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# Onboard Prediction of Propagation Loss in Shallow Water

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*Applied Ocean Acoustics Branch  
Acoustics Division*

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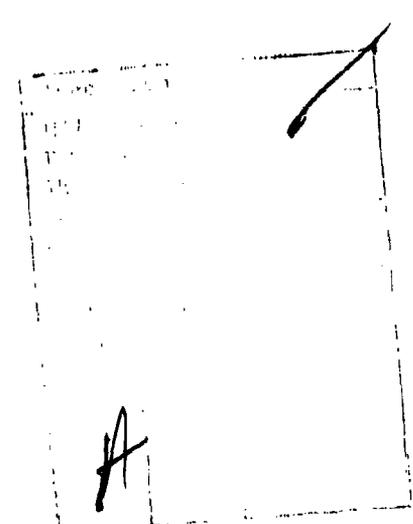
be made when a positive gradient is present.); 3. Details of the sound-speed profile can cause significant changes in propagation loss, therefore even if bottoms were well characterized, sophisticated computer models would be required to predict propagation loss; 4. Virtually all propagation loss curves can be described to within a fraction of a dB by the function  $PL = B + 15 \log R + AR + CR^2$  with the  $C$  coefficient usually zero. Thus, the output field can be described by two or, at most, three free parameters. Since there are no fewer than 24 input parameters it is thus easy to explain observed propagation loss and very difficult to predict it. Moreover, it is doubtful that propagation loss experiments can uniquely define bottom parameters; 5. Certain aspects of the theory remain unverified and/or inadequately treated. These include: (1) surface and bottom roughness, (2) shear in the sediment, (3) substrate roughness, (4) modal coupling, and (5) biological scatterers; 6. Grain size distribution is not an adequate predictor of acoustical properties; hence currently existing sediment charts are of little or no value in performance prediction; and 7. Many input parameters are very poorly known. These include: a. bottom roughness, b. wave height spectrum, c. sediment shear-wave speed, d. sediment shear attenuation, e. shear and sound speed and attenuation gradients in the sediment, and f. distribution and effective attenuation of biologics. In most cases the theory is not certain enough to determine the uncertainty in propagation loss caused by uncertainty in these parameters.

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## ONBOARD PREDICTION OF PROPAGATION LOSS IN SHALLOW WATER

### INTRODUCTION

One of the most important objectives of the Navy's ocean acoustic research program is to provide the fleet with information that can be used to improve its ASW capability. While the state of the art in underwater acoustics will, no doubt, always continue to improve, it is nevertheless incumbent upon the research community to periodically examine the state of the art to see if it has reached a point where a profitable transfer of information to the Fleet is possible. This report is an attempt at such an assessment. Specifically, it examines the state of the art in the prediction of propagation loss in shallow water as it pertains to the Fleet's need for an onboard performance prediction capability in shallow water.

There is a general requirement for an onboard performance prediction capability, which includes shallow water, but the Fleet has yet to articulate specific requirements. Thus, it is not possible to achieve the ideal of weighing capabilities against requirements. We shall, instead, present the state of the art as we feel it relates to the problem of onboard performance prediction. The problem may be formulated as follows. Given the limited environmental information available on board a naval vessel and various operational constraints, does the state of the art in shallow water acoustics permit one to establish reasonable bounds on sonar performance? The environmental data available onboard is limited to temperature-pressure profile, water depth, sea-state and a rudimentary idea of bottom type. Operational constraints include limited computational facilities and lack of onboard acoustical expertise.

This report considers only propagation loss which is but one component of a performance prediction model. A complete prediction model would include other effects such as ambient noise and, for active systems, reverberation which might also require special consideration in shallow water. We restrict ourselves to depth averaged propagation loss. This is not always very restrictive since for near isospeed shallow water, the acoustic field tends to be uniform except near the surface and bottom. Although shallow water is notorious for its spatial variability, we will also restrict our considerations to the range independent case, recognizing this as clearly unrealistic but the most amenable to theoretical treatment and most suitable for analysis using input parameters potentially available onboard. Normal mode programs are currently capable of handling the range dependent case only within the adiabatic approximation which has not been demonstrated to be adequate for all cases. Moreover, the added difficulty of obtaining the environment inputs required for range dependent calculations and the computational complexity of the range dependent case make it very difficult to use a range dependent model in the onboard situation. In any case, little is lost by restricting our considerations to the depth-averaged-range-independent case since we shall show that we cannot accurately predict propagation loss in even this simplest case. We can probably do no better for the range and depth dependent case. If however, the variability in bottom parameters is sufficiently rapid, a statistical approach could possibly yield accurate predictions for mean propagation loss.

By shallow water propagation we mean propagation that is dominated by repeated interaction with the bottom. Generally, this restricts our considerations to the continental shelf (depths less

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than about 200 m which include 7.5% of the total ocean area). We shall adhere rigidly to this definition, and not consider propagation, in otherwise shallow water, when a strong positive sound speed gradient is present since such propagation is not strongly influenced by the bottom. It is reasonable to assume that positive gradients in shallow water are not fundamentally different than surface ducts in deep water. Onboard models for surface ducts in deep water are probably applicable to shallow water positive gradient cases as well.

### GENERAL COMMENTS

Shallow water acoustics is plagued by an overabundance of input parameters. Table 1 lists 24 separate inputs to a "Universal Propagation Loss Model" for a range independent shallow water environment with just a single sediment layer. An additional 11 inputs would be required for each additional sediment layer. It should be noted that each item listed was considered at some time to be important by at least one worker in the shallow water field. With the exception of water depth and temperature profile, all of these inputs are more or less indeterminate.

With such a complex input field it is perhaps surprising to find that the output field, the depth averaged propagation loss, is a very simple monotonically increasing function of range.

Table 1 — Inputs to Universal (Range Independent) Shallow Water Propagation Loss Model

1. WATER DEPTH	
2. SOUND SPEED PROFILE	
A. TEMPERATURE	
B. SALINITY	
3. ACOUSTIC ATTENUATION IN WATER	} X NUMBER OF SEDIMENT LAYERS
4. DENSITY OF SEDIMENT	
5. SOUND SPEED IN SEDIMENT	
6. SHEAR SPEED IN SEDIMENT	
7. ACOUSTIC ATTENUATION IN SEDIMENT	
8. SHEAR ATTENUATION IN SEDIMENT	
9. SOUND SPEED GRADIENT IN SEDIMENT	
10. SHEAR SPEED GRADIENT IN SEDIMENT	
11. ATTENUATION GRADIENT IN SEDIMENT	
12. DENSITY GRADIENT IN SEDIMENT	
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16. DENSITY OF BASEMENT	
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19. SURFACE ROUGHNESS	
20. BOTTOM ROUGHNESS	
21. SUBBOTTOM ROUGHNESS	
22. ENTRAINED GAS BUBBLES	
23. FISH AND OTHER BIOLOGICAL SCATTERERS	
24. WIND VECTOR	

In fact, we have found that virtually all computed (depth averaged) propagation loss curves can be fit, to within a fraction of a dB, to the following simple function:

$$PL = 15 \log R + AR + B + CR^2$$

(some examples of this will be given in the Appendix). For most cases the  $C$  parameter can be taken to be zero. Thus we have more than 20 input parameters which determine two or, at most, three output parameters. It is thus not very surprising that theory and experiment can be made to agree using "reasonable" values for the input parameters. This process of *explaining* observed propagation loss is, however, something quite removed from *predicting* it. All existing models consider only certain subsets of the complete input field, yet they are usually successful in explaining data. It is doubtful that even a broad band experiment with a diversity of source and receiver depths would be capable of uniquely determining the correct set of input parameters, since many input parameters have the same effect on propagation-loss-versus-frequency and the dependence of acoustic pressure on depth is relatively weak and dominated by the sound-speed profile. The dependence of propagation loss on most of these input parameters is a strong function of the exact nature of the sound-speed profile; hence, a set of bottom parameters that works for one speed profile is not guaranteed to work with a different (albeit similar) profile.

The imbalance between inputs and outputs also makes it hard to verify the mathematical models themselves. It is extremely difficult to isolate any one parameter; hence, it is difficult to determine whether the model treats the influence of a particular parameter correctly. This is especially true of the harder-to-model parameters such as bottom roughness. It is quite possible (by shifting critical parameters to incorrect but plausible values) for a model to treat many effects incorrectly and still agree with data.

#### SEDIMENT SOUND SPEED AND DENSITY

Probably the single most important bottom parameter is the speed of sound in the surficial sediment. At best, charts of surface sediments will describe the bottom as consisting of one of the sediment types shown in Table 2. Often the descriptors are far less specific than those shown in the table, consisting of vague categories such as "mud" or "sand" or ill-defined ones such as "mud and sand." Sometimes bottoms are indicated that fall outside the categories indicated in Table 2, such as "gravel" or "shell" whose acoustical properties are not well known. Moreover, shallow water areas are notorious for their spatial variability with a patchwork of bottom types the rule rather than the exception, so that the occurrence of a significant propagation path over a single bottom type becomes unlikely. Furthermore, the accuracy of the sediment charts is unknown and in addition there is always the possibility that the surficial sediment layer may be so thin that its properties may not be the controlling ones. In view of the above, the very best situation that we could ever have is to know that for a certain propagation path the bottom falls into one of the nine sediment types listed in Table 2. We shall consider this to be the case.

The sediment types are determined by the relative amounts of the three constituents, sand, silt, and clay. The sediment types are listed in order of decreasing mean grain diameter. Table 1 which deals specifically with the properties of the sediments of the continental terrace, is the latest in a series of such tables compiled by Hamilton [1-3]. The primary acoustical quantity of interest is the velocity ratio. Since this quantity is known to be virtually independent of the sound-speed in the overlying water [2] the sound speed of the sediment, in a given situation, can be found by first determining the sound speed of the bottom water (by  $BT$  calculation or velocimeter) and multiplying by the sound speed ratio.

Table 2 — Sediment Parameters for Continental Terrace (After Hamilton 1980)[1]

Sediment Type	No. Samples	Mean Grain Dia.		Sand (%)	Silt (%)	Clay (%)	Density (g/cm <sup>3</sup> )		Sound-Speed Ratio	
		(mm)	( $\phi$ )				Avg.	$\sigma$	Avg.	$\sigma$
Sand										
Coarse	2	0.5285	0.92	100.0	0.0	0.0	2.034	—	1.201	—
Fine	22	0.1593	2.65	90.9	4.9	4.2	1.941	0.11	1.145	0.028
Very fine	12	0.0960	3.38	81.9	10.5	7.6	1.856	0.08	1.115	0.041
Silty sand	27	0.0490	4.35	57.6	28.9	13.5	1.772	0.10	1.078	0.031
Sandy silt	26	0.0308	5.02	28.0	59.2	12.8	1.771	0.17	1.080	0.036
Silt	19	0.0237	5.40	7.8	80.1	12.1	1.740	0.29	1.057	0.022
Sand-silt-clay	23	0.0172	5.86	32.3	41.6	26.1	1.596	0.11	1.033	0.024
Clayey silt	62	0.0077	7.02	7.3	60.0	32.7	1.488	0.13	1.014	0.023
Silty clay	19	0.0027	8.52	4.8	41.2	54.0	1.421	0.07	0.994	0.009

The measure of error given in Hamilton's tables is the standard error of the mean. It can be demonstrated, however, that the standard deviation ( $\sigma = \sqrt{n} SE \gg SE$ ) is a better measure of the expected variation of the *mean* sound-speed ratio from one location to another. What this means, for example, is that given a propagation path over a clayey-silt bottom, all we know is that there is a 68% probability that the mean sound-speed ratio over the path falls somewhere between 1.037 and 0.991. As we shall see this implies that we know nothing whatsoever about the propagation loss for this bottom type. The percent error in the sound-speed ratio is small (a few percent at most) but the quantity which determines propagation loss is not sound-speed ratio but one minus sound-speed ratio. The difference between a ratio of 1.01 and 1.03 is not 2% but 300% as it affects propagation loss.

The situation for the sediment density is similar to that for sediment sound speed. Propagation loss, however, is a much stronger function of sediment sound speed than sediment density, particularly when the sound-speed ratio is near unity. This will be discussed in more detail later.

#### OTHER SEDIMENT PARAMETERS

As was the case for sound speed and density, estimates of other bottom parameters are likely to be based solely on a sediment type determined from a sediment chart. Again we assume, as a "best case" that a sediment type as given in Table 2 is known.

#### Acoustic Attenuation Coefficient

Hamilton [2] has shown that the plane wave acoustic attenuation,  $\alpha$  (in dB/m) is more or less a linear function of frequency. The acoustic attenuation coefficient  $K$  is the constant of proportionality ( $\alpha = Kf$ ). The presence of acoustic attenuation in the sediment affects propagation loss for two reasons; first, it permits sound at angles more grazing than the critical angle to enter the sediment, and, second, it provides an attenuation mechanism for the sound which thus enters the sediment. Hamilton presents [4] his results in the form of scatter diagrams and regression curves for  $K$  versus porosity or mean grain size as shown, for example, in Fig. 1. As can be seen from the figure there is a great deal of uncertainty in  $K$  for a given value of porosity. For most sediment types, this uncertainty would be accentuated by the additional uncertainty in the porosity of a

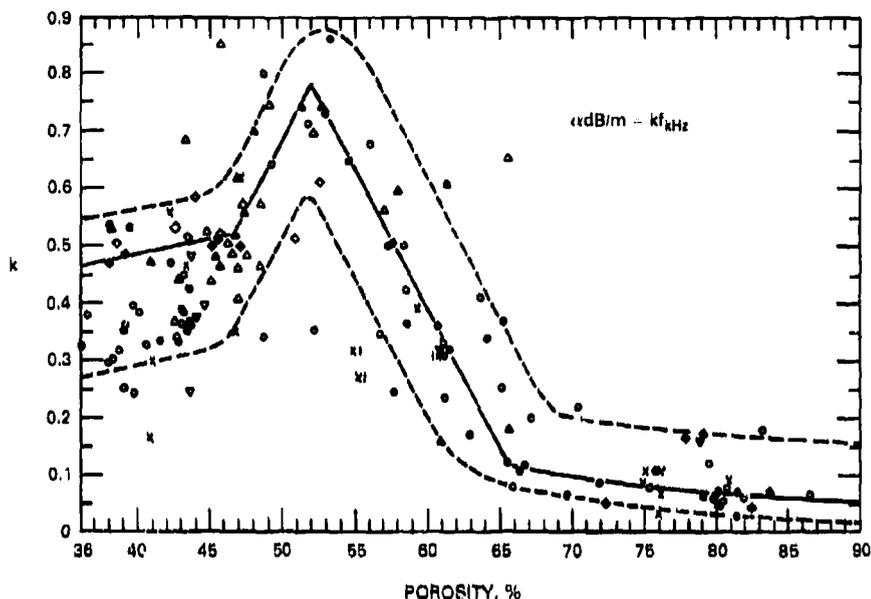


Fig. 1 — Attenuation coefficient  $K$  ( $Kf = \alpha \text{dB/km}$ ) vs sediment porosity (From Ref. 2)

given sediment, coupled with the substantial slope in some regions of the  $K$  vs porosity curves. To give some rough idea of the importance of  $K$  in determining propagation loss we note that the acoustic attenuation in dB/m in isospeed shallow water is proportional to  $K$  for small values of  $K$ . Thus the large percentage uncertainty in  $K$  for high porosity sediments can cause a large error in propagation loss. It has been shown [4] that  $K$  changes significantly as a function of depth in the sediment. Hawker [5] has shown that this can have a significant effect on propagation loss. If the attenuation gradient were known, its effect could easily be accounted for in normal mode models, but this is not usually done.

#### Sound-Speed Gradients in the Sediment

The speed of sound in sediment [6] is known to increase substantially with depth in the sediment. Typical values for the gradient lie between  $0.5$  and  $2.0 \text{ s}^{-1}$ . As we shall see, such gradients can have a profound effect on propagation loss for low speed sediments. This is due to the fact that the gradient tends to decrease the depth of penetration of sound into the lossy sediment. Neglect of the sediment sound-speed gradient can thus result in overestimating the propagation loss over low speed sediments though it has practically no effect on propagation over high speed sediments where the attenuation is higher and the degree of penetration is governed primarily by the impedance mismatch.

#### Sediment Shear Velocity

Hamilton [2] has compiled tables of Shear Sound Speeds for the Shallow Water sediment types listed in Table 2. The values, which notably are not monotonic in sediment type, are listed in Table 3 along with the standard deviation (which would again appear to be a better indicator of expected variation than standard error of the mean given by Hamilton).

Table 3 — Shear Sound Speed for Various Sediment Types

Sediment	Shear Sound Speeds (m/s)	Standard Deviation
coarse sand	250	143
fine sand	417	143
very fine sand	472	138
silty sand	447	97
sandy silt	363	169
silt	270	81
sand-silt-clay	412	72
clayey-silt	324	84
silty clay	263	52

For a number of reasons one suspects that these mean values are likely to be very unreliable:

1. The computed standard deviations are large.

2. The shear-wave speed is calculated by subtracting measured values of  $\rho c^2$  from computed values of the bulk modulus to obtain a shear modulus. One is thus subtracting two large uncertain numbers to obtain a small number. This is always a very inaccurate procedure. This is attested to by the fact that for a number of cases Hamilton obtains a negative shear modulus (he does not include these in his average).

3. Direct measurement of shear speeds in high porosity sediments have been reported [7] which yield shear speeds an order of magnitude smaller than those reported by Hamilton.

4. Kuperman [8] required a value of 600 m/s for fine sand to get good agreement for propagation loss with his model. This is 50% higher than the tabulated values.

Some measurements are also available for shear-speed gradients [9] and attenuation [10] but these too are probably unreliable.

It appears that shear in the sediment is an important factor in determining propagation loss, but no direct theoretical assault on the problem has yet been attempted. Shear is important for two reasons: 1) Shear waves are very lossy, hence conversion to shear waves either at interfaces or due to gradients will cause energy to be lost from the wave. 2) a shear-wave speed which is lower than the highest sound speed in the overlying water column results in otherwise trapped modes becoming leaky. The NRL normal mode program [11-13] allows for shear waves in the basement only and restricts the shear-wave speed to values large enough to completely trap the wave. Though shear speeds in rock meet this criterion shear speeds in unconsolidated sediments do not. It is therefore impossible to use the NRL model to assess the true importance of shear waves in the sediment. Kuperman, [8] at SACLANT, has modified the NRL program to account for low shear speeds in the subbottom. The shear is treated as a perturbation which affects the boundary condition at the water sediment interface. This is unsatisfactory for several reasons: 1) Since the sediment now becomes the subbottom, all structure in the sediment including wave-speed gradients and layering is ignored. 2) Shear may not be a "perturbation" at all, it may have a significant effect on the eigenfunctions. 3) According to calculations made by Vidmar [14] of ARL/UT the most important effect of shear in the sediment is shear wave conversion at the sediment-basement interface; the SACLANT model only allows for conversion at the sediment water interface.

Vidmar [14] has modelled bottom reflection for a sediment with shear and structure and finds that shear has a significant effect when the sediment is thin enough for sound to reach the sediment basement interface. However, his model has not been applied to the propagation loss problem. Probably the most satisfactory approach taken to date is that of McDaniel [15] of ARL/PSU who has apparently integrated a Haskell-Thompson model of the sediment with a normal mode program. This approach however would not allow for the wave-speed gradients in the sediment that appear to be so important in Vidmar's work.

## WATER COLUMN PARAMETERS

In some ways, it is easier to obtain good water column inputs in shallow water than in deep water, since one can obtain bathythermogram data right down to the bottom rather than having to rely on archival data as is customarily done for great depths in deep water. On the other hand the large temporal and spatial variability of the sound-speed profile and the importance of salinity gradients largely offsets this advantage.

### Sound Speed Profile

It is sometimes assumed that a detailed knowledge of the sound speed profile is not required to be able to predict propagation loss in shallow water [16]. That is, it is assumed that describing the water column as having an average sound velocity gradient of plus or minus so many  $s^{-1}$  (or in the Marsh-Schulkin Model a "layer depth") would be sufficient to characterize the sound-speed profile for the purpose of predicting propagation loss. We shall show later on that this is not so. In particular we will demonstrate that, even in the absence of a sound channel, the average sound speed gradient does not determine the propagation loss. Propagation losses for two sound velocity profiles with the same average velocity gradient can differ by more than 20 dB at 100 km even with a relatively fast bottom (fine sand) and low frequency (200 Hz).

It has been shown [17] that in many shallow water areas, ignoring salinity gradients can lead to gross errors in sound-speed profile. Large salinity gradients are capable of turning a strong negative gradient into strong positive and even relatively small gradients can determine the absence or presence of a surface duct, which in turn can cause significant changes in propagation loss. Such gradients are likely to be present in the vicinity of river outlets and in arctic and subarctic shallow water areas. In such regions a bathythermograph is clearly inadequate for determining sound velocity profile and a velocimeter would be required.

All of the sophisticated shallow water propagation computer models are capable of utilizing detailed sound-speed profiles, if available; simple algebraic models or empirical models obviously are not.

### Water Depth

Water depth is a relatively easy parameter to obtain; however, it should be pointed out that small errors in water depth can cause significant errors in propagation loss at low frequencies due to modal cut off. Urick [18] has observed significant changes in propagation with tidal changes in mean water depth.

### Surface and Bottom Roughness

Surface and bottom roughness can increase propagation loss at higher frequencies. As one might expect, surface roughness effects are accentuated by the presence of a positive velocity gradient and bottom roughness effects by negative velocity gradients. We shall show later on that details of the sound-speed profile can also have a pronounced effect on the sensitivity of propagation loss to boundary roughness. The surface roughness is determined by the wind speed or, more properly, by the sea state. To accurately model the effects of surface roughness some estimate of the wave height spectrum is required. Such an estimate is apparently lacking for shallow water. In its place modellers are forced to obtain wave height spectra from models like the Pierson-Moskowitz model. This model gives wave height spectra as a function of wind speed for a fully developed deep sea. Its applicability to shallow water wave spectra has not, to my knowledge, been verified and due to fundamental differences between shallow water and deep water waves and the limited fetch in shallow water it is not unlikely that there are substantial differences between shallow water and deep water cases.

The bottom roughness is virtually unknown as far as information on charts is concerned. Bottom roughness information could be obtained with a high resolution depth sounder; however, it is often treated as a "free parameter" by modellers [8].

According to mathematical models the propagation loss due to boundary roughness in dB/km is a quadratic function of rms wave height (or the equivalent bottom roughness); thus, in cases where boundary roughness has a significant effect the prediction of propagation loss depends critically on having a good value for the rms roughness. It does not appear that such an estimate is available on board.

A mathematical model of the effects of boundary roughness has been developed by Kuperman and Ingenito [19] and has been incorporated into the NRL propagation program in the Kirchhoff approximation. The model has been extended to allow for detailed knowledge of the bottom roughness at ARL/UT [20]. This model treats the surface roughness as a change in the boundary condition for the coherent part of the wave. McDaniel [21-23] of ARL/PSU treats boundary roughness as a modal coupling mechanism. Both approaches seem reasonable but appear to be incompatible; each approach retaining what the other seems to be discarding.

The crucial point, however, is that none of the theoretical models has been experimentally verified. One or both may, in fact, be quite good, but it is impossible to know for sure at this time.

### Biological Scatterers

It has been suggested on several occasions, by Weston [24,25] that fish may in some cases be the dominant cause of propagation loss in shallow water. This is not an unreasonable hypothesis in view of the relative abundance of marine life in shallow water and there is some experimental evidence which supports it. If the hypothesis were true, prediction of propagation loss would be virtually impossible since it would require accurate estimates of fish populations as a function of time and direction.

### ALGEBRAIC PROPAGATION LOSS MODEL

Given the limited environmental inputs and computational facilities available on board, it is natural to consider the use of simple algebraic formulae to describe propagation loss. A number

of such algebraic models have been developed; some derive from theoretical considerations [26-28] while others are purely empirical [29]. The models have been tested against measured values for propagation loss and found to be successful [16] or unsuccessful [30] depending on how much fiddling with input parameters is tolerated. We have developed yet another algebraic model based on a combination of some ideas of Weston, [24] Urick [26,31] and McPherson and Daintith [27]. An algebraic model which is based on theoretical considerations, such as this one, should properly be compared against sophisticated theoretical models rather than against experiment. That is, we wish to know whether the expression properly models what it intends to model, not whether it happens to fit propagation loss data since such data usually are the product of a number of effects many of which are not considered by the model. We will find that our model handles what it claims to model much better than the others, but it is somewhat less effective in predicting actual propagation loss. This is because the other models (unintentionally) overestimate the losses they model, which partially compensates for the losses they do not model. Such serendipitous agreement with experiment is a poor foundation for constructing a model for Fleet use.

The system which we model is illustrated in Fig. 2a. A water column of uniform depth  $H$  overlies a homogeneous semi-infinite sediment. The water has density  $\rho_w$ , volume attenuation  $\alpha_w$  and a sound-speed profile with a linear gradient given by

$$c_w = c_w^{(0)} + g_w \left( \frac{z}{H} \right)$$

where  $z$  is measured upwards from the sediment water interface. We will consider here only zero or positive values of  $g_w$  (isospeed or negative gradient sound-speed profiles) although we note that we have been successful in modeling positive sound-speed gradients using a similar approach.

The sediment is assumed to have density  $\rho_s$ , sound-speed  $c_s > c_w^{(0)}$ , and a plane wave attenuation coefficient given by

$$\alpha_s = k_s f,$$

that is, an attenuation which is proportional to frequency.

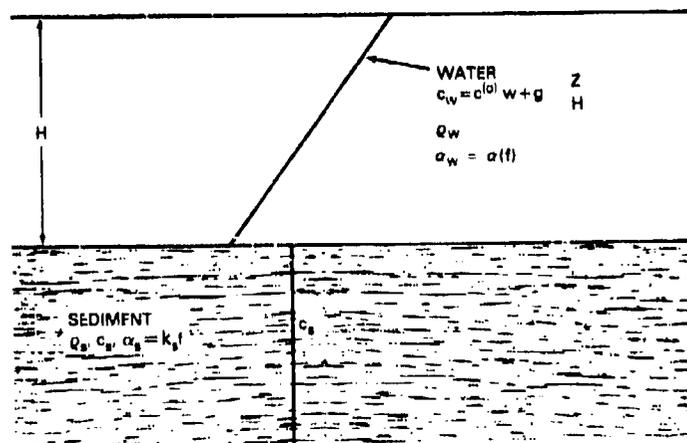


Fig. 2a — Velocity profile for shallow water propagation model

We will not present a derivation of the model since it has no real bearing on the subject of the report. Briefly however the model is an energy conservation model based on the following (see Fig. 2b).

1. SRBR rays have an attenuation that is proportional to the square of the grazing angle (since the attenuation per bounce is proportional to  $\theta_g$  [ $\alpha = \beta\theta_g$ ] as is the number of bounces per meter). Such rays (and their associated normal modes) are subject to mode stripping which results in  $15 \log R$  attenuation [24].

2. RBR rays have an attenuation that is independent of grazing angle (since, again, the attenuation per bounce is proportional to  $\theta_g$  but with a linear sound-speed gradient the number of bounces per meter is inversely proportional to  $\theta_g$ ). Such modes are not subject to mode stripping and attenuate as a group with an attenuation less than that of any SRBR ray.

3. The mode stripping process continues until the effective angle of the last mode stripped

$$\theta_s = \sqrt{\frac{1.7H}{\beta R}} \quad (1)$$

is equal to  $\theta_L$ , the larger of either the maximum grazing angle for an RBR mode,

$$\theta_{g \max} = \sqrt{\frac{2g}{c_w^{(o)}}} \quad (2)$$

or the cutoff angle of the lowest mode

$$\theta_c = \frac{c_w^{(o)}}{2fH} \quad (3)$$

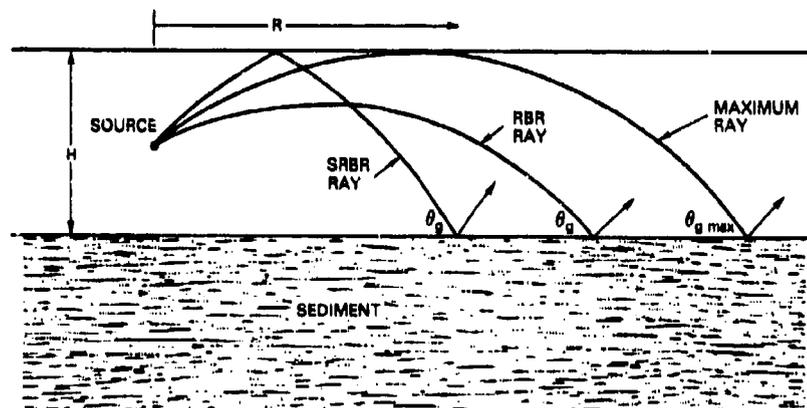


Fig. 2b — Definition of grazing angles and ray types for propagation model

where  $f$  is the frequency (i.e.  $\theta_L = \max\{\theta_{g, \max}, \theta_c\}$ ). As long as mode stripping continues the geometrical loss is  $15 \log R$ ; thereafter it is  $10 \log R$ .

4. In addition to the 10- or 15- $\log R$  loss, the wave suffers an attenuation due to bottom loss whose magnitude is determined by  $\theta_L$  and a mainstream attenuation given by  $\alpha_w$ . Thus, the propagation loss is given by

$$PL = 15 \log R + 5 \log(H\beta) + \frac{\beta R \theta_L^2}{4H} - 7.18 + \alpha_w R \quad (4)$$

when  $\theta_s > \theta_L$

and

$$PL = 10 \log R + 10 \log\left(\frac{H}{2\theta_L}\right) + \frac{\beta R \theta_L^2}{4H} + \alpha_w R \quad (5)$$

for  $\theta_s < \theta_L$ , where  $R$  is the range in meters.

The coefficient  $\beta$  is determined from the Rayleigh reflection coefficient

$$\beta = \frac{10d}{d\theta} \left[ \log R_R \right]_{\theta=\theta_L/\sqrt{2}} \approx \frac{10 \log R_R}{\theta} \Big|_{\theta=\theta_L/\sqrt{2}} \quad (6)$$

where  $R_R$  is the reflection coefficient including the effects of bottom attenuation. For small values of  $\theta_L$  (or for isospeed water)

$$\beta = 12.282 N_o \left[ \frac{\sqrt{1 + \frac{N_o K_s}{(1 - N_o^2)18.19}} - 1}{(1 - N_o^2) \left( 1 + \left\{ \frac{N_o k_s}{(1 - N_o^2)18.19} \right\}^2 \right)} \right]^{1/2} \quad (7)$$

where  $N_o = \frac{c_w^{(o)}}{c_s}$  and  $M_o = \frac{\rho_s}{\rho_w}$ .

For most cases of interest  $\beta$  is given to sufficient accuracy [32] by

$$\beta \approx \frac{0.477 M_o N_o K_s}{(1 - N_o^2)^{3/2}} \quad (8)$$

The mainstream water attenuation is given by [13]

$$\alpha_w = 0.001936 \left[ \frac{0.1f^2}{1+f^2} + \frac{40f^2}{4100+f^2} \right] \quad (9)$$

Note that  $\beta$ , and hence the effective attenuation coefficient at long range, is linearly dependent on the density ratio, sediment attenuation coefficient, and sound-speed ratio for small values of  $N_o$ . For values of  $N_o$  near unity, however,  $\beta$  is extremely sensitive to the precise value of  $N_o$ . Since for many sediment types  $N_o \approx 1$  it is necessary to know  $c_s$  to great precision in order to be able to predict propagation loss. Also note that for isospeed water the effective attenuation coefficient is inversely proportional to the frequency squared, while for negative sound-speed gradients the effective attenuation coefficient is independent of frequency over a wide range of frequency (except for the relatively weak effect due to the frequency dependence of  $\alpha_w$ ).

In Figs. 3 to 8 we compare propagation loss as calculated from Eqs. 5 to 9 with depth averaged propagation loss calculated using the NRL normal mode program with a mid-depth source. For fine sand we use nominal values of  $N_o = 0.872$ ,  $M_o = 1.957$  and  $K_s = 0.51$ , for sandy silt we use  $N_o = 0.919$ ,  $M_o = 1.767$  and  $K_s = 0.76$  and for clayey silt we use  $N_o = 0.989$ ,  $M_o = 1.469$  and  $K_s = 0.08$ . For the isospeed case  $c_w$  was 1500 m/s and for the negative gradient case the surface sound speed was 1520 m/s and the bottom water wave speed was 1500 m/s ( $g = 20$  m/s). The agreement between the algebraic formulae and the normal mode calculation is seen to be quite good — especially for higher frequencies and faster bottoms.

## ONBOARD PREDICTION OF PROPAGATION LOSS

We have shown that simple algebraic models are indeed capable of essentially duplicating normal mode calculations for homogeneous sediments and linear water column sound-speed gradients. To assess our capability to actually predict propagation loss with this (or a more sophisticated algebraic) model we must address three major issues:

1. Are the input parameters known to sufficient accuracy to enable us to set reasonable bounds on propagation loss?
2. Is propagation loss very sensitive to details of the environmental input (such as the detailed sound-speed profile) which would require a sophisticated computer model to handle?
3. Are the best models currently available capable of predicting propagation loss even with complete knowledge of the environment?

We have already touched upon the third question. Although the answer may well be negative we shall assume in what follows that the NRL normal mode program [13] represents ground truth. This assumes that unverified aspects of the model (such as bottom roughness) are valid and that neglected phenomena (such as shear in the sediment) are unimportant.

### Uncertainty in Propagation Loss Due to Uncertainty in Input Parameters

In the section on Sediment Sound Speed and Density we showed that the proper measure of the uncertainty of the sound-speed ratio for a known sediment type is the standard deviation rather than the standard error of the mean ( $\sigma = \sqrt{n} SE$ ). In Figs. 9 and 10 the algebraic model is used to show

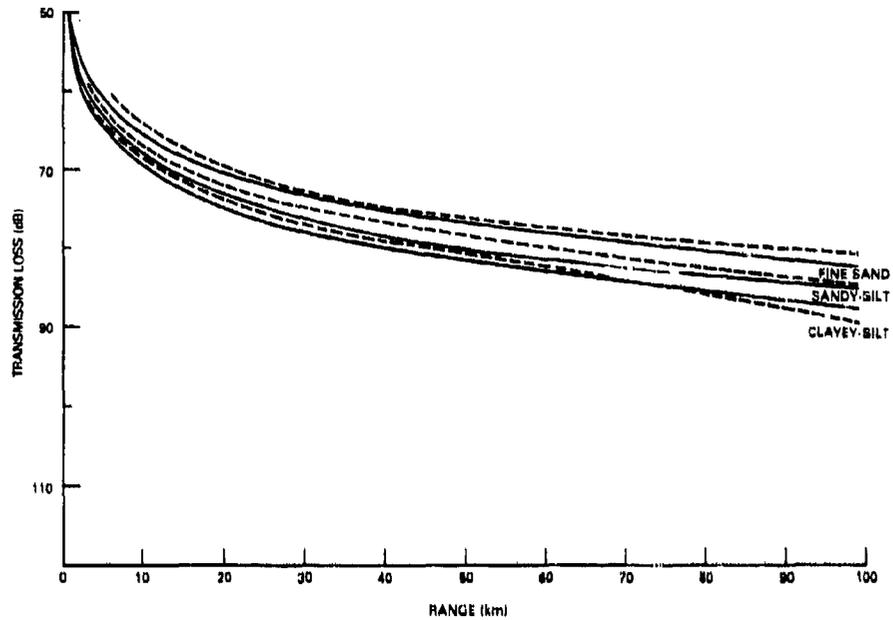


Fig. 3 — Depth averaged propagation loss vs range at 200 Hz for isospeed water, 100 m deep and three different sediment types. Solid line from algebraic formulas (Eqs. 4 through 9); dashed line from NRL normal mode program (incoherently summed).

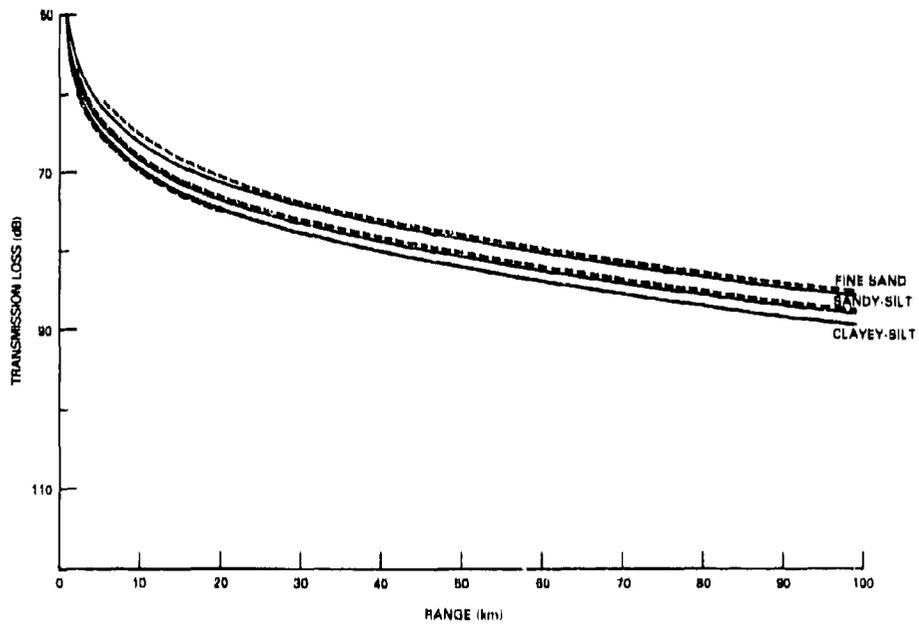


Fig. 4 — Depth averaged propagation loss vs range at 800 Hz for isospeed water, 100 m deep and three different sediment types. Solid line from algebraic formulas (Eqs. 4 through 9); dashed line from NRL normal mode program (incoherently summed).

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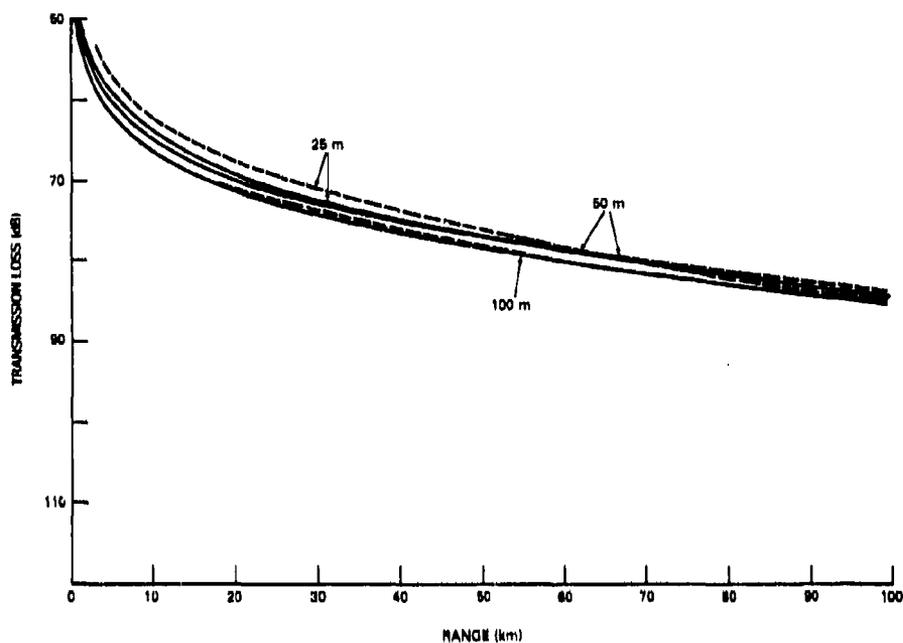


Fig. 5 — Depth averaged propagation loss vs range for isospeed water at 800 Hz over a fine sand bottom for three different water depths. Solid line from algebraic formulas (Eqs. 4 through 9); dashed line from NRL normal mode program. Compare with Fig. 6.

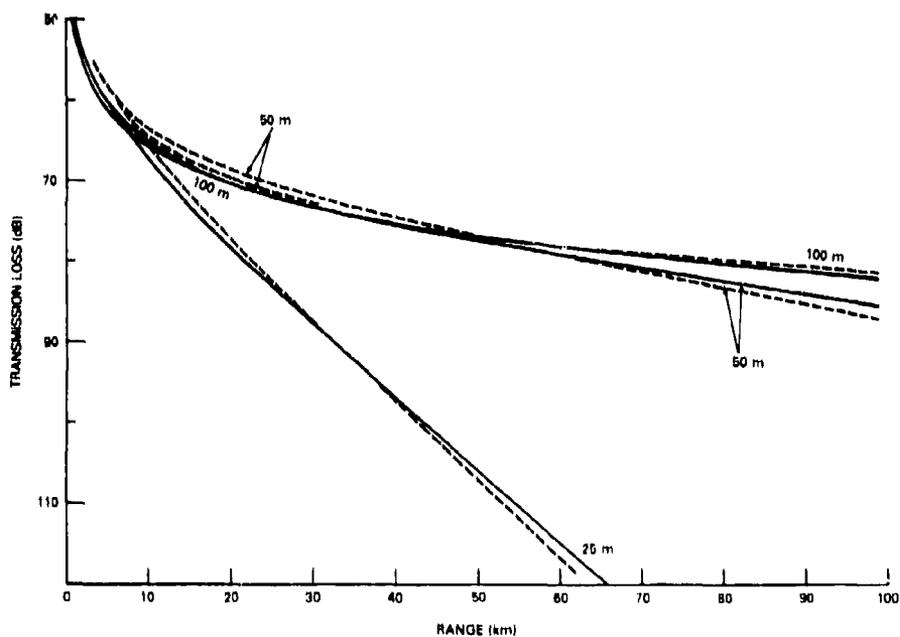


Fig. 6 — Depth averaged propagation loss vs range for isospeed water at 200 Hz over a fine sand bottom for three different water depths. Solid line from algebraic formulas (Eqs. 4 through 9); dashed line from NRL normal mode program. Compare with Fig. 5.

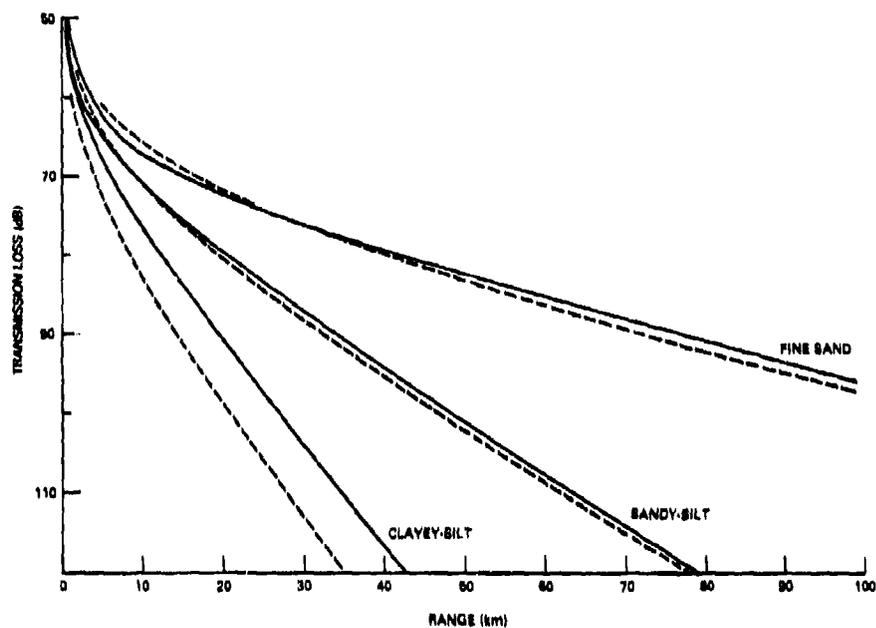


Fig. 7 -- Depth averaged propagation loss vs range for water with a  $0.2 \text{ s}^{-1}$  negative velocity gradient at 200 Hz and 100 m water depth for three different sediment types. Solid line from algebraic formulas (Eqs. 4 through 9); dashed line from NRL normal mode program. Compare with Fig. 8.

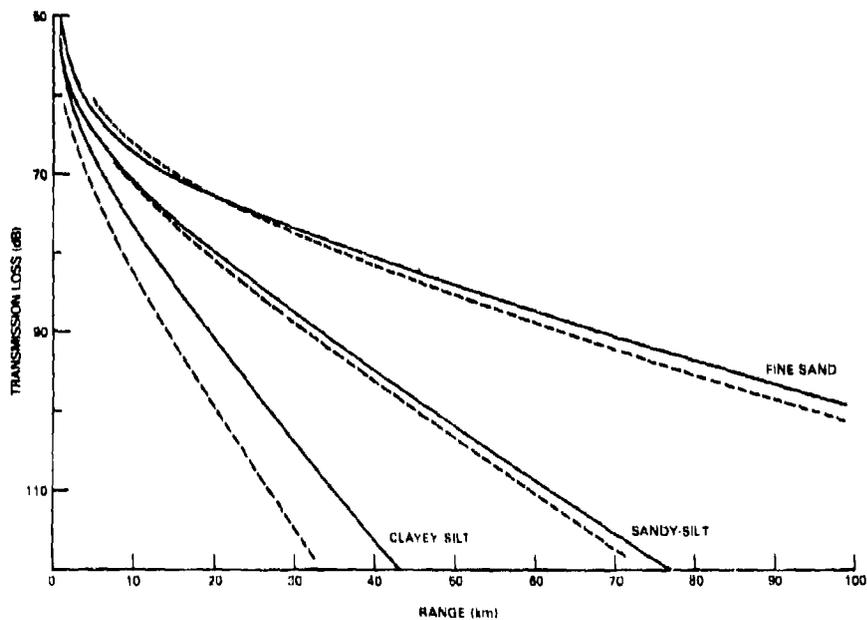


Fig. 8 -- Depth averaged propagation loss vs range for water with a  $0.2 \text{ s}^{-1}$  velocity gradient at 800 Hz and 100 m water depth for three different types. Solid line from algebraic formulas (Eqs. 6 through 9); dashed line from NRL normal mode program.

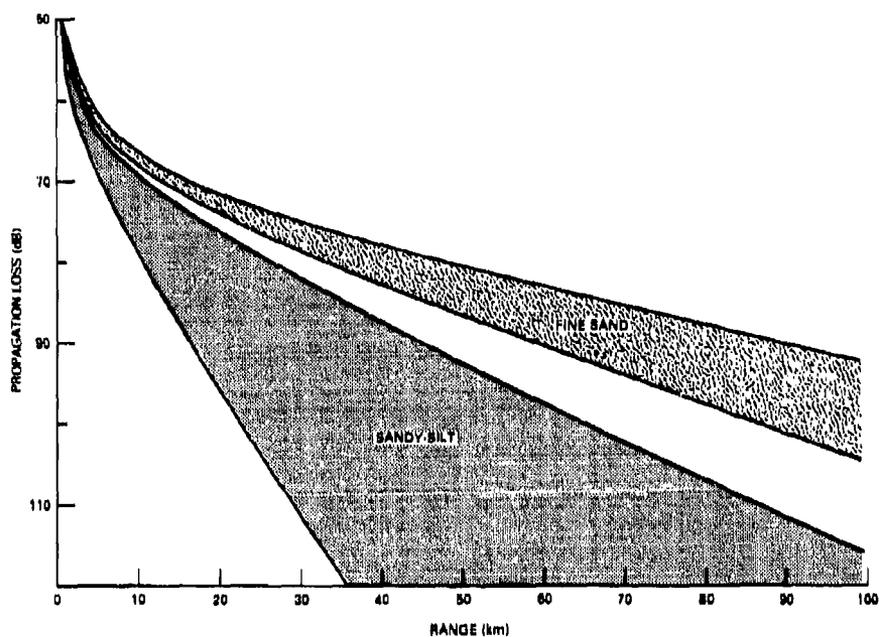


Fig. 9 — Propagation loss vs range for fine sand and sandy silt bottom with a  $0.2 \text{ s}^{-1}$  negative velocity gradient. Shaded area shows spread of calculated transmission loss as sediment velocity ranges between  $c_H - \sigma$  and  $c_H + \sigma$  where  $c_H$  the sediment sound speed and  $\sigma$  its standard deviation are taken from Table 2. Frequency 200 Hz, water depth 100 m.

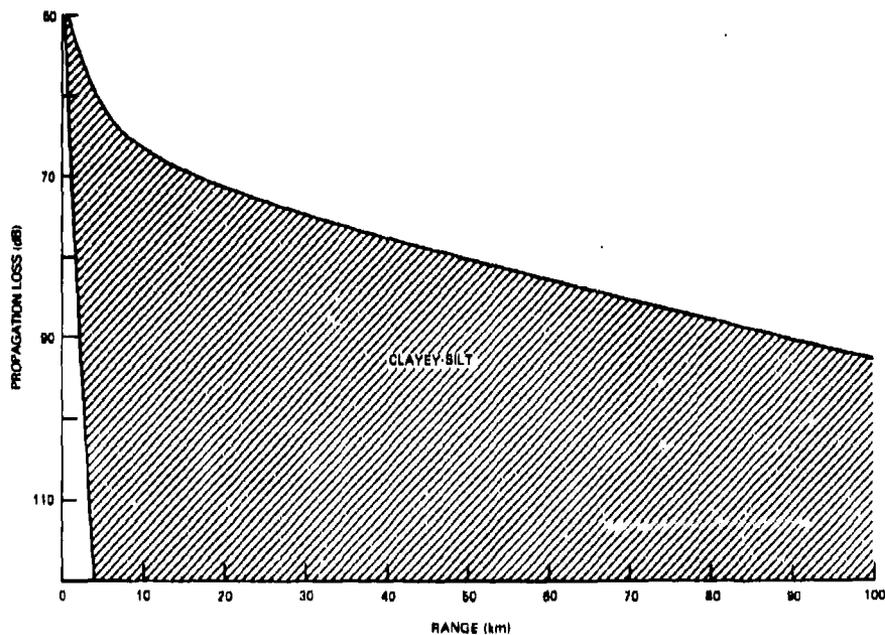


Fig. 10 — Propagation loss vs range for clayey silt bottom with a  $0.2 \text{ s}^{-1}$  negative velocity gradient. Shaded area shows spread of calculated transmission loss as sediment velocity ranges between  $c_H - \sigma$  and  $c_H + \sigma$  where  $c_H$  is the sediment sound speed and  $\sigma$  its standard deviation taken from Table 2. Frequency 200 Hz, water depth 100 m.

the effect of adding  $\pm\sigma$  to Hamilton's value for the sound speed in fine sand, sandy-silt and clayey-silt for the negative sound-speed gradient test case. Essentially the same results were obtained using the NRL normal mode program. As might be expected the error range is much larger for the slower sound-speed sediments due to the  $(1 - N_0^2)^{-3/2}$  dependence of  $\beta$ . It is probably worth discussing in some detail the meaning of the shaded areas in Figs. 9 and 10. Given the uncertainty in the sound speed alone, only 68% of all sandy-silt propagation loss curves would be expected to fall within the shaded area. Uncertainty in  $k_p$  and  $\rho_p$  would add an additional uncertainty of  $\pm 3$  dB. Although the spread of values for fine sand is quite a bit less than that of sandy-silt it is important to note that due to the smaller slope of the fine sand propagation loss curve, the uncertainty in maximum range will be about the same for both fine sand and sandy-silt. Using a nominal value of 95 dB for an FOM the uncertainty in maximum range is about 35 km for both sediment types. For clayey-silt the maximum range (for a 95 dB FOM) could be anywhere from 2 km to over 100 km. This probably overstates the case if the ambient noise is determined from the calculated propagation loss, since an increase in propagation loss will decrease ambient noise which will increase the figure of merit. This reduces the sensitivity of detection range to changes in predicted propagation loss.

The question arises as to whether the extremely large uncertainty in the propagation loss for clayey-silt might be an artifact of an oversimplified model for the bottom. After all, we have assumed an infinite homogeneous bottom so that all rays that penetrate the bottom are forever lost, whereas, in fact, the sound-speed gradient in the sediment could refract such rays back to the water column. Figure 11 shows the effect of sound-speed gradients of 0.5, 1.0, and 2.0  $\text{s}^{-1}$  on the propagation loss for clayey-silt as calculated as using the NRL normal mode program with  $c_p$  given by Hamilton's average value. As one might expect, the steepness of the propagation loss curve is moderated by presence of the sound-speed gradient in the sediment. In Fig. 12, however, we see that this factor actually increases the spread of propagation loss curves somewhat. (It is also interesting to note that the propagation loss does not always decrease with increasing gradient in the sediment.) We conclude that, for negative sound-speed gradients, propagation loss for clayey-silt is virtually impossible to predict and for faster bottom types predictions are, at best, unreliable.

The negative gradient case is, of course, the most demanding. In Fig. 13, however, we show that the situation for clayey-silt is equally hopeless for isospeed water. In Fig. 13 we have plotted propagation loss-curves for isospeed water and a clayey-silt bottom varying the sediment sound speed, sediment attenuation, and sediment sound-speed gradient by plus or minus one standard deviation. This spread of propagation loss-curves is enormous. Maximum range estimates would vary from a few kilometers to several hundred kilometers! (It might also be pointed out that just about any conceivable propagation loss curve could be fitted using "reasonable" values for a clayey-silt sediment.)

Next, we address the question of whether propagation loss is too sensitive to details of the environment to permit algebraic modelling. We note that our algebraic model requires a linear sound-speed gradient. Figure 14 shows eight different hypothetical sound-speed profiles each of which has a value of 1520 m/s at the surface and decreases monotonically to 1500 m/s at the fluid-sediment interface. Case A, is the specific case which Eqs. 5 through 9 model. Cases B through H which have the same average gradient, would perforce have to be modeled as being the same as Case A. In Fig. 15 normal mode propagation loss-curves for 200 Hz and a fine sand bottom are plotted for each of the eight velocity profiles shown in Fig. 14. The spread in propagation loss-curves due to the differences in these profiles is even larger than the uncertainty due to uncertainty in bottom parameters. Moreover, there is no reason to believe that these randomly chosen 0.2  $\text{s}^{-1}$  average gradient profiles bracket the entire range of possible 0.2  $\text{s}^{-1}$  gradient loss curves. We conclude that even for monotonically decreasing sound-speed profiles, the average gradient is not a good predictor of propagation loss and hence Eqs. 5 through 9 (though they do model the linear gradient correctly) are generally not useful for predicting propagation loss even if the bottom parameters were homogeneous and precisely known.

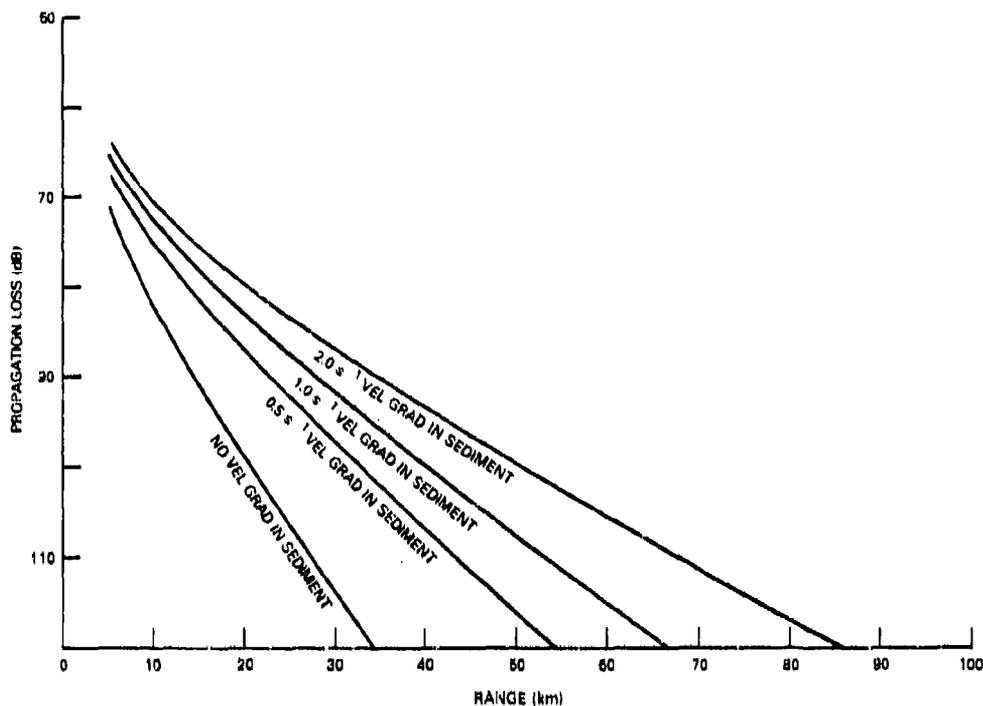


Fig. 11 — Propagation loss vs range for clayey-silt bottom with negative velocity gradient in the water column and four different values for  $g_s$ , the sound speed gradient in the sediment. Frequency 200 Hz; water depth 100 m.

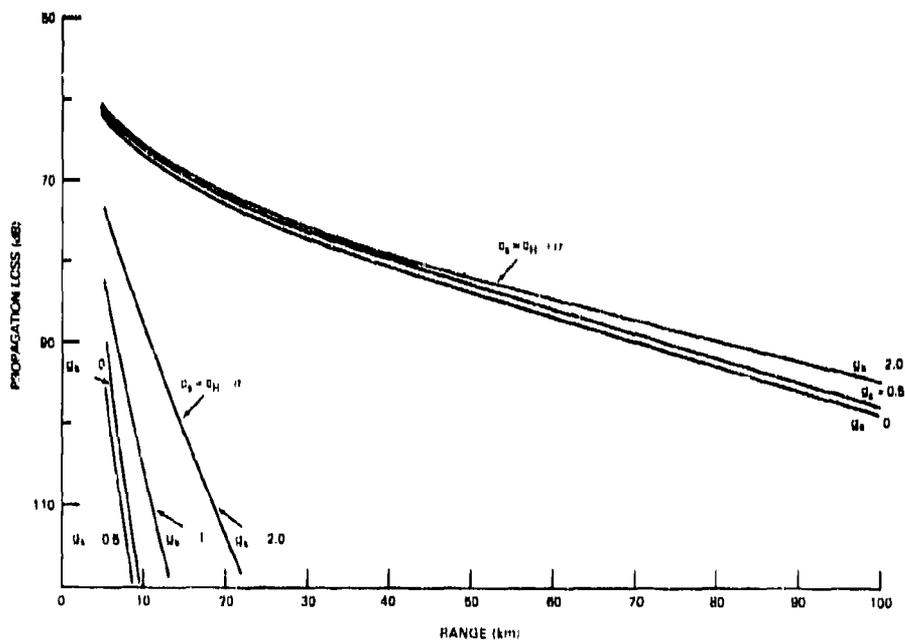


Fig. 12 — Propagation loss vs range for clayey-silt bottom assuming a sediment velocity of  $c_H + \sigma$  a negative velocity gradient for several different values of  $g_s$ , the sound speed gradient in the sediment. Water depth 100 m, frequency 200 Hz.

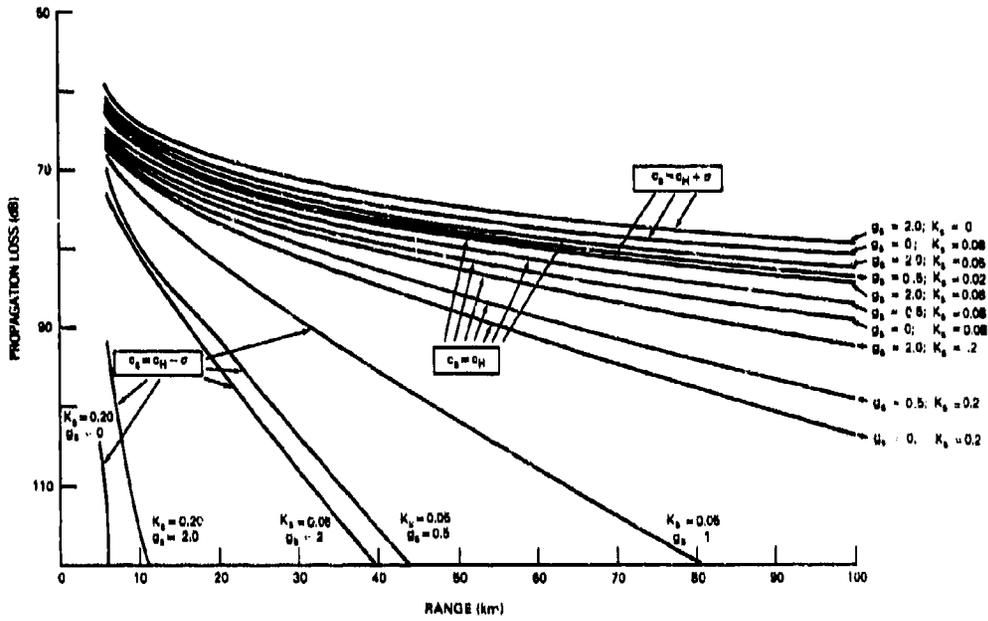


Fig. 13 — Propagation loss vs range for clayey-silt bottom and isospeed water assuming the attenuation coefficient is 0.05, 0.08 or 0.2 dB/Hz/km, the gradient in the sediment  $g_s$  is 0, 0.5, 1 or  $2 \text{ s}^{-1}$  and that the sound speed is  $c_H - \sigma$ ,  $c_H$  or  $c_H + \sigma$ . Water depth 100 m; frequency 200 Hz.

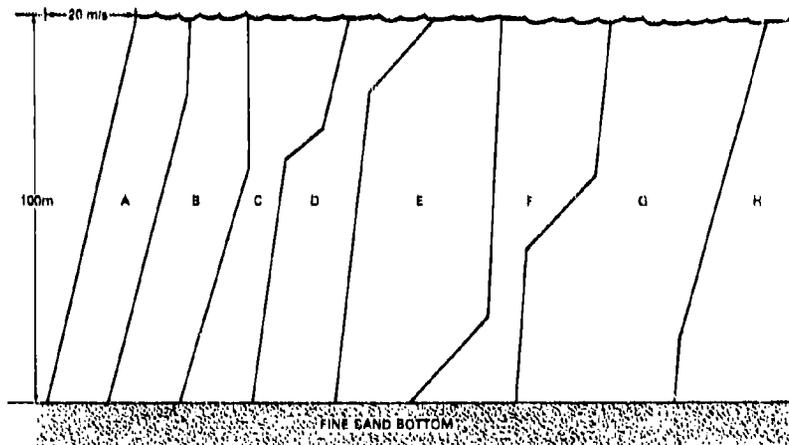


Fig. 14 — Eight different sound speed profiles for which sound speed decreases monotonically with depth and the average gradient is  $0.2 \text{ s}^{-1}$

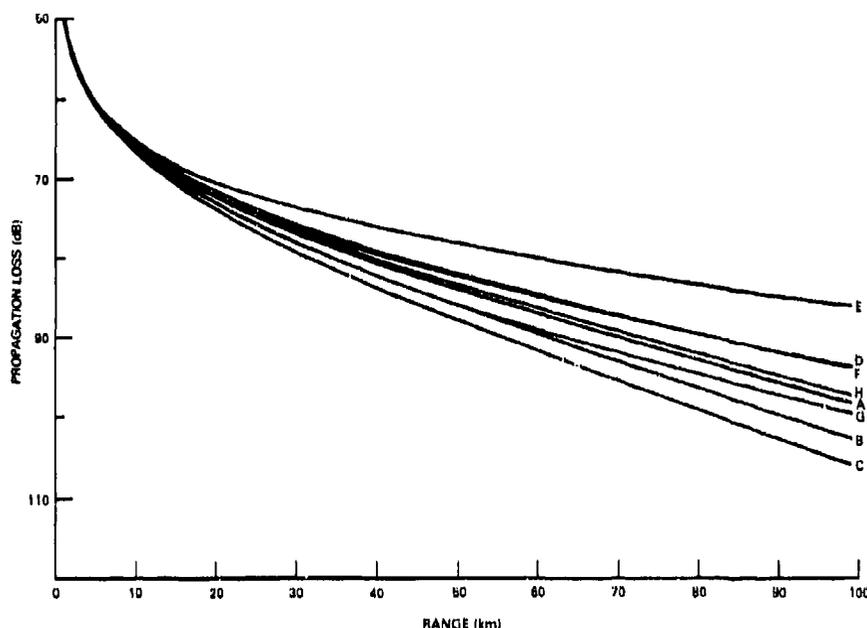


Fig. 15 — Propagation loss vs range for sound speed profiles in Fig. 14

We also considered the possibility that some simple property of the velocity distribution other than average velocity gradient could be used in a simple model to predict propagation loss. At the very least perhaps the order of lossiness ( $E < D < F < H < A < G < B < C$ ) could be predicted. Table 4 indicates that even this is apparently not possible since the order is evidently a strong function of frequency.

We have also shown that the effects of bottom roughness and surface roughness are strongly influenced by details of the sound-speed profile, the propagation loss for the A type profile being three times as sensitive to bottom roughness as the D or C type and six times as sensitive as the E type.

We conclude that, although propagation loss in shallow water can be expressed in the simple form,  $L = 15 \log R + AR + B + CR^2$  (see Appendix) the coefficients of expansion are extraordinarily complicated, intertwined functions of the input parameters.

## CONCLUSIONS

1. For simple cases, i.e., homogeneous liquid bottom, linear sound-speed gradient, no surface or bottom roughness, a simple algebraic model for depth averaged propagation loss works as well as the more complex normal mode model.

2. The uncertainty in bottom parameters for a given sediment type, particularly sound-speed and attenuation, makes it impossible to set meaningful bounds on propagation loss particularly for negative gradients or slow bottoms. (Useful predictions, however, can probably be made when a positive gradient is present.)

Table 4 — Loss at 100 km for Eight  $0.2 \text{ s}^{-1}$  Profiles

	100 Hz	200 Hz	400 Hz	800 Hz
Least Lossy	E	E	E	E
	D	D	F	C
↓	H	F	D	F
	A	H	C	D
	G	A	A	G
	F	G	H	B
	Most Lossy	B	B	A
	C	C	B	H

3. Details of the sound-speed profile can cause significant changes in propagation loss, therefore even if bottoms were well characterized, sophisticated computer models such as the NRL normal mode program would be required to predict propagation loss.

4. Virtually all propagation loss curves can be described to within a fraction of a dB by the function

$$L = B + 15 \log R + AR + CR^2,$$

with the  $C$  coefficient usually zero. Thus the output field can be described by two or, at most, three free parameters. Since there are no fewer than 24 input parameters, it is thus easy to *explain* observed propagation loss and very difficult to *predict* it. Moreover, it is doubtful that propagation loss experiments can uniquely define bottom parameters.

5. Certain aspects of the theory remain unverified and/or inadequately treated. These include:

- 1) Surface and bottom roughness
- 2) Shear in the sediment
- 3) Substrate roughness
- 4) Modal coupling
- 5) Biological scatterers

6. Grain size distribution is not an adequate predictor of acoustical properties; hence, currently existing sediment charts are of little or no value in performance prediction.

7. Many input parameters are *very* poorly known. These include:

- 1) Bottom roughness
- 2) Wave height spectrum
- 3) Sediment shear
- 4) Sediment shear attenuation

- 5) Shear and longitudinal wave speed and attenuation gradients in the sediment
- 6) Distribution and effective attenuation of biologics

In most cases the theory is not certain enough to determine the uncertainty in propagation loss caused by uncertainty in these parameters. Accurate onboard prediction of transmission loss for shallow water is, in general, not possible within the present state of the art.

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## Appendix

### EMPIRICAL FORMULA FOR PROPAGATION LOSS

In the course of this study, we found that it was possible to fit propagation loss to the formula

$$PL = 15 \log R + AR + B + CR^2 \quad (A1)$$

using a Tchebychev fit. Table A1 indicates the coefficients for this fit for a number of test cases along with the size of the maximum deviation between Eq. (A1) and the actual propagation loss curve between 5 and 100 km at 200 Hz. Coefficients are given for a three parameter and a two parameter ( $C = 0$ ) fit. Sediment properties are from Hamilton.

Table A1 — Coefficients for Empirical Formula for Propagation Loss Curve

Sediment	Profile Type	Avg. Vel. Grad. (s)	Three Parameter Fit				Two Parameter Fit		
			A	B	C x 10 <sup>4</sup>	Maximum Deviation	A	B	Maximum Deviation
Fine sand	A	0.2	0.18	49.18	+1.098	0.017	0.197	48.98	0.11
" "	B	0.2	0.22	49.04	+1.772	0.043	0.239	48.73	0.18
" "	C	0.2	0.27	49.02	+0.461	0.042	0.260	48.96	0.066
" "	D	0.2	0.149	49.04	+0.020	0.047	0.149	49.04	0.047
" "	E	0.2	0.0729	49.61	-0.801	0.042	0.065	49.72	0.088
" "	F	0.2	0.146	49.76	-0.587	0.051	0.141	49.84	0.081
" "	G	0.2	0.252	48.8	-4.36	0.046	0.203	49.76	0.46
" "	H	0.2	0.173	49.14	+0.09	0.021	0.183	48.98	0.091
" "	A	0.12	0.12	49.45	+0.765	0.012	0.128	49.27	0.083
" "	A	0.24	0.217	49.04	+1.49	0.036	0.233	48.78	0.15
" "	A	0	0.0233	49.84	-1.083	0.060	0.0127	50.00	0.122
Clayey-silt	A*	0.2	0.418	52.57	+5.18	0.164	0.471	51.73	0.53
" "	A	0.2					0.761	53.87	0.20

\* $g_1 = 0.5$ .