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LIMITING PAYLOAD DECELERATION DURING GROUND IMPACT

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Abstract

This paper considers the problem of limiting the deceleration of balloon and sounding-rocket instrument payloads during ground impact by adding energy-dissipating, deformable structures. The basic physics is analyzed in a simple force and energy-balance relationships are developed. The pertinent equations are organized in a form that is conducive for computer computation. Two example problems, one of a sounding-rocket payload and the other of a balloon payload are calculated and the results presented in the form of a performance map.

Nomenclature

\( A_{cp} \)  
.crushpad cross-sectional area

\( A_p \)  
.payload cross-sectional area

\( a_{max} \)  
.maximum payload deceleration

\( F_c \)  
.dimensionless crush force, defined in equation (12)

\( F_{cr} \)  
.honeycomb crush strength

\( F_{sg} \)  
.dimensionless ground force, defined in equation (13)

\( g \)  
.acceleration due to gravity

\( K_{sg} \)  
.subgrade modulus of ground

\( L \)  
.dimensionless depth of ground depression under payload

\( l_{cp} \)  
.undeformed length of crushpads

\( m \)  
.payload mass

\( V_p \)  
.payload velocity at ground level

\( \epsilon_{cp} \)  
.deformation of crushpad

\( \delta \)  
.depth of ground depression under payload

\( \gamma \)  
.dimensionless deformation of crushpad

Introduction

The increase in cost and sophistication of balloon and sounding-rocket instrument payloads makes it imperative that they be recovered undamaged. The payload descends to the earth by parachute, but limitations in parachute size result in the payload impacting the ground with a substantial velocity. This paper considers the use of energy-absorbing, deformable structures attached to the base of the payload which limit the maximum deceleration during ground impact to a prescribed level.

Mechanical Behavior of the Ground

The mechanical behavior of the ground during compression is assumed to be that of a linear-elastic or Hooke material and can be characterized by a coefficient called the subgrade modulus, \( K_{sg} \), which has units of lbf per square inch per inch. The ground develops a resisting force to a body of cross-sectional area, \( A \), which is linearly proportional to the depression of the ground caused by the body, \( g^r \).

\[ F = K_{sg} A^r g \]  
(1)

The equation of motion of the body of mass \( m \) during impact neglecting gravity, is

\[ F = -K_{sg} A^r = m \frac{d^2 x}{dt^2} \]

where the negative sign shows that the force resists the motion. Solution of the differential equation results in a quarter-sine-wave deceleration pulse of duration \( \pi/\sqrt{2 K_{sg} A} \) and maximum deceleration \( V_p \sqrt{K_{sg} A/m} \) where \( V_p \) is the impact velocity at \( t = 0 \).

The kinetic energy of the payload, \( 1/2 m V_p^2 \), is dissipated by doing work in deforming the ground. The energy absorbed by the ground, \( \Delta E \), is

\[ \Delta E = \int_0^t Fdy = \frac{1}{2} K_{sg} A^2 \gamma \]  
(2)

Effect of Payload Impact Area on Deceleration

Consider two payloads having the same mass and impact velocity but different-size impact areas. What is the effect of impact area on the maximum deceleration if both payloads impact ground having the same subgrade modulus? From equation (1),

\[ \left( \begin{array}{c} a_{max 1} \\ \ldots \\ a_{max 2} \end{array} \right) = \left( \begin{array}{c} F_{1} \\ \ldots \\ F_{2} \end{array} \right) = \left( \begin{array}{c} A_1 \gamma \\ \ldots \\ A_2 \gamma \end{array} \right) \]  
(3)

The ground must dissipate the same kinetic energy for both payloads, neglecting the small gravitational potential energy change undergone by the payload in depressing the ground. Then, from equation (2)

\[ A_1^2 \gamma = A_2^2 \gamma \]  
(4)
Eliminating the ratio of depressions between equations (3) and (4) gives,

\[
\left( \frac{\delta_{\text{max}}}{\delta_{\text{max}}} \right) = \sqrt{\frac{A_1}{A_2}}
\]

Equation (5) shows the important result that during impact the smaller impact-area body experiences a smaller maximum deceleration. This result will be exploited to limit the deceleration of the payload during impact. Of course, the smaller impact area body must cause a deeper depression in the ground in dissipating the same energy as the larger impact-area body.

**Potential Energy Change of Payload During Impact**

When the potential energy change of the payload is included the energy balance becomes

\[
\frac{1}{2} K (A_1)^2 - \frac{1}{2} m v_f^2 + mg \delta_{\text{max}}
\]

The maximum decelerating force is

\[
F_{\text{max}} = K (A_1)^2 = ma_{\text{max}}
\]

Thus,

\[
\delta_{\text{max}} = \sqrt{1 + \frac{K (A_1)^2}{mg}}
\]

**Energy-absorbing Crushpads**

The maximum deceleration can be limited by adding energy-absorbing crushpads. The crushpad material is hexagonal cell, multicolumnar honeycomb which dissipates energy by plastic deformation parallel to the cell axes. The energy absorbing property of the honeycomb is characterized by a coefficient called the crush strength, \( f_{\text{cr}} \), which has units of lbf per square inch. The honeycomb offers a constant resisting force to compression of

\[
F = f_{\text{cr}} A_{\text{cp}}
\]

where \( A_{\text{cp}} \) is the cross-sectional area of the honeycomb crushpad. The honeycomb crushpad absorbs energy,

\[
E = f_{\text{cr}} A_{\text{cp}} \delta_{\text{cp}}
\]

when crushed a distance \( \delta_{\text{cp}} \). It is assumed that the crushpad is always long enough so that it is does not "bottom out" and become nearly rigid.

**Maximum Deceleration of Payload**

The ideas and equations just developed may be used to predict the maximum deceleration of a payload with honeycomb crushpads attached to its base during impact with the ground. Consider Fig. 1 in which the base of the payload impacts the ground.

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**Fig. 1 Impact geometry.**

\( \delta_{\text{cp}} \) is the undeformed length of the crushpad, \( \delta_{\text{cp}} \) is the depth of ground depression caused by the payload, \( \delta \) is the depth of ground depression under the crushpad, and \( \delta_{\text{cp}} \) is the crushpad deformation. The various lengths may be nondimensionalized with respect to \( \delta_{\text{cp}} \):

\[
\lambda = \frac{\delta_{\text{cp}}}{\delta_{\text{cp}}}, \quad L = \frac{\delta}{\delta_{\text{cp}}}, \quad \text{and} \quad l = \frac{\delta_{\text{cp}}}{\delta_{\text{cp}}}
\]

There are two situations which must be analyzed separately:

1. The base of the payload does not contact the ground, \( L = 0 \). The ground under the crushpad deforms and the crush pad may deform, but the combined deformation is less than the undeformed length of the crushpad, \( 1 + \lambda < 1 \).

2. The base of the payload impacts and deforms the ground, \( L = 0 \). The payload is assumed rigid during the impact. The crushpad may deform and is completely buried. The lengths are related by

\[
\lambda = 1 + L - l
\]

We will analyze the two situations in the next two sections and determine the matching conditions between them.

1. **Payload does not contact ground, \( L=0, \lambda<1 \)**

Consider the behavior of a crushpad which impacts the ground. Initially, the ground deforms until its compressive stress reaches the crushpad strength, \( f_{\text{cr}} \). Then the ground does not deform any farther, and the crushpad begins to deform with a constant decelerating force. Physically, the ground and crushpad are in series.

Let us develop the appropriate equations describing this two-phase energy-dissipation process. The ground deforms a distance \( \delta \) and develops a maximum resisting force, \( F_{\text{max}} \), which is limited by the crushpad strength,
\[ F = K_A \delta = f_{cr} \delta = m \]  

(10)

The crushpad limits the deceleration to
\[ \left( \frac{a_{max}}{g} \right) = \left( \frac{f_{cr} A_{cr}}{mg} \right) \left( \frac{K_{sg} A_{cp} \delta_{cp}}{mg} \right) \]  

(11)

It is convenient to introduce two dimensionless forces:

- Dimensionless crush force,
  \[ F_{cr} = \left( \frac{f_{cr} A_{cr}}{mg} \right) \]  
  \[ (12) \]

- Dimensionless ground force,
  \[ F_{sg} = \frac{K_{sg} A_{cp} \delta_{cp}}{mg} \]  
  \[ (13) \]

Equation (11) becomes
\[ \left( \frac{a_{max}}{g} \right) = F_{cr} \delta_{cr} \]  

(14)

The depth of penetration into the ground is
\[ \delta = \frac{f_{cr}}{g} \]  

or \[ \xi = \frac{f_{cr}}{F_{sg}} \]  

(15)

The energy absorbed by the ground is
\[ 1/2 K_A \delta^2 \]  

The crushpad deforms a length \( \delta_{cp} \) and absorbs energy, \( f_{cr} A_{cp} \delta \). The sum of these two energies is equal to the payload kinetic energy, \( 1/2 mv^2 \), and potential energy change, \( mg(\delta + \delta_{cp}) \).

The energy balance is
\[ 1/2 K_A \delta^2 + f_{cr} A_{cp} \delta = 1/2 mv^2 + mg(\delta + \delta_{cp}) \]  

(16)

A dimensionless kinetic energy \( T \), may be defined
\[ T = \left( \frac{v_{\infty}^2}{2K_A} \right) \]  

(17)

The energy balance, equation (16), becomes in dimensionless form,
\[ 1/2 F_{cr} \xi^2 + F_{cr} \delta = T + \lambda + \lambda \]  

(18)

There are two limiting cases:

1. The crush strength of the honeycomb is so high that the ground does not develop a sufficient force to deform it, \( \lambda = 0 \); all the energy is removed by deformation of the ground.

2. The ground has such a high subgrade modulus that it deforms only infinitesimally before the crush strength of the honeycomb is reached, \( \xi = 0 \); all the energy is removed by deformation of the honeycomb.

Equation (18) may be used to determine the value of \( K_{sg} \) below which a given crush-strength crushpad does not deform, limiting case 1. The crushpad acts as a rigid protuberance, and the only energy dissipation is caused by the deformation of the ground under it. Introducing equation (15) in equation (18) and setting \( \lambda = 0 \) gives
\[ K_{sg} = \left( \frac{1/2 F_{cr} - 1}{T} \right) \frac{f_{cr}}{\xi} \]  

(19)

In limiting case 1, the subgrade modulus has a very large value but is not infinite, and the deformation of the ground is very small but not zero; equation (14) must still be satisfied. The crushpad deformation is obtained from equation (18) by introducing equation (15) and letting \( \lambda \rightarrow 0 \),
\[ \lambda = \frac{T}{f_{cr} - 1} \]  

(20)

Case 1 results in the maximum possible crushpad deformation.

2. Payload contacts the ground, \( L > 0 \), \( \lambda = 1 + L - \xi \)

The maximum decelerating force for this situation is
\[ F_{max} = f_{cr} A_{cr} + K_{sg} (A_{cp} - A_{cr}) \delta_{cp} = m \]  

or
\[ \left( \frac{a_{max}}{g} \right) = F_{cr} + F_{sg} \left[ \frac{A_{cr}}{A_{cp}} \right] \frac{L}{\lambda} \]  

(21)

The force acting on the crushpads is
\[ f_{cr} A_{cr} = K_{sg} A_{cp} \delta_{cp} \]  

(22)

The energy balance is
\[ 1/2 K_{sg} \delta_{cp}^2 + 1/2 K_{sg} (A_{cp} - A_{cr}) \delta_{cp}^2 + f_{cr} A_{cp} \delta_{cp} = 1/2 mv_{\infty}^2 + mg(\delta_{cp} + \delta_{cr}) \]  

(23)

Equation (23) becomes in dimensionless form
\[ 1/2 F_{cr} \xi^2 + \left( \frac{A_{cr}}{A_{cp}} \right) \frac{L}{\lambda} + F_{cr} \delta = T + L + 1 \]  

(24)

There are also two limiting cases for this situation:

1. The payload just contacts the ground, \( L = 0 \).

2. The crushpads do not deform, \( \lambda = 0 \). The crushpads act as rigid protuberances.

Equation (24) may be used to determine the value of \( K_{sg} \) above which the payload does not contact the ground, limiting case 1. This is the value of \( K_{sg} \).
at which situation 1 and situation 2 must match. Setting \( L = 0 \) in equation (24) and calling this value of \( K_{sg} \), \( K_{sg}^* \)

\[
K_{sg} = \left( \frac{F_{cr}}{2(1 + 1) \frac{mg}{A_{cp}}} \right) \frac{f_{cr}}{A_{cp}}
\]

(25)

The denominator of equation (25) must be positive. Setting the denominator equal to zero and solving for \( f_{cr} \) results in the value of crush strength, below which the payload contacts the ground, regardless of the value of \( K_{sg} \). For crush strengths greater than this value, the payload may or may not contact the ground, depending on the value of \( K_{sg} \). Call this value of crush strength \( f_{cr}^* \).

\[
\frac{1}{2} \left( 1 - \frac{1}{2} \right) \frac{mg}{A_{cp}} L^2 = L + 1 \quad (26)
\]

Limiting case ii is obtained by setting \( L = 0 \) and solving for \( L \). The maximum deceleration is obtained by substituting equation (22) into equation (26).

\[
\left( \frac{a_{max}}{g} \right) = F_{sg} \left[ 1 + \left( \frac{mg}{A_{cp}} \right) L \right] \quad (27)
\]

Setting \( L = 0 \) in equation (27) gives the maximum \( K_{sg} \) at which the payload contacts the ground, \( K_{sg}^* \).

\[
K_{sg}^* = 2 \left( 1 + 1 \right) \frac{mg}{A_{cp} \kappa_{cp}} \quad (28)
\]

This is the value of \( K_{sg} \) at which limiting case i of situation 1 must match limiting case ii of situation 2.

The crush strength corresponding to \( f_{cr}^* \) is the crush strength above which the crushpads act as rigid protuberances when the payload is in contact with the ground. \( f_{cr}^* \) is calculated by setting \( \lambda = 0 \) and \( L = 0 \), giving \( f = 1 \).

\[
f_{cr}^* = \frac{1}{2} \frac{K_{sg}^*}{L_{cp} \kappa_{cp}} = \frac{2f_{cr}^*}{\kappa_{cr}} \quad (29)
\]

Example 1, Sounding-rocket payload

Consider a sounding rocket payload 38 inches in diameter \( (A_{pt} = 1134 \text{ in}^2) \) which has a mass, \( m = 1325 \text{ lb} \). The parachute delivers the payload to the ground at 26 ft/s. The size of the crushpad structure is limited, because of instrumentation in the base of the payload and the length of the transition section between the payload and booster, to a total surface area of \( A_{cp} = 80 \text{ in}^2 \) and length \( \kappa_{cp} = 9.5 \text{ inches} \) beyond the base. We wish to determine the maximum deceleration of the payload during impact with the ground, \( (a_{max}/g) \), as a function of the subgrade modulus, \( K_{sg} \), for various crush strengths \( (f_{cr}) \) crushpads. The results are presented in Fig. 2 in the form of a crushpad performance map.

First, let us consider some limiting cases and envelopes. Equation (6) can be used to calculate the maximum deceleration of the payload without crushpads by setting \( A = A_p \). See Fig. 2.

The limiting case of rigid crushpads \((L = 0)\) is calculated in two parts which must match at the value of \( K_{sg} \) given by equation (29). Substituting numerical values into equation (29) gives \( K_{sg}^* = 49.8 \text{ psi/in.} \) For values of \( K_{sg} \) less than \( K_{sg}^* \), equation (27) is solved for \( L \) and used in equation (28) to find the corresponding \( (a_{max}/g) \). For values of \( K_{sg} \) greater than \( K_{sg}^* \), equation (18) is solved for \( L \) which is substituted into equation (14) to find the maximum deceleration. A rigid crushpad lowers the maximum deceleration compared with that of no crushpad because of the area effect discussed previously.

The envelope of \( (a_{max}/g) \) vs \( K_{sg} \) for the payload just contacting the ground \((L = 0)\) when crushpad deformation is present \((L > 0)\) is obtained by substituting equation (15) and \( f = 1 \) into equation (18) resulting in

\[
1/2 F_{sg} \left( 2 + L^2 \right) = L + 1 \quad (30)
\]

Equation (31) is solved for \( L \) for values of \( K_{sg} \) greater than \( K_{sg}^* \). The corresponding maximum deceleration is obtained from equation (14). This envelope is shown in Fig. 2.

Now, let us calculate the maximum deceleration of various crush-strength crushpads as a function of subgrade modulus. We must first determine the value of the subgrade modulus, \( K_{sg}^* \), for the specific crush strength of the crushpad below which the crushpads act as a rigid protuberance, \( L = 0 \). There are two possibilities: The payload is not in contact with the ground (case i of situation 1) or the payload contacts the ground (case ii of situation 2). If \( f_{cr} > f_{cr}^* \), the payload does not contact the ground \((L = 0)\) and it is in situation 1. \( K_{sg} \) is given by equation (19). If \( f_{cr} < f_{cr}^* \), the payload contacts the ground \((L > 0)\) and it is in situation 2. To calculate the appropriate \( K_{sg}^* \), for situation 2, use equation (27) in the form

\[
F_{cr} = F_{sg} \left( 1 + L \right) \quad (32)
\]

to eliminate \( F_{sg} \) in equation (27) giving

\[
F_{cr} \left[ 1/2 + L + 1/2 \left( \frac{A_{pt}}{A_{cp}} \right) L^2 \right] = T (1 + L) + (1 + L)^2 \quad (33)
\]

Solve equation (33) for \( L \) and substitute it back into equation (32) to find \( K_{sg}^* \).


As an example consider crushpads made of 

\( f_{cr} = 750 \text{ psi crush-strength honeycomb.} \) From equation (30), \( f_{cr}^* = 472.8 \text{ psi.} \) Since \( f_{cr} > f_{cr}^* \) it is situation 1. Equation (19) gives \( K_{sg}^* = 128.7 \text{ psi/in.} \)

As another example suppose the crushpads are constructed of 300 psi crush-strength honeycomb, \( f_{cr} < f_{cr}^* \) and it is situation 2. Substituting numerical values into equation (33) and using the quadratic formula gives \( L = 0.192. \) Substituting this value of \( L \) into equation (32) gives \( K_{sg}^* = 26.5 \text{ psi/in.} \)

Now let us calculate the value of \( K_{sg} \) at which the 300 psi crushpads result in the payload just contacting the ground. Substituting numerical values into equation (25) gives \( K_{sg} = 74.5 \text{ psi/in.} \)

At values of \( K_{sg} \) greater than \( K_{sg}^* \), the payload does not contact the ground and we have situation 1.

These calculations can be summarized by the following procedure.

1. **Limiting case of rigid crushpads.**
   - Calculate \( K_{sg}^* \) using equation (29).
   - \( K_{sg} < K_{sg}^* \): Solve equation (27) for \( L \) and substitute into equation (28) to find the corresponding \( (a_{max}/g) \).
   - \( K_{sg} > K_{sg}^* \): Solve equation (18) for \( L \) and substitute into equation (14) to find the corresponding \( (a_{max}/g) \).

2. **Limiting values of subgrade modulus for specific crush-strength honeycomb.**
   - Calculate \( f_{cr}^* \) using equation (30).
   - \( f_{cr} < f_{cr}^* \): Solve equation (33) for \( L \) and substitute into equation (32) to obtain \( K_{sg} \). Use equation (25) to calculate \( K_{sg} \).
   - \( f_{cr} > f_{cr}^* \): Use equation (19) to calculate \( K_{sg}^* \).

3. **Variation of \( (a_{max}/g) \) with \( K_{sg} \) for specific crush-strength honeycomb.**
   - \( f_{cr} < f_{cr}^* \): 0 < \( K_{sg} < K_{sg}^* \).
     - Use equation (26) to calculate \( L \) and substitute into equation (21) to find corresponding \( (a_{max}/g) \).
   - \( K_{sg} > K_{sg}^* \): Use equation (14) to calculate crushpad-limited \( (a_{max}/g) \).
   - \( f_{cr} > f_{cr}^* \): 0 < \( K_{sg} < K_{sg}^* \).
     - Use equation (16) to calculate crushpad-limited \( (a_{max}/g) \).

Consider the curves of constant crush strength less than \( f_{cr}^* = 216.4 \text{ psi.} \) The base of the payload contacts the ground for all values of subgrade modulus. As the subgrade modulus increases, the crushpads deform more and dissipate a larger proportion of the total energy. The 100 psi crush-strength crushpads have so little energy-absorbing capacity that they are fully deformed over most of the range of subgrade modulus shown in Fig. 2.

Next, consider the curves of constant crush strength in the range \( f_{cr} < f_{cr}^* = 472.8 \text{ psi.} \)

At the lower values of \( K_{sg} \), the crushpads act as rigid protuberances and their deceleration performance is given by the curve labeled rigid crushpad. At the intermediate values of \( K_{sg} \), the crushpads increasingly deform with increasing subgrade modulus. Finally, for values of \( K_{sg} > K_{sg}^* \), the payload does not contact the ground and all the energy is dissipated by deformation of the crushpads. The crushpads limit the deceleration to a constant value regardless of the specific value of \( K_{sg} \).

Finally, consider the curves of constant crush strength, \( f_{cr} > f_{cr}^* \). For values of \( K_{sg} < K_{sg}^* \), the crushpads act as rigid protuberances, and their deceleration performance is given by the curve labeled rigid crushpad. For \( K_{sg} > K_{sg}^* \), the payload contacts the ground, and for \( K_{sg} > K_{sg}^* \), the payload does not impact the ground. For values of \( K_{sg} > K_{sg}^* \), the crushpads limit the deceleration to a constant value.

The sounding rocket is launched at the White Sands Missile Range in New Mexico. Soil surveys of the anticipated impact area indicate a coarse-grained, sandy silt(1) which has a subgrade modulus range of 100 to 400 psi/in. Fig. 2 shows that the payload deceleration during ground impact can be limited to less than 20g's by using 300 psi crush-strength honeycomb.

**Example 2. Balloon payload**

Balloon payloads do not have the severe size limitations on the crushpad structure that sounding-rocket payloads do. Consequently, the maximum deceleration of the balloon payload during ground impact can be limited to a much lower value than that for a sounding rocket.

Consider a balloon payload which has a cross-sectional area of 3,000 in$^2$ and which has a mass of 1,250 lb. The crushpad area is 500 in$^2$ and 12 inches long. The parachute delivers the payload to the ground at 20 ft/s.

Fig. 3 shows a crushpad performance map for the balloon payload. The calculation procedure is identical to that for the sounding-rocket payload described in example 1. It is seen that the performance map for the balloon compared with that of the sounding-rocket is shifted to more than an order-of-magnitude lower subgrade modulus. Impact deceleration in any but the softest of soils (\( K_{sg} < 10 \text{ psi/in.} \)) can be limited to less than 10g's by using 20 psi crush-strength crushpads.

**References**

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![Fig. 2 Crushpad performance map for sounding-rocket payload.](image1)

![Fig. 3 Crushpad performance map for balloon payload.](image2)