

AD A104413

14 NPL MK-46

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

9 REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL Memorandum Report #636	2. GOVT ACCESSION NO. AD-A104413	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) INVESTIGATION OF A NONLINEAR APPLICATION OF THE NAVY'S DYNAMIC DESIGN ANALYSIS METHOD FOR SOUND ISOLATED EQUIPMENT.	5. TYPE OF REPORT & PERIOD COVERED Interim report on a continuing NRL problem.	
7. AUTHOR(s) W. W./Webbon	6. PERFORMING ORG. REPORT NUMBER	
8. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, DC 20375	9. CONTRACT OR GRANT NUMBER(s) 16 S0293 SL	
11. CONTROLLING OFFICE NAME AND ADDRESS	10. PROGRAM ELEMENT PROJECT, TASK AREA & WORK UNIT NUMBERS 635011/S0293 SL 58-0270-0-1	12. REPORT DATE Sept 28 1981
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 12 22	13. NUMBER OF PAGES 21	
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		15. SECURITY CLASS. (of this report) UNCLASSIFIED
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
18. SUPPLEMENTARY NOTES This research was sponsored by the Naval Sea Systems Command under the Improved Performance Machinery Program.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Structural dynamics Dynamic design Shipboard shock Submarines		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The Navy's dynamic design-analysis method (DDAM) is used to predict the peak forces and deflections expected in items of equipment on combat ships subjected to underwater-explosion attacks. The DDAM cannot be applied directly to sound-isolated equipment because of the nonlinearity in the mounting. A method of linearizing the nonlinear elements of a structure by iterative analysis is described. The (Continues)		

DD FORM 1473 1 JAN 73

EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

251950

20. ABSTRACT (Continued)

average elements are continuously adjusted during the iterative analysis until convergence occurs by matching peak forces and deflections of the nonlinear elements. Trial analyses on a structure consisting of a pair of beams with four or five nonlinear elements show that the method converges within a reasonable number of iterations.

**INVESTIGATION OF A NONLINEAR APPLICATION OF THE NAVY'S DYNAMIC
DESIGN ANALYSIS METHOD FOR SOUND ISOLATED EQUIPMENT**

INTRODUCTION

The Navy's dynamic design-analysis method (DDAM) is used to predict the peak forces and deflections expected in items of equipment on combat ships subjected to underwater-explosion attacks. It proceeds by analyzing the equipment to determine its fixed-base normal modes of vibration and uses empirical formulas for assessing the response of each normal mode to the shock from an explosion. The required modal analysis limits the DDAM to treatment of linear elastic structures. Although many items of shipboard equipment can be considered linear elastic, there are a number of important exceptions. Sound-isolation mounts, in particular, are designed to be soft for small deflections to minimize transmission of sound from operating machinery, and to be much stiffer for larger deformations in order to limit deflections under shock.

The DDAM cannot be applied directly to sound-isolated equipment because of the nonlinearity in the mounting. A method of linearizing the structure by iterative analysis is described. The method represents nonlinear elements of a structure by average linear elements chosen to match peak forces and peak deflections which can be obtained from the nonlinear elements.

The Navy's DDAM utilizes "in- lb -sec" units. This report addresses the problem of nonlinear anti-noise mounts in relationship to a type of linear system which is commonly analyzed by DDAM. Therefore in an effort to make applications of this method possible to the commercial engineering community, "in- lb -sec" units are used throughout. Table 1 shows conversion factors for some of the units used in this report to SI units.

Table 1 - Conversion factors for units of measurements

MEASUREMENT	UNITS	MULTIPLY BY	TO GET
Length	inches, in.	0.0254	meters, m
Force, weight	pounds, lb	4.4482	newtons, N
Mass	lb-sec ² /in	175.13	kilograms, kg
Velocity	in/sec, ips	0.0254	m/s
Acceleration	gravities, g	9.8044	m/s ²
Pressure, stress	lb/in ² , psi	6894.7	pascals, Pa

Manuscript submitted August 20, 1981.

BACKGROUND

Linearizing of nonlinear elements by a "brute force" method of solution has been done at least twice before with shipboard-shock application. Reference 1 cites a case where a propulsion shaft was analyzed using DDAM with the assumption that there was enough clearance in the bearings so that each bearing could be modeled as a pinned support. Results were combined over the modes to find that the estimated peak rotation at most of the bearings was larger than the clearance. So rotational stiffnesses of the bearing pedestals were figured and each support remodeled to have an average stiffness appropriate to its peak rotation. Then the analysis was redone.

The piping specification of Reference 2 is a similar example. A piping system is to be modeled using the small-deflection stiffness for nonlinear sound-isolation hangers. A complete DDAM analysis is performed and the deflection of each hanger calculated. If the deflection of any hanger is calculated as larger than the allowed deflection, the analysis must be redone with a new position for the hanger taken at its bottomed-out position and a new flexibility representing only the structure supporting the hanger.

In both of these cases, there is only one iteration and it is done by hand. The initial analysis is inspected, stiffnesses are changed and the analysis is then redone.

APPROACH

This "brute force" method of solution could be automated. The result would be more precise and far less laborious than if done by hand. The resulting linear elastic system would then be as close as practical to the real nonlinear system. The methods of such an automated iterative analysis procedure are developed in this report. A mathematical model is analyzed to determine the feasibility of the procedure and to uncover problems. A FORTRAN program was written to perform the analysis.

METHOD

The design spectrum method is briefly reviewed as an aid in understanding the approach used in the report. The basic equations are taken from Reference 3.

Suppose a structure, initially at rest, is attached to a base which undergoes a motion, $Z(t)$, which is a known function of time. Hooke's law for an elastically distorted structure is:

$$X_i = \sum_j D_{ij} F_j \quad (1)$$

where X_i is the displacement of point i relative to the base, D_{ij} is the displacement of point i that would arise due to a unit force at j (influence coefficient), and F_j is the force applied at j . d'Alembert's principle says that $F_j = -m_j \ddot{Y}_j$ in the absence of external forces, where m_j is the mass at point j , and Y_j is the absolute motion of the mass. The absolute and relative motions of the mass

are related as $X_i = Y_i - Z$ for translation of a single base in one direction. Hence, the equation describing the dynamic response of the system is,

$$X_i(t) = - \sum_j D_{ij} m_j (\ddot{X}_j + \ddot{Z}) \quad (2)$$

This set of differential equations has the solution,

$$X_i(t) = \sum_a X_{ia}(t) \quad (3)$$

where $X_{ia}(t)$ is the displacement response (relative to the base) of mass i when the structure responds in mode a which has a circular natural frequency, w_a . The summations in equations 2 and 3 (and equations 5, 7 and 9 to follow) are over the same number of elements. This is the number of masses in the structure which is also the number of natural modes of vibration (for 1 dimension). Each displacement response, $X_{ia}(t)$, is given by,

$$X_{ia}(t) = \bar{X}_{ia} \bar{P}_a H_a(t), \quad (4)$$

with \bar{X}_{ia} the mode shape amplitude of mass i in mode a relative to the other masses, and where

$$\bar{P}_a = \frac{\sum_j m_j \bar{X}_{ja}}{\sum_j m_j \bar{X}_{ja}^2}, \quad (5)$$

is the participation factor for mode a ; and where

$$H_a(t) = - \frac{1}{w_a} \int_0^t \ddot{Z}(T) \sin w_a(t-T) dT,$$

the so called "Duhamel Integral," is the displacement (relative to the base) of a linear oscillator of frequency, w_a , subjected to a base motion. The Navy's DDAM uses "shock design values" obtained for different locations and directions on different types of ships by analysis of records from shipboard shock tests. Results have been fitted to formulas showing the largest absolute value of $H_a(t)$ as a function of the ship, direction, mode frequency, w_a and the modal weight,

$$\bar{w}_a = \frac{g(\sum_j m_j \bar{X}_{ja})^2}{\sum_j m_j \bar{X}_{ja}^2}. \quad (7)$$

The shock design value of $H_a(t)$ from the empirical formulas can be used to determine the extreme value of X_{ia} in equation 4 for each mode of vibration of the structure. A statistical combination of the extreme modal responses must be performed to obtain a response value. For instance, to determine the force response of a spring between two mass points i and j or between a mass and the base, the force in each mode must first be determined. The force in mode a in the spring is,

$$P_a = (U_{ia} - U_{ja})K, \quad (8)$$

where the U_{ra} ($r=1$, number of masses) are the set of modal displacements which occur when the design value for $H_a(t)$ is used in equation 4, and K is the stiffness of the spring. For springs attached to ground, $U_{ja}=0$.

Once the force in each mode is determined, then the combined force response is given by,

$$P = \left| P_q \right| + (-P_q^2 + \sum P_a^2)^{1/2}, \quad (9)$$

where q is the mode number at which the largest absolute value of the force in the spring occurs.

The deflection (stretch) of each spring may be calculated in a similar manner or may be obtained from equations 8 and 9 as,

$$S = P/K \quad (10)$$

since, $S_a = (U_{ia} - U_{ja})$.

NONLINEAR SOLUTION

Based on the preceding discussion, the procedure for the nonlinear application of the DDAM used in this study is as follows:

1. Choose starting spring constants for the nonlinear springs in the structure.
2. Solve the resulting linear structure for natural frequencies and mode shapes.
3. Perform a DDAM analysis and determine the forces and deflections in the springs.
4. Use the force from the DDAM calculation to find the deflection which each nonlinear spring would need to produce that force; or, use the DDAM deflection to find the force in the nonlinear spring.
5. Calculate a new average stiffness as the ratio of the force to the deflection as obtained from step 4, and take that as a better approximation to the spring constant.
6. Repeat steps 2 thru 5 until the forces calculated in step 3 are not much different from those calculated in the previous step 3. At this point the solution has converged. Should convergence not occur before a specified number of iterations, stop the analysis.

Figure 1 is taken from a particular calculation showing convergence to one of the nonlinear springs. This figure illustrates the method of steps 4 and 5 used to select a new average stiffness for each spring. The position labeled 1 is a point on the non-

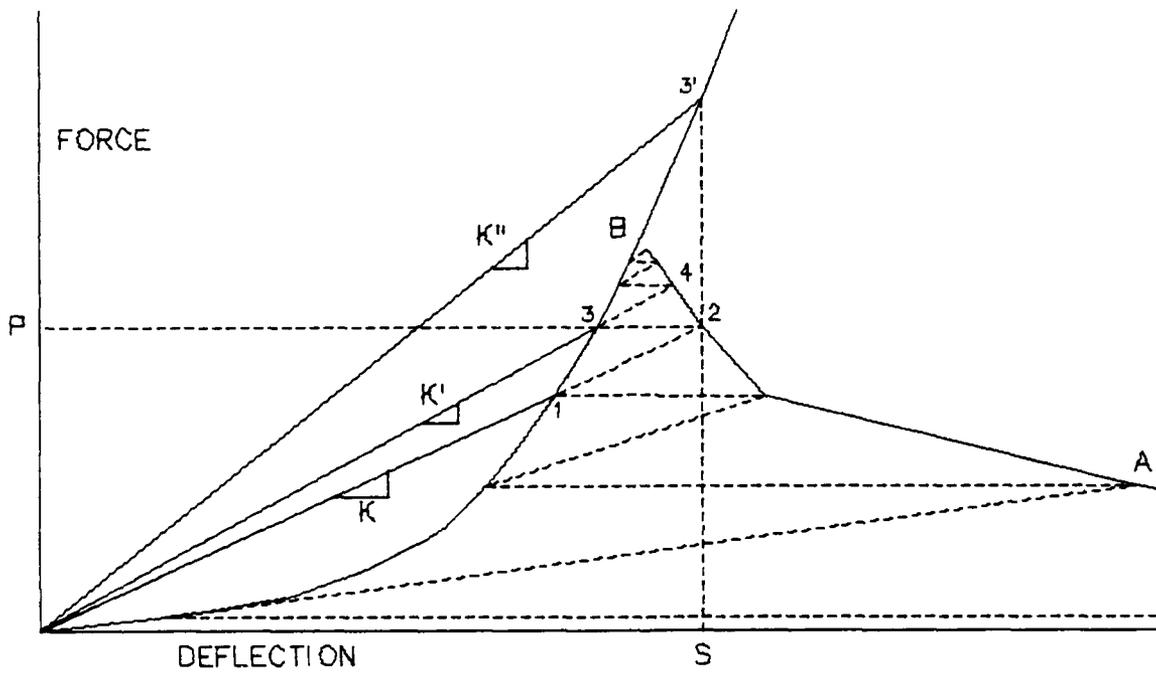


Fig. 1 — Spring constant calculation

linear force-deflection function obtained from a previous iteration. (The iterations leading to it are indicated by broken lines.) The slope of the line K from the origin to point 1 is taken as the stiffness of the spring for the next DDAM calculation. The position labeled 2 is the force and deflection obtained for the spring of stiffness K by the DDAM.

Point 3 returns the calculated point to the nonlinear curve by correcting the deflection. Then the slope of the line K through point 3 is taken as the spring constant for the next DDAM calculation which produces the force and deflection indicated by point 4. The process continues (broken lines) until the average spring constant used in a DDAM calculation produces a force and deflection falling sufficiently near the nonlinear curve. All of the nonlinear springs in the structure are approximated simultaneously in the same way as in the example shown here.

Iteration by this procedure can occur at constant force (correcting the deflection to intercept the curve, as explained) or at constant deflection (correcting the force). For the case shown, the iteration at constant force produces faster convergence. This is suggested by the slope of the line, K", from the origin to point 3' which would be selected if the force were corrected to match the DDAM deflection.

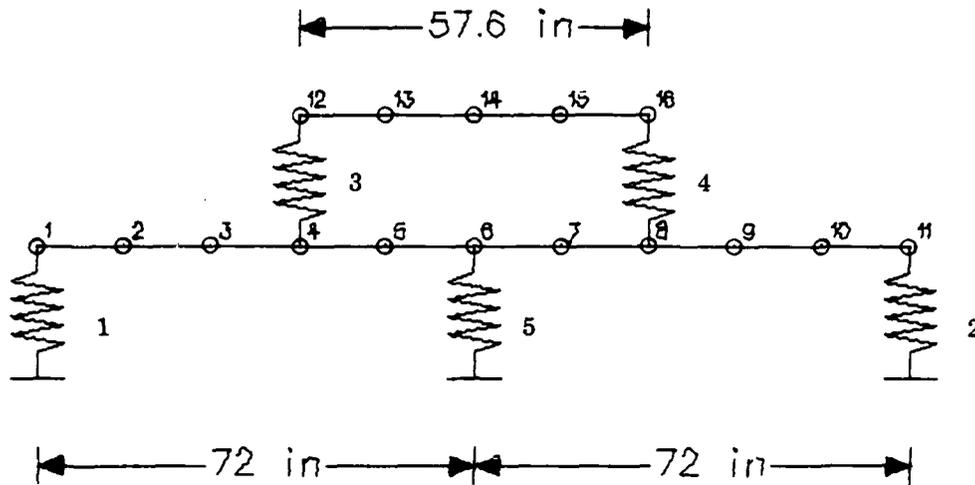
The criteria chosen for convergence of the structure is as follows:

1. Obtain the change in the spring force for each spring by subtracting from the spring force in the current iteration, the spring force in the previous iteration.
2. Find the largest absolute value of the changes in the spring forces.
3. Compare the value obtained in step 2 to a supplied positive constant convergence force which may be adjusted to the level of accuracy desired. If the value is less than the convergence force, the solution is considered converged.

This convergence criteria was chosen to insure that the convergence of the most energetic springs would be assured. Weaker springs with less energy should not seriously affect structural response. Hence, stringent convergence criteria on these weaker springs might be unduely restrictive.

MATHEMATICAL MODEL

A mathematical model of a simple structure was developed to test the convergence of the proposed nonlinear method. The model consists of two symmetric beams with equally spaced lumped masses. These beams are connected to the base and to each other through nonlinear springs as shown in Figure 2. The model as shown is statically indeterminate to degree 1. A statically determinate



CROSS SECTION MOMENTS OF INERTIA
 UPPER BEAM: 200 in⁴
 LOWER BEAM: 1200 in⁴

ELASTIC MODULUS : 30 x 10⁶ psi

MASS VALUES BY MASS NUMBER:

1	0.5 lb-sec ² /in	9	1.0
2	1.0	10	1.0
3	1.0	11	0.5
4	2.0	12	2.0
5	1.0	13	3.0
6	1.0	14	4.0
7	1.0	15	3.0
8	2.0	16	2.0

TOTAL MASS: 26.0 lb-sec²/in
 TOTAL WEIGHT: 10036 lb

Fig. 2 - Mathematical model

variation of this model was also analyzed by removing spring 5.

The forces in springs 1 thru 5 may be single valued functions of deflection which can be represented by straight line segments. This includes linear and bilinear springs, for example. A straight line segment representation was chosen since many force-deflection functions for nonlinear mounts are specified in this manner. For convenience, three spring equations were developed to generate straight line segment representations of nonlinear functions:

$$P(S) = C_1 S + C_2 S^3. \quad (\text{Duffing's spring}) \quad (11)$$

$$P(S) = \frac{C_1 S}{(1-S/C_2)(1+S/C_2)} \quad (\text{asymptotic}), \quad (12)$$

$$P(S) = C_1 S + C_2 \frac{S^7}{|S|} \quad (\text{6th power}). \quad (13)$$

The discrete force-deflection curves of these functions are developed by specifying the number of points desired, the maximum deflection expected in the spring, and the constants C_1 and C_2 . Force values are then calculated at equal deflection intervals. Alternately, any general curve can be described point by point. All of the springs considered were single valued, had initially zero force and deflection and were considered symmetric in tension and compression.

COMPUTER PROGRAM

A simplified flow chart of the Fortran program and subroutines used to analyze the model and test the nonlinear method is shown in Figure 3.

The program starts by calling subroutine THURSDY which reads in parameters similar to the actual DDAM parameters necessary to calculate the response in each mode. Control passes to subroutine NONLIN which reads in the model starting parameters. These parameters consist of the following: printout controls; the number of nonlinear springs in the model; the maximum allowed number of iterations before quitting without convergence; the convergence force; a description of each spring including type, number of points, constants and connectivity; and finally the initial spring constants to start the solution.

The next subroutines in the flow chart constitute a solution block. Hence, although specially written here, these subroutines could be replaced by a more general program such as NASTRAN. The sequence of operations here is similar to what would occur in any solution block which solves for natural frequencies and mode shapes. The stiffness matrix is generated from the model geometry which includes elastic moduli, moments of inertia and spring constants. On subsequent passes, only the spring constant values of the nonlinear springs will change and the stiffness matrix must be accordingly updated. For the 1st pass only, the mass matrix is assembled.

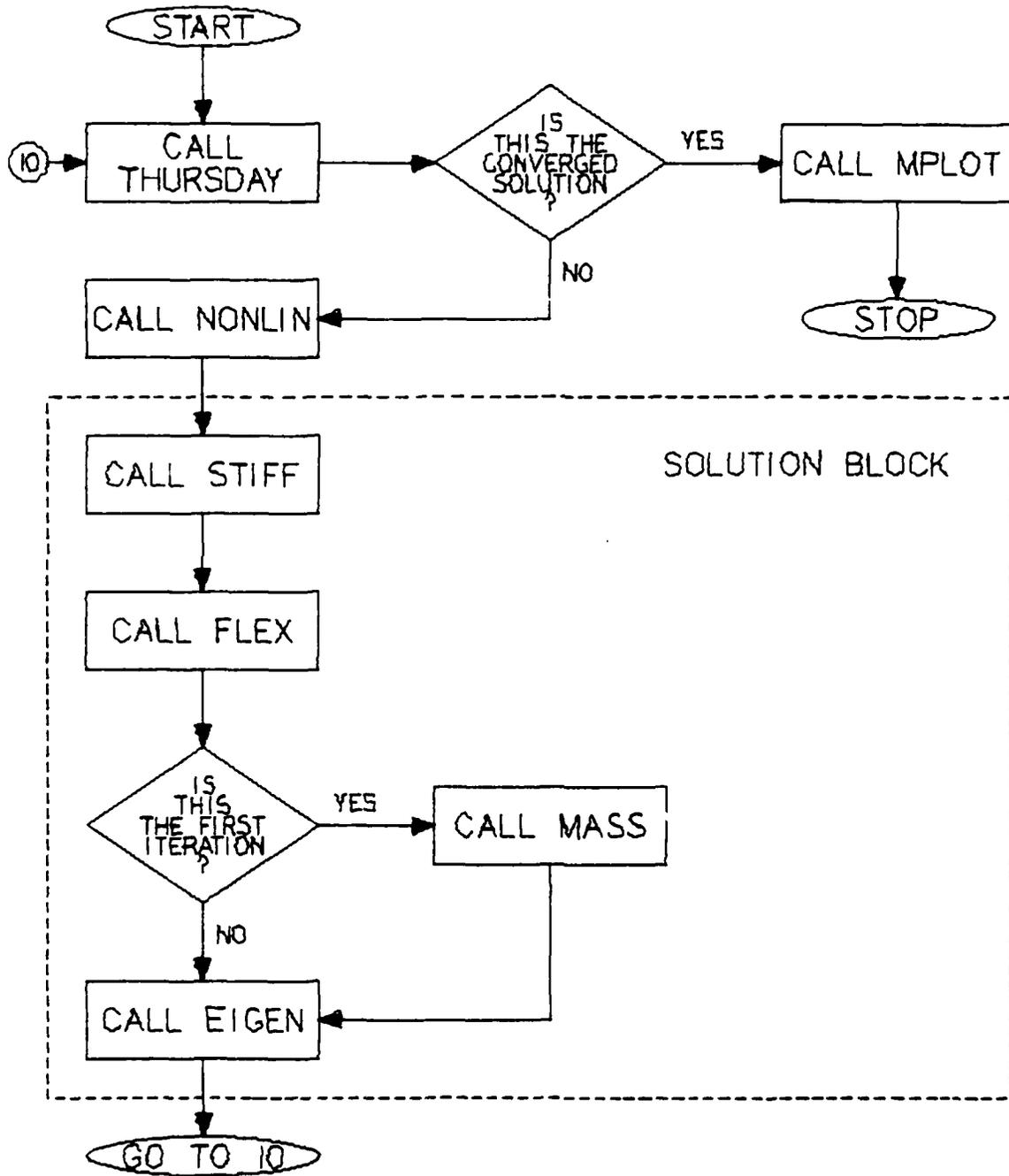


Fig. 3 - Main program flow

For subsequent iterations the mass matrix does not change. Subroutine EIGEN extracts the mode shapes and natural frequencies. Then control returns to THURSDY.

When THURSDY is called for the second and subsequent times, it generates the displacement responses, U_{ra} , of the masses for each mode. If the solution has not converged, control then passes again to NONLIN where the bulk of the work of this method occurs.

When NONLIN is called for the second and subsequent times, a complete evaluation of the status of the solution is performed. Figure 4 contains a simplified flow chart of NONLIN. First the modal forces in each spring are calculated using equation 8. Then the total force combining modes is determined using equation 9. If a previous iteration exists, then convergence is checked. If convergence has occurred, control is again passed to the solution block and the converged solution is printed out.

If the solution has not converged or if this is the first iteration, the next step is to determine spring constants for the next iteration. This is done by entering the force-deflection curve of each spring and finding the new deflection for the calculated force. A new spring constant is then generated for the next iteration by dividing the force by the new deflection.

For some situations convergence will not occur by this method of selection of the spring constant. The particular solution will characteristically "bounce" about the true value indefinitely. These situations are due to a nearly horizontal slope of the force-deflection curve of a particular spring. For these cases the spring constant is more accurately selected from the displacement intercept (point 3' in figure 1). Logic has been included to detect "bouncing" and switch to the alternate selection method for the particular spring. The spring is detected by choosing the largest reversing, repetitive force difference between iterations among all the nonlinear springs whose absolute value is within 10% of the previous force difference absolute value.

Once a converged solution has been found, control passes from THURSDY to a plotting routine which generates graphical output for review. In a more general case, this could of course represent a post-processor for combining modal results and/or obtaining structural response plots.

SAMPLE CASE

The solution method described was applied to the mathematical model for a variety of nonlinear springs and initial spring constants. Results for a typical case are summarised in figures 5 to 10. The redundant model with 5 nonlinear springs was used.

Figure 5 shows the highly nonlinear springs which were used in this test case. Springs 1 thru 4 are all asymptotic springs (equation 12) with asymptotes at 1.0, 2.0, 4.0 and 8.0 inches, respectively. Initial spring rates, C_1 , are 100000, 50000, 25000 and 12500 lbs/in, respectively. Hence, each spring is approximately half the strength of the previous. Spring 5 is represented by the

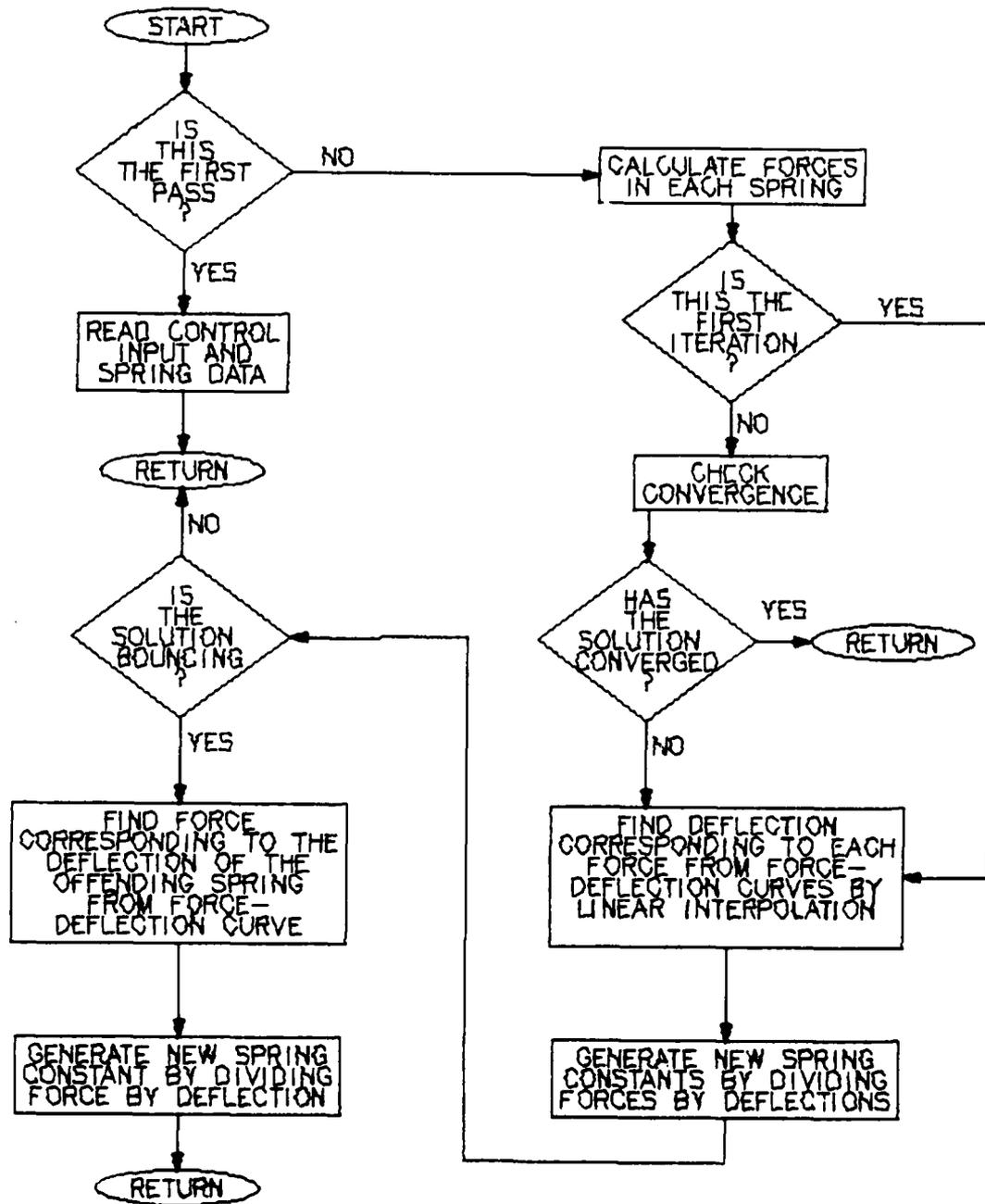


Fig. 4 - Subroutine NONLIN

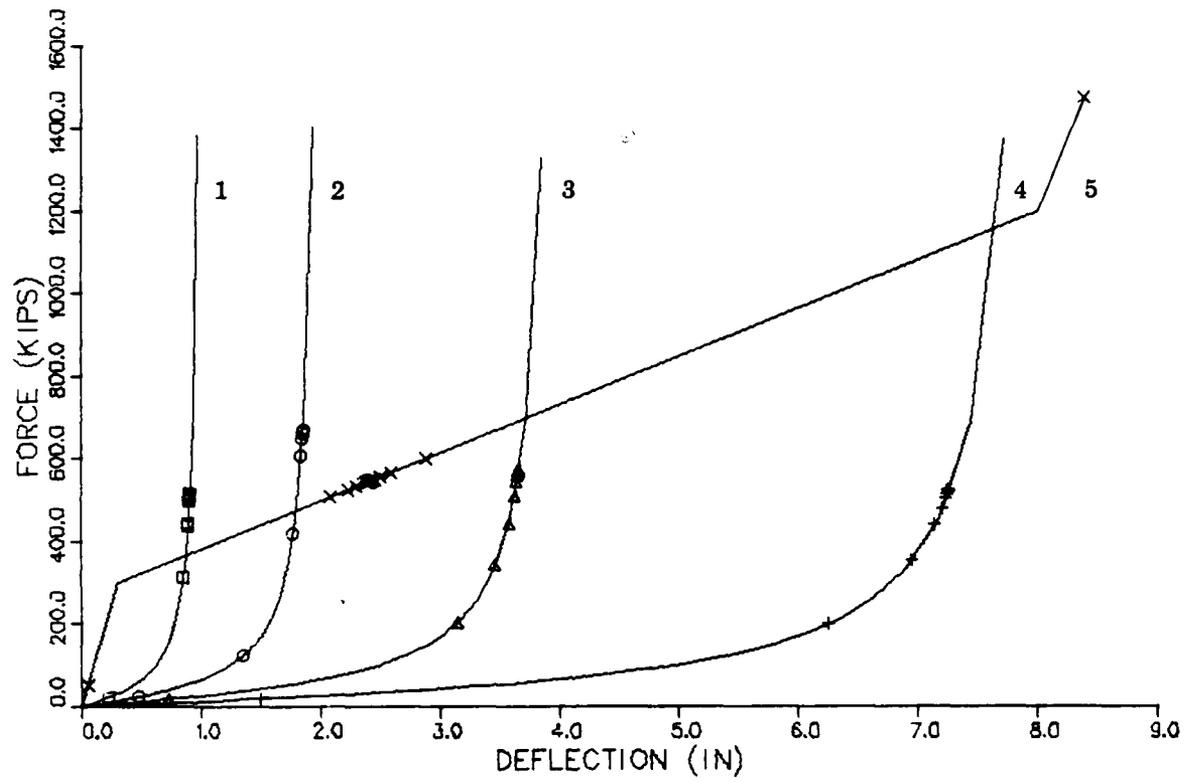


Fig. 5 - Force-deflection convergence

three segment force-deflection curve, as shown. Symbols plotted on the curves represent the P intercepts for each iteration.

For this particular case, starting spring constants were chosen to be below two kips/in. For all of the asymptotic springs a steady climb up the force-deflection curve occurs as the iterations progress. Forces along spring 5 are seen to "bounce" about the final converged position. The "bouncing," however, is not severe enough to force selection of the next spring constant by the deflection intercept (figure 1, K").

Figure 6 illustrates the change in spring constants as the solution progresses. This solution, which was an attempt to diverge, converged after 25 iterations with a convergence force difference of 100 lbs. A flatter version of spring 5 tested in another case converged in only 11 iterations because the deflection intercept method was turned on.

Figure 7 illustrates the significant natural frequencies of the system for each iteration. The significant mode shapes are illustrated in figures 8 thru 10. These three modes contribute 98% of the modal weight.

EMPIRICAL RESULTS

A total of 21 combinations of springs and starting spring constants were evaluated. Convergence was achieved in every case. The largest number of iterations required was 25 for the example cited. Other samples converged within five to 12 iterations including some which converged to a force difference of only one pound. Study of the graphs typically showed that the spring constants and mode shapes tended to stabilize in less than half the converged iterations for practical purposes.

Convergence occurred for starting spring constants ranging from 10^{-6} to 10^6 times the converged values, with the starting values affecting only the number of iterations required. The relative stiffnesses of nonlinear springs caused no problems. Ratios of highest to lowest converged spring rate as high as 1200 posed no problem in convergence. Both statically determinate and indeterminate structures converged. Symmetrically placed springs with identical force-deflection curves converged without "see-sawing." Discontinuities in frequency curves (see Figure 7) indicate that mode shape cross-overs can occur without preventing convergence.

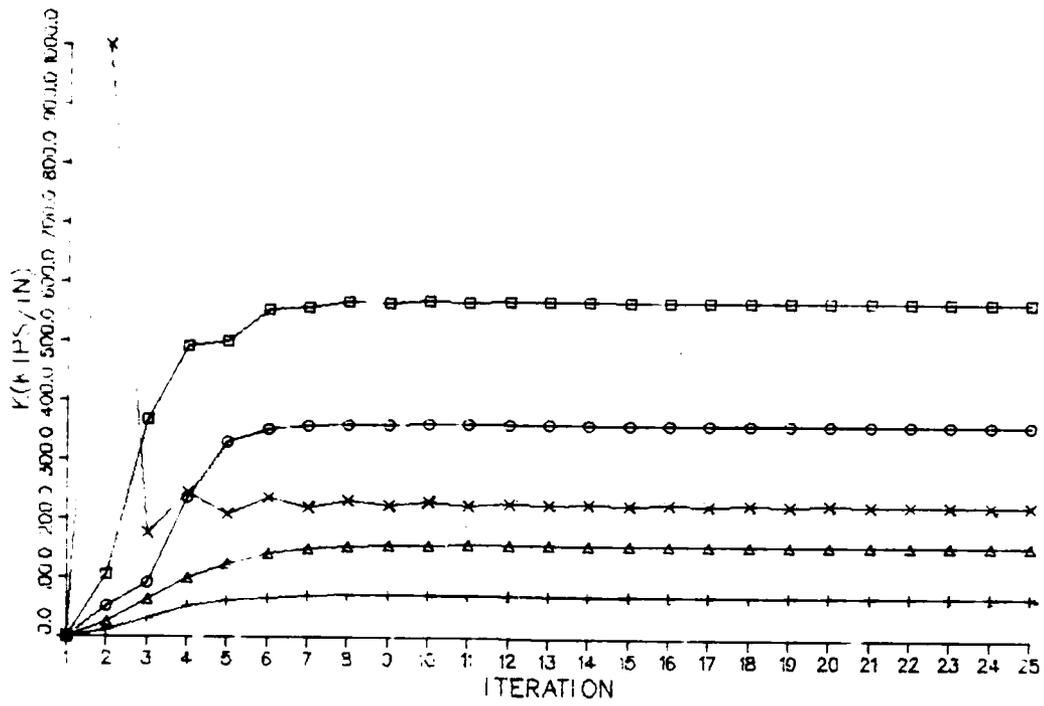


Fig. 6 — Spring constants versus iteration

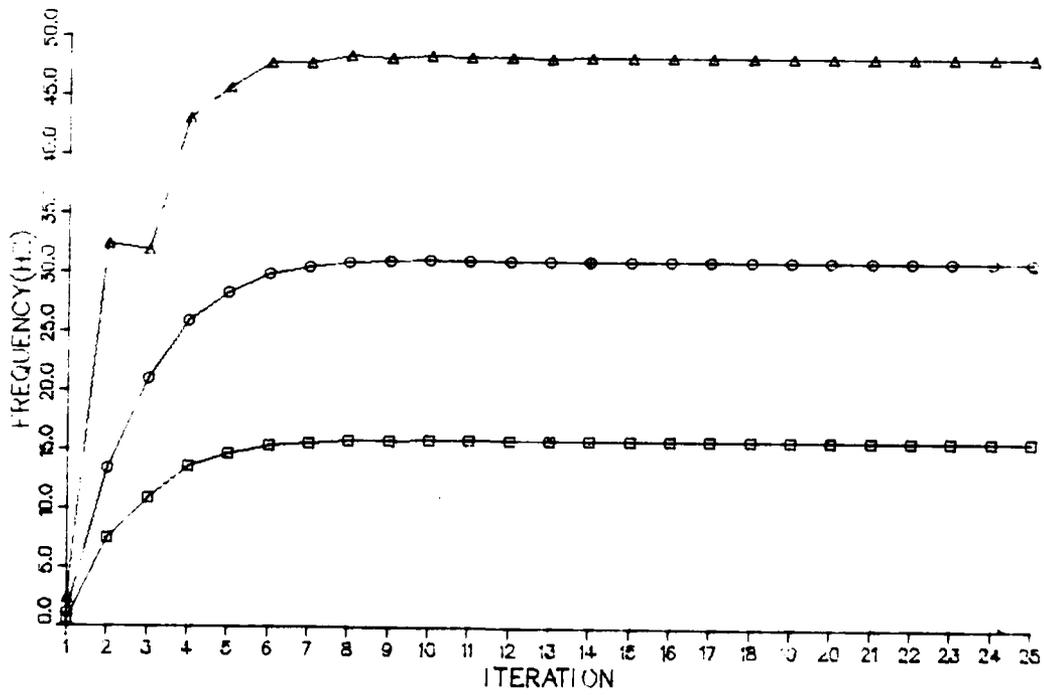


Fig. 7 — Significant frequencies versus iteration

Mode 1, Freq 15.9, Max Defl 6.63 Inches

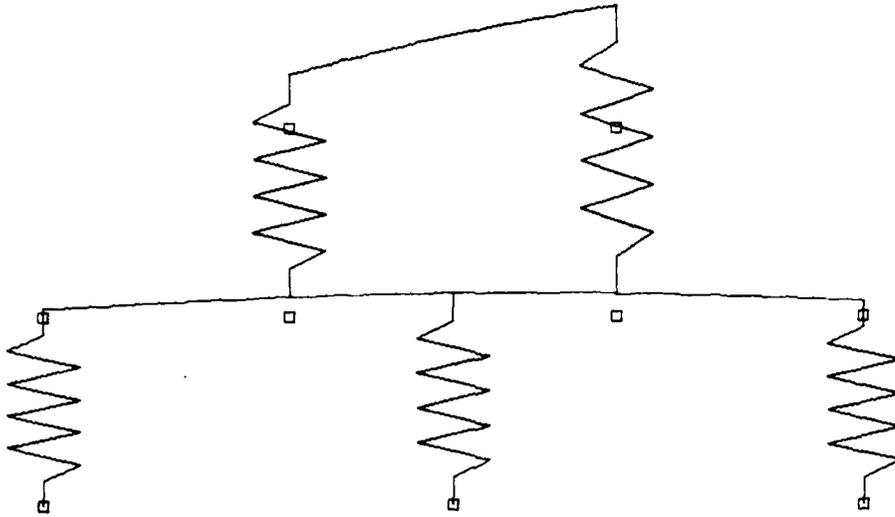


Fig. 8 -- Significant mode shapes

Mode 2, Freq 31.1, Max Defl 1.57 Inches

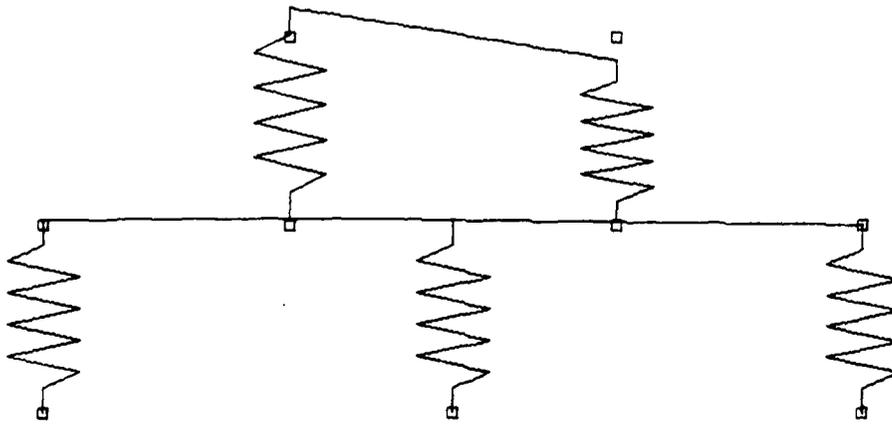


Fig. 9 -- Significant mode shapes

Mode 3, Freq 48.4, Max Defl 1.24 Inches

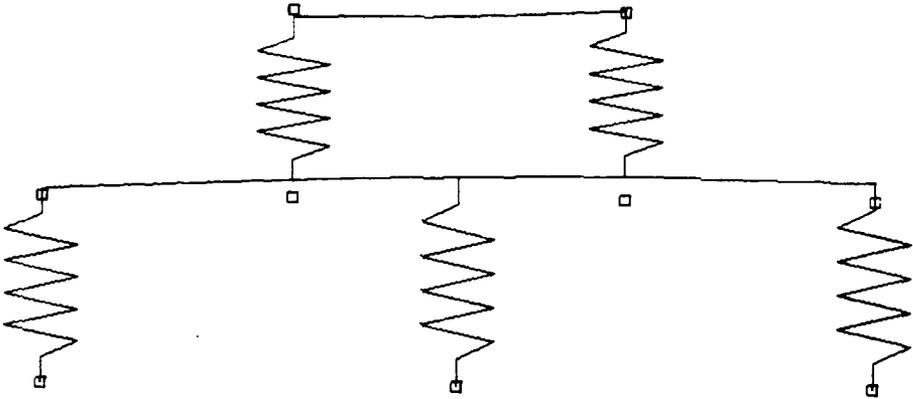


Fig. 10 - Significant mode shapes

CONCLUSIONS

1. The empirical results show that for this problem the non-linear application of DDAM converges for a variety of adverse and unusual conditions.

2. Convergence is rapid for most cases even for a poor choice of initial spring constants. The empirical results suggest, however, that faster convergence rates can be achieved by using a better selection rule for the spring constants using both the P and S intercepts of figure 1. The rates might also be improved by a consideration of the trajectory of solution points (line AB in figure 1).

3. It appears that the method demonstrated could provide a numerically improved and much less labor intensive method for solving the problems posed in reference 1 and 2 and other problems where the response of nonlinear shipboard equipment to shock must be estimated by using a normal mode approach.

4. The present study shows that the method is mathematically practicable. Physical accuracy of the approximation and the applicability of linear design-spectrum methods to nonlinear systems in a meaningful way were not investigated.

RECOMMENDATIONS

Based upon the results of this study, the following steps are recommended:

1. Perform comparisons of structures analyzed with this method and known solutions or test results. Use these comparisons to determine the range of validity of the method and to guide selection of parameters within the method to insure best solution.

2. Link the demonstrated method to an existing model building and equation solving computer program such as NASTRAN. This will permit a wider variety of test problems, three-dimensional models and realistic solution time information for real problems.

3. Expand the demonstrated method to include:

A. Faster convergence rates as mentioned in item 2 of conclusions.

B. Unsymmetrical springs for tension and compression.

C. Calculation of the loaded position of the nonlinear springs from a consideration of static equilibrium.

D. The effects of nonlinear coupling of springs to other co-ordinate directions.

E. A library of standard noise mounts.

REFERENCES

1. H. Bamford, G. W. Bishop, R. Ravenhall, and I. Schmack, "Addendum to Stress Report Main Propulsion Shafting for ATLANTA Class Cruiser Dynamic Shock Analysis," Princeton, New Jersey. Bishop Engineering Company Report 194-3 Addendum A (July 20, 1968).
2. Anonymous, "Technical Manual Structural Analysis Procedure for Sound-Isolated Piping Systems," Naval Ship Systems Command NAVSHIPS 0948-105-3010, 1 April 1974.
3. G.J. O'Hara, P.F. Cunniff, "Elements of Normal Mode Theory," NRL Report 6002, 15 November 1963.

ATE
LMED
— 8