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COMPUTATIONAL PARAMETRIC STUDY OF THE AERODYNAMICS OF SPINNING SLENDER BODIES AT SUPersonic SPEEDS

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Computational Parametric Study of the Aerodynamics of Spinning Slender Bodies at Supersonic Speeds

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Three dimensional finite-difference flow field computation techniques have been employed to generate a parametric aerodynamic study at supersonic speeds. Computations for viscous turbulent and inviscid flow have been performed for cone-cylinder, secant-ogive-cylinder, and tangent-ogive-cylinder bodies for a Mach number range of $1.75 < M < 5$. The aerodynamic coefficients computed are pitching moment, normal force, center of pressure, Magnus moment, Magnus force, Magnus center of pressure, form drag, viscous drag, roll damping and pitch damping. All aerodynamic coefficients are computed in a...
20. ABSTRACT (Continued)

conceptually exact manner. The only empirical input is that required for turbulence modeling. Computed results are compared to experimental data from free flight aerodynamic ranges and wind tunnels in order to validate the computational techniques. Parametric comparisons illustrate the effects of body configuration and Mach number for the ten aerodynamic coefficients. The results for Magnus and pitch damping are of particular interest.
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I. INTRODUCTION

Recent trends in projectile design have led to shapes with greater length and more slender ogives. Unexpected flight stability problems have been encountered due to decreased aerodynamic stability of these new shapes. Clearly, conventional aerodynamic predictive capabilities were not adequate. In an effort to avoid these problems in the future, the Ballistic Research Laboratory has been developing advanced numerical computational techniques for computing projectile aerodynamic characteristics to improve shell design technology.

Substantial progress has been made in the past 10 years in the development of aerodynamic computational techniques and in the availability of high speed digital computers. This progress has made it possible to begin to use advanced finite-difference computational techniques to perform parametric aerodynamic studies for evaluation of proposed design concepts.

The use of advanced numerical computational techniques for a parametric study is difficult to justify to compute only static aerodynamic parameters since cheaper, less complex techniques such as Ref. (1), (2) and (3) are available. However, if dynamic derivatives such as Magnus and pitch damping are considered important and if viscous drag is of interest, then the advanced computational techniques are justified and, in fact, must be used. This paper reports the initial results of an ongoing research effort at BRL to form an advanced aerodynamic computation capability that will provide the shell designer with a complete package of static and dynamic aerodynamic coefficients for use in design studies.

II. COMPUTATIONAL TECHNIQUES

A. Scope of Effort

Three dimensional finite-difference flow field computational techniques for inviscid and turbulent viscous flow have been applied to generate a comprehensive set of aerodynamic coefficients for cone-cylinder (CC), tangent-ogive-cylinder (TOC), and secant-ogive-cylinder (SOC) body configurations. The model geometries considered in this study are shown in Figure 1. Body lengths up to seven calibers and ogive lengths of two, three, and four calibers have been considered. The aerodynamic coefficients computed are pitching


moment, normal force, center of pressure, Magnus moment, Magnus force, Magnus center of pressure, form and viscous drag, roll damping and pitch damping. The sign convention for the pitch plane and Magnus forces is shown in Figure 2. All aerodynamic coefficients are computed in a conceptually exact manner. The only empirical input is that required for the modeling of turbulent eddy viscosity.

The computations have been carried out for a Mach number range of 1.75 < M < 5. These computations were all performed for an angle of attack of 1°, a nondimensional spin rate (PD/V) of 0.19, and for sea level atmospheric free-stream conditions. Specific comparisons to wind tunnel data were made for the tunnel operating conditions.

B. Coupled Inviscid-Viscous Computations

The sequence of computations which are run in order to compute the static aerodynamic parameters, including turbulent viscous effects, is shown in Figure 3. Each block represents a separate computer code. These codes have been combined using the overlay technique on the BRL Cyber computer. The two main codes are those which compute three dimensional turbulent boundary layer development and three dimensional inviscid flow.

The computation of the effects of viscosity is of crucial importance when such parameters as roll damping, Magnus, and drag are of interest. The technique employed here is a fully implicit, finite difference numerical scheme developed by Dwyer. This technique takes into consideration the changes in direction of the cross-flow velocity that occur on the side of the shell where the inviscid cross-flow opposes the surface spin.

The equations of motion solved are the basic equations defining the three-dimensional compressible, turbulent boundary-layer flow over a body of revolution described by the relation \( r = r(x) \). The coordinate system is shown in Figure 4.

Continuity

\[
\frac{\partial}{\partial x} (r\rho u) + \frac{\partial}{\partial y} (r\rho v) + \frac{\partial}{\partial \phi} (\rho w) = 0
\]  

(1)

\( x \) momentum

\[
\rho \left[ \frac{\partial u}{\partial x} + \frac{v}{r} \frac{\partial u}{\partial y} + \frac{w}{r} \frac{\partial u}{\partial \phi} - \frac{w^2}{r} \frac{\partial r}{\partial x} \right] =
\]

\[
- \frac{\partial p_e}{\partial x} + \frac{\partial}{\partial y} \left[ \frac{\partial u}{\partial y} - \frac{u}{\partial y} \right]
\]  

(2)

\[ \phi \text{ momentum} \]
\[ \bar{p}[u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial r} + \frac{\partial \bar{u}w}{\partial r}] = - \frac{1}{r} \frac{\partial \tilde{p}}{\partial \phi} + \frac{3}{r} \left[ \mu \frac{\partial w}{\partial y} - \frac{\rho v^* w^*}{\partial y} \right] \]

\[ \text{Energy} \]
\[ \bar{p}[u \frac{\partial \bar{h}}{\partial x} + v \frac{\partial \bar{h}}{\partial y} + w \frac{\partial \bar{h}}{\partial r}] = \bar{u} \frac{\partial \tilde{p}}{\partial w} + \frac{w}{r} \frac{\partial \tilde{p}}{\partial \phi} + \mu \left[ (\frac{\partial \bar{u}}{\partial y})^2 + (\frac{\partial \bar{w}}{\partial y})^2 \right] - \rho u^* \frac{\partial w^*}{\partial y} - \rho v^* \frac{\partial \bar{w}}{\partial y} \]
\[ + \frac{3}{r} \left[ \frac{\rho \bar{v}^*}{\partial y} \frac{\partial \bar{h}}{\partial y} - \frac{\rho v^* h^*}{\partial y} \right] \]

where \( v = \tilde{v} + \rho v^*/\bar{p} \) and the bar indicates a time-averaged quantity.

In order to obtain closure of this system of equations, the following models of the turbulence terms have been introduced:

**Turbulent shear stress**
\[ - \rho u^* v^* = - \rho v^* w^* = \rho \lambda^2 \left[ (\frac{\partial \bar{u}}{\partial y})^2 + (\frac{\partial \bar{w}}{\partial y})^2 \right] \]
\[ = \varepsilon \left[ (\frac{\partial \bar{u}}{\partial y})^2 + (\frac{\partial \bar{w}}{\partial y})^2 \right] \]

where \( \varepsilon \) is introduced as the turbulent viscosity and the mixing length, \( \lambda = 0.09 \delta \tanh [(0.4/0.09)(y/\delta)] \). Van Driest damping is used to account for the effect of the laminar sublayer.

**Turbulent heat transfer**
\[ - \rho v^* h^* = \frac{k_t}{c_p} \frac{\partial \bar{h}}{\partial y} \]

The turbulent Prandtl number is introduced as
\[ Pr_t = c_p \varepsilon /k_t = 0.90 \]

The three-dimensional displacement surface is not merely the vector sum of the longitudinal and circumferential components of the boundary-layer displacement thickness. Instead, the differential equation derived by Moore\(^5\):

---

\[
\frac{\partial}{\partial x} [\rho e u x (\delta_{3D} - \delta^*)] + \frac{\partial}{\partial \phi} [\rho e w (\delta_{3D} - \delta^*)] = 0
\] (5)

must be solved for \(\delta^*\), the three-dimensional boundary-layer displacement thickness where
\[
\delta^* = \int_0^\delta (1 - \frac{\rho u}{\rho e u e}) dy
\]
\[
\delta^* = \int_0^\delta (1 - \frac{\rho w}{\rho e w e}) dy
\]

With a body fixed coordinate system, the gas dynamic equations for inviscid flow can be written as
\[
E_z + Fr + G_\phi + H = 0
\] (6)

where the flux vectors \(E\), \(F\), \(G\), and \(H\) are

\[
E = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho w \\ \rho v w \end{bmatrix} \quad F = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho vw \end{bmatrix}
\]

\[
G = \frac{1}{r} \begin{bmatrix} \rho w \\ \rho uw \\ \rho w^2 + p \\ \rho vw \end{bmatrix} \quad H = \frac{1}{r} \begin{bmatrix} \rho v \\ \rho uv \\ \rho (v^2 - w^2) \\ 2 \rho vw \end{bmatrix}
\]

These equations are solved using MacCormack's\(^6\) two-step, predictor-corrector finite difference scheme. The unique feature of the program used here, which was developed by Sanders\(^7\), for the Magnus problem, is that the flow field is computed about an axisymmetric model plus displacement surface. Due to the distortion of the viscous layer caused by interaction of the surface spin, the effective aerodynamic shape has no plane of symmetry.

---


The flow field variables resulting from these computation steps have been
developed to yield the following aerodynamic coefficients—pitching moment,
normal force, center of pressure, Magnus force, Magnus moment, Magnus center
of pressure, form drag, viscous drag, and roll damping. The computational
time for a single body configuration and flow field condition is approximately
ten minutes on a CDC 7600 computer.

C. Coning Motion Computations

In order to compute the effective pitch damping, the technique developed
by Schiff is used. This computational technique relates the side moment on a
body undergoing a steady coning motion about the CG location to the pitch
damping \( C_M \), see Figure 5.

The numerical technique is MacCormack's predictor-corrector, explicit
marching scheme. This computation involves the solution of the Euler
equations including terms for Coriolis \( \lambda \) and centrifugal \( \mu \) forces in a body fixed coordinate system. For this case, the \( H \) vector in
equation 6 becomes

\[
H = \frac{1}{r} \begin{bmatrix}
\rho v \\
\rho uv + pr[2(\omega_2 w - \omega_3 v) + \omega_1 w_2 r - z(\omega_2^2 + \omega_3^2)] \\
\rho (v^2 - w^2) + pr[2(\omega_3 u - \omega_1 w) + \omega_1 w_2 z - r(\omega_1^2 + \omega_3^2)] \\
2\rho vw + pr[2(\omega_1 v - \omega_2 u) + \omega_3 (w_2 r + \omega_1 z)]
\end{bmatrix}
\]

where \( \omega_1, \omega_2, \omega_3 \) are the components of the angular velocity vector
\( \omega \) resolved in the \( z, r, \phi \) directions, respectively.

For the case of a steady coning motion, the flow field is time-invariant
in the body-fixed coordinate system. The effective pitch damping
\( C_{Mq} + C_{Ma} \) is determined using the relation

\[
C_{n\dot{\phi}} = \sin \sigma (C_{Mq} + C_{Ma})
\]

where \( C_{n\dot{\phi}} \) is side moment at coning rate \( \dot{\phi} \) and effective angle of attack \( \sigma \),
which is valid for small values of \( \sigma \) and \( \dot{\phi} \). Thus a dynamic aerodynamic
parameter is determined using a steady flow field computation. This is a
potentially very useful tool for the exterior ballisticsian. The computation
time is approximately 90 seconds on a CDC 7600 computer for the body configu-
rations in this study.

8. Schiff, L.B., "Nonlinear Aerodynamics of Bodies of Coning Motion", ATAA
III. RESULTS

A. Comparisons to Experiment

Detailed comparisons of the computations to experimental data for turbulent boundary layer profile characteristics, wall pressure measurements and Magnus force are reported in Ref. 9. Comparisons shown here will be limited to the aerodynamic coefficients of interest.

Charters and Kent\textsuperscript{10} have shown that roll damping can be related to the skin friction drag for a cylinder according to the relation

$$C_{\Delta p} = -0.25 C_{DBL} \quad (8)$$

Murphy\textsuperscript{11} has shown good agreement with this relation in a series of free flight range tests firings of two caliber tangent ogive cylinder models with total lengths of 5, 7, and 9 calibers.

The results of this computational study confirm that equation (8) is a good engineering relation for estimating the roll damping coefficient. A summary of the computed results are compared to the Charters-Kent relation in Figure 6. The computational results show a spread which is due to the effect of ogive configuration. This effect is better illustrated in Figure 7 where roll damping is plotted versus projectile length for four ogive configurations at Mach 2.75. In general, this computational study shows that roll damping is linear with respect to body length for a particular flight velocity and that the zero offset is a function of ogive configuration.

Examples of comparisons of the computed results to experimental data are presented in Figures 8 through 12. The comparisons for pitch plane static parameters shown in Figures 8, 9 and 10 indicate excellent agreement for $M > 2.5$. The results for the supersonic marching computational technique used here have indicated a reduced accuracy for flow over shell with short ogives at low supersonic velocities. The limited comparison for Magnus in Figure 11


indicates acceptable agreement if allowance is made for the small magnitude of the Magnus effect and the variance between the wind tunnel and range experimental measurements. A comparison between computation and experiment for pitch damping is shown in Figure 12. The experimental point, which is for an L/D of 5.12 and cone angle of 9.52°, shows excellent agreement with the trend of the computed results. A similar comparison for pitch damping is shown in Figure 13 for a 10° cone. This comparison includes both wind tunnel and free flight range data. The pitch damping is very small for a cone; but the agreement shown is considered to be very good. In general, it is felt that the numerical computations do provide an accuracy for the aerodynamic coefficients that is within the uncertainty of our ability to determine these coefficients experimentally. However, it is felt that a broader scope of comparison for the aerodynamic coefficients between experiment and computation is of interest and increased effort to accomplish this is underway.

B. Parametric Comparisons

Examples illustrating the parametric results are shown in Figures 14 through 30. The series of comparisons shown in Figures 14 through 23 illustrates an example for each aerodynamic coefficient computed in this study. The case chosen is the SOC model for a total length of six calibers and for ogive lengths of two, three, and four calibers. The aerodynamic coefficients are plotted versus Mach number for atmospheric free stream launch conditions assuming an adiabatic wall temperature boundary condition. These comparisons show, for a fixed body length, that configurations with long slender ogives have reduced pitch damping, less drag, and a reduced Magnus moment compared to bodies with shorter ogive lengths. The development of the Magnus force over the full length of the shell is shown in Figure 24 for two ogive configurations and a total length of six calibers. This figure shows that the Magnus effect is strongly dependent on the length of the cylindrical afterbody. Only a small portion of the Magnus force is generated on the ogive. Examples are shown in Figures 25 through 27 illustrating the effects of variations in ogive shape for fixed forebody and total projectile lengths. These comparisons show that pitching moment, Magnus moment, and pitch damping are increased as ogive bluntness is increased. The final sequence of parametric comparisons is shown in Figures 28 through 30 where the effect of varying the body length is shown for a fixed ogive shape. These figures show that pitching moment, Magnus moment, and pitch damping are all increased as the body length is increased.

The comparisons shown represent a small fraction of the potential comparisons possible from the total data base generated. The intent here has been to illustrate the capability of the computation techniques rather than develop any conclusion as to the relative superiority of any particular configuration. This study is part of a continuing effort that is being expanded to include boattail configurations and a wider Mach number range--transonic velocities are of particular interest.

IV. SUMMARY

A computational aerodynamics parametric study has been described in which advanced numerical techniques for computing three-dimensional inviscid and turbulent viscous supersonic flow fields have been used. A comprehensive data
base has been generated for cone-cylinder, tangent-ogive-cylinder, and secant-ogive-cylinder configurations. Of particular interest are the computations of Magnus effects, which are accomplished in a conceptually exact manner, and the computations of pitch damping. Comparisons between the computed results and experiment have provided verification of the computational techniques. Comparisons of the computed results for differing body configurations have established the ability of the computational techniques to distinguish the effects of body configuration on the aerodynamic coefficients.
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LIST OF SYMBOLS

\( q \)  
free stream dynamic pressure = \((\rho_\infty V_\infty^2)/2\)

\( r \)  
local radius of model

\( u, v, w \)  
velocities in boundary-layer coordinates

\( x \)  
surface coordinate in longitudinal direction

\( y, Y \)  
coordinate perpendicular to local surface

\( z \)  
cyliindrical coordinate along model axis

\( A \)  
reference area = \(\pi D^2/4\)

\( C_{DBL} \)  
viscous drag = \(\int \int \tau_x \cos \theta_B \, dS)/qA\)

\( C_{DF} \)  
total drag = \(\int \int p_w \sin \theta_B \, dS)/qA + C_{DBL}\)

\( C_{\phi \rho} \)  
roll damping = \(\int \int \tau_\phi \, dS)/(qAD PD/V)\)

\( C_m \)  
pitching moment = \(\int \int zp_w \cos \phi \cos \theta_B \, dS)/qAD\)

\( C_{M\alpha} \)  
slope of pitching moment coefficient = \(dC_m/d\alpha\)

\( C_{M\rho\alpha} \)  
slope of Magnus moment coefficient = \((dC_n/\alpha)/(PD/V)\)

\( C_{Mq} + C_{M\alpha} \)  
pitch damping = \(C_{n\theta}/\sin \theta\)

\( C_n \)  
normal force = \(\int \int p_w \cos \phi \cos \theta_B \, dS)/qA\)

\( C_{n}\)  
Magnus moment = \(\int \int (zp_w \sin \phi \cos \theta_B + z\tau_\phi \cos \phi \cos \theta_B + z\Delta \sin \phi \cos \theta_B + z\tau_x \sin \phi \sin \theta_B) \, dS)/(qAD)\)

\( C_{n\theta} \)  
side moment in coning motion = \(\int \int (z-z_{cg})p_w \sin \phi \cos \theta_B \, dS)/(qA\theta)\)

\( C_{PN} \)  
center of pressure = \(C_m/C_N\)

\( C_{PY} \)  
Magnus center of pressure = \(C_n/C_Y\)

\( C_Y \)  
Magnus force = \(\int \int (p_w \sin \phi \cos \theta_B + \tau_\phi \cos \phi \cos \theta_B + \Delta \sin \phi \cos \theta_B + \tau_x \sin \phi \sin \theta_B) \, dS)/qA\)

\( D \)  
diameter of model
LIST OF SYMBOLS
(CONTINUED)

LN  length of nose in calibers (shown as L1 in Figure 1)
P  spin rate, rad/s
Re  Reynolds number based on model length
S  surface area
V  velocity along model trajectory

\( \Delta p \)  centrifugal pressure gradient contribution to side force
\( \sigma \)  effective angle of attack for coning motion
\( \tau_x \)  longitudinal velocity wall shear
\( \tau_\phi \)  circumferential velocity wall shear
\( \theta_B \)  local slope of body surface
\( \dot{\theta} \)  magnitude of the angular velocity of the shell
\( \Omega \)  angular velocity of the body-fixed coordinates measured with respect to an inertial system
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