EXPERIMENTS WITH WAVES ON RELATIVISTIC ELECTRON BEAMS

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J. L. Hirshfield
Professor of Applied Science
Principal Investigator
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I. OBJECTIVE OF RESEARCH

This research has been directed towards a basic understanding of small-signal gain mechanisms in a free-electron laser (FEL).* The experiments five years ago at Stanford University, in which 10 μm radiation was amplified on a 24 MeV electron beam have been explained in detail using single-particle models.1 Experiments at Columbia University reported in 19782 in which 400 μm radiation was generated have been explained using a collective model. For a cold electron beam, the full comprehensive theory3 indicates that these mechanisms are not necessarily separable, so that the full description may be indispensible. Moreover the elementary theories often omit the effects of momentum spread on the electron beam, input coupling loss due to multi-mode propagation on the beam, and non-ideal electron orbits due to the actual magnetic fields of the wiggler with the guide field. For a beam with finite momentum spread, a geometric optics collective theory4 indicated yet another gain mechanism, originating with a wave-particle resonance. Furthermore, as shall be described in detail below, the presence of the axial guide field introduces additional complexity by permitting a large enhancement in gain if certain resonance conditions are fulfilled.

It is clear that careful experiments are required to sort out all these competing gain mechanisms. Our objective in this research therefore has been to assemble an apparatus to allow gain measurements to be performed on a FEL with detailed independently measured knowledge of both the equilibrium electron orbits in the wiggler and the momentum spread on the electron

* Many types of so-called free electron lasers have been discussed. In this work we restrict ourselves to a configuration in which a relativistic electron beam interacts with electromagnetic radiation while moving under the influence of a stationary periodic magnetic field and a uniform axial magnetic field.
beam. We furthermore have intended to study methods of altering the beam momentum distribution and beam density, so as to explore the parameter regimes where the limiting-case gain mechanisms may be identified.

In the first year of this work, good progress has been made towards these goals, as will be detailed below. In addition, important theoretical advances have been made in understanding the very complex orbits which electrons follow in a wiggler with an axial guide magnetic field. A small-signal gain theory has been developed which takes into account this axial guide field, and which indicates that large enhancements in gain may be realized if the parameters allow resonance between the cyclotron and undulatory motions.

We anticipate continuation of this research under joint AFOSR/ONR support.

II. RESULTS

Most of the substantive results achieved to date have either already been published, are in the process of preparation for publication, and/or have been presented at Scientific Conferences. We shall limit the detail presented here therefore, and refer the reader to the archival works themselves.

A. Experimental

Installation of the Febetron electron accelerator, vacuum chamber, axial guide magnetic field, and pulsed magnetic wiggler is complete. Diode current and voltage diagnostics have been developed. Cathode designs are under development to produce low current (100's of amperes) beams of low emittance. A novel 90° deflection momentum analyzer has been designed to determine the momentum spread for a sample of the beam. Low current cw
beams in the energy range 4-20 keV were employed to determine electron orbits in the wiggler/guide field combination. Beam analyzers were designed and employed to distinguish orbit deflection and helicity. A summary of one result obtained in this experiment is shown in Fig. 1, where the measured threshold for transition between helical and non-helical orbits has been plotted versus a universal parameter which embodies beam energy and guide field. Predictions of the idealized theory are also shown. Considering the approximation in the theory, we take the agreement to be remarkable. To our knowledge this is the first attempt to measure orbit dynamics in a FEL. This work was stimulated by theory performed by our collaborator L. Friedland [Phys. Fluids 23, 2376 (1980)]. Since the theory showed the stability threshold to persist for weakly-relativistic beams, we were able to perform the experiment at low energy. This work was presented recently at a conference [P. Avivi, F. Dotan, A. Fruchtman, A. Ljudmirsky, and J. L. Hirshfield, Bull. APS 25, 910 (1980)]. This abstract is appended. The work is currently in draft form in preparation for publication.

B. Theoretical

One result was the discovery of a synergism between the undulatory motion induced by the wiggler, and the helical motion intrinsic to the orbit in the uniform guide field. Together these combine to enhance FEL gain. This work is now published [L. Friedland and J. L. Hirshfield, Phys. Rev. Lett. 44, 1456 (1980)]; a reprint is appended.

A second result is the extension of the small-signal gain theory published earlier [I. B. Bernstein and J. L. Hirshfield, Phys. Rev. A 20, 1661 (1979)] to include finite axial momentum spread. A representative result is shown in Fig. 2, where gain values for a FEL modeled using the
parameters of our experiment are used. The gain spectra for a cold beam, and for a beam with a parallel momentum spread between 5-20% is shown.

This work was reported at a recent conference [A. Fruchtman and J. L. Hirshfield, Bull. APS 25, 911 (1980)]. The abstract is appended. The work is currently in manuscript form in preparation for publication.

A third result is concerned with a description of the non-helical orbits in a wiggler in a uniform axial magnetic field. For the "idealized" wiggler field $B_w = (A \cos k z + \delta \sin k z)B_w$, Friedland has computed orbits numerically. But S. Y. Park at Yale has shown that these orbits can be determined as well from an analytic theory; this allows greater physical insight to this complex problem than does the numerical approach. This analytic theory is in preparation for publication. It appears, however, that the strongly non-helical orbits may entail off-axis departures which are so large, as to call into question the validity of the idealized field. To examine this question, R. A. Smith and S. Y. Park at Yale have derived the exact field of a bi-filar helix. One objective is to assess the errors inherent in using the idealized field for the non-helical orbits. The ultimate objective of this work is a calculation of the spontaneous emission from electrons with strongly non-helical orbits. It appears that this radiation is rich in harmonics of the basic undulatory frequency; thus one might anticipate devising a FEL working on one or more of these "space harmonics", and thus furnishing shorter wavelength radiation than a conventional FEL with the same parameters.

Continuation of this research for at least two more years is anticipated by joint AFOSR/ONR support through ONR Contract N00014-79-C-0588, which was established originally on June 1, 1979 to support FEL theory. Co-Principal Investigators of the combined experimental/theoretical program are J. L. Hirshfield and I. B. Bernstein.
REFERENCES


FIGURE CAPTIONS

FIG. 1. Threshold values of $B_w/B_z$ for onset of orbit instability, as a function of the universal parameter for a helical wiggler. Data points are shown for beam energies between 4-14 keV. The inset illustrates one of the analyzers used to determine helicity.

FIG. 2. Theoretical results for FEL power gain for the planned experiment, showing the effect of electron momentum spread.
\[
\left[(5.89V^{1/2}/B_z)^{2/3} - 1\right]^{3/2}
\]
FEL Amplifier Using Febeutron Accelerator

\[ \gamma = 1.78 \]

\[ \lambda_0 = 3.6 \text{ cm} \]

\[ s = 1.0 \]

\[ \Delta u / u = 0 \]

\[ j = 100 \text{ A/cm}^2 \]

\[ 0.05 \]

\[ 0.10 \]

\[ 0.15 \]

\[ 0.20 \]

\[ \text{Power Gain} \]

\[ \text{Frequency (GHz)} \]

FIG. 2
Free-Electron Laser with a Strong Axial Magnetic Field

L. Friedland and J. L. Hirshfield
Department of Engineering and Applied Science, Mason Laboratory, Yale University, New Haven, Connecticut 06520
(Received 27 February 1980)

A small-signal theory is given for gain in a free-electron laser comprising a cold relativistic electron beam in a helical periodic transverse, and a strong uniform axial, magnetic field. Exact finite-amplitude, steady-state helical orbits are included. If perturbed, these orbits oscillate about equilibrium, so that substantial gain enhancement can occur if the electromagnetic perturbations resonate with these oscillations. This gain enhancement need not be at the cost of frequency upshift.

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Intensive activity is underway to exploit the gain properties of a relativistic electron beam undulating in a periodic transverse magnetic field. Such free-electron laser (FEL) configurations have provided oscillation at 3.4 (Ref. 1) and 400 μm, and amplification at 10.6 μm. Theory has advanced apace, and elaborate schemes have been proposed for obtaining high FEL efficiency.

A factor which limits the practical application of this interaction at wavelengths shorter than perhaps a few microns is the rapid decrease in small-signal gain $G_0$ as the electron energy increases. This is shown explicitly in the well-known result for $G_0$ in the single-particle limit (i.e., when collective effects are negligible)

$$G_0 = (\omega_\gamma / k c)^2 (k L / 2 \gamma)^2 F'(0).$$

(1)

Here $\omega_\gamma$ and $\gamma$ are the beam plasma frequency $Ne^2/mc^2$ and normalized energy $W/mc^2$, $k$ and $\xi$ are the helical transverse magnetic field wave number $2\pi/L$ and normalized strength $eB_z/mck_0$, $L$ is the interaction length, and $F'(0) = (d/d\theta)(\sin^2 \theta)$ is the line-shape factor, with $\theta = [k v_{50} - \omega (1 - \nu_{50}/c)] (L/2c)$, where $\nu_{50}$ is the unperturbed electron axial velocity. The peak gain occurs at $\theta = 1.3$, where $F'(0) = 0.54$. For example, with $\gamma$...
A suggestion has appeared for enhancing the small-signal gain above values given by Eq. (1), or for achieving comparable gains with smaller $B_{\perp}$ by employing a strong axial magnetic field so as to exploit resonance between the cyclotron frequency and the undulatory frequency. The present Letter presents a single-particle derivation for the small-signal gain of a FEL in a uniform axial magnetic field $B_{\parallel}$. We shall demonstrate that careful adjustment of the system parameters will allow enhancement of the FEL small-signal gain by an order of magnitude or more for the above examples without increasing the undulatory velocity. This result goes beyond that predicted by Sprangle and Granatstein who have suggested an enhancement of an order of magnitude or more in a reduction in frequency upshift, since branches $A$ and $C$ have orderly helical orbits. Stability is insured if $\mu^2 = a^2 - \beta d > 0$, where $a = k_{\parallel} u_{\perp} / \gamma u_{20}$, $b = \Omega u_{20} / \gamma t_{20}$, and $d = k_{\parallel} \xi / \gamma$. The quantity $\mu$ is the natural resonance frequency in response to small perturbations of the orbit. We shall show that strong resonance response of the electrons to electromagnetic perturbation can lead to enhanced FEL gain for small $\mu$, i.e., for $\Omega$ close to $\Omega_{e}$.

The derivation of FEL gain proceeds by solving the single-particle equations of motion, subject to weak electromagnetic perturbing fields $\mathbf{E} = \mathbf{E}_0 \cos(kz - \omega t)$ and $\mathbf{B} = \mathbf{B}_0 (kc / \omega) \mathbf{E}_0 \cos(kz - \omega t)$.

The orbits, which have been the subject of recent study, can possess more than one steady state, depending upon $\gamma$, $B_{\parallel}$, $B_{\perp}$, and $k_{\parallel}$. These steady states are characterized by the normalized velocity components (i.e., $u_{\perp} = u_{\perp} / c$)

$$u_{10} = 0, \quad u_{20} = k_{\parallel} \xi (\Omega_{t} - \Omega / c),$$

$$u_{30} = (1 - u_{20}^2 - \gamma^2 \gamma^2 - \gamma^2)^{1/2},$$

where the basis vectors $\hat{z}(z) = -\hat{x} \sin \theta + \hat{y}$, $\hat{y}(z) = -\hat{x} \cos \theta + \hat{y}$, and $\hat{z}(z) = \hat{z}$. These orbits, which have been the subject of recent work, are nonhelical, highly anharmonic motions, while branches $A$ and $C$ have orderly helical orbits.

Prior workers have not considered this effect. In this work $\gamma u_{\perp} / c = 1$ without the axial magnetic field, then a given gain enhancement $\eta$ achieved through this resonance alone would result in a reduction in frequency upshift by a factor $(1 + \eta^2) / 2$. The process we describe in this Letter will be shown to permit significant gain enhancement without undue sacrifice in frequency upshift. The gain enhancement originates with the natural frequency of oscillation of electrons on finite amplitude equilibrium helical orbits. Prior workers have not considered this effect.

A full derivation of our result will be presented elsewhere. Exact unperturbed relativistic orbits are considered in the customary FEL model magnetic field

$$\mathbf{B}(\mathbf{r}) = B_{\parallel} \hat{z}_r + B_{\perp} \hat{y}_r \cos \theta + \hat{y}_r \sin \theta.$$

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The derivation of FEL gain proceeds by solving the single-particle equations of motion, subject to weak electromagnetic perturbing fields $\mathbf{E} = \mathbf{E}_0 \cos(kz - \omega t)$ and $\mathbf{B} = \mathbf{B}_0 (kc / \omega) \mathbf{E}_0 \cos(kz - \omega t)$.
about the equilibrium orbits on either branch A or C as discussed above. These equations are

\[ \dot{u}_1 = (\beta \mu_2 - \Omega / \gamma) u_2 - (\beta \mu_2 \gamma / \gamma) u_1 + (\mu E / mc\gamma)(k c u_2 / \omega - 1), \]

\[ \dot{u}_2 = - (\beta \mu_2 - \Omega / \gamma) u_1 - (\beta \mu_2 \gamma / \gamma) u_2 + (\mu E / mc\gamma)(k c u_2 / \omega - 1), \]

\[ \dot{u}_3 = (\beta \mu_2 - \Omega / \gamma) u_1 + (\beta \mu_2 \gamma / \gamma), \]

where \( \gamma = (\mu E / mc)(\gamma + u_2)^2 \) and

\[ 2(E_1 + iE_2) = -E_0 \exp\{i[N_0 + k c u_2 - \omega t + \alpha]\} \]

with \( \alpha \) the random initial electron phase. When time variations and electromagnetic fields are absent, Eqs. (4)-(6) lead to the exact steady states given by Eq. (3). To linearize Eqs. (4)-(6), we introduce the velocity perturbations \( \dot{w}_1 = u_1 - u_{10} \ll u_{10} \) and retain only the lowest-order quantities. This results in

\[ w_1 = \frac{4E_0}{\mu_2 k c} [\cos(\beta t + \alpha) - \cos(\alpha) \sin(\beta t + \alpha)] + \mu^{-1} \dot{w}_1(0) \sin t, \]

where

\[ A = \alpha + \beta(1 - u_{10}) + u_{20}, \beta = \mu_2 \gamma / \gamma, \omega = \gamma \mu_2, \dot{w}_1(0) = (\mu E / 2mc\gamma)(1 - u_{10}) \sin(\alpha), \]

and \( w_1(0) = 0 \). The other components follow from

\[ \dot{w}_2 = -aw_1 + (\mu E / 2mc\gamma)(1 - u_{10} - u_{20}^2) \cos(\beta t + \alpha), \quad w_2(0) = 0; \]

and

\[ \dot{w}_3 = dw_1 + (\mu E / 2mc\gamma)u_{20}(1 - u_{30}) \cos(\beta t + \alpha), \quad w_3(0) = 0. \]

Equation (7) for \( w_1 \) exhibits the aforementioned natural resonance at frequency \( \mu_2 \), while the electromagnetic perturbation drives the transverse motion at frequency \( \beta \). Gain enhancement can be expected when \( \mu_2 \) is close to \( \beta \).

The energy gain for an electron is calculated from

\[ \frac{\mu c}{e} \frac{dy}{dt} = -w_1 E_{10} - w_2 E_{20} - \frac{1}{2} \frac{E_3}{k + k_0} \frac{\sin(\beta t + \alpha)}{\sin(\beta t + \alpha)} \int dt \dot{w}_3(t'). \]

The third term in Eq. (10) is much larger than the other two on account of the factor \((k + k_0)\). The dominant single-particle energy transfer in the FEL (even with an axial magnetic field) is seen to be work \( e cu_2 E_3 \) done along the transverse undulatory motion, enhanced by the strong variation in \( E_3 \) as its phase varies through \( u_3 \). The energy variation [Eq. (10)] is averaged over random phase \( \alpha \) to give

\[ \langle \frac{\mu c}{e} \frac{dy}{dt} \rangle, \]

which in turn leads to the gain through

\[ G = \frac{2(\mu_2 E_3)^2}{Nc^2} \int dt \langle \frac{dy}{dt} \rangle, \]

where \( N \) is the beam electron density and \( T = L/c \) is the total interaction time for the electrons in a system of length \( L \).

The final result is

\[ G = \frac{\omega^2 k_c c}{16\gamma} u_{20}^2 \left[ \left( \frac{1 + 2a}{\mu_2} \left( \mu + 2 \frac{u_2}{1 - u_{20}} \right) \right) F' \left( \theta = \frac{F(\theta + \phi) - F(\theta - \phi)}{2\phi} \right) \right. \]

\[ \left. + \frac{F(\theta + \phi) - F(\theta - \phi)}{2\phi} - \frac{a}{\mu_2} \left[ P'(\theta) - P(\theta + \phi) - P(\theta - \phi) \right] \right], \]

where \( \theta = \beta T / 2, \phi = \mu T / 2, F(x) = (\sin x / x)^2, \) and

\[ P(x) = F'(x) / 2; \]

and where we have approximated \((k + k_0)(1 - u_{20}) \approx k_0\). We shall examine Eq. (11) in several limits.

For \( \mu \ll \beta \), only the terms involving \( F'(\theta) \) and \( P'(\theta) \) in Eq. (11) are significant, and on branch A the latter of these is smaller than the former by at least a factor \( 2\phi \). Thus to a good approxi-

\[ G(\mu \gg \beta) = G(\mu_2 k_c c u_{20} \phi^2 F'(\phi)), \]

where \( Z = 2 + \mu^2 \left( 2d + 4d \right)(1 - u_{10}) \), and in the case where the axial magnetic field is absent, \( \Omega = 0 \), \( \mu = a = k_c c u_{20} \gg \beta \), and \( u_{20} = k_0 + \gamma \). Thus, \( Z = 2 \) and Eq. (12) goes over to Eq. (1). When \( \Omega \neq 0 \) and \( \mu \)
gain enhancement can be achieved as claimed by the prior workers, due to resonant enhancement of \(n_{g0}\), but not without sacrificing frequency upshift, as discussed above.

However a more attractive possibility exists when \(i\) is small, and approaches \(i\). Here one can approximate \(Z = \mu^{-1}bd(1 - i_{g0})^{-1} \gg 1\); this results from resonance between the electromagnetic perturbation which gives oscillatory motion to the electron at a frequency \(i\), close to its natural oscillation frequency \(\mu\). Gain enhancement due to large \(Z\) is seen to be possible without simultaneously increasing \(i_{g0}\), so that the desirable frequency upshift property of the FEL need not be sacrificed.

We define a gain enhancement factor \(\eta = G/G_0\) to compare two free-electron lasers, identical except that one has a strong axial magnetic field, while the second does not. In the first laser, the transverse magnetic field \(B_t\) is reduced so that \(\omega_{g0}\) is the same for both lasers. (This assures that both enjoy the same frequency upshift.) Then

\[
\eta = Z\{1 - |F(\theta + \psi) - F(\theta - \psi)|/2|F(\theta)|\}.
\]  

We have evaluated Eq. (13) for two examples with magnetic field parameter \(\xi\). The values \(\xi_{0} = 0.5\) and 1.0 are for the FEL without axial field, and provide the same \(\xi_{0}\) as do the indicated (smaller) values of \(\xi\) for the FEL with the indicated axial field strength. Example is for \(g = 10\), \(k_0 = 6.0\ cm^{-1}\), and \(L = 130\ cm\). Solid curves, orbits on branch \(A\); dashed curves, orbits on branch \(A\). For high enhancement values, such as on the \(\xi_{0} = 1.0\) branch \(A\) example, the numerical precision required to compute accurate results suggests that the phenomenon is very sensitive to the system parameters.

![FIG. 2. (a) Gain enhancement \(\eta\) and (b) corresponding normalized axial magnetic field \(\Omega/c\), vs transverse magnetic field parameter \(\xi\). The values \(\xi_{0} = 0.3\) and 1.0 are for the FEL without axial field, and provide the same \(\xi_{0}\) as do the indicated (smaller) values of \(\xi\) for the FEL with the indicated axial field strength. Example is for \(g = 10\), \(k_0 = 6.0\ cm^{-1}\), and \(L = 130\ cm\). Solid curves, orbits on branch \(A\); dashed curves, orbits on branch \(A\). For high enhancement values, such as on the \(\xi_{0} = 1.0\) branch \(A\) example, the numerical precision required to compute accurate results suggests that the phenomenon is very sensitive to the system parameters.](image)

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10. L. Friedland, to be published.
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Abstract Submitted
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Experimental Study of Electron Orbit Stability in a FEL Magnetic Wiggler

P. AVIVI, F. DOTAN, A. FRUCHTMAN, A. LJUDHIRSKY, and J. L. HIRSHFIELD, Center for Plasma Physics, Hebrew University of Jerusalem, Israel, and L. FRIEDLAND, Yale Univ.--Free electron laser experiments are being widely pursued in which a relativistic electron beam propagates along the axis of a helical magnetic wiggler. An axial guide magnetic field is usually present for beam collimation, and a recent theory shows how both steady-state and dynamical resonant effects due to this guide field can lead to gain enhancement. Theory for equilibrium orbits in this magnetic field configuration indicates that stable helical orbits are possible, but that strongly non-helical ("unstable") orbits are easily produced. Results of an experiment are presented in which novel orbit analyzers were employed to search for the threshold for orbit stability, as a function of beam energy, guide field, and wiggler field. The observations are consistent with theory.

*Sponsored by AFOSR and ONR.
**Also Yale University.

2L. Friedland, Phys. Fluids (to be published).

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J. L. Hirshfield

(same name typewritten)

Yale University, P.O. Box 2159,

(address) Yale Station

New Haven CT 06520

This form, or a reasonable facsimile, plus two Xerox Copies must be received NOT LATER THAN WEDNESDAY, AUGUST 13, 1980 at the following address:

Division of Plasma Physics Annual Meeting
Ms. Diane Miller
Jayco
P.O. Box 970
Del Mar, California 92014
Influence of Electron Energy Spread on Amplification in a Free Electron Laser* A. FRUCHTMAN and J. L. HIRSHFIELD**, Center for Plasma Physics, Hebrew Univ. of Jerusalem, Israel—A collective theory has been published for small-signal amplification in a free electron laser comprising a relativistic electron beam propagating along the axis of a static helical pump magnetic field. However, detailed analysis has heretofore been limited to the case of a cold electron beam, i.e. $f_0(\alpha, \beta, \gamma) = \text{const.} \delta(\alpha) \delta(\beta) \delta(\gamma - \gamma_0)$, where $\alpha$ and $\beta$ are the transverse components of the canonical angular momentum and $\gamma$ is the total energy normalized to $m c^2$. The present work extends the above analysis to the case of a beam with finite energy spread, i.e. $f_0(\alpha, \beta, \gamma) = \text{const.} \delta(\alpha) \delta(\beta) \{H(\gamma - \gamma_1) - H(\gamma - \gamma_2)\}$, where $H$ is the step function. This model accounts for reductions in amplification due to phase mixing, but not enhancement of amplification due to wave-particle coupling. Results will be shown for several cases of practical importance.

*Sponsored by AFOSR and ONR.
**Also Yale University.
Particle Orbits in a Magnetic Wiggler With a Uniform Guide Magnetic Field

R.A. SMITH and J.L. HIRSHFIELD, Yale. Univ., and S.Y. PARK+ and J.M. BAIRD†, Naval Research Lab.—An exact analytic solution is given for the orbit of a charged particle moving in the combined magnetic field of a bifilar helical wiggler and a uniform solenoid. The axial velocity is shown to satisfy an anharmonic oscillator equation whose solutions are Jacobian elliptic functions. Other velocity components are readily determined from the axial velocity. Solutions are classified according to the type of Jacobian elliptic function, which in turn is determined by magnetic field parameters and initial particle conditions. The exact solution allows calculation of spectra of single-particle radiation, and allows formulation of the linearized Vlasov-Maxwell theory for an ensemble.

*Supported by AFOSR and ONR.
†B-K Dynamics, Rockville, MD.
Installation of the Febetron electron accelerator, vacuum chamber, axial guide magnetic field, and pulsed magnetic wiggl er is complete. Diode current and voltage diagnostics have been developed. Cathode designs have begun to produce low current (100's) of amperes) beams of low emittance. A novel 90 degree deflection momentum analyzer has been designed to determine the momentum spread for a sample of the beam. Low current cw beams in the energy range 4-20 keV were employed to determine electron orbits in the wiggler/guide field combination. Beam analyzers were designed and employed to distinguish orbit deflection and helicity.