THE H-R SUB X METHOD FOR PREDICTING TRANSITION, (U)

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I. INTRODUCTION

This paper gives a brief description of a shortcut method, the H-R_x method, for predicting transition in a wide class of boundary layer flows, including the effects of pressure gradient, surface heat transfer and suction. Here H and R_x are the body shape factor and the Reynolds number, based on distance x measured in the direction of the flow, respectively. The method is extremely simple to use and a good substitute to the well known but rather complicated e_9 method.

II. THE e_9 METHOD OF FORECASTING TRANSITION

The H-R_x method has not been correlated with test data but rather has been justified in terms of Tollmien-Schlichting waves and e_9 type calculations. Data are gradually being accumulated that show that the e_9 method is the best all around method that now exists for predicting boundary layer transition. Since the H-R_x method is rooted in the e_9 method, we proceed to make some comments about the latter.

Transition, although it may commence with the amplification of Tollmien-Schlichting waves as described by linear stability theory, is dominated in its late stages by three dimensional and non-linear effects. Why then does transition occur at a disturbance amplification ratio a_t of about a_t = e_9 [1] as computed from linear instability theory?

The e_9 method is rooted in the following observations: Liepmann [2] hypothesized that at the breakdown to turbulence, the Reynolds stress \( \tau = -\rho u'v' \) due to the amplified fluctuations becomes comparable in magni-
tude to the maximum mean laminar shear stress $\tau_L = \mu(\partial u/\partial y)$. The ratio $\tau/\tau_L$ is given by

$$\frac{\tau}{\tau_L} = \frac{2}{c_{f_L}^2} \left[ kb \left( \frac{u_n'}{u_e} \right) \right] a(x)^2_{\max}$$

where $c_{f_L}$ is the skin friction coefficient, $u_n'$ is the disturbance amplitude at the neutral point, $b = v'/u'$ and $k = \frac{u'v'/uv}{a}$, $a(x)$ is the amplification factor with respect to the neutral point, $u$ and $v$ are velocities in the $x$ and $y$ direction respectively and prime indicates disturbance values. Obremski et al. [3] observes that in a low turbulence environment the critical Tollmien-Schlichting mode at the beginning of amplification may possess an amplitude $u_n'$ of the order of $0.001\% u_e$, or perhaps even less; $u_e$ is the edge velocity. Furthermore, Klebanoff et al. [4] found, in detailed examination of flat plate measurements, that disturbance growth via the laminar instability mechanism ceases to be valid when the rms velocity fluctuation $v'$ in the boundary layer reaches $(u'/u_e)_{\max} = 0.015$, but that the first appearance of turbulent spots is expected at about $(u'/u_e)_{\max} = 0.20$. Assuming that these figures represent disturbance growth not only on a flat plate but also on airfoils and bodies of revolution, all at low free stream turbulence level, we find that amplification in the linear regime $[(u'/u_e)_{\max}/(u_n'/u_e)] = 0.015/0.00001 = 1500 \approx 10^6$ substantially exceeds the amplification in the nonlinear regime $[0.20/0.015 = 13 \approx 10^2]$. Total amplification $(= 0.20/0.00001)$ at transition is seen to be about $20,000 \approx 10^5$; this amplification is of the same order as that reported by Michel [5], $e^{9.2}$, and computed by Smith et al. [6] and van Ingen [7] from linear instability theory. This amplification factor is consistent with the hypothesis of Liepmann. If we set $\tau/\tau_L = 1$, $(u_n'/u_e) = 0.00001$, $c_{f_L} = 0.664 R_L^{-1/2}$ with $R_L = 3 \times 10^6$ (for transition on a flat plate), $(u'/v')_{tr} = 0(10)$ and $(uv/u'v')_{tr} = 0(10)$ at transition, then eq. (1) gives an amplification at transition $a(x_{tr}) = 1.4 \times 10^4 = e^{9.5}$. This suggests that
in boundary layer flows, where free-stream turbulence is very low, the
growth of Tollmien-Schlichting waves controls the development to turbulent
flow and so linear instability theory can be used as a basis for forecast-
ing transition.

III. SOME PREDICTIONS FROM STABILITY THEORY AND THE $\text{e}^9$ METHOD

The factors being considered in this note, for either two-dimensional
or axisymmetric low speed flow, are (1) effect of pressure gradient, (2) ef-
f ect of suction, and (3) effect of wall heating or cooling. Figure 1 shows
the results of calculations of the neutral stability point for wedge and
other flows under a large variety of conditions. It contains results for
an adiabatic flat plate with varying degrees of mass transfer, including
the asymptotic suction case. The largest number of points are for various
wedge flows in water with and without heat. The significant fact is that the
neutral stability point when plotted in the form of the critical Reynolds
number, $R_{\text{c}}$ vs. $H$, the shape factor, this rather considerable variety
of flows forms a single well defined curve. Shape factor is the immediate
determinant of neutral stability rather than some more remote measure such
as Hartree's $\beta$ or Pohlhausen's $\lambda$.

The present authors proceeded to determine if a similar correlation
existed at transition as predicted by the $\text{e}^9$ method. Suction was not in-
cluded in the study but a considerable number of wedge flows with various
wall temperatures were studied for water. The results are shown in Figure
2, which also includes the data of Figure 1. But in Figure 2 the calcul-
ated values are plotted in terms of $R_x$ vs. $H$ rather than $R_{\text{c}}$ vs. $H$. Again,
except for the highest temperature differentials the data form well defined
curves; the scatter is so small that suction effects would undoubtedly
fall along the same curve. The correlation indicates that the neutral and
transition points are primarily a function of $H$. Pressure gradient, heating
and suction are only methods of influencing \( H \). The results of Figure 2 are only for two-dimensional and similar flows, but they suggest a method of predicting transition: for the conditions of the problem, make a plot of \( H \) vs. \( R_x \) from ordinary boundary layer calculations, as one proceeds back along the body. The plot will always start beneath the two loci of Figure 2. When the curve crosses the \( e^9 \) locus, transition should occur. An equation that fits the \( e^9 \) locus very well is (for bodies of revolution \( R_x \) is replaced with \( R_s \) where \( s \) is the distance measured along the surface of the body):

\[
\log[R_x(e^9)] = -40.4557 + 64.8066H - 26.7538H^2 + 3.3819H^3, \quad (2)
\]

\[
2.1 < H < 2.8
\]

IV. THE H-R Xavier METHOD FOR PREDICTING TRANSITION OVER A BODY OF REVOLUTION

Because the correlations of Figures 1 and 2 are for two-dimensional similar flows, as a sample problem we chose a heated Reichardt body of revolution moving in 67°F, 19.44°C, water at a velocity that gives \( u_\infty/\nu = 6 \times 10^6/ \text{ft}, 20 \times 10^6/\text{m}, \) Figure 3. Because there is a stagnation point the flow starts out very stable (low \( H \)). For the case of no heating (\( \Delta T = 0°C \)) the neutral point is crossed at \( R_x \approx 10^6 \) and transition occurs between \( R_x = 3 \) and \( 4 \times 10^6 \). The circle is for the full \( e^9 \) calculation, so the agreement is perfect. With 10°F (5.5°C) of heating, the boundary layer is more stable and now the neutral point is crossed at about \( R_x = 1.2 \times 10^6 \) and transition is moved back to about \( R_x = 9 \times 10^6 \). Again the agreement with the full \( e^9 \) method is very good. At a heating of 20°F (11.1°C) transition is moved still further back but the agreement with the full \( e^9 \) calculation is not quite as good. But the example shows clearly the method of prediction and that the correlation seems to work even when the flow is nonsimilar and non-two-dimensional.

For a practical shape such as in this example, the H-R Xavier trace meanders. For a pure similar flow the trace would be vertical. If there is a sudden
change in pressure gradient, suction or heat the trace would be horizontal for some distance. Note that only ordinary but accurate boundary layer calculations of $H$ vs. $R_x$ are needed for transition prediction, not lengthy stability calculations.

V. DISCUSSION

Other simple methods of predicting transition lack the generality of the $H-R_x$ method because they are correlations based on parameters that are more removed from the best measure of stability, $H$. Michel's method for instance is an excellent correlation method, correlating $R_\theta$ vs. $R_x$ at transition for two-dimensional incompressible flow, e.g., airfoils. But when bodies of revolution, heat, or suction are considered the correlation is no longer applicable. The same kind of trouble applies to methods like Granville's [8], which correlates with a pressure gradient parameter such as Pohlhausen's $\Lambda$. Obviously this kind of method fails if other means than pressure gradients are used to affect transition. Consider a fixed point on a particular body. That point has one value of $\Lambda$ regardless of the degree of suction or heating. But if $H$ is used as the correlating parameter, changes at that point in the boundary layer flow, as by heating or suction, manifest themselves in changes in $H$. However, we must stress that when there is heat transfer more than the shape factor is involved, particularly if temperature differences are high. For instance, in the full $\epsilon^9$ method using the extended Orr-Sommerfeld equation [1], additional terms such as $d^2u/dy^2$ enter the stability problem in addition to the basic velocity profile data that determines $H$. Note that the correlation is poor at high heating rates where $H < 2.1$.

Figure 2.

The $H-R_x$ method for predicting transition gives reasonable answers so long as the flow does not vary too much from nearly similar flow, i.e., local
similarity. Also it is applicable only so long as the effects of surface roughness, vibrations and freestream turbulence level are sufficiently low, just as required for the basic $e^9$ method. The method is also restricted to heating rates where $T_w - T_m$ does not exceed about 23°C.

The correlation has been developed entirely theoretically using the $e^9$ method as if it were exact for predicting transition. No attempt has been made to develop the correlation from experimental data, partly because of the labor involved but partly because good test data are quite scarce for unusual conditions such as heating in water. The main fact that can be claimed therefore is that the H-R$^x$ method is a very convenient substitute for calculating nearly the same results as the full $e^9$ method. Also of course this sample case is not the only one that has been studied; the H-R$^x$ method has been used extensively in other studies with plausible predictions, and a number of other comparisons with the full $e^9$ method have been made. Where the flow differs widely from the conditions of the correlation, a check should be made by the full $e^9$ method. But for a large range of practical conditions, the H-R$^x$ method seems accurate, convenient and quite general.
Figure 1
HEATED WEDGE FLOWS IN WATER
$T_e = 67^\circ F, 19.44^\circ C$

Figure 2
Figure 3
REFERENCES


LIST OF FIGURES

Figure 1  Critical Reynolds number $R_e_{crit}$ vs. $H$.

Figure 2  Critical and transition (computed) Reynolds number for adiabatic and heated wedge flows in water. $\beta$ is Hartree's $\beta$.

Figure 3  Paths of boundary-layer development ($H$ vs. $R_e$) and predicted transition for a 13:1 Reichardt body. Comparison with heated wedge-flow predictions. Circles denote point where $Re_s(s^9)$ is reached, $u_0/v = 6 \times 10^6$/ft, $20 \times 10^6$/m. $s$ is length measured along the body surface.