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A NEW WAY FOR CONSTRUCTING HIGHER ORDER ACCURACY SPLINE SMOOTHI--ETC(U)

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A NEW WAY FOR CONSTRUCTING HIGHER ORDER ACCURACY SPLINE SMOOTHING FORMULAS

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A NEW WAY FOR CONSTRUCTING HIGHER ORDER ACCURACY SPLINE SMOOTHING FORMULAS

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ABSTRACT

In this paper the author introduces the operator $\bar{\Delta}^{(n)} := P_n(\mu)\bar{\Delta}$ with higher order accuracy for approximation to the differential operator D, where $\bar{\Delta}$ denotes centered difference operator, μ denotes averaging operator,

$$P_n(\mu) = \sum_{m=0}^{n} C_m(\mu - I)^m, \quad C_m = -\frac{m}{2m+1} C_{m-1}, \quad C_0 = 1 \ .$$

A class of new many-knot spline basis $\Omega_{k,n} := (P_n(\mu))^{\ell} N_k$ was suggested. The smoothing formulas

$$f_{k,n} = \frac{1}{h}\int_{-\infty}^{\infty} \Omega_{k,n}\left(\frac{\cdot - t}{h}\right) f(t)\,dt \quad \text{and} \quad S_{k,n}f = \sum f_i \Omega_{k,n}$$

are discussed.

AMS (MOS) Subject Classification: 41A15

Key Words: Spline, smoothing, many-knot, Higher order accuracy

Work Unit Number 3 - Numerical Analysis and Computer Science

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SIGNIFICANCE AND EXPLANATION

I. J. Schoenberg studied B-splines and established some smoothing formulas for fitting data. In particular the smoothing approximation $S_k f = \sum f_i N_{i,k}$ (where $N_{i,k}$ are B-splines and f is an arbitrary function) has been successfully used in curve fitting. The paper proposes a new class of spline function denoted $\Omega_{i,k}$ instead of $N_{i,k}$. The new approximation $S_{k,n} f = \sum f_i \Omega_{i,k}$ achieves higher order accuracy. To construct $\Omega_{i,k}$, we first introduce the averaging operator $P_n(\mu)$, $P_n(x) = \sum_{m=0}^{n} C_m (x-1)^m$, $C_m = -\frac{m}{2m+1} C_{m-1}$, $C_0 = 1$, and then define $\Omega_{i,k} := [P_n(\mu)]^{\ell} N_{i,k}$. The smoothing formulas for function f are given by $f_{k,n} = \frac{1}{h} \int_{-\infty}^{\infty} \Omega_{k,n}(\frac{\cdot - t}{h}) f dt$ and $S_{k,n} f = \sum f_i \Omega_{k,n}$.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

A NEW WAY FOR CONSTRUCTING HIGHER ORDER ACCURACY SPLINE SMOOTHING FORMULAS

Dong-Xu Qi*

The modern mathematical theory of spline approximation was introduced by I. J. Schoenberg in 1946. In the paper [6] he studied so-called "B-spline basis". A B-spline basis can be normalized in various ways. One of them is the so called normalized B-spline, see [2], denoted by $N_{i,k}$ for the B-spline function of degree $k - 1$ having support (x_i, x_{i+k}). The spline smoothing formula for degree $k - 1$ to an arbitrary function f can be represented by $S_k f = \sum f_i N_{i,k}$. This approximation has been used in curve fitting successfully [1], [4].

In order to improve accuracy of the smoothing operator S_k, the author in this paper suggests a new spline basis denoted $\Omega_{i,k,n}$ instead of $N_{i,k}$. Thus, a new way for the construction of spline smoothing formulas is introduced. I prefer calling $S_{k,n} f = \sum f_i \Omega_{i,k,n}$ a <u>smoothing operator with grade</u> n <u>and order</u> k. In here when $n = 0$, $\Omega_{i,k,0}$ is just $N_{i,k}$ and $S_{k,0}$ is the same as S_k. Since $S_{k,n} f \in \varphi_k + \varphi_k^*$, this is a class of many-knot splines.

Concerning higher order accuracy spline smoothing formulas, I. J. Schoenberg [1946] has already discussed in [6] and Z. S. Liang studied the many-knot spline smoothing [4]. My main attempt in this paper is to suggest a new way for constructing them.

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1. The smoothing operator

Denote the centered difference operator by $\bar{\Delta}_h$, defined by

$$\bar{\Delta}_h f(x) := f(x + \frac{h}{2}) - f(x - \frac{h}{2}) \quad .$$

For simplicity let $h = 1$, and $\bar{\Delta} := \bar{\Delta}_1$.

The B-spline of order k with equally spaced knots are denoted by N_k, and it can be represented by

$$N_k(x) = \left(\bar{\Delta} D^{-1}\right)^k \delta(x) \quad , \tag{1.1}$$

$$N_{i,k}(\cdot) := N_k(\cdot - i) \quad , \tag{1.2}$$

where D^{-1} is the integral operator, δ is Dirac δ-function.

It is our purpose to find a more exact difference approximation to the operator D. I would like to choose following ready-made identity.

Fact 1.1 ([5] p. 43)

$$\log(y + \sqrt{1 + y^2}) = \sqrt{1 + y^2} \sum_{m=0}^{\infty} (-1)^m \frac{2^{2m}(m!)^2}{(2m+1)!} y^{2m+1} \quad . \tag{1.3}$$

Fact 1.2 The following expansion

$$x = \mathrm{sh}\, x \sum_{m=0}^{\infty} c_m(\mathrm{ch}\, x - 1)^m \tag{1.4}$$

holds. Set

$$(2m+1)!! := (2m+1)(2m-1)\ldots 3.1 \quad ,$$

then

$$c_m = -\frac{m}{2m+1} c_{m-1} = (-1)^m \frac{m!}{(2m+1)!!}, \; c_0 = 1 \quad .$$

Proof From (1.3)

$$\log(y + \sqrt{1 + y^2}) = \sqrt{1 + y^2} \sum_{m=0}^{\infty} (-1)^m \frac{m!}{(2m+1)!!} 2^m y^{2m+1}$$

$$= \sqrt{1 + y^2} \sum_{m=0}^{\infty} c_m 2^m y^{2m+1} \quad .$$

Let $y = \operatorname{sh} \frac{x}{2}$, then $\operatorname{ch} \frac{x}{2} = \sqrt{1 + y^2}$, $x = 2 \log(y + \sqrt{1 + y^2})$. Thus

$$x = 2 \operatorname{ch} \frac{x}{2} \sum_{m=0}^{\infty} c_m 2^m \left(\operatorname{sh} \frac{x}{2}\right)^{2m+1}$$

$$= 2 \operatorname{ch} \frac{x}{2} \operatorname{sh} \frac{x}{2} \sum_{m=0}^{\infty} c_m \left(2 \operatorname{sh}^2 \frac{x}{2}\right)^m$$

$$= \operatorname{sh} x \sum_{m=0}^{\infty} c_m (\operatorname{ch} x - 1)^m \quad .$$

∎

Introduce operators E and μ_α defined by

$$E^\alpha f(x) := f(x + \alpha) \quad ,$$

$$\mu_\alpha f(x) := \frac{1}{2} \left(f(x + \frac{\alpha}{2}) + f(x - \frac{\alpha}{2})\right), \quad \mu := \mu_1 \quad ,$$

and notice the relationships between those operators (see [3], p. 230)

$$E = e^D, \qquad \operatorname{ch} \frac{D}{2} = \mu \quad ,$$

$$2 \operatorname{sh} \frac{D}{2} = e^{\frac{D}{2}} - e^{-\frac{D}{2}} = E^{\frac{1}{2}} - D^{-\frac{1}{2}} = \bar{\Delta} \quad .$$

Use $\frac{D}{2}$ and I instead of x and 1 in (1.4)

$$D = 2\,\mathrm{sh}\,\frac{D}{2}\sum_{m=0}^{\infty} C_m\left(\mathrm{ch}\,\frac{D}{2} - I\right)^m$$

$$= \sum_{m=0}^{\infty} C_m(\mu - I)^m\overline{\Delta}$$

$$= \sum_{m=0}^{n} C_m(\mu - 1)^m\overline{\Delta} + R_n \quad , \tag{1.5}$$

where

$$R_n := 2\,\mathrm{sh}\,\frac{D}{2}\sum_{m=n+1}^{\infty} C_m\left(\mathrm{ch}\,\frac{D}{2} - I\right)^m \quad . \tag{1.6}$$

Define $\overline{\Delta}^{(n)}$ as the first part of (1.5), i.e.,

$$\overline{\Delta}^{(n)} := \sum_{m=0}^{n} C_m(\mu - I)^m\overline{\Delta} = P_n(\mu)\overline{\Delta} \quad ,$$

where

$$P_n(\mu) = \sum_{m=0}^{n} C_m(\mu - I)^m = \sum_{j=0}^{n} 2^{-j}\sum_{m=j}^{n}(-1)^{m-j}\binom{m}{j}C_m\sum_{i=0}^{j}\binom{j}{i}E^{\frac{j}{2}-i} \quad . \tag{1.7}$$

In the general case, define

$$\overline{\Delta}_h^{(n)} := P_n(\mu_h)\overline{\Delta}_h \quad . \tag{1.8}$$

This is a collection of operators approximate to D. Beyond doubt $P_n(1) = 1$, $P_0(\mu) = I$.

<u>Fact 1.3</u> If k is any nonnegative integer, then the sum of all coefficients of items $(\mu_h)^j$ in the expansion $(P_n(\mu_h))^k$ equals to 1.

Notice (1.6), the first term in R_n for any h

$$C_{n+1}D\left[\frac{1}{2!}\left(\frac{hD}{2}\right)^2\right]^{n+1} = 2^{-3(n+1)}C_{n+1}h^{2(n+1)}D^{2n+3} \quad . \tag{1.9}$$

This implies the following:

Theorem 1.1 Assume that $f \in C^{2n+3}$. Then

$$\bar{\Delta}_h^{(n)} f(x) = Df(x) - 2^{-3(n+1)} C_{n+1} f^{(2n+3)}(\xi) h^{2(n+1)}$$

where $\xi \in [x - \frac{n+1}{2} h, x + \frac{n+1}{2} h]$.

Definition We call the operator $(\bar{\Delta}_h^{(n)} D^{-1})^{\ell}$ a smoothing operator with grade n and degree ℓ.

It is to be noted that $(\bar{\Delta}_h^{(0)} D^{-1})^k$ is just as with I. J. Schoenberg's. Here it is the smoothing operator of grade 0 and degree k.

Fact 1.4 From Theorem 1.1, if $g \in P_{2n+1}$ on $[a,b]$, then $\bar{\Delta}_h^{(n)} D^{-1} g = g$, all $x \in [a + \frac{n+1}{2} h, b - \frac{n+1}{2} h]$.

2. A class of many-knot splines

As has been already pointed out, the B-spline N_k with equally spaced knots $(h = 1)$ is the result of the 0-th grade smoothing operator applied to the Dirac δ-function

$$N_k = (\bar{\Delta}D^{-1})N_{k-1} = (\bar{\Delta}D^{-1})^k\delta \quad . \tag{2.1}$$

Now we use the smoothing operator $\bar{\Delta}^{(n)}D^{-1}$ of grade n for the δ-function repeatedly. We can define a class of spline functions which as more knots than N_k:

$$\Omega_{k,n} := (P_n(\mu))^{\ell} N_k \tag{2.2}$$

and

$$\Omega_{i,k,n}(\cdot) := \Omega_{k,n}(\cdot - i) \quad .$$

If $\ell = k$, then $\Omega_{k,n}(x) = (\bar{\Delta}^{(n)})^k\{\frac{x_+^{k-1}}{(k-1)!}\}$ which has knots

$$\xi_j^{(n,k)} = -\frac{(n+1)k-j}{2},\ j = 0,1,\ldots,2(n+1)k,\ n > 0 \quad .$$

We often take $\ell = k$ if without note.

The following facts can be proved easily in the same way as the corresponding facts for N_k.

<u>Fact 2.1</u>:

(1) $\Omega_{k,n}(x) = \Omega_{k,n}(-x)$;

(2) $\Omega_{k,n}(x) = 0$ for all $|x| > \frac{(n+1)k}{2}$;

(3) $D^m\Omega_{k,n}(x) = (\bar{\Delta}^{(n)})^m\Omega_{k-m,n}(x),\ 0 < m < k$;

(4) $D^{-m}\Omega_{k,n}(x) = (\bar{\Delta}^{(n)})^{\ell}\{x_+^{k+m-1}/(k+m-1)!\},\ m > 0$;

(5) $\sum_{j=-\infty}^{\infty} \Omega_{k,n}(x + j) = 1,\ \int_{-\infty}^{\infty} \Omega_{k,n}(x)dx = 1$;

(6) $\Omega_{k,n}$ can be represented by the convolution integral

$$\Omega_{k,n}(\cdot) = \int_{-\infty}^{\infty} \Omega_{k-1,n}(\cdot-t)\Omega_{0,n}(t)dt \quad ;$$

(7) $$\Omega_{k,n}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{\Omega}_{k,n}(\xi)e^{i\xi x}d\xi$$

$$\hat{\Omega}_{k,n}(\xi) = \int_{-\infty}^{\infty} e^{-i\xi x}\Omega_{k,n}(x)dx$$

$$= [\frac{\sin(\xi/2)}{\xi/2} P_n(\cos \frac{\xi}{2})]^k$$

(8) Integration by parts:

$$\int_{-\infty}^{\infty} \Omega_{k,n}(x)f(x)dx = (\bar{\Delta}^{(n)}D^{-1})^k f(0).$$

From the above mentioned facts we have the following theorems:

Theorem 2.1 Assume f is a continuous function or with discontinuity of the first kind on $[a,b]$, and is extended with period $b-a$ to $(-\infty,\infty)$, then

$$\lim_{h\to 0} \int_{-\infty}^{\infty} \delta_h(x-t)f(t)dt = \frac{1}{2}(f(x+0) + f(x-0)) \quad ;$$

If f is a function whose derivatives of order ℓ is continuous or is a discontinuity of the first kind on $[a,b]$, then

$$\lim_{h\to 0} \int_{-\infty}^{\infty} \frac{d^\ell}{dx^\ell} \delta_h(x-t)f(x)dt = \frac{1}{2}(f^{(\ell)}(x+0) + f^{(\ell)}(x-0)) \quad ,$$

where

$$\delta_h(x) := \frac{1}{h} \Omega_{k,n}(\frac{x}{h}) \quad .$$

This Theorem shows that the many-knot spline function δ_h converges weakly to the Dirac δ-function.

Theorem 2.2 Given the function f, define its many-knot spline smoothing function by

$$f_{k,n} := \bar{\Delta}_h^{(n)}D^{-1}f_{k-1,n} = (\bar{\Delta}_h^{(n)}D^{-1})^k f \quad . \tag{2.3}$$

Then

$$f_{k,n} = \frac{1}{h} \int_{-\infty}^{\infty} \Omega_{k,n}\left(\frac{\cdot - t}{h}\right) f(t)\,dt \quad . \tag{2.4}$$

<u>Theorem 2.3</u> If $f \in C^{\ell}(-\infty,\infty)$, then

$$\int_{-\infty}^{\infty} |f_{k,n}^{(\ell)}(x)|^2 dx \le \int_{-\infty}^{\infty} |f^{(\ell)}(x)|^2 dx \quad . \tag{2.5}$$

<u>Proof</u> Take the derivative of order ℓ for (2.4), and the integration by parts, and notice that

$$\left|\frac{\sin x}{x} P_n(\cos x)\right| \le 1 \quad .$$

If f is a discrete valued function $y_i = f(x_i)$, $x_i = x_0 + ih$, then a numerical smoothing formula is as follows:

$$S_{k,n} f := \sum_j y_j \Omega_{k,n}\left(\frac{\cdot - x_j}{h}\right) \quad . \tag{2.6}$$

Formula (2.6) can be efficiently applied to the problems of curve fitting for discrete data.

3. Examples

In this section some discussions which are helpful for applications in practice will be given.

From (2.2), with $\ell = k$, $n = 1$, $k = 1,2,3,4$, we show the particular representations as follows:

$$\Omega_{1,1}(x) = \begin{cases} \frac{7}{6}, & |x| < \frac{1}{2}, \\ \frac{1}{2}, & |x| = \frac{1}{2}, \\ -\frac{1}{6}, & \frac{1}{2} < |x| < 1, \\ -\frac{1}{12}, & |x| = 1, \\ 0, & |x| > 1; \end{cases}$$

$$\Omega_{2,1}(x) = \begin{cases} \frac{50}{36} - \frac{65}{36}|x|, & |x| < \frac{1}{2}, \\ \frac{42}{36} - \frac{49}{36}|x|, & \frac{1}{2} < |x| < 1, \\ -\frac{22}{36} + \frac{15}{36}|x|, & 1 < |x| < \frac{3}{2}, \\ \frac{2}{36} - \frac{1}{36}|x|, & \frac{3}{2} < |x| < 2, \\ 0, & |x| > 2; \end{cases}$$

$$\Omega_{3,1}(x) = \begin{cases} \frac{462}{432} - \frac{878}{432}x^2, & |x| < \frac{1}{2}, \\ \frac{858}{432} - \frac{1584}{432}|x| + \frac{706}{432}x^2, & \frac{1}{2} < |x| < 1, \\ \frac{471}{432} - \frac{810}{432}|x| + \frac{319}{432}x^2, & 1 < |x| < \frac{3}{2}, \\ -\frac{627}{432} + \frac{654}{432}|x| - \frac{169}{432}x^2, & \frac{3}{2} < |x| < 2, \\ \frac{141}{432} - \frac{114}{432}|x| + \frac{23}{432}x^2, & 2 < |x| < \frac{5}{2}, \\ -\frac{9}{432} + \frac{6}{432}|x| - \frac{1}{432}x^2, & \frac{5}{2} < |x| < 3, \\ 0, & |x| > 3, \end{cases}$$

$$\Omega_{4,1}(x) = \begin{cases} \frac{7920}{7776} - \frac{20556}{7776}x^2 + \frac{13059}{7776}|x|^3, & |x| < \frac{1}{2}, \\ \frac{8444}{7776} - \frac{3144}{7776}|x| - \frac{14268}{7776}x^2 + \frac{8867}{7776}|x|^3, & \frac{1}{2} < |x| < 1, \\ \frac{25212}{7776} - \frac{53448}{7776}|x| + \frac{36036}{7776}x^2 - \frac{7901}{7776}|x|^3, & 1 < |x| < \frac{3}{2}, \\ \frac{4152}{7776} - \frac{11328}{7776}|x| + \frac{7956}{7776}x^2 - \frac{1661}{7776}|x|^3, & \frac{3}{2} < |x| < 2, \\ -\frac{22440}{7776} + \frac{28560}{7776}|x| - \frac{11988}{7776}x^2 + \frac{1663}{7776}|x|^3, & 2 < |x| < \frac{5}{2}, \\ \frac{9060}{7776} - \frac{9240}{7776}|x| + \frac{3132}{7776}x^2 - \frac{353}{7776}|x|^3, & \frac{5}{2} < |x| < 3, \\ -\frac{1308}{7776} + \frac{1128}{7776}|x| - \frac{324}{7776}x^2 + \frac{31}{7776}|x|^3, & 3 < |x| < \frac{7}{2}, \\ \frac{64}{7776} - \frac{48}{7776}|x| + \frac{12}{7776}x^2 - \frac{1}{7776}|x|^3, & \frac{7}{2} < |x| < 4, \\ 0, & |x| > 4. \end{cases}$$

From (2.2) the following tables are given:

Table 1:

x	$N_1(x)$	$\Omega_{1,1}(x)$
0	1	$\frac{7}{6}$
$\pm\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
± 1		$-\frac{1}{12}$

Table 2:

0	$N_2(x)$	$\Omega_{2,1}(x)$	
		$\ell = 1$	$\ell = 2$
0	1	$\frac{7}{6}$	$\frac{100}{72}$
$\pm\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{35}{72}$
± 1		$-\frac{1}{12}$	$-\frac{14}{72}$
$\pm\frac{3}{2}$			$\frac{1}{72}$

Table 3

x	$N_3(x)$	$\Omega_{3,1}(x)$			$\Omega'_{3,1}(x)$
		$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 3$
0	$\frac{3}{4}$	$\frac{40}{48}$	$\frac{270}{288}$	$\frac{1848}{1728}$	0
$\pm \frac{1}{2}$	$\frac{1}{2}$	$\frac{25}{48}$	$\frac{156}{288}$	$\frac{970}{1728}$	$\mp \frac{878}{432}$
± 1	$\frac{1}{8}$	$\frac{4}{48}$	$\frac{8}{288}$	$-\frac{80}{1728}$	$\mp \frac{172}{432}$
$\pm \frac{3}{2}$		$-\frac{1}{48}$	$-\frac{12}{288}$	$-\frac{105}{1728}$	$\pm \frac{147}{432}$
± 2			$\frac{1}{288}$	$\frac{20}{1728}$	$\mp \frac{22}{432}$
$\pm \frac{5}{2}$				$-\frac{1}{1728}$	$\pm \frac{1}{432}$

Table 4

x	$N_4(x)$	$\Omega_{4,1}(x)$				$\Omega'_{4,1}(x)$	$\Omega''_{4,1}(x)$
		$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$	$\ell = 4$	
0	$\frac{2}{3}$	$\frac{210}{288}$	$\frac{1392}{1728}$	$\frac{9332}{10368}$	$\frac{63360}{62208}$	0	$-\frac{13704}{2592}$
$\pm \frac{1}{2}$	$\frac{23}{48}$	$\frac{144}{288}$	$\frac{902}{1728}$	$\frac{5647}{10368}$	$\frac{35307}{62208}$	$\mp \frac{14349}{10368}$	$-\frac{645}{2592}$
± 1	$\frac{1}{6}$	$\frac{40}{288}$	$\frac{176}{1728}$	$\frac{545}{10368}$	$-\frac{808}{62208}$	$\mp \frac{6772}{10368}$	$\frac{8222}{2592}$
$\pm \frac{3}{2}$	$\frac{1}{48}$	0	$-\frac{39}{1728}$	$-\frac{480}{10368}$	$-\frac{4359}{62208}$	$\pm \frac{1771}{10368}$	$\frac{321}{2592}$
± 2		$-\frac{1}{288}$	$-\frac{8}{1728}$	$-\frac{26}{10368}$	$\frac{256}{62208}$	$\pm \frac{752}{10368}$	$-\frac{1340}{2592}$
± 5			$\frac{1}{1728}$	$\frac{16}{10368}$	$\frac{155}{62208}$	$\mp \frac{265}{10368}$	$\frac{323}{2592}$
± 3				$\frac{1}{10368}$	$-\frac{24}{62208}$	$\pm \frac{28}{10368}$	$-\frac{30}{2592}$
$\pm \frac{7}{2}$					$\frac{1}{62208}$	$\mp \frac{1}{10368}$	$\frac{1}{2592}$

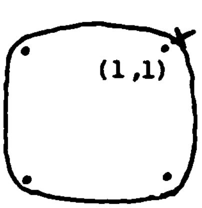
From (2.6), set $n = 1$, $\ell = k$, $k = 3,4$. We obtain

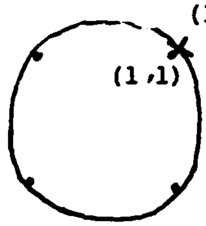
$$S_{3,1}f(x_i) = y_i + \frac{5}{432} \overline{\Delta}^4 y_i \quad ,$$

$$S_{4,1}f(x_i) = y_i + \frac{4}{7776} \overline{\Delta}^4 y_i - \frac{3}{7776} \overline{\Delta}^6 y_i \quad .$$

Assume four points in the plane are given:

$$(0,0), \quad (1,0), \quad (1,1), \quad (0,1) \ .$$





	$S_{3,1}f$		$S_{4,1}f$	
t	x(t)	y(t)	x(t)	y(t)
1.6	0.65648	1.11870	0.61804	1.13213
1.8	0.89167	1.08685	0.83711	1.09813
2.0	1.02315	1.02315	1.00514	1.00514
2.2	1.08685	0.89167	1.09813	0.83711
2.4	1.11870	0.65648	1.13213	0.61804

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this paper the author introduces the operator $\bar{\Delta}^{(n)} := P_n(\mu)\bar{\Delta}$ with higher order accuracy for approximation to the differential operator D, where $\bar{\Delta}$ denotes centered difference operator, μ denotes averaging operator,

$$P_n(\mu) = \sum_{m=0}^{n} C_m(\mu - I)^m, \quad C_m = -\frac{m}{2m+1} C_{m-1}, \quad C_0 = 1 .$$

A class of new many-knot spline basis $\Omega_{k,n} := (P_n(\mu))^{\ell} N_k$ was suggested. The

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ABSTRACT (continued)

smoothing formulas

$$f_{k,n} = \frac{1}{h}\int_{-\infty}^{\infty} \Omega_{k,n}\left(\frac{\cdot - t}{h}\right) f(t)\,dt \quad \text{and} \quad S_{k,n}f = \sum f_i \Omega_{k,n}$$

are discussed.