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There is an observed variation in body wave magnitude of approximately ± 0.2 units for explosions in different areas of Yucca Flat, NV. The variation appears to correlate with the Cenozoic basin structure at Yucca Flat. The basin has been modeled geophysically by Ferguson (unpublished manuscript). In this study it is hypothesized that the variation is caused by scattering or resonance effects within the local geologic structure. This conjecture has been investigated by computation of synthetic teleseismic P-wave amplitude responses for the Yucca Flat geophysical mode. The tech. due to Aki & Larner (1970) was used. Good quantitative agreement with observations was found.
Far Field Seismic Response Functions for Explosive Sources in Yucca Flat, Nevada

by

John Ferguson
ABSTRACT

There is an observed variation in body wave magnitude of approximately ± 0.2 units for explosions in different areas of Yucca Flat, Nevada. The variation appears to correlate with the Cenozoic basin structure at Yucca Flat. The basin has been modeled geophysically by Ferguson (unpublished manuscript). In this study it is hypothesized that the variation in $m_b$ is caused by scattering or resonance effects within the local geologic structure. This conjecture has been investigated by computation of synthetic teleseismic P-wave amplitude responses for the Yucca Flat geophysical model. The technique due to Aki and Larner (1970) was used. Good quantitative agreement with observations was found.
Body wave magnitude estimates for a number of nuclear explosions at Yucca Flat, Nevada have been analyzed for spatial variation (Alewine, personal communication). Each explosion was corrected to a common datum by the use of well established scaling laws for yield and depth of burial (Mueller and Murphy, 1971). These data, when plotted on the map of Yucca Flat in figure 22, show a progressive decline of about 0.5 magnitude units from the middle of the valley toward the east over a distance of four kilometers. This variation complicates the analysis and calibration of seismic yield estimation and source discrimination techniques.

The magnitude variation is correlative with the structure contours of the top of the Paleozoic units as presented in the FIB model of Yucca Flat. The systematic nature of the variation suggests a deterministic explanation of the phenomena in terms of scattering of the outgoing body waves by the local structure.

This problem is not soluble in any simple form for realistic geophysical models of Yucca Flat. A numerical solution might be computed by any of several techniques such as finite difference, finite element or some form of ray tracing. After a review of the literature on numerical wave propagation techniques, the above alternatives were rejected in favor of the Aki and Larner (1970)
Fig. 22. Contours of body wave magnitude variation ($m_b$) superimposed on a contour map of the Tertiary-Paleozoic interface for Yucca Flat, Nevada. The $m_b$ contours were modified from Alewine (personal communication).
collocation method. This technique promised to have high accuracy and reasonable computation speed. It is not limited to plain strain theory like the finite difference calculation or high frequency like the asymptotic ray theory; also the solution is not restricted to a limited set of rays or modes.
DERIVATION OF THE METHOD

In this section a technique introduced into seismology by Aki and Larner (1970), Larner (1970), Bouchon (1973) and Bouchon and Aki (1977), will be developed in a general form. The method assumes a solution in each homogeneous layer or region in terms of a series of plane waves. The boundary conditions between the regions are matched at a finite number of points on the boundaries. In numerical analysis this type of calculation is called an orthogonal basis boundary value collocation scheme. The solution is expanded in an orthogonal basis of known solutions to the partial differential equation and forced to satisfy the boundary conditions at a finite number of points. The interpolatory characteristics of the solution are used to provide an approximate satisfaction of the boundary conditions at unsampled boundary points. Due to certain requirements of the reciprocal method that will be used to apply the explosion source, it is more convenient to develop the method in terms of potentials. From Lame's theorem (Aki and Richards, 1980, p 68), the equations of motion for a homogeneous elastic medium will yield solutions in terms of a scalar and a vector potential.

1a) \( \underline{u} = \nabla \phi + \nabla \times \psi \),

1b) \( \nabla \cdot \psi = 0 \),
each $k$ and only three scalar potentials are required (Lamer, 1970, p 25-44). The new propagation coordinates are:

3a) $K = (k^2 + \gamma^2)^{1/2}$,

3b) $|K'| = |k + \gamma\gamma|$,

3c) $K = K \cos (\Omega)$,

3d) $r' = \frac{R}{z} = \begin{pmatrix} x' \\ y' \\ z \end{pmatrix} = \begin{pmatrix} \cos (\Omega) & \sin (\Omega) & 0 \\ -\sin (\Omega) & \cos (\Omega) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

The displacements are written in terms of the required potentials as:

4a) $U_1 (x', y', z) = \frac{\partial \phi}{\partial x'} - \frac{\partial \psi}{\partial z}$,

4b) $U_2 (x', y', z) = \frac{\partial x}{\partial x'}$,

4c) $U_3 (x', y', z) = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x'}$. 
These equations include separate wave equations for compressional, \( \lambda_c \), and shear, \( \lambda_d \), waves. In general the potentials are ambiguously determined and hence are useful only when certain symmetries are present in the formulation.

Plane wave solutions with harmonic time dependence,

\[ \psi = A \exp \left( -i(\omega t + kx + \gamma y \pm \nu z) \right), \]

will be assumed. In the case of a wave incident on a horizontal interface all scattered wave vectors will be contained in the same vertical plane as the incident wave. An alternative statement is that \( k \) and \( \gamma \) are constant for all waves satisfying the boundary conditions. Thus the coordinates may be rotated to eliminate one wavenumber component and the potentials reduced to three scalar potentials, with horizontally polarized shear (SH) motion uncoupled from compressional (P) and vertically polarized shear (SV) motion (Aki and Richards, 1980, p 215). In the case of an interface with topography variable in the \( x \)-direction only and incident waves making the angle \( \Omega \) with that direction, the \( y \) wavenumber component, \( \gamma \), will remain constant, but the \( x \) wavenumber component, \( k \), will couple over the entire wavenumber spectral range. The constancy of \( \gamma \) permits the specification of a vertical plane for each value of \( k \) which will contain the scattered waves. The P-SV and SH motions will uncouple for
In each homogeneous region the general solution will be expanded in an infinite number of plane wave contributions:

5a) \( \phi (x,y,z,k) = \left[ A_1 (k) \exp (-i\omega z) + A_2 (k) \exp (i\omega z) \right] \exp \left( i(kx + \eta y) \right) \),

5b) \( \psi (x,y,z,k) = \left[ A_3 (k) \exp (-i\gamma z) + A_4 (k) \exp (i\gamma z) \right] \exp \left( i(kx + \eta y) \right) \),

5c) \( \chi (x,y,z,k) = \left[ A_5 (k) \exp (-i\tau z) + A_6 (k) \exp (i\tau z) \right] \exp \left( i(kx + \eta y) \right) \).

The term, \( \exp(-i\omega t) \), is suppressed in all the following equations.

The boundary conditions to be satisfied are continuity of displacement and traction normal to the boundary at each internal interface. The normal traction is set to zero on the free surface. The general solutions for the displacement and stress components are:

(a) \( U_i (x,y,z) = \int R_{ki} \ e_{kd} \ A_d \ exp (-ikx) \ dk \),

(b) \( \sigma_{ij} (x,y,z) = \int R_{ki} \ R_{jm} \ f_{kma} \ A_d \ exp (-ikx) \ dk \)

\( i,j,k,m = 1,2,3, \ \ \ell = 1,2,...,\omega \).

The normal tractions at an interface are obtained from the inner product of the unit normal vector and the stress tensor or the operator,

\( F_{ik} = \hat{F}_{ij} \hat{R}_{j} (\xi) \), \( i,j = 1,2,3, \ \ \ell = 1,2,...,\omega \).

The unit normal is a function of the interface topography, \( \xi (x) \).

The operators \( \hat{g} \), \( \hat{\xi} \) and \( \hat{\eta} \) are exponential functions of \( y, z, k \) and \( \eta \), as well as the elastic constants and density. They are wavenumber domain spacial derivative operators as required by 4. The internal
boundary conditions require that these quantities be evaluated on each side of the interface. In matrix form this is expressed;

\[ \begin{bmatrix} \frac{\sigma}{\Xi} \nabla \end{bmatrix} = \int \begin{bmatrix} \frac{\mathbf{R}^T \mathbf{F}}{\Xi} \\ \frac{\mathbf{R}^T \mathbf{F}}{\Xi} \end{bmatrix} \begin{bmatrix} \mathbf{A} \end{bmatrix} \exp(-ikx) \, dk . \]

The integration over k must be performed numerically, so the Fourier transform is replaced by a Fourier series of finite length (rectangle rule integration or discrete Fourier transform). This results in a periodic repetition of the interface topography with fundamental wavelength, L. The collocation technique evaluates the displacements and tractions at a finite number of points in the interval [0,L]. The space and spectral discretizations are:

\[ \begin{align*}
\eta \equiv & \Delta x \ n , \quad n = 0, 1, \ldots, N - 1 , \\
\kappa \equiv & \frac{2\pi}{L} \ m , \quad m = 0, \pm 1, \ldots, \pm \frac{(M-1)}{2} \\
\end{align*} \]

At this point the notation required becomes rather complex. In addition to the tensor indices we now have indices associated with the series summation and collocation points. The differing layers of the model must also be distinguished. Figure 23 is introduced to clarify some of the additional notation. The superscripts, i, will serve to identify interfaces starting with the free surface, i=1. The superscripts, l, refer to the layers, where layer 1 is just below
Fig. 23. An introduction to the indicial notation used to identify the homogeneous regions and interfaces used in the Aki-Larner technique derivation.
INTERFACE INDICES
\( i = 1, 2, 3, 4, 5 \)

LAYER INDICES
\( J = 1, 2, 3, 4, 5 \)

INTERFACE TOPOGRAPHY
\( \xi \) (\( \xi \) function)

P AND S WAVE VELOCITIES AND DENSITIES FOR EACH LAYER
\( V_p, V_s, \rho \)

MODEL REPETITION DISTANCE
\( L \)

INTERFACE TOPOGRAPHY IS INvariant IN THE Y-DIRECTION (OUT OF THE PAGE)

MODEL STRUCTURE AND NOTATION
the 1th interface. There are P layers and interfaces. For the
discrete space specified by 9, equation 8 may be written,
10) \[ \mathbf{u}^i = \mathbf{G}^i \mathbf{A}^i. \]

In 10, \( \mathbf{u}^i \) is the 6\( \cdot \)N vector of displacements and tractions evaluated
at the discrete interface points, \( \mathbf{G}^i \) is the 6 \( \cdot \) M \( \times \) 6 \( \cdot \) N matrix
which transforms the discrete k, 6 \( \cdot \) M vector of potential
coefficients, \( \mathbf{A}^i \), in layer i. In the simplest application we will
set M=N for a square linear system of equations. A similar equation
exists in layer i + 1 at the same interface, which would be the
(i + 1)th interface. If the \( \mathbf{u}^i \) and \( \mathbf{u}^{i+1} \) vectors evaluated at
\( t^{i+1} (x) \) are set equal the result may be solved for \( \mathbf{A}^i \) in terms of
\( \mathbf{A}^{i+1} \).
11) \[ \mathbf{A}^i = \left[ \mathbf{G}^i \right]^{-1} \mathbf{G}^{i+1} \mathbf{A}^{i+1}. \]

The product matrix is the propagator matrix for this problem. The
product of each such propagator at all interfaces,
12) \[ \mathbf{J} = \mathbf{G}^1 \begin{pmatrix} t' \end{pmatrix} \mathbf{P} \begin{pmatrix} j \end{pmatrix} \mathbf{G}^{j-1} \begin{pmatrix} t' \end{pmatrix} \mathbf{G}^j \begin{pmatrix} t' \end{pmatrix}, \]
results in a relationship between the displacement and stress at the
free surface and the wave field in the half-space, layer P, given by
13) \[ \mathbf{u}' = \mathbf{J} \mathbf{A}^P. \]

An incident body wave is specified in the half space, then the free
surface boundary conditions of vanishing stress are used to solve for
the reflected waves in the halfspace thus completely specifying \( \mathbf{A}^P \).
The $A^i$ vectors in each other layer are found by application of the propagator matrices in equation 11.

Larner (1970) introduced a variation of the collocation method which is of considerable importance. The partitions of $U$ and the columns of the partitions of $G$ are Fourier transformed over $N$ space samples. The boundary conditions are then specified in the wavenumber domain. The resulting Fourier series is then truncated to $M \leq N$ points and the equations solved as before, except for an inverse transform to convert the displacements and tractions back to the space domain. For $M=N$ this procedure is identical to the space domain collocation, otherwise it is a least squares solution of the collocation equation.

The wavenumber spectrum for this problem possesses poles corresponding to surface wave modes. Integration along the real wavenumber axis will result in numerical instability. The inclusion of viscoelastic attenuation will remove the poles from the real $k$-axis and permit integration along contours parallel to that axis. The attenuation may be specified as an average temporal $Q$ for the model with a complex frequency and wavenumber or as an average spatial $Q$ with complex velocity and wavenumber. In order that the proper phase relationships exist across the incident wavefront the imaginary part of the horizontal wavenumber, in all layers, must equal the imaginary part of the horizontal wavenumber of the incident wave (Aki and Larner, 1970). The imaginary part of the frequency is
determined by,

(4) \( \Im(\omega) = i 2\pi |f| / (2Q) \),

for a temporal \( Q \).

The numerical accuracy of the solution technique is very high. If the interface topography is adequately sampled, so that no aliasing occurs (i.e., the Fourier expansions are convergent), then the solutions are computed with accuracy equivalent to the rounding error of the computer. The simultaneous equations in 11 and 13 are usually well conditioned, so that little loss of significance occurs in their solution. A different form of error results from the so called Rayleigh ansatz (Aki and Larner, 1970). For interface slopes greater than about 60°, the solution should contain multiply reflected waves which are not provided for in the specification. In practice this error has not been very important.
1.1 SOURCE-RECEIVER RECIPROCITY

The Aki-Larner formulation may be used to calculate the response of an irregularly layered model to an incident plane body wave. The theory must be extended to include point explosive sources. Bouchon and Aki (1977) demonstrate how a discrete wavenumber representation can be used to calculate the near field response of complex sources in irregularly layered structures. Because of our interest in teleseismic measurements and the far field response, a more efficient procedure results from the use of the seismic reciprocity theorem (Bouchon, 1976) and the plane wave response. The reciprocity theorem, after White (1965), states:

If, in a bounded, inhomogeneous, anisotropic elastic medium, a transient force \( f(t) \) applied in some particular direction at some point \( P \) creates at a second point \( Q \) a transient displacement whose component in some direction is \( u(t) \), then the application of the same force \( f(t) \) at point \( Q \) in direction will cause a displacement at point \( P \) whose component in the direction is \( u(t) \).

The response of a point dilatational source, located at \( P \) in layer \( j \), at a point \( Q \), in the half space layer \( n \), can be derived from the consideration of a force \( F \) at \( Q \). The force, \( F \), is directed along the ray connecting \( P \) and \( Q \) and results in a vector displacement \( U(P) \) at \( P \). By the reciprocity theorem the same force at \( P \), in the displacement direction, will produce a displacement \( U(Q) \) in the ray direction (P-wave motion). The dilatational source is modeled by three orthogonal force dipoles. To obtain the displacement at \( Q \), in
ray direction, we compute the dilatation at \( P \) and modulate by \( M \), the desired source moment spectrum (von Seggern and Blanford, 1972).

\[
\psi(Q) = \frac{M}{f} \left( \frac{\partial \xi_1(r)}{\partial x} + \frac{\partial \xi_2(r)}{\partial y} + \frac{\partial \xi_3(r)}{\partial z} \right).
\]

We substitute the potential at \( P \) due to a plane wave incident along the desired ray into 15 to obtain,

\[
\psi(Q) = \frac{\omega^2}{\xi_3^2} \frac{M}{f} \left( A_1 \exp(-i\omega t) + A_2 \exp(i\omega t) \right) \exp(-i(kx + \gamma y)).
\]

Equation 16 provides the teleseismic P-wave motion due to the explosive source. The efficiency of this method derives from the ability to calculate the response from any source location for a given ray with only one model plane wave response calculation. In contrast, the discrete wavenumber source representation would require a re-evaluation of the propagator matrices for each source location.
A THEORETICAL RESPONSE PROFILE

The procedure outlined in the previous sections has been coded in FORTRAN IV. The program has been tested on a number of simple problems including flat layered and gently perturbed structures. Working versions exist for both the Control Data Corporation 6600 and the Digital Equipment Corporation VAX 11-780.

The scattering hypothesis can be tested on a profile taken from the Yucca Flat model. The first calculation has been done on an east-west profile at a latitude of 37° 02' N. The basin has very little variation parallel to strike and good structural control is provided by geophysical surveys in that area. In figure 24, the observed $m_b$ variation south of 37° 05' is plotted along with a profile taken from Goforth, et. al. (1979). The calculated, one Hertz P-wave amplitude variation for sources distributed along the profile in figure 24 should agree with this profile if the hypothesis is correct.

The geologic structure of the profile can be approximated by four layers. The Paleozoic limestone can be modeled as a half space which is faulted at a low angle ($\sim 50^\circ$) on the west by the Carpetbag fault system so that an asymmetrical basin is formed. The basin is filled with Tertiary volcanics and Quaternary alluvium. The water table at a depth of about 500 meters divides the volcanics into saturated and dry units. The alluvium is entirely free of water.
Fig. 24. A profile across Yucca Flat, Nevada at a latitude of $37^\circ \ 02'\ N$ showing the observed $m_b$ variation. The geological structure was taken from Goforth, et. al (1979).
Average velocities and densities for these units were derived from well logs as contained in Table III. The profile is shown at the bottom of figure 25.

The response due to sources at the four locations shown in figure 25, for P-wave angles of emergence between -40 and 40 degrees (west and east) and propagation vectors within the model plane, were computed. An average Q of 50 was assumed at a frequency of one Hertz and a source depth of 0.8 kilometer was utilized. This calculation resulted in a polar radiation pattern of amplitude vs. emergence angle for each shot point as shown at the top of figure 25.

In order to model the averaging process inherent in the magnitude calculation the median amplitude of each shot will be compared. The median is plotted as a semi-circular arc on each radiation pattern. The ratio of median amplitudes between shots on the west (near the Carpetbag fault) and those on the east is 2.4. This corresponds to an $m_b$ variation of 0.4 magnitude units, in good agreement with the profile in figure 24.

The results of figure 25 are limited in several important ways. No out of plane rays are considered and only a single narrow frequency band was studied. Future calculations will include the out of plane rays and a time domain synthesis over a broader (one or two octave) frequency band. The radiation patterns display a rather complicated variation which may be observable in teleseismic observations. This phenomena can only be studied by examination of a number of specific shots and receivers, with spectral and time domain modeling of the observed P-waves.
TABLE III

VELOCITY STRUCTURE OF YUCCA FLAT, NEVADA

<table>
<thead>
<tr>
<th>Stratigraphic Unit</th>
<th>P-Wave Velocity</th>
<th>S-Wave Velocity</th>
<th>Poisson's Ratio</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>QAL</td>
<td>1.34</td>
<td>0.64</td>
<td>0.35</td>
<td>1.80</td>
</tr>
<tr>
<td>TV (Dry)</td>
<td>2.14</td>
<td>1.14</td>
<td>0.30</td>
<td>1.80</td>
</tr>
<tr>
<td>TV (Wet)</td>
<td>3.00</td>
<td>1.60</td>
<td>0.30</td>
<td>1.80</td>
</tr>
<tr>
<td>PZ</td>
<td>4.57</td>
<td>2.64</td>
<td>0.25</td>
<td>2.50</td>
</tr>
</tbody>
</table>

Units are km/sec and gm/cc.
Fig. 25. Polar radiation diagrams of teleseismic P-wave amplitude as a function of emergence angle for shots distributed across Yucca Flat, Nevada.
P-WAVE AMPLITUDE VARIATION

A

B

C

D

W

E

QAL

TVd

TVs

PZ

Q=50

f=kHz
CONCLUSIONS

The results of this investigation indicate that it is possible to model the effects of near-source scattering on teleseismic signals from Yucca Flat, Nevada. This is made possible by the existence of a reasonably accurate geophysical model of the geologic structure developed from gravity, borehole and seismic exploration. The Aki-Larner technique is a computationally effective means of performing response calculations for quite complicated models, such as the one exhibited here. Future efforts will concentrate on time domain waveform synthesis and detailed source receiver pair models. It is also possible to model other profiles across the valley. Similar studies on other test sites can guide the improvement of earthquake-explosion discrimination and yield estimation.
REFERENCES


