION FACT(K)
FACT = 1.0
DO 20 I = 1, K
FACT = FACT * I
RETURN
END

SUBROUTINE TRACE (A, B, K1A)
DIMENSION A(K1A, K1A)
SUM = 0.0
DO 20 I = 1, K1A
SUM = SUM + A(I, I)
CONTINUE
B = SUM
RETURN
END

SUBROUTINE OUTPUT (A, K1A, K2A)
DIMENSION A(K1A, K2A)
DO 20 I = 1, K1A
WRITE (6, 30) (A(I,J), J = 1, K2A)
CONTINUE
30 FORMAT (' ', 12X, 6 (2X, E15.8))
RETURN
END

SUBROUTINE OVER (RMATRX, CHK, K1, K2, IFLAG)
DIMENSION RMATRX(K1, K2)
DO 20 I = 1, K1
DO 20 J = 1, K2
IF (RMATRX(I, J) .GE. CHK) IFLAG = 1
CONTINUE
RETURN
END

SUBROUTINE SETCDF (X, K1X, K2X, Y)
DIMENSION X(K1X, K2X)
DOUBLE PRECISION Y(K1X, K2X)
DO 20 I = 1, K1X
DO 20 J = 1, K2X
Y(I, J) = DBLE (X(I, J))
CONTINUE
RETURN
SUBROUTINE DPTOSP (X,K1X,K2X,Y)
DOUBLE PRECISION  X(K1X,K2X)
DIMENSION  Y(K1X,K2X)
DO 20 I=1,K1X
DO 20 J=1,K2X
Y(I,J)=X(I,J)
20 CONTINUE
RETURN
END

SUBROUTINE SCALDP (A,B,C,K1A,K2A,K1C,K2C)
DOUBLE PRECISION  A(K1A,K2A),C(K1A,K2A)
DOUBLE PRECISION  B
DO 10 I=1,K1A
DO 10 J=1,K2A
10 C(I,J)=B*A(I,J)
K1C=K1A
K2C=K2A
RETURN
END

SUBROUTINE TRANSD (A,B,K1A,K2A,K1B,K2B)
DOUBLE PRECISION  A(K1A,K2A),B(K2A,K1A)
DO 20 I=1,K1A
DO 20 J=1,K2A
20 B(J,I)=A(I,J)
K1B=K2A
K2B=K1A
RETURN
END

SUBROUTINE NLGCP (A,B,K1A,K2A,K1E,K2E)
DO0J
EPFPICN  A(K1A,K2A),B(K1A,K2A)
DO 10 I=1,K1A
DO 10 J=1,K2A
10 B(I,J)=B*0.0-A(I,J)
K2E=K2A
RETURN
END

SUBROUTINE ADCDP (A,B,C,K1A,K2A,K1C,K2C)
DOUBLE PRECISION  A(K1A,K2A),B(K1A,K2A),C(K1A,K2A)
DO 20 I=1,K1A
DO 20 J=1,K2A
20 C(I,J)=A(I,J)+B(I,J)
K1C=K1A
K2C=K2A
RETURN
END
SUBROUTINE MULTDP(A,B,D,K1A,K2A,K1E,K2E,K1D,K2D)
DOUBLE PRECISION A(K1A,K2A),B(K1B,K2E),D(K1A,K2B)
IF(K2A .NE. K1E) GO TO 40
DO 20 I=1,K1A
DO 20 J=1,K2B
L=0
DO 20 K=1,K2A
L=L+1
IF(L .NE. 1) GO TO 20
D(I,J)=DBLE(0.0)
20 D(I,J)=D(I,J)+(A(I,K)*B(K,J))
K1E=K1A
K2B=K2B
GO TO 80
40 WRITE(6,60)
60 FORMAT(1HO,'MATRICES A AND B ARE NOT CONFORMABLE')
80 RETURN
END
C
SUBROUTINE SUBDP(A,B,C,K1A,K2A,K1C,K2C)
DOUBLE PRECISION A(K1A,K2A),B(K1A,K2A),C(K1A,K2A)
DO 20 I=1,K1A
DO 20 J=1,K2A
C(I,J)=A(I,J)-B(I,J)
K1C=K1A
K2C=K2A
RETURN
END
C
SUBROUTINE TRACED(A,B,K1A)
DOUBLE PRECISION A(K1A,K1A)
DOUBLE PRECISION SUM,B
SUM=DBLE(0.0)
DO 20 I=1,K1A
SUM=SUM+A(I,I)
20 CONTINUE
B=SUM
RETURN
END
C
SUBROUTINE OUTDBL(A,K1A,K2A)
DOUBLE PRECISION A(K1A,K2A)
DO 20 I=1,K1A
DO 20 J=1,K2A
WRITE(6,30) A(I,J)
20 CONTINUE
30 FORMAT( * ,12X,D15.8)
RETURN
END
C
SUBROUTINE OVERDP(RMATRIX,CHK,K1,K2,IFLAGO)
DOUBLE PRECISION RMATRIX(K1,K2)
DOUBLE PRECISION CHK
DO 20 I=1,K1
DO 20 J=1,K2
IF (RMATRX(I,J) .GE. CHK) IFLAG=1
20 CONTINUE
RETURN
END

DOUBLE PRECISION FUNCTION FACDTP(K)
DO 20 I=1,K
REALI=I
DR,&ALI=DBLE(REALI)
20 FACTDTP=FACTDTP*EREALI
RETURN
END

SUBROUTINE OUTDP(A,K1A,K2A)
DO 20 I=1,K1A
WRITE(6,30) (A(I,J) ,J=1,K2A)
20 CONTINUE
30 FORMAT(' 3,12X,6(2X,D15.9)')
RETURN
END

SUBROUTINE PLOT(X,Y,IPINT)
DIMENSION X(IPINT),Y(IIJINT)
WRITE(6,20)
20 FORMAT(/', '40X, 'STATE ESTIMATES')
CALL WPLT1(X,Y,IPINT,4,XHAT)
RETURN
END

SUBROUTINE VMAX(X,KX,RMIN,RMAX)
DIMENSION X(KX)
RMN=0.0
RMAX=0.0
DO 100 I=1,KX
IF (X(I) .GT. RMAX) RMAX=X(I)
IF (X(I) .LT. RMIN) RMIN=X(I)
100 CONTINUE
RETURN
END

SUBROUTINE PLOHAT(Y1,Y2,K1Y1,K2Y1,K1Y2,K2Y2,XVECT1,YVECT1,
$YVECT2,DT,ITIT1,JLAB1)
DIMENSION ITIT1(10)
DIMENSION JLAB1(10)
DIMENSION Y1(K1Y1,K2Y1)
DIMENSION Y2(K1Y2,K2Y2)
DIMENSION YVECT1(K2Y2)
DIMENSION YVECT2(K2Y2)
DATA ICHARA,ICHARE/'A','E'/
DO 20 I=1,K2Y2
SUBROUTINE PLXSEN(DXHAT,K1X,K2X,XVECT1,YVECT1,DT,ITIT2,JLAB2)
DIMENSION DXHAT(K1X,K2X)
DIMENSION XVECT1(K2X)
DIMENSION YVECT1(K2X)
DIMENSION ITIT2(10)
DIMENSION JLABEL(10)
DO 20 I=1,K2X
XVECT1(I)=DT*I
20 CONTINUE
DO 100 J=1,K1X
DO 50 I=1,K2X
YVECT1(I)=DXHAT(J,I)
50 CONTINUE
WRITE(6,60)ITIT2
60 FORMAT('11,4DX,10A4)
CALL WPLT4(XVECT1,YVECT1,K2X,40,JLAB2)
100 CONTINUE
RETURN
END

SUBROUTINE CPLCT(X,K1X,K2X,XVECT1,YVECT1,YMIN,YMAX,DT,JLABEL,JTITLE)
DIMENSION X(K1X,K2X)
DIMENSION XVECT1(K2X)
DIMENSION YVECT1(K2X)
DIMENSION JLABEL(10)
DIMENSION JTITLE(10)
DATA ICHAR/*9/
DO 20 I=1,K2X
XVECT1(I)=DT*I
20 CONTINUE
DO 200 J=1,K1X
DO 40 I=1,K2X
YVECT1(I) = X(J,I)
CONTINUE
WRITE(6,100) ITITLE
100 FORMAT(*1',40X,10A4)
CALL WPLOT2(XVECT1(K2X),XVECT1(I),YMAX,YMIN)
CALL WPLOT3(ICHAR,XVECT1,YVECT1,K2X)
CALL WPLOT4(40,JLABEL)
200 CONTINUE
RETURN
END
Appendix F

LIST OF APL FUNCTIONS USED TO GENERATE WHEELSET SIMULATED DATA
\begin{verbatim}
\* WHLSIN

[1] A+SYSMTX
[2] B+INFMTX
[3] C+ 3 3 F0
[4] D+ 3 1 F0
[5] H+ 3 3 F 1 0 0 0 1 0 0 0 1
[6] 'RANDOM INPUT STATISTICS'
[7] $\Omega$ 'ENTER SUM LENGTH MEAN MNSQVALUE'
[8] STATRI+[0]
[9] W+WHNOISE STATRI
[10] 'DETERMINISTIC INPUT STATISTICS'
[11] $\Omega$ 'ENTER SUM LENGTH MEAN MNSQVALUE'
[12] STATDI+[0]
[13] WT+WHNOISE STATDI
[14] U+(STATRI[2],1,1)+WT
[15] DYN
[16] XMES+X
[17] 'MEASUREMENT NOISE STATISTICS'
[18] $\Omega$ 'ENTER SUM LENGTH MEAN MNSQVALUE'
[19] STATMN+[0]
[20] V1+WHNOISE STATMN
[21] V2+WHNOISE STATMN
[22] V3+WHNOISE STATMN
[23] V+(3,STATMN[2])+V1,V2,V3
[24] Z+(H+,XMES)+V

\* F+SYSMTX

[1] F11+1x(((3THETAxCOEF22)+(B0xVEL))xMASSxVEL)
[2] F12+2xCOEF22xTHETAxMASS
[3] F13+1xK1xMASS
[4] F21+0
[5] F22+1xK51xVEL+2xTHETAxCOEF11xLxL
[6] F23+1xALPHAxVEL+LxR0
[7] F31+1
[8] F32+0
[9] F33+0

\* G+INFMTX

[1] G11+R0xMASS
[2] G21+0
[3] G31+1
\end{verbatim}
\[ Y + WHTNOISE \times A + B \times MN \times MS \]

[1] \[ A + (? \times [1] \times [2]) / 100000 \]
[2] \[ A + ([1], [2]) \times A \]
[3] \[ B + (1, [2]) \times A \]
[4] \[ MN + (+/+) \times (x / FB) \]
[5] \[ B + (? \times (MN - X[3])) / x \]
[6] \[ MS + MN SQVAL x \]
[7] \[ B + ((X[4] + MS) \times 0.5) / x \]
[8] \[ Y + B \]

\[ Y + MN SQVAL x \]

[1] \[ Y + (+/+(X+, XQX))/ (X/(FX)) \]

\[ Y + COV \times MN \]

[1] \[ MN + (0F X) \times (+/+) \times 1FX \]
[2] \[ MN + MN \]
[3] \[ Y + (X - MN) + , x + (X - MN) / 1FX \]

\[ DYN \]

[1] \[ T + N \times DT \times N \]
[2] \[ INITIALIZE \]
[3] \[ PHI + DT STM A \]
[4] \[ DMATRIX \]
[5] \[ M + 0 \]
[6] \[ LO + M + M + 1 \]
[7] \[ XX[+ (M+1)] + (PHI +, X:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\\] \]
[8] \[ YY[+ (M+1)] + (C+, X:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\\] \]
[9] \[ (M(N-1)) / LO \]
[10] \[ XX + (MN+H) + x + \]
[11] \[ YY + (LL, H) + x + \]

\[ Y + x \]
\begin{verbatim}
\$ INITIALIZE
[1]  HH=f(f,a)*0.5  
[4]  XX+(HH+1)+0
[5]  YY+(LL+1)+0
[6]  UU+(MM+1)+0
[7]  \texttt{ENTRY (O)''}
[8]  II[1]+0
\$ 

\$ FHI+DT STM A;R;H;FACTOR;INDEX
[1]  RF+(f,0)*0.5  
[2]  II+(iR)*1=R
[3]  H*0
[4]  FHI+I
[5]  FACTOR+I
[6]  L0:HH+1
[7]  FACTOR+FACTOR,XX+DT=H
[8]  FHI+FHI+FACTOR
[9]  INDEX+(*1,FACTOR)+R*2
[10]  +(INDEX)(EPS)/L0
\$

\$ DMATRIX;H
[1]  H=DT+4
[2]  DD+(H-3)x((0 STM A)+(4xH STM A)+(2x(2xH) STM A)+(4x(3xH) STM A)+(DT STM A))+,IP
\end{verbatim}
Appendix G

LIST OF THE PROGRAM USED TO OPERATE THE A/D CONVERTER
0001 FTN4.L
0002 PROGRAM DMAD3(3,40)
0003 C PROGRAM TO DIGITIZE WATER CHANNEL HOT WIRE DATA.
0004 C THE PROGRAM READS NCH CHANNELS, NPPC SAMPLES PER CHANNEL FROM THE
0005 C GMAD-1 A/D CONVERTER. THE DATA READ IS WRITTEN ON TAPE.
0006 C VERSION OF 18 JULY 1978
0007 DIMENSION IBUF(512), IBUF(512)
0008 INTEGER PARS(5)
0009 EQUIVALENCE (PARS(1), LU), (PARS(2), IPRINT), (PARS(3), MAG),
0010 (PARS(4), LU)
0011 C GET LOGICAL UNITS
0012 CALL AMPAR(PARS)
0013 C DEFAULT THEM IF NECESSARY
0014 IF (LUCRT .LE. 0) LUCRT=1
0015 IF (MAG .LE. 0) MAG=0
0016 IF (LU .LE. 0) LU=17
0017 N=0
0018 NCH=2
0019 NPPC=256
0020 2222 FORMAT("ENTER THE NUMBER RECORDS PER FILE *")
0021 ICLAS=0
0022 C AT PRESENT S.A.M. (2048 WORDS), IBUF MUST EQUAL 512 OR LESS
0023 IBUF=NCH*NPPC
0024 C GET A CLASS NUMBER
0025 CALL EXEC(18,0, IBUF, IBUF, 0, 0, ICLAS)
0026 C TELL USER WHAT CLASS NUMBER WE GOT
0027 NUC=1AND(ICLAS, 777778)
0028 WRITE(LUCRT, 1203) NUMB
0029 1203 FORMAT("CLASS NUMBER ", 6, "B")
0030 C SET BIT 15 IN CLASS WORD FOR NO WAIT AND BIT 13 TO KEEP CLASS NR.
0031 ICLAS=ICLAI8(ICLAS, 1200000)
0032 WRITE(LUCRT, 1201)
0033 1201 FORMAT("ENTER DESIRED SAMPLING RATE IN HZ. *")
0034 C READ(LUCRT, *) HERTZ
0035 C COMPUTE CLOCK DIVISOR
0036 ICDV=IFIX(1.0E+07/HERTZ)
0037 HERTZ=1.0E+07/FLOAT(ICDV)
0038 WRITE(LUCRT, 1202) HERTZ
0039 1202 FORMAT("ACTUAL SAMPLING RATE IS ", C.4, " HERTZ")
0040 C CONSTRUCT CONTROL WORD FOR SETTING CLOCK DIVIDER
0041 ICLK=LU+30000
0042 C SET CLOCK DIVIDER
0043 CALL EXEC(3, ICLK, IDC)
0044 15 CONTINUE
0045 WRITE(LUCRT, 2222)
0046 C READ(LUCRT, *) MAXREC
0047 IF (MAXREC .LE. 0) GO TO 30
0048 C MAXREC IS DIVIDED BY 2 BECAUSE TWO RECORDS
0049 C ARE WRITTEN EACH TIME THROUGH THE LOOP
0050 5MAXREC+MAXREC/2
0051 DO 25 J=1,MAXREC
0052 C READ FROM A/D INTO BUFA AND SUSPEND PROGRAM
0053 CALL EXEC(I,U,IBUF,A,IBUFL)
0054 C DO CLASS GET. INITIALLY IT WILL (I HOPE) CLEAR THE CLASS REQUEST
0055 C THAT GOT US THE CLASS NUMBER. AFTERWARDS IT SHOULD CLEAR ONE OF
0056 C THE CLASS WRITES TO TAPE.
0057 CALL EXEC(21,ICLAS,IBUF,B,IBUFL)
0058 C BEGIN WRITING BUFA TO MAG TAPE, THEN
0059 C SIMULTANEOUSLY READ FROM A/D TO BUFB
0060 CALL EXEC(18,MAG,IBUF,A,IBUFL,0,0,ICLAS)
0061 CALL ABREG(IRETN,18)
0062 IF(IRETN.NE.0) WRITE(LUCRT,1204) IRETN
0063 1204 FORMAT("CLASS WRITE ERROR ",I2)
0064 CALL EXEC(I,U,IBUF,B,IBUFL)
0065 C CHECK IBUF WRITE FOR COMPLETION
0066 CALL EXEC(21,ICLAS,IBUF,B,IBUFL)
0067 C START WRITING BUFB TO TAPE
0068 CALL EXEC(18,MAG,IBUF,B,IBUFL,0,0,ICLAS)
0069 CALL ABREG(IRETN,18)
0070 IF(IRETN.NE.0) WRITE(LUCRT,1204) IRETN
0071 25 CONTINUE
0072 E NDFILE MAG
0073 N=N+1
0074 WRITE(LUCRT,1003)N
0075 1003 Format("FILE NUMBER ",I4," HAS BEEN READ")
0076 GO TO 15
0077 30 CONTINUE
0078 C CLEAR BIT 13 IN CLASS WORD. WE ARE DONE WITH THE CLASS NUMBER.
0079 ICLAS=ICLAS.1377778
0080 C NOW DO GET TO CLEAR FINAL REQUEST AND RELEASE CLASS NUMBER.
0081 49 CALL EXEC(21,ICLAS,IBUF,B,IBUFL)
0082 CALL ABREG(IRETN,18)
0083 WRITE(LUCRT,1205) IRETN
0084 1205 FORMAT("FINAL GET RETURNED ",96)
0085 IF(IRETN.LT.0) GO TO 49
0086 E NDFILE MAG
0087 S TO P
0088 E N D

FTN4 COMPILER: HP3200-16092 REV. 1005 (780318)

** NO WARNINGS ** NO ERRORS ** PROGRAM = 81450 COMMON = 00000
Appendix H

PLOTS OF THE LIKELIHOOD FUNCTION, OBSERVATION TERM AND BIAS TERM FOR TEST CASES 1 THROUGH 9
Figure H.1 Plot of the likelihood function for test case 1.
Figure H.2 Plot of the observation term for test case 1.
Figure H.3 Plot of the bias term for test case 1.
Figure H.4 Plot of the likelihood function for test case 2.
Figure H.7 Plot of the likelihood function for test case 3.
Figure H.8 Plot of the observation term for test case 3.
Figure H.9 Plot of the bias term for test case 3.
Figure H.12 Plot of the bias term for test case $h$. 
Figure H.13 Plot of the likelihood function for test case 5.
Figure II.17 Plot of the observation term for test case 6.
Figure H.18 Plot of the bias term for test case 6.
Figure H.19 Plot of the likelihood function for test case 7.
Figure H.22 Plot of the likelihood function for test case 3.
Figure H.23 Plot of the observation term for test case 8.
Figure H.26 Plot of the observation term for test case 9.