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ON DIFFERENTIATING HYPEROSCULATORY ERROR TERMS.(U)
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Research Report CCS 408

ON DIFFERENTIATING HYPEROSCILATORY ERROR TERMS

by

J. Barzilai

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ABSTRACT

We generalize Ralston's result on differentiating error terms to the hyperosculatory and nonpolynomial cases.

KEY WORDS

Interpolation errors, Hyperosculatory interpolation, Nonpolynomial interpolation

We generalize Ralston's result [5] on differentiating error terms to the hyperosculatory and nonpolynomial cases. This result is used implicitly in [6,7] and explicitly in [1,2,3].

<u>Theorem 1</u>. Let $f: R \to R$ have continuous derivatives in an interval J. Let $x_j \in J$ $j = 0,1,\ldots,n$. Let $P(x)$ be the unique hyperosculatory interpolation polynomial of degree $< r = \sum_{j=0}^{n} \gamma_j$ satisfying

$$(1) \qquad \left.\begin{array}{l} P^{(k_j)}(x_j) = f^{(k_j)}(x_j) \\ \\ k_j = 0,1,\ldots,\gamma_{j-1}\ ,\ \gamma_j \geq 1 \end{array}\right\} \quad j = 0,1,\ldots,n\ ,$$

with $x_k \neq x_\ell$ for $k \neq \ell$. Then for $x \in J$, $x \neq x_j$ $j = 0,1,\ldots,n$, we have

$$(2) \qquad f'(x) - P'(x) = \frac{f^{(r)}(\xi)}{r!} W'(x) + \frac{f^{(r+1)}(\eta)}{(r+1)!} W(x)\ ,$$

where $W(x) = \prod_{j=0}^{n} (x-x_j)^{\gamma_j}$, and ξ and η are in the interval spanned by $x, x_0, x_1, \ldots, x_n$.

<u>Proof</u>. The error term in the hyperosculatory interpolation (1) is given by

$$(3) \qquad f(x) - P(x) = \frac{f^{(r)}(\xi)}{r!} W(x)$$

with ξ and $W(x)$ as above (see e.g. [4]). To simplify notation we will prove the theorem for the case $\gamma_j = s$, $j = 0,\ldots,n$.

Let $\psi_{ij}(x)$ be the unique interpolation polynomial of degree $< r$ satisfying

$$\psi_{ij}^{(k)}(x_\ell) = \delta_{ik} \cdot \delta_{j\ell}$$

for $j, \ell = 0, \ldots, n$; $i, k = 0, \ldots, s-1$. Then

$$P(x) = \sum_{i=0}^{s-1} \sum_{j=0}^{n} f^{(i)}(x_j)\psi_{ij}(x) ,$$

and

$$\frac{d}{dx}\frac{f(x)}{W(x)} = \sum_{i=0}^{s-1} \sum_{j=0}^{n} f^{(i)}(x_j)\frac{d}{dx}\frac{\psi_{ij}(x)}{W(x)} + \frac{d}{dx}\frac{f^{(r)}(\xi)}{r!} . \tag{4}$$

Let $x_{n+1} \neq x_j$ $j = 0, \ldots, n$ and define interpolation polynomials $\bar{\psi}_{ij}(x)$ by

$$\bar{\psi}_{ij}^{(k)}(x_\ell) = \psi_{ij}^{(k)}(x_\ell) = \delta_{ij}\delta_{j\ell} ,$$

$$\bar{\psi}_{ij}(x_{n+1}) = 0 ,$$

$$\bar{\psi}_{0,\,n+1}^{(k)}(x_\ell) = 0 ,$$

$$\bar{\psi}_{0,n+1}(x_{n+1}) = 1 ,$$

for $j, \ell = 0, \ldots, n$ and $i, k = 0, \ldots, s-1$.

The polynomial $\bar{P}(x) = \sum_{i=0}^{s-1} \sum_{j=0}^{n} f^{(i)}(x_j)\bar{\psi}_{ij}(x) + f(x_{n+1})\bar{\psi}_{0,n+1}(x)$ of degree $\leq r$ satisfies (1) and $P(x_{n+1}) = f(x_{n+1})$. Therefore we have $f(x) - \bar{P}(x) = W(x)(x - x_{n+1})\frac{f^{(r+1)}(\eta)}{(r+1)!}$ with η in the interval spanned by $x, x_0, \ldots, x_n$.

The uniqueness of the interpolation polynomials implies

$$(5) \qquad \begin{cases} \bar{\psi}_{ij}(x) = \psi_{ij}(x) - \dfrac{\psi_{ij}(x_{n+1})}{W(x_{n+1})} W(x) \quad \text{for} \quad j = 0, \ldots, n;\ i = 0, \ldots, s-1 , \\[2ex] \bar{\psi}_{0,n+1}(x) = \dfrac{W(x)}{W(x_{n+1})} . \end{cases}$$

Hence

$$\frac{f(x)}{W(x)} = \sum_{i=0}^{s-1} \sum_{j=0}^{n} f^{(i)}(x_j) \frac{\bar{\psi}_{ij}(x)}{W(x)} + \frac{f(x_{n+1})}{W(x_{n+1})} + \frac{f^{(r+1)}(\eta)}{(r+1)!}(x-x_{n+1})$$

and

$$(6) \quad \frac{1}{x-x_{n+1}}\left(\frac{f(x)}{W(x)} - \frac{f(x_{n+1})}{W(x_{n+1})}\right) = \sum_{i=0}^{s-1} \sum_{j=0}^{n} f^{(i)}(x_j) \frac{1}{x-x_{n+1}} \cdot \frac{\bar{\psi}_{ij}(x)}{W(x)} + \frac{f^{(r+1)}(\eta)}{(r+1)!} .$$

For $x \neq x_j$ $j = 0,\ldots,n$ we now let $x_{n+1} \to x$. Since $W(x_{n+1}) \neq 0$ and $\bar{\psi}_{ij}(x_{n+1}) = 0$ we have

$$(7) \quad \lim_{x_{n+1}\to x} \frac{1}{x-x_{n+1}} \cdot \frac{\bar{\psi}_{ij}(x)}{W(x)} = \lim_{x_{n+1}\to x} \frac{1}{x-x_{n+1}}\left(\frac{\bar{\psi}_{ij}(x)}{W(x)} - \frac{\bar{\psi}_{ij}(x_{n+1})}{W(x_{n+1})}\right)$$

$$= \frac{d}{dx} \frac{\bar{\psi}_{ij}(x)}{W(x)} = \frac{d}{dx} \frac{\psi_{ij}(x)}{W(x)}$$

where the last equality holds by (5).

From (6) and (7) we have

$$(8) \quad \frac{d}{dx} \frac{f(x)}{W(x)} = \sum_{i=0}^{s-1} \sum_{j=0}^{k} f^{(i)}(x_j) \frac{d}{dx} \frac{\psi_{ij}(x)}{W(x)} + \frac{f^{(r+1)}(\eta)}{(r+1)!} .$$

Comparing (4) and (8) we conclude

$$(9) \quad \frac{d}{dx} \frac{f^{(r)}(\xi)}{r!} = \frac{f^{(r+1)}(\eta)}{(r+1)!} .$$

Differentiating (3) using (9) finally yields (2). $\square$

<u>Theorem 2</u>. If $T \in C^{(r+1)}(J)$ is a hyperosculatory interpolating function for f satisfying (1) (i.e. (1) holds with P replaced by T), then Theorem 1 holds with (2) replaced by

$$f'(x) - T'(x) = \frac{f^{(r)}(\xi) - T^{(r)}(\xi)}{r!} W'(x) + \frac{f^{(r+1)}(\eta) - T^{(r+1)}(\eta)}{(r+1)!} W(x) \quad (2')$$

Proof. Replace f in Theorem 1 by $h = f - T$, and note that the unique interpolation polynomial satisfying (1) for h is $P = 0$. □

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